

FINITE ELEMENT STRUCTURAL REDESIGN BY LARGE ADMISSIBLE PERTURBATIONS

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ABSTRACT

In structural redesign, two structural states are involved; the baseline (known) State S1 with unacceptable performance, and the objective (unknown) State S2 with given performance specifications. The difference between the two states in performance and design variables may be as high as 100% or more depending on the scale of the structure. A Perturbation Approach to Redesign (PAR) is presented to relate any two structural states S1 and S2 that are modeled by the same finite element model and represented by different values of the design variables. General perturbation equations are derived expressing implicitly the natural frequencies, dynamic modes, static deflections, static stresses, Euler buckling loads and buckling modes of the objective State S2 in terms of its performance specifications, and State S1 data and FEA results. Large Admissible Perturbation (LEAP) algorithms are implemented in code RESTRUCT to define the objective State S2 incrementally without trial and error by postprocessing FEA results of State S1 with no additional FEAs. Systematic numerical applications in redesign of a 10-element 48-d.o.f. beam, a 104-element 192-d.o.f. offshore tower, a 64-element, 216-d.o.f. plate, and a 144 element 896-d.o.f. cylindrical shell show the accuracy, efficiency, and potential of PAR to find an objective state that may differ 100% or more from the baseline design.

I. INTRODUCTION

Several problems in analysis, design, and modification of a structure or a structural design can be stated as redesign problems. Those are two-state problems involving the baseline State S1 and the objective State S2. S1 is known and has been modeled and analyzed by FEM. In the event that the performance of State S1 is unacceptable, the objective State S2 must be defined to satisfy performance specifications. The Perturbation Approach to Redesign (PAR) developed in this work can relate any two structural states that can be modeled and analyzed by the same FE model. PAR has the potential to perform redesign in the sense of resizing, reshaping, and reconfiguration to satisfy any performance requirements that can be predicted by FEA including modal dynamics, static deflections and stresses, and global buckling. LEAP algorithms implemented in code RESTRUCT (REdesign of STRUCTures) [3] presently can handle resizing for natural frequencies, mode shapes and static deflections.

Figure 1 shows several two-state problems that appear in the analysis-design-redesign process following a basic FE analysis. In analysis, the following two-state problems are encountered: (P1) Model correlation [28], (P2) Derivation of global failure equations [1, 14], (P3) Failure point identification [14], (P4) Redundancy [14], (P5) Reliability, [4], (P6) Non-Destructive-Testing [24]. In design, the following two-state problems are encountered: (P7) Redesign for target performance [1, 2, 11, 12, 24, 26, 27], (P8) Redesign for target redundancy, (P9) Redesign for target reliability.

LEAP theory was developed during the past seven years from the linear perturbation techniques introduced by Stetson in 1975 [26, 27] and modified by Sandstrom et al [24]. They redesigned a structure for both natural frequency and mode shape objectives but allowed only small differences between the baseline and objective states. In that respect, linear perturbation methods are equivalent to design sensitivity methods. Nonlinear perturbation methods [11, 12] allow for large differences between the two states. The objective state is found by postprocessing

data of the baseline structure only, using an incremental prediction correction scheme [1]. Presently, research efforts are directed towards two goals. The first one is to redesign larger scale structures as far away from the baseline structure as possible before a second FEA is needed. Large admissible perturbations [1] updating only cognate modes [2] in an incremental process are used towards that end. Substructuring is also investigated for that purpose, as well as for reshaping and reconfiguration. The second goal is to implement more and different objectives and derive the corresponding general perturbation equations. LEAP algorithms are under development for static stress, global buckling load, and buckling mode objectives.

The problem of redesign by large admissible perturbations is analyzed in Chapter II. Several two-state problems mentioned above are stated as redesign problems in Section II.1. The Perturbation Approach to Redesign (PAR) is presented in Section II.2 and LEAP theory for development of solution algorithms is summarized in Section II.3. Many numerical applications using four different structures are presented in Chapter III to assess the present status of code RESTRUCT, and the potential and limitations of PAR.

II. REDESIGN BY LARGE ADMISSIBLE PERTURBATIONS

A simple modeling-analysis-design-redesign process for structures using FEM is shown in Figure 1. Rectangular blocks indicate two-state problems which can be formulated as redesign problems using PAR and solved efficiently by a LEAP algorithm. Shaded blocks indicate problems already solved in some form by code RESTRUCT. Some of those problems are discussed below.

II.1. Redesign and Other Two-State Problems

The classical structural redesign problem appears in Figure 1 after analyzing either the original or the correlated FE model. Undesirable response – such as a natural frequency in the range of wave excitation, a dynamic mode with high amplitudes near the free surface where wave and current loads are maximum, or high stresses and deflections – makes redesign mandatory. The performance specifications of the objective design are desirable values of those response particulars.

After placing a structure in service, tests are performed to measure its performance and compare it to FEM predictions. In the modeling process, simplifying assumptions, uncertainty, and ignorance result in discrepancies between measurements and predictions particularly for marine structures which have large manufacturing tolerances. The process of finding a FE model of a physical structure that will correctly predict measured structural response is called model correlation. The initial FE model is the known State S1. The objective State S2 represents the unknown correlated FE model. The Perturbation Approach to Redesign presented in the following section preserves element connectivity and changes geometric properties so that the correlated model represents a real structure [2]. That is, PAR does not change simply numbers in the mass and stiffness matrices. PAR can also solve the problem of model correlation for geometry dependent hydrodynamic load [28].

The problem of failure point identification can also be formulated by PAR and solved by a LEAP algorithm. S1 represents the initial structural state and S2 the unknown failure point (design point in reliability terminology) on a limit surface [10, 20]. The advantage of PAR is that it can provide an implicit expression for a global failure criterion by relating State S2 to S1.

Related is the problem of reserve and residual redundancy. In the literature, several different aspects of redundancy are presented as definitions depending on the type of structure and analysis performed [5, 6, 21]. PAR remedies this lack of invariant and consistent redundancy definition by introducing a redundancy injective mapping [14] defining the difference between the initial intact or damaged structure and the design point.

