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# Critical Problems of Computational Aeroacoustics

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The discipline of aeroacoustics is concerned with the propagation of sound in air. This is a fluid phenomenon, for the description of which the Navier-Stokes equations (or the Euler equations in the inviscid case) have been shown to be adequate. Hence the possibility arises of solving these equations as a predictive tool for determining sound fields due to particular sources, and also as a tool for the design and control of sources of sound. The application of computational techniques to the solution of equations governing aeroacoustic phenomena is known as computational aeroacoustics (CAA).

A special difficulty facing CAA is the fact that the sound levels of interest generally comprise only a small perturbation on the underlying flow field. For example, rather noisy environments are characterized by acoustic pressures which are only about $10^{-5}$ times the ambient atmospheric pressure [1]. Accordingly, extremely accurate solutions of the governing equations are needed, so that the sound field is not lost in the error in computing the whole field.

A particular simplification which occurs in many, but by no means all, aeroacoustic problems is that Mach numbers are low and there are no shocks. From the numerical point of view, flows without shocks are more regular than those with shocks, and it is possible to compute them with greater accuracy than would be the case for shocked flows, for the same computational effort.

We have been guided by an internal NASA report [2] on special problems which are considered to distinguish CAA from the discipline of computational fluid dynamics. The report cited advocates the establishment of research tools to be used in the discipline of CAA. In the spirit of the report, our effort this summer has focused initially on critical problems of CAA, solutions of which would be part of a stable of tools at the disposal of researchers in CAA. The problems we have settled on are listed in Figure 1.

The first problem listed is that of aeroacoustics in the absence of rigid boundaries. Lighthill [3] gave a formulation of this problem in which he showed that the sources of the acoustic field were quadrupole in nature. We have preferred a different formulation of the problem, in which the quadrupoles are sources for a nonlinear wave equation, as opposed to the linear one used by Lighthill. This is given in Figure 2. The figure also gives further details of a solution procedure for the Euler equations appropriate for the aeroacoustic problem and motivated by the analysis of Crow [4]. In accordance with our formulation, we contend that an algorithm for accurate solutions of an inhomogeneous nonlinear wave equation is of prime importance.

The second problem concerns the effects of rigid boundaries. In this case one has to distinguish two types of flows: those immersed in a stream with upstream and downstream boundaries, and those which are not. A flow of the latter type would be flow in a shock tube, whereas pipe flows and jet flows would be of the former type. For pipe and jet flows the proper choice of upstream and downstream boundary conditions has not always been clear-cut [5]. Now there is a theory for inviscid incompressible flows which treats the prescription of boundary conditions as a control problem [6,7]. We consider the numerical implementation of boundary conditions in accordance with this theory to be a high-priority item for CAA, and also the extension of the theory to viscous and compressible subsonic flows is considered to be important.

A third problem would be computation of the interference and diffraction of sound waves. In this case one needs very precise information about the phases of the waves. A capability in this area would enable one to design devices to reduce noise by canceling out sound waves.

Further problems listed in Figure 1 are the scattering of sound waves and the control of noise.

The subsequent effort of the summer has focused on algorithms to solve the problems just described, beginning with the first one, aeroacoustics in free space. To date this has been the only one of the problems which has received special attention during the reporting period. Figures 3, 4, and 5 describe technical aspects of the numerical treatment proposed for an inhomogeneous nonlinear wave equation in the acoustic regime. Figure 3 describes a time discretization ("semigroup" formulation) of the problem; Figure 4 discusses the solution of the time-discretized equations by means of a further spatial discretization; and Figure 5 shows how the time-discretized equations can be given an integral formulation, in a manner close to the treatment of the linear wave equation.

References


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4. S. C. Crow, Aerodynamic Sound Emission as a Singular Perturbation Problem, Studies in Applied Mathematics 49 (1970).

5. J. C. Hardin and D. S. Pope: Sound Generation by a Stenosis In a Pipe, AIAA 90-3919 (1990).

6. Joel C. W. Rogers: Boundary Conditions and Stability of Inviscid Plane-Parallel Flows, Quart. Appl. Math. 31 (1973).

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1. Free space problems: nonlinear wave equation.

2. Boundary effects: numerical implementation of control theory for inviscid incompressible flows; extension of theory to viscous and subsonic flows.

3. Diffraction of sound waves: accurate phase calculations.

4. Scattering of sound waves.

5. Control of noise.

Figure 1. Critical Problems of Computational Aeroacoustics.

Given $\rho$ and u at t = 0. $\rho_t$ is obtained at t = 0 from $\rho_t + \nabla \cdot (\rho u) = 0$.

Update $\rho$ by solving

$$\rho_{tt} - \nabla^2 p(\rho) = \sum \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j)$$

Write $u = V + \nabla\phi$, where $\nabla \cdot V = 0$. V is a functional of $\omega = \nabla \times u$: $V = \mathcal{F}\{\omega\}$.

