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RESEARCH ON INVERSE, HYBRID AND OPTIMIZATION PROBLEMS IN ENGINEERING SCIENCES WITH EMPHASIS ON TURBOMACHINE AERODYNAMICS: REVIEW OF CHINESE ADVANCES

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ABSTRACT

A brief review of advances in the inverse design and optimization theory in the following engineering fields in China is presented: I) Turbomachine aerodynamic inverse design: including mainly: (1) two original approaches---image-space approach and variational approach , (2) improved mean-streamline (stream surface) method, (3) optimization theory based on optimal control. II) Other engineering fields: inverse problem of heat conduction, free-surface flow, variational cogeneration of optimal grid and flow field, optimal meshing theory of gears.

I. INTRODUCTION

Up to now, most(over 95%) of the technical literature deals only with the direct (analysis) problem due to possibly the fact that the inverse(design) problem(finding the unknown boundary shape) is, in general, much more difficult to formulate as well as to solve than the direct one, though the inverse problem is more important for , and directly related to, practical design. As a result, for instance, almost all turbomachine bladings are still designed by repeated use of direct problem methods in a cut-and-try manner, which is of course not only inconvenient and time-consuming, but also incapable of providing very good results. So in the 1950's in China we have tried to apply the mean-streamline method for inverse problem of Wu & Brown[30] to cascade design and some improvements of this method were suggested[2]. In the 1960's a new image-plane approach to the inverse problem was proposed [5]. It was realized, however, that also the inverse problem can not be successfully used for practical blade design, because it often leads to blade configurations that are either unfeasible from consideration of stress, vibration, cooling and technology or even unrealizable(e.g. giving profiles unclosed or with negative thickness). Therefore the traditional direct and inverse problems can not keep up with the development of modern turbomachinery(TM) and it was suggested in Refs.[18,56] to extend the scope of aerodynamic problems and reclassify them into four categories: direct, inverse, hybrid and optimization problems. Then the image-plane approach was extended to hybrid problem in Refs.[6,56], and another new approach to inverse and hybrid problems based on variational principles (VPs) was also suggested in Refs. [18,21]. Since then, a lot of variants of the image-plane approach, the variational approach and the mean-streamline method have been developed in China and extended to 3-D case.

The Chinese research on the optimization problem of bladings started with the problem of optimal radial distribution of flow parameters in TM with long twisted blades in 1963[52]. Later, advances in this area are characterized and facilitated considerably by the introduction of modern optimal control theory.

II. RECLASSIFICATION OF ENGINEERING PROBLEM SETTING

Generally speaking, any problem of engineering sciences can be posed in different ways, resulting in four problem categories: direct, inverse, hybrid and optimization problems. Specifically, for the aerodynamic problem of blade cascades these problems are defined conceptually in Table I. The aerodynamic problem for S_2-stream surface can be classified similarly as shown in Table II.

The hybrid problem is a unification as well as a generalization of the direct and inverse problems, encompasses a wide variety of types(see Table III for cascades on arbitrary streamsheet of revolution) and hence is very flexible and capable of meeting various design requirements. It provides design engineers with a series of new rational versatile ways for blade design. In addition, the inverse and hybrid problems also constitute an important ingredient of the optimization problem.

Table I. Problem Classification of Cascade Flow

	problem	Given	To be sought
1	Direct (D)	cascade geometry	velocity field
2	Inverse(I)	surface velocity distribution	cascade geometry
3	Hybrid (H)	'partly geometric conditions' & 'partly flow condition'	the remainding unknown geometry & flow field
4	Optimization (Opt)	1)objective functional 2)design inequality constraints	optimal cascade geometry & flow field

Table II. Problem Classification of S_2 -Flow

problem		Given	
		on S_2	on hub & casing
Direct	complete	Shape	geometry
Direct	semi-	Shape	pressure distribution
Inverse	complete	$V_\varphi r$	pressure distribution
Inverse	semi-	$V_\varphi r$	geometry
Hybrid		Shape or $V_\varphi r$	'partly geometry'+ 'partly pressure distribution'
Optimization		1) objective function, 2) inequality design constraints	

Table III. Hybrid Problem Types of Cascades

Types	Given conditions	
	geometric	aerodynamic
H_A	part of airfoil form	velocity distribution along the remainding part of profile contour
H_B	airfoil thickness distribution	blade loading distribution $(p_p - p_s)$
H_C	airfoil thickness distribution	velocity distribution along the suction surface
H_D	airfoil thickness distribution	distribution of velocity difference $(W_s - W_p)$

Since the hybrid problem of fully 3-D flow may have a wide variety of types, depending on the manner in which the boundary conditions (B.C.) on the blade surface are combined with those on the annuclar walls, it is necessary to employ some properly defined compound symbols to designate them as proposed in Refs. [46, 47]. For instance, the symbol $(I \times H_A)$ designates such a hybrid problem type in which an inverse problem is posed on the blade surface, while a H_A-problem is posed on the annular walls(Fig. 1a). In other words, the symbol before 'X' characterizes the problem type on the blade surface, while the symbol behind 'X'---that on the annular walls. It is easy to see that the scope of possible hybrid problem types can be made even much broader by posing different problem types on different portions of the blade(and/or annular) walls (Fig. 1b).

III. INVERSE & HYBRID PROBLEMS OF BLADE-TO-BLADE FLOW IN TURBOMACHINES

The research on inverse and hybrid problems in China has been going basically along the following three lines: (i) universal approach based on image-plane concept; (ii) unified approach based on variational principles (VPs) and the related finite element method (FEM); (iii) improvement of the mean-streamline method and of other well-known methods.

1) Universal Approach Based on Image-Plane Concept
   Two different image planes $\xi\psi$ and $\xi\eta$ have been introduced:
1-1. Methods based on image-plane $\xi\psi$.
   The first universal image-plane method for solving the inverse problem of 2-D compressible cascade flow was suggested by Liu in 1964 [5] and extended to cascades on arbitrary streamsheet of revolution by Liu & Tao in 1967 [6] and to hybrid problem H_A by Liu & Tao