Finally, a new methodology for reliability analysis and design of large scale structures is under development based on PAR [4]. The Perturbation Approach to Reliability provides an alternative to the systems approach [5, 21, 29] and the stochastic FEM [19, 30] which are the two most popular methods in structural reliability. PAR makes possible the introduction of advanced structural analysis in the reliability computations without simplifying the structure. PAR also

allows randomness in geometry, material, and load. There is no limitation to the number of random variables used and the random load need not be applied incrementally until structural failure. The reliability analysis problem is a two-state problem where S1 is the initial structure and S2 the design point.

PAR can also address the very difficult problems of target redundancy and target reliability design. S1 is the initial structural design of inadequate redundancy or reliability and S2 is the objective structure of specified redundancy or reliability [4]. These are difficult design problems because redundancy and reliability are not computed by FEM. PAR can solve these problems because of the introduction of an injective mapping relating S1 to S2.

II.2. Perturbation Approach to Redesign (PAR)

The PAR methodology has been developed to solve the above two-state problems. It has five major steps: **Step 1:** A Structure (S1) is modeled and analyzed by a general purpose FE code; MSC/NASTRAN is used in our work. So far, four types of analysis have been considered in PAR and the governing equations are listed below. For modal dynamics the free vibration equations for S1 are

$$([k] - \omega_j^2 [m]) \{\psi\}_j = \{0\} \quad \text{for } j=1,2,\dots,n, \quad (1)$$

where the n eigenvalues ω_j , $j=1,2,\dots,n$ satisfy equation $\det([k] - \omega_j^2 [m]) = 0$. In equation (1), damping may be included only in Rayleigh's form and added mass is included in $[m]$. For the static deflections and stresses of S1 we have

$$[k] \{u\} = \{f\} \quad (2)$$

$$\text{and} \quad \{\sigma\} = [S] [k]^{-1} \{f\}, \quad (3) \quad [S] = [G] [D] [N], \quad (4)$$

where $[G]$, $[D]$, $[N]$ are the stress-strain, strain-displacement, and shape function matrices. The governing equation for global buckling in finite elements is

$$([k_o] + [k_\sigma]) \{\psi_b\} = \{0\}, \quad (5)$$

where $[k_o]$ and $[k_\sigma]$ are the small displacement and initial geometric stiffness matrices.

Step 2: The following perturbation relations are introduced relating State S2 to S1:

$$[k'] = [k] + [\Delta k], \quad (6) \quad [m'] = [m] + [\Delta m], \quad (7)$$

$$[\omega'^2] = [\omega^2] + [\Delta(\omega^2)], \quad (8) \quad [\phi'] = [\phi] + [\Delta\phi], \quad (9)$$

where unprimed and primed symbols refer to the baseline (S1) and the objective State S2, respectively, and prefix Δ indicates difference between counterpart quantities of states S1 and S2.

$[\phi] = [\{\psi\}_1, \{\psi\}_2, \dots, \{\psi\}_n]$, is the matrix of eigenvectors of S1 and $[\omega^2]$ is the diagonal matrix of the corresponding eigenvalues. Perturbation relations pertaining to equations (2) and (3) are

$$\{u'\} = \{u\} + \{\Delta u\}, \quad (10) \quad \{f'\} = \{f\} + \{\Delta f\}, \quad (11)$$

$$\{\sigma'\} = \{\sigma\} + \{\Delta\sigma\}, \quad (12) \quad [S'] = [S] + [\Delta S]. \quad (13)$$

For the global buckling eigenvalue problem we have

$$[k'_o] = [k_o] + [\Delta k_o], \quad (14) \quad [k'_\sigma] = [k_\sigma] + [\Delta k_\sigma], \quad (15)$$

$$\begin{bmatrix} P'_{cr} \end{bmatrix} = \begin{bmatrix} P_{cr} \end{bmatrix} + \begin{bmatrix} \Delta P_{cr} \end{bmatrix}, \quad (16) \quad \begin{bmatrix} \phi'_b \end{bmatrix} = \begin{bmatrix} \phi_b \end{bmatrix} + \begin{bmatrix} \Delta \phi_b \end{bmatrix}. \quad (17)$$

Further, in Step 2, desirable values of some response particulars of S2 such as natural frequencies, static deflections and mode shapes are specified. An incomplete set of mode shapes may be used and only some degrees of freedom may be defined in each mode.

Step 3: The differences in structural properties between S1 and S2 are expressed in terms of the fractional changes $\alpha_e, e=1,2,\dots,p$ of p properties of elements or groups of elements as:

$$\begin{bmatrix} \Delta k \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} \Delta k_e \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} k_e \end{bmatrix} \alpha_e, \quad (18) \quad \begin{bmatrix} \Delta m \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} \Delta m_e \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} m_e \end{bmatrix} \alpha_e, \quad (19)$$

$$\begin{bmatrix} \Delta S \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} \Delta S_e \end{bmatrix} = \sum_{e=1}^p \begin{bmatrix} S_e \end{bmatrix} \alpha_e. \quad (20)$$

Several α_e s may refer to the same element but different properties such as bending, torsion, and stretching. The unknowns in the process of defining S2 from its specifications and S1 are the fractional changes α_e . When the α_e s are defined it is ensured that element connectivity in the FE model is preserved and S2 represents a real structure.