To update V, update $\omega$ from

$$\frac{D}{Dt}\left(\frac{\omega}{\rho}\right) = \left(\frac{\omega}{\rho}\right)_t + u \cdot \nabla\left(\frac{\omega}{\rho}\right) = \frac{\omega}{\rho} \cdot \nabla u.$$

Let $h_1$ be given by

$$\nabla^2 h_1 = \nabla \cdot (u \times \omega) = -u \cdot \nabla \times \omega.$$

Then $\phi$ is updated by solving

$$\phi_t + \frac{1}{2}u^2 + h - h_1 = 0,$$

where

$$h = h(\rho) = \int \frac{dp(\rho)}{\rho}$$

Figure 2. Solution Procedure in Free Space

Rewrite as:

$$\hat{\rho}_t + \nabla \cdot q = 0$$
$$q_t + \nabla \hat{p} = F$$
$$\rho = \rho_0 + \hat{\rho}, p(\rho) = p_0 + \hat{p}, p_o = p(\rho_0)$$
$$\frac{d}{dt}\int \hat{\rho} dx = 0, \frac{d}{dt}\int q dx = \int F\, dx$$
$$\hat{E}(\hat{\rho}) = \int_{\rho_0}^{\rho_o+\hat{\rho}} p(\rho) d\rho - p_o\hat{\rho}$$
$$\frac{d}{dt}\int \left(\hat{E} + \frac{q^2}{2}\right) dx = \int q \cdot F\, dx$$

"Semigroup" formulation:

$$\frac{\hat{\rho}^{n+1} - \hat{\rho}^n}{\Delta t} + \frac{1}{2}\nabla \cdot \left(q^{n+1} + q^n\right) = 0$$
$$\frac{q^{n+1} - q^n}{\Delta t} + \nabla\left(\frac{\hat{E}^{n+1} - \hat{E}^n}{\hat{\rho}^{n+1} - \hat{\rho}^n}\right) = \frac{1}{2}\left(F^{n+1} + F^n\right)$$
$$\int \hat{\rho}^{n+1} dx = \int \hat{\rho}^n dx, \int q^{n+1} dx = \int q^n dx + \frac{1}{2}\Delta t \int \left(F^{n+1} + F^n\right) dx$$
$$\int \left[\hat{E}^{n+1} + \frac{\left(q^{n+1}\right)^2}{2}\right] dx = \int \left[\hat{E}^n + \frac{\left(q^n\right)^2}{2}\right] dx$$
$$+ \frac{1}{4}\Delta t \int \left(q^{n+1} + q^n\right) \cdot \left(F^{n+1} + F^n\right) dx$$

Figure 3. Treatment of Nonlinear Wave Equation

$$\hat{\rho}^{n+1} = \hat{\rho}^{n} - \Delta t\ \nabla \cdot q^{n} + \frac{1}{2}(\Delta t)^2 \nabla^2 \left( \frac{\hat{E}^{n+1} - \hat{E}^{n}}{\hat{\rho}^{n+1} - \hat{\rho}^{n}} \right)$$

$$-\frac{1}{4}(\Delta t)^2\ \nabla \cdot \left( F^{n+1} + F^{n} \right)$$

Spatial discretization: $\Delta x = \Delta y = \Delta z = h$,

$$\left(\nabla^2 f\right)_{ijk} \cong (f_{i+1,j,k} + f_{i-1,j,k} + f_{i,j+1,k} + f_{i,j-1,k}$$

$$+ f_{i,j,k+1} + f_{i,j,k-1} - 6 f_{ijk}) / h^2$$

$$\xi^{(0)} = \hat{\rho}^{n}$$

$$G(\xi) = \xi + \frac{3(\Delta t)^2}{h^2} \frac{\hat{E}(\xi) - \hat{E}\left(\xi^{(0)}\right)}{\xi - \xi^{(0)}}$$

G monotone nonlinear, can solve $G(\xi) = g$ easily. Iterative solution:

$$G\left(\xi^{(i)}\right) = g\left(\xi^{(i-1)}\right)$$

$$\xi^{(i)} \xrightarrow[i \to \infty]{} \hat{\rho}^{n+1}$$

Figure 4. Solution of equations.

$$\nabla^2 f(x) \cong \frac{3}{2\pi}\int_{|x-x^1|=c(x^1)\Delta t} \frac{f(x^1)}{\left(c(x^1)\Delta t\right)^3} \frac{1}{\left|x^1 - x - (\Delta t)^2 c(x^1)\nabla^1 c(x^1)\right|} dS^1$$

$$-\frac{6}{(\Delta t\, c(x))^2} f(x)$$

Inserting into equation at top of Figure 4, we get new

$$\tilde{G}(\xi) = \xi + \frac{3}{(c(x))^2} \frac{\hat{E}(\xi) - \hat{E}\left(\xi^{(0)}\right)}{\xi - \xi^{(0)}}.$$

$\tilde{G}$ is monotone and nonlinear

Iterative solution of equations:

$$\tilde{G}\left(\xi^{(i)}\right) = \tilde{g}\left(\xi^{(i-1)}\right)$$

$$\xi^{(i)} \xrightarrow[i \to \infty]{} \hat{\rho}^{n+1}$$

Formulation is close to Huyghens' construction in linear case $\frac{dp}{d\rho} = c_0^2$.

Figure 5. Integral Representation of $\nabla^2 f$.