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Implementation Of The Lanczos Eigen-Solver For The CSI Code On High Performance Computers

by

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EXTENDED ABSTRACT

Lanczos algorithm for the solution of generalized eigenvalue problems have been receiving a lot of attention in recent years [1 - 7] due to its computational efficiency.

The basic steps involved in the Lanczos algorithm are summarized in Table 1 [6]. In Table 1, M is the structural mass matrix, and K_σ is defined as

$$K_\sigma = K - \sigma M$$

were σ is the shift factor and K is the structural stiffness matrix.

The focus of this research work is to implement a Lanczos algorithm for the Control-Structure Interaction (CSI) code which can exploit both parallel and vector capabilities provided by modern, high performance computers (such as the Alliant, and Cray-YMP). A partial restoring orthogonality scheme is also developed and incorporated into the basic Lanczos algorithm.

From Table 1, major computational time in the Lanczos algorithm can be identified as:

I. Matrix-vector multiplication (see steps 1b, 1d, and 2e)
II. Matrix factorization (see step 2a)
III. Forward/Backward elimination (see step 2a)

Step 2a (in Table 1) involves the solution for system of simultaneous linear equations. It should be emphasized here that factorization of the matrix K_σ need be done only once. The forward/backward elimination phase, however, needs to be done repeatedly.

The solution for system of simultaneous linear equations in step 2a can be obtained very effectively by incorporating the newly developed Parallel Vector equation SOLVEr [8], PVSOLVE, into the Lanczos procedure. Furthermore, to be consistent with the data structure for PVSOLVE, the structural mass matrix M also needs to be stored in a row-oriented, variable-band fashion.Effective (mass) matrix-vector multiplication which exploits

both parallel and vector speed has also been developed.

The numerical performance (in terms of accuracy and efficiency) of the proposed parallel-vector Lanczos algorithm is demonstrated by solving for the frequencies and modeshapes of the Phase Zero CSI model [9, 10] as shown in Figure 1 on the Alliant and Cray-YMP multi-processor high performance computers. The superior performance of the Lanczos algorithm is illustrated in Table 2.

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TABLE 1: The Lanczos Algorithm
Given an Arbitrary vector r_0 then:

1. Set

   a. $q_0 = 0$

   b. $\beta_1 = (r_0^T M r_o)^{1/2}$

   c. $q_1 = \frac{r_0}{\beta_1}$

   d. $p_1 = M q_1$

2. For j = 1, 2, . . ., repeat:

   a. $\bar{r}_j = K_\sigma^{-1} p_j$

   b. $\hat{r}_j = \bar{r}_j - q_{j-1} \beta_1$

   c. $a_j = q_j^T M \hat{r}_j = p_j^T \hat{r}_j$

   d. $r_j = \hat{r}_j - q_j \alpha_j$

   e. $\bar{p}_j = M r_j$

   f. $\beta_{j+1} = (r_j^T M r_j)^{1/2} = (\bar{p}_j^{-T} r_j)^{1/2}$

   g. if enough vectors, then terminate the loop

   h. $q_{j+1} = \frac{1}{\beta_{j+1}} r_j$

   i. $p_{j+1} = \frac{1}{\beta_{j+1}} \bar{p}_j$

Figure 1: Phase Zero CSI Finite Element Model

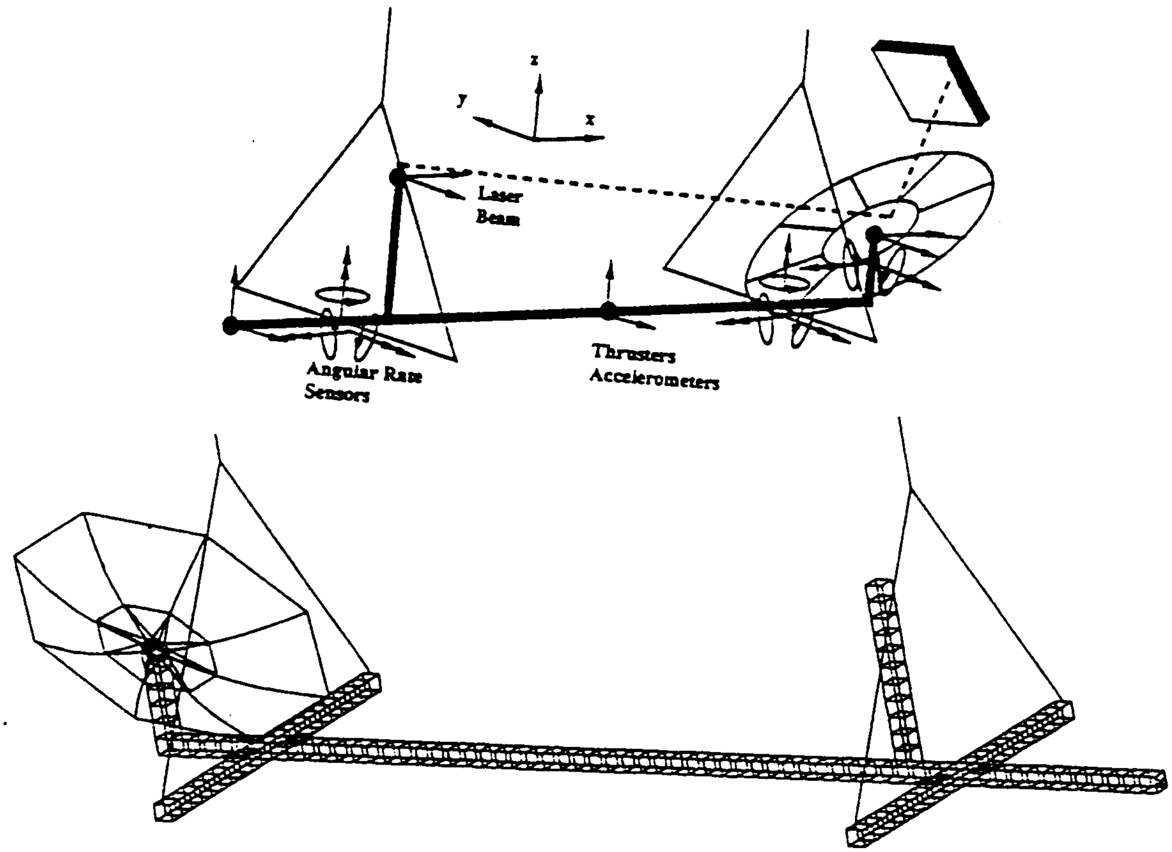

TABLE 2: Alliant CPU Time Comparison of Subspace and Lanczos algorithms for the Phase Zero CSI Model

No. of Requested Eigen-pairs	**Subspace Elapsed Time (Use PVSOLVE) on ALLIANT**	**Lanczos Elapsed Time (Use PVSOLVE) on ALLIANT**
10	50.3^{sec}	38.3^{sec}
15	68.3	41.3
20	79.2	46.9
50	421.2	89.1
100	2083.9	209.5
	λ_{100} = 193.878334	λ_{100} = 193.875783