Step 4: The differences in structural response between states S1 and S2 are expressed implicitly in terms of the α_e s by the general perturbation equations. For modal dynamics we have

$$\sum_{e=1}^p \left(\{ \psi' \}_i^T \begin{bmatrix} k_e \end{bmatrix} \{ \psi' \}_i - \omega_i'^2 \{ \psi' \}_i^T \begin{bmatrix} m_e \end{bmatrix} \{ \psi' \}_i \right) \alpha_e = \omega_i'^2 \{ \psi' \}_i^T \begin{bmatrix} m \end{bmatrix} \{ \psi' \}_i - \{ \psi' \}_i^T \begin{bmatrix} k \end{bmatrix} \{ \psi' \}_i, \quad (21)$$

$$\sum_{e=1}^p \{ \psi' \}_j^T \begin{bmatrix} k_e \end{bmatrix} \{ \psi' \}_i \alpha_e = - \{ \psi' \}_j^T \begin{bmatrix} k \end{bmatrix} \{ \psi' \}_i, \quad (22)$$

$$\sum_{e=1}^p \{ \psi' \}_j^T \begin{bmatrix} m_e \end{bmatrix} \{ \psi' \}_i \alpha_e = - \{ \psi' \}_j^T \begin{bmatrix} m \end{bmatrix} \{ \psi' \}_i, \quad (23)$$

for $i = \overline{1,2,\dots,n}, j = i+1, i+2, \dots, n$ [1, 2]. Equation (21) represents the n diagonal terms of the energy balance equation $\begin{bmatrix} K' \end{bmatrix} - \begin{bmatrix} M' \end{bmatrix} \omega'^2 = 0$ for S2, that is, the Rayleigh quotients for $\omega_i'^2$. Equations (22) and (23) represent the orthogonality conditions of modes $\{ \psi' \}_i$ with respect to $\begin{bmatrix} k' \end{bmatrix}$ and $\begin{bmatrix} m' \end{bmatrix}$. Theoretically, orthogonality of modes with respect to one of $\begin{bmatrix} k' \end{bmatrix}$ or $\begin{bmatrix} m' \end{bmatrix}$ implies orthogonality with respect to the other. Numerically, however, both conditions must be forced if $\{ \psi' \}_j, j=1,2,\dots,n$, are to represent modes of a real structure.

The general perturbation equations for static deflections are derived from the counterpart of equation (2) for structure S2 based on the modal dynamic expansion of $\{ u' \}$ in terms of the unknown modes $\{ \psi' \}_j, j=1,2,\dots,n$. Thus, inversion of matrix $\begin{bmatrix} k' \end{bmatrix}$ is avoided. Linearizing only the explicit dependence on the α_e s, we have [1, 15]

$$u'_i = \sum_{m=1}^{n_r} \left(\frac{\phi'_{im} A_m}{B_m} \right) - \sum_{e=1}^p \left(\sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m^2} C_{me} \right) \alpha_e, \quad \text{where} \quad (24)$$

$$A_m = \sum_{j=1}^n (\phi'_{jm} f'_j), \quad B_m = \{ \psi' \}_m^T \begin{bmatrix} k \end{bmatrix} \{ \psi' \}_m, \quad C_{me} = \{ \psi' \}_m^T \begin{bmatrix} k_e \end{bmatrix} \{ \psi' \}_m.$$

The general perturbation equations for static stresses are derived in a similar manner [14]

$$\{ \Delta \sigma \} = - \{ \sigma \} + \left(\begin{bmatrix} S \end{bmatrix} + \sum_{e=1}^p \begin{bmatrix} S_e \end{bmatrix} \alpha_e \right) \left[\sum_{m=1}^n \frac{\phi'_{im} A_m}{B_m} - \sum_{e=1}^p \left(\sum_{m=1}^n \frac{\phi'_{im} A_m}{B_m^2} C_{me} \right) \alpha_e \right]. \quad (25)$$

For global buckling, the general perturbation equations are derived using the same method as in the case of the modal dynamics eigenvalue problem [14]

$$\sum_{e=1}^p \{\psi'_b\}_i^T ([k_{c_e}] - P_i[k_{\sigma_{0e}}]) \{\psi'_b\}_i \alpha_e = \{\psi'_b\}_i^T (P_i[k_{\sigma_0}] - [k_c]) \{\psi'_b\}_i, \quad (26)$$

$$\sum_{e=1}^p \{\psi'_b\}_j^T [k_{c_e}] \{\psi'_b\}_i \alpha_e = -\{\psi'_b\}_j^T [k_c] \{\psi'_b\}_i, \quad (27)$$

$$\sum_{e=1}^p \{\psi'_b\}_j^T [k_{\sigma_{0e}}] \{\psi'_b\}_i \alpha_e = -\{\psi'_b\}_j^T [k_{\sigma_0}] \{\psi'_b\}_i, \quad (28)$$

for $i = 1, 2, \dots, n$, $j = i + 1, i + 2, \dots, n$, where $[k_c] = [k_0] - [k_{\sigma F}]$, $k_{\sigma F}$ includes the body force, and $[k_{\sigma}] = -P_i[k_{\sigma_0}] - [k_{\sigma F}]$.

Step 5: In this final step, the problem of finding State S2 based on its specifications and results of FEA for S1 is formulated and solved for the p unknown α_e s using the LEAP algorithm presented in the next section. The problem formulation is as follows:

$$\text{Minimize } \|\alpha\|_2 \in \mathcal{R}^p, \quad (29)$$

subject to n_ω natural frequency objectives $\omega_i'^2$, $i = 1, 2, \dots, n_\omega$; n_ϕ normal mode objectives ϕ'_{ki} , number of $(k, i) = n_\phi$; n_u static deflection objectives u'_i , $i = 1, 2, \dots, n_u$; n_σ static stress objectives σ'_i , $i = 1, 2, \dots, n_\sigma$; n_b global buckling eigenvalues P'_i , $i = 1, 2, \dots, n_b$; $n_{\phi b}$ buckling mode objectives $\phi'_{b_{ki}}$, number of $(k, i) = n_{\phi b}$; $2p$ lower and upper bounds on the redesign variables α_e , $-1 < \alpha_e^- \leq \alpha_e \leq \alpha_e^+$, $e = 1, 2, \dots, p$; n_a admissibility constraints extracted from

equations (22) and (23), where $n_a = 2 \sum_{i=1}^{n_\omega} (n_r - i) = n_\omega [(2n_r - 1) - n_\omega]$; and n_{ab}

admissibility constraints extracted from equations (27) and (28), where

$$n_{ab} = 2 \sum_{i=1}^{n_b} (n_r - i) = n_b [(2n_r - 1) - n_b].$$

All of the above redesign objectives are substituted in the appropriate general perturbation equations (21)-(28). The remaining unused general perturbation equations may be used to predict the unspecified performance particulars of the objective State S2. Accuracy of those predictions, however, is not as high as those of the redesign objectives. All the constraints of the above problem may result in an empty, non-empty, or countable feasible domain. In the first case, the redesign objectives cannot be achieved for the selected set of redesign variables, in which case a minimum error solution in satisfaction of the redesign objectives is achieved by a generalized inverse algorithm [1, 2, 11, 15]. In the second case, an optimum solution is achieved using an optimality criterion (29).

II.3. Large Admissible Perturbation (LEAP) Algorithm

The redesign problem formulated by PAR in Section II.2 can be solved by a LEAP algorithm. Many LEAP algorithms have been developed to solve a variety of two-state problems [1, 2, 14, 15, 28] and have been documented in detail. Suffice to present here the basic steps and difficulties of the solution algorithm. The LEAP algorithm developed to solve the redesign problem is outlined in Figure 2. It starts from the baseline structure (S1) and reaches incrementally the

objective S2 by prediction and correction. In the prediction phase of the algorithm, the small perturbation method [24, 26, 27] is used. The modal dynamics general perturbation equations are linearized. For that purpose, increments are limited to 7% differences between S2 specifications and the corresponding S1 properties. Predictions are small but inadmissible because admissibility conditions (22) and (23) are linearized. In the correction phase, perturbations are corrected by satisfying the nonlinear general perturbation equations and are forced back into the admissible space by satisfying the nonlinear admissibility conditions. The total CPU time for redesign may be reduced by a factor occasionally as high as 4 when in the first increment the space of cognate modes is identified and thereafter all computations are performed in that space. Such is the case for torsional redesign [2] of the offshore tower in Figure 5. Torsional modes (3, 18, 19) constitute one cognate subspace with very weak interaction with other modal subspaces such as those for bending and stretching.

In each increment, in both phases the resulting problem may be underdetermined or overdetermined depending on the relation between the number p of redesign variables α_e , the number of equality constraints (S2 specifications) $n = n_\omega + n_\phi + n_u + n_\sigma + n_b + n_{\phi b} + n_a + n_{ab}$, and the $2p$ bounds on the α_e s. When the problem is overdetermined, a minimum error solution in satisfaction of the S2 specifications is produced by a generalized inverse algorithm. When the problem is undetermined, it is solved by optimization using the minimum change criterion in equation (29). To achieve this global objective, at each increment the following objective is minimized

$$\min \sum_{e=1}^p \left[(1 + \ell \alpha_e) \prod_{q=1}^{\ell-1} (1 + q \alpha_e) - 1 \right]^2 \quad (30)$$

The problem is solved by quadratic programming [8] or sequential quadratic programming [7] depending on whether the expression for $[\Delta k]$ is linear as in equation (18), or nonlinear as in the case of plate and shell redesign. In those cases, the plate or shell thickness is selected as redesign variable resulting in a cubic expression for $[\Delta k]$ in terms of the α_e s. $[\Delta S]$ is always a nonlinear expression of the α_e s because $[S_e]$ depends on the distance of the point where the stress is computed from the neutral axis. The LEAP algorithm is implemented in code RESTRUCT (REdesign of STRUCTures) [3]. It is 27,000 FORTRAN 77 commands and may serve as a postprocessor to any special or general purpose FE code. We presently use it to postprocess MSC/NASTRAN.V64 data on the secondary (UB) main frame computer (IBM-3090) of the University of Michigan.

The LEAP algorithm outlined above finds the optimum objective structure S2 without trial and error and with no additional FEAs. The $[k]$ matrix inversion required in static deflection and stress redesign is avoided by using modal expansions as shown in equations (24) and (25). Thus, an accurate modal basis is mandatory even as S2 moves far away from S1. LEAP algorithms can surmount the following three difficulties as well. All general perturbation equations (which become equality constraints in the optimization problem) are strongly nonlinear implicit expressions of the redesign variables α_e . The static force vector $\{f\}$ may depend on the structure's geometry (e.g. hydrodynamic loads) and consequently change in the redesign process. Finally, the set of specifications provided for S2 are usually incomplete and only some d.o.f.s of specified modes are defined.

III. NUMERICAL APPLICATIONS

A total of 42 numerical applications are presented in this section on optimal redesign of four different structures [22, 9, 31]. Results are summarized in Tables 1, 4, 5, 6 and show the accuracy of code RESTRUCT for applications with number of redesign variables ranging from 8 to 21; natural frequency and mode shape redesign objectives changing by a factor ranging from 0.3 to 2.0; degrees of freedom ranging from 48 to 896. For each redesign objective, Tables 1, 4, 5, 6 show the objective value, the value actually achieved as computed by reanalysis with MSC/NASTRAN and the corresponding relative error. CPU time and numbers of extracted modes n_r , admissibility conditions n_a , and redesign variables are also shown. The values of the redesign variables of the optimum solution are not shown. The optimal solution appears in the

form of optimal Euclidian norm of the α_e s in Tables 5 and 6; and in the form of the Hasover-Lind reliability index [10] in Tables 1 and 4.

10-element 48-d.o.f. beam: The clamped-hinged beam in Figure 4 is subjected to a uniform load in the y direction and a concentrated load applied at node 7 in the z direction. $\omega_1 = 183.092$ rad/sec, the horizontal and vertical deflections at node 7 are $v_7 = 12.151$ mm and $w_7 = 17.733$ mm as computed by MSC/NASTRAN. Redesign variables and structural groups are shown in Table 2. The accuracy of the redesign process is shown in Table 1 for one, two or three simultaneous redesign objectives. The problem of reliability analysis is studied assuming randomness in geometric properties, A (area), I_y , I_z (moments of inertia), and material properties E (Young's modulus) and ρ (density). The fractional changes α_e are assumed to be independent normal random variables of zero mean. Standard deviations are selected as $\sigma_{\alpha_{EI}} = 0.40$ for bending rigidities EI_y and EI_z , and $\sigma_{\alpha_{\rho A}} = 0.30$ for mass per unit length ρA . In order to compute the probability of failure to first (FORM) or second (SORM) order [20], computation of individual and joint design points and the corresponding Hasover-Lind reliability index β is required as shown in Figure 3. Computation of β is achieved by transforming the α_e s to independent standard normal random variables through the Rosenblatt transformation [13]. These numerical applications as well as those following on the offshore tower show that large admissible perturbation methods can introduce sophisticated structural analysis in reliability without simplifying the structural model and without repeated FEAs [4].

104-element 192-d.o.f. offshore tower: The offshore tower shown in Figure 5 is 69.95 m high and operates in 45.72 m water depth. The tower at the base is square with a 38.10m side and tapers linearly to 22.86 m at the deck. The FE model of the tower is composed of 104 circular tubular beam elements and has 192 dofs. Loading on the tower is due to: (i) 240 tonnes deck load which is applied to the structure as uniformly distributed load at the deck nodal points. (ii) Wave hydrodynamic forces calculated for a design wave of 182.88 m length and 6.10 m height using Morison's equation. The wave propagates in the x -direction. (iii) Wind generated water current in the x -direction with linear velocity profile of 1.03 m/sec at the mean free surface waterline and zero at the sea bed. $\omega_1 = \omega_2 = 4.695$ rad/sec for the first bending modes in the XZ and the YZ planes. $\omega_3 = 5.353$ rad/sec for the first torsional mode with respect to axis Z . Redesign variables and structural groups are shown in Table 3.

Failure states are defined by deterioration factors in the first and third eigenvalues of 1.54 and 2.00. Geometric and material properties are random. The fractional changes α_e s, shown in Table 3, are assumed to be independent normal random variables with zero mean. Standard deviations are selected as $\sigma_{\alpha_{EI}} = 0.40$ for bending rigidity EI and $\sigma_{\alpha_{\rho A}} = 0.30$ for mass per unit length ρA . Design points are again computed by postprocessing FE analysis results for the baseline design only. It should be noted that both in Tables 1 and 4 the computed β are very high because the external load is deterministic and limit states were pushed as far away from the baseline design as possible in order to demonstrate the accuracy and limitations of code RESTRUCT.

64-element 216-d.o.f. plate: The clamped-free-free-free plate in Figure 6 is subjected to a uniform load p and has the dimensions and properties shown in the figure. Its response is computed by MSC/NASTRAN and redesign is performed by RESTRUCT. The incremental optimization problem is nonlinear and solved by sequential quadratic programming [7] because $[\Delta k]$ is a cubic expression of the α_e s which represent fractional changes of the plate thickness [17, 22]. The plate is subdivided into 8 structural groups each containing 8 finite elements. Results of redesign are summarized in Table 5 and show very high accuracy even for changes by a factor of 2 in eigenvalues and maximum deflection.

144-element 896-d.o.f. cylindrical shell: The simply supported shell shown in Figure 7 is subjected to hydrostatic pressure load p due to 286 meters submergence in salt water [23]. Dimensions [25] and properties are also shown in the figure. Its modal dynamic and static deflection response is computed by MSC/NASTRAN [16, 18]. The optimization problem in each increment is nonlinear and solved by sequential quadratic programming [7]. The cylindrical shell is subdivided into 5 structural groups and even though symmetry is not forced by linking

symmetric groups (1 and 5, 2 and 4) as was done in the plate redesign problem, symmetry was preserved in the redesign process. Results of code RESTRUCT are summarized in Table 6 and show good accuracy even for changes by a factor of 2 in eigenvalues and deflection.

In all of the above applications, the LEAP algorithm in RESTRUCT can be pushed further by taking additional incremental steps if higher errors are considered acceptable. For higher accuracy, however, one more FE analysis may be used after about 10 increments.

CONCLUDING REMARKS

Several two-state problems in structural analysis, design, and redesign can be formulated by PAR (Perturbation Approach to Redesign) and solved by a LEAP (Large Admissible Perturbation) algorithm. The objective structural design is found incrementally without trial and error or repeated FEAs for differences in response from the baseline design of the order of 100% or more. In structural reliability, PAR provides an attractive alternative to Stochastic Finite Elements and the Systems approach.

Computer code RESTRUCT which implements the large admissible perturbation methodology, is being developed since 1983, has been tested thoroughly and has generated confidence in its potential to solve two-state problems. Several theoretical and numerical developments are under way. New types of finite elements are being introduced; new structures are being redesigned, such as stiffened plates and shells; new two-state problems are studied, e.g. submarine acoustic noise reduction, redesign for buckling objectives, redesign for stress objectives; a perturbation approach to reliability analysis and design is being developed; larger scale structures are being redesigned by postprocessing FEA results by MSC/NASTRAN.V66 which has superelement capability. For that purpose, a supercomputer version of RESTRUCT running on the San Diego supercomputer has been developed.

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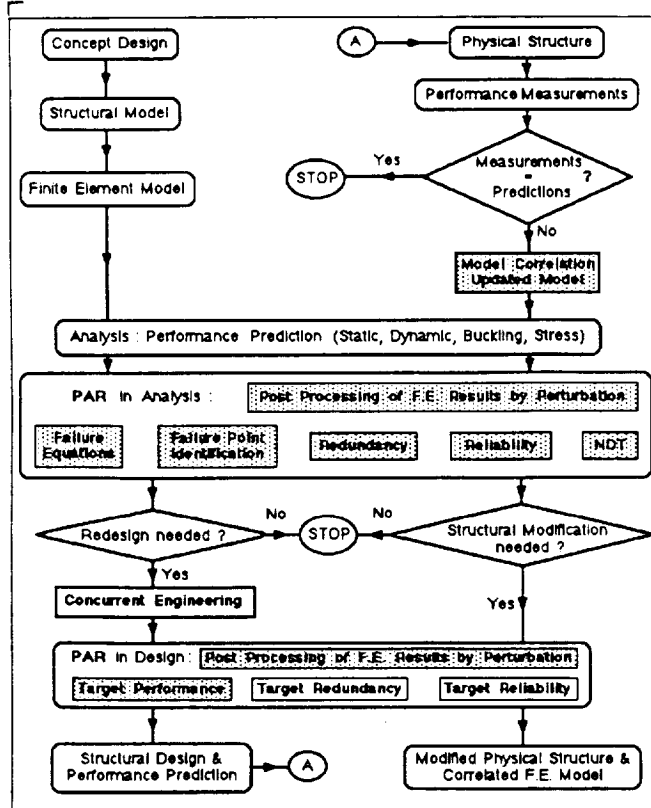


Figure 1. Structural analysis and design problems solved by PAR (rectangular blocks indicate problems that can be solved by LEAP theory; shaded blocks indicate problems already solved in some form)

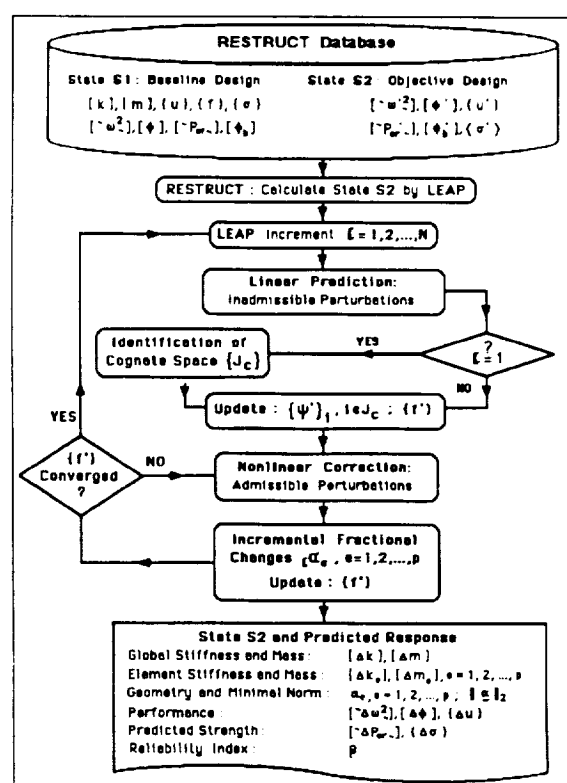


Figure 2. Implementation of LEAP (Large Admissible Perturbation) algorithm in code RESTRUCT

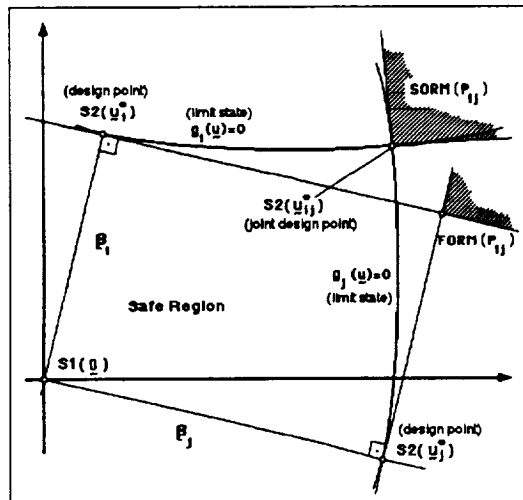


Figure 3. First and second order methods for reliability computations

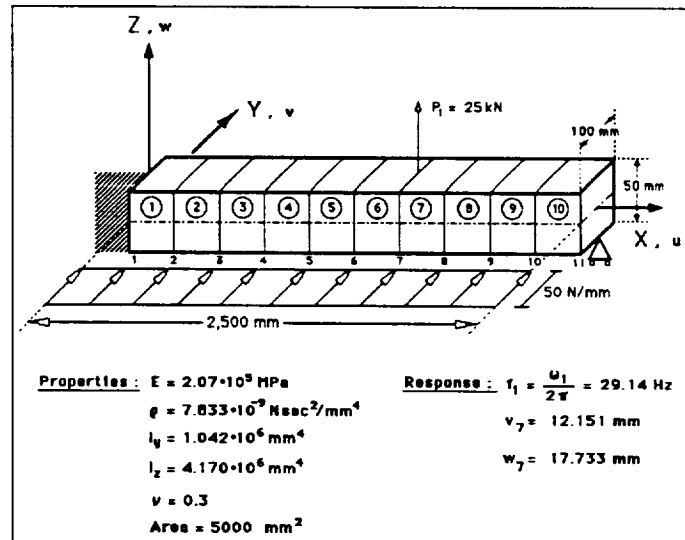


Figure 4. 10-element, 48-d.o.f. clamped hinged beam

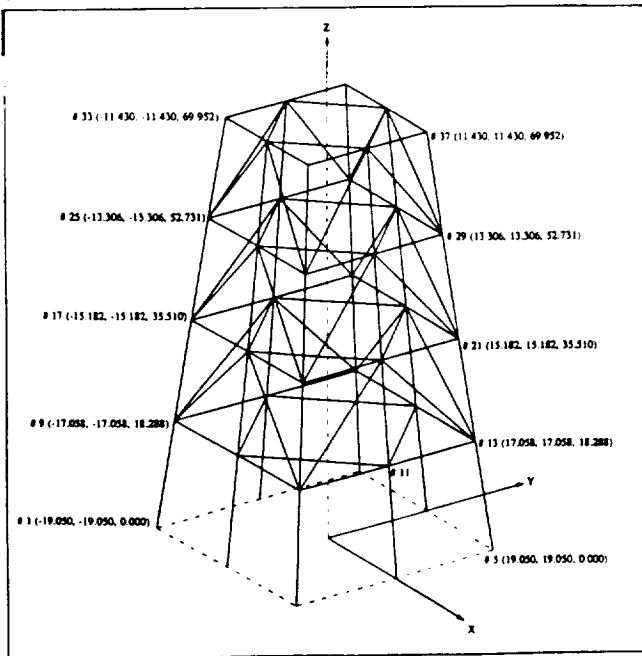


Figure 5. 104-element, 192-d.o.f. offshore tower

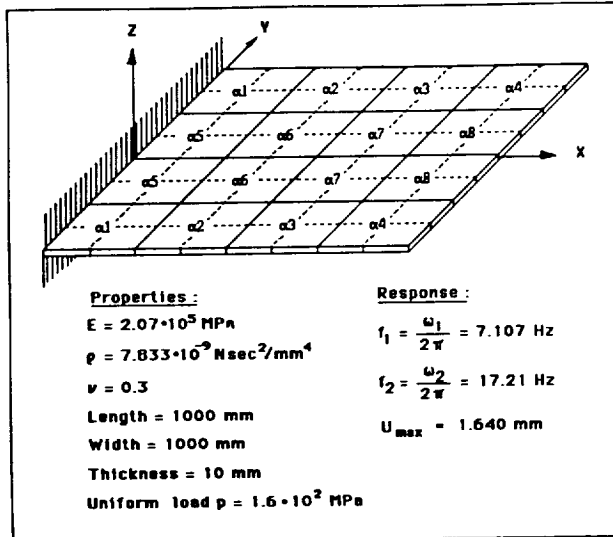


Figure 6. 64-element, 216-d.o.f. plate

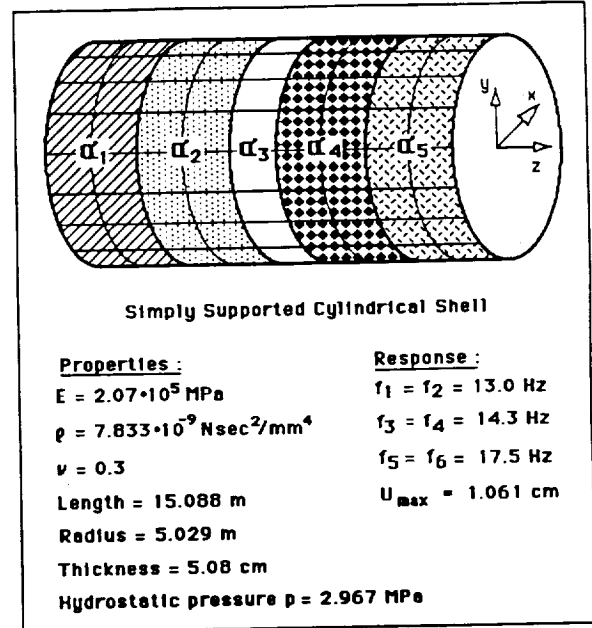


Figure 7. 144-element, 896-d.o.f. cylindrical shell

Table 1. Redesign and reliability of clamped-hinged beam

Case #	ω_1^2 / ω_1^2			v_1^2 / v_1^2			w_1^2 / w_1^2			β	CPU (msec)	n_r	n_a	P
	F.S.*	Reanalysis	Error(%)	F.S.*	Reanalysis	Error(%)	F.S.*	Reanalysis	Error(%)					
1	0.4588	0.4578	-0.205	—	—	—	—	—	—	7.65	31050	9	8	14
2	—	—	—	2.0358	2.0440	0.405	—	—	—	11.40	52620	10	12	14
3	—	—	—	—	—	—	2.0358	2.0458	0.489	7.39	48378	9	8	14
4	0.4588	0.4570	-0.401	2.0358	2.1124	3.762	—	—	—	20.83	75520	9	12	21
5	0.4588	0.4554	-0.736	—	—	—	2.0358	2.0568	1.033	7.36	68486	11	10	14
6	—	—	—	2.0358	2.1125	3.769	2.0358	2.0701	1.684	15.09	90435	9	9	14
7	0.4588	0.4545	-0.931	2.0358	2.1893	7.542	2.0358	2.0737	1.863	18.03	105928	10	12	21
11	0.3	0.2892	-3.586	—	—	—	—	—	—	13.02	81131	11	10	14
12	—	—	—	3.0	3.0122	0.407	—	—	—	19.52	80156	10	12	14
13	—	—	—	—	—	—	3.0	3.2152	7.173	11.31	108912	11	12	14
14	0.3	0.2959	-1.379	3.0	3.1554	5.179	—	—	—	39.65	116314	9	12	21
15	0.3	0.2841	-5.271	—	—	—	3.0	3.0997	3.325	15.63	99915	11	10	14
16	—	—	—	3.0	3.2042	6.801	3.0	3.2969	9.898	25.40	101394	7	5	14
17	0.3	0.2940	-2.002	3.0	3.3282	10.942	3.0	3.1003	3.342	33.59	167822	10	12	21

cases 4, 7, 13, 15, and 17 are solved by the generalized inverse algorithm. *F.S. = Failure State

Table 2. 10-element, 48-d.o.f. clamped-hinged beam : structural groups and redesign variables

Structural Group #	Redesign Variables, α_e ; $p = 21$	Elements #
1	$\alpha_1 (\alpha_{EI_x}), \alpha_2 (\alpha_{EI_z}), \alpha_3 (\alpha_{EA})$	1, 2
2	$\alpha_4 (\alpha_{EI_x}), \alpha_5 (\alpha_{EI_z}), \alpha_6 (\alpha_{EA})$	3, 4
3	$\alpha_7 (\alpha_{EI_x}), \alpha_8 (\alpha_{EI_z}), \alpha_9 (\alpha_{EA})$	5
4	$\alpha_{10} (\alpha_{EI_x}), \alpha_{11} (\alpha_{EI_z}), \alpha_{12} (\alpha_{EA})$	6
5	$\alpha_{13} (\alpha_{EI_x}), \alpha_{14} (\alpha_{EI_z}), \alpha_{15} (\alpha_{EA})$	7
6	$\alpha_{16} (\alpha_{EI_x}), \alpha_{17} (\alpha_{EI_z}), \alpha_{18} (\alpha_{EA})$	8
7	$\alpha_{19} (\alpha_{EI_x}), \alpha_{20} (\alpha_{EI_z}), \alpha_{21} (\alpha_{EA})$	9, 10

Table 3. 104-element, 192-d.o.f. offshore tower : structural groups, redesign variables, and dimensions

Structural Group #	Redesign Variables α_e	Description	D_e (m)	D_i (m)	Number elements
1	$\alpha_1 (\alpha_{EI}), \alpha_2 (\alpha_{EA})$	Legs below first bracing	0.762	0.737	8
2	$\alpha_3 (\alpha_{EI}), \alpha_4 (\alpha_{EA})$	Legs between first and second bracing	0.616	0.584	8
3	$\alpha_5 (\alpha_{EI}), \alpha_6 (\alpha_{EA})$	Legs above second bracing	0.616	0.584	16
4	$\alpha_7 (\alpha_{EI}), \alpha_8 (\alpha_{EA})$	Horizontal bracing	0.483	0.464	32
5	$\alpha_9 (\alpha_{EI}), \alpha_{10} (\alpha_{EA})$	Horizontal cross bracing	0.508	0.489	16
6	$\alpha_{11} (\alpha_{EI}), \alpha_{12} (\alpha_{EA})$	Vertical cross bracing	0.610	0.591	24

Table 4. Redesign and reliability of offshore tower

Case #	$\omega_1'^2/\omega_1^2$			$\omega_3'^2/\omega_3^2$			β	CPU (msec)	n_r	n_a
	F.S.*	Reanalysis	Error(%)	F.S.*	Reanalysis	Error(%)				
1	0.6598	0.6531	-1.018	—	—	—	4.43	973814	18	8
2	—	—	—	0.6598	0.6530	-1.030	3.8	925711	19	8
3	0.6598	0.6547	-0.786	0.6598	0.6541	-0.871	9.37	985832	18	8
11	0.5000	0.4871	-2.572	—	—	—	8.03	1589439	18	8
12	—	—	—	0.5000	0.4844	-3.112	21.47	1529708	19	8
13	0.5000	0.4895	-2.100	0.5000	0.4877	-2.462	13.7	1617425	18	8

$p = 12$; cases 12 and 13 are solved by the generalized inverse algorithm. *F.S. = Failure State

Table 5. Redesign of 64-element 216-d.o.f. plate

Case #	$\omega_1'^2/\omega_1^2$			$\omega_2'^2/\omega_2^2$			u_{max}'/u_{max}			$\sum \alpha_e^2$	CPU (sec)
	Goal	Reanalysis	Error(%)	Goal	Reanalysis	Error(%)	Goal	Reanalysis	Error(%)		
1	1.2867	1.2844	-0.177	—	—	—	—	—	—	0.0267	263
2	2.0000	1.9818	-0.909	—	—	—	—	—	—	0.2354	713
3	—	—	—	—	—	—	0.7579	0.7633	0.718	0.0358	420
4	—	—	—	—	—	—	0.5000	0.5069	1.374	0.2794	1044
5	1.2867	1.2842	-0.195	—	—	—	0.7772	0.7818	0.594	0.0295	440
6	2.0000	1.9801	-0.997	—	—	—	0.5000	0.5064	1.289	0.2816	1199
7	1.2867	1.2848	-0.144	1.1589	1.1572	-0.140	—	—	—	0.0275	381
8	2.0000	1.9848	-0.760	1.5000	1.4875	-0.831	—	—	—	0.2407	1037
9	1.2867	1.2840	-0.204	1.2867	1.2788	-0.621	0.7772	0.7821	0.631	0.0549	562
10	2.0000	1.9747	-1.264	2.0000	1.9198	-4.011	0.5000	0.5077	1.531	0.4378	1532
11	1.3195	1.3157	-0.287	1.3195	1.3103	-0.697	0.6598	0.6649	0.787	0.1561	833
12	2.0000	1.9721	-1.395	2.0000	1.9341	-3.397	0.3536	0.3600	1.831	1.2394	2072
13	1.2867	1.2844	-0.173	1.1589	1.1574	-0.125	0.7772	0.7817	0.577	0.0297	562
14	2.0000	1.9806	-0.971	1.5000	1.4941	-0.395	0.5000	0.5063	1.259	0.2817	1528
15	1.3195	1.3093	-0.774	1.1761	1.1691	-0.591	0.6598	0.6678	1.224	0.1618	832
16	2.0000	1.9104	-4.479	1.5000	1.4408	-3.950	0.3536	0.3710	4.945	1.3025	2065

In all cases, $n_r = 7, n_a = 5, p = 8$.

Table 6. Redesign of Simply Supported Cylindrical Shell

Case #	$\omega_1'^2/\omega_1^2$			$\omega_2'^2/\omega_2^2$			u_z'/u_z			$\sum \alpha_e^2$	CPU (sec)
	Goal	Reanalysis	Error(%)	Goal	Reanalysis	Error(%)	Goal	Reanalysis	Error(%)		
1	1.3310	1.3200	-0.900	—	—	—	—	—	—	0.5867	549
2	1.5700	1.5300	-2.700	—	—	—	—	—	—	1.3300	715
3	1.9171	1.7800	-7.000	—	—	—	—	—	—	2.4800	1077
4	1.9171	1.7800	-7.000	1.6700	1.5000	-9.000	—	—	—	2.486	2031
5	1.331	1.3200	-0.900	—	—	—	0.6480	0.6610	2.000	0.5790	940
6	1.9171	1.7800	-7.000	—	—	—	0.648	0.646	0.00	2.486	2131

In all cases, $n_r = 5, n_a = 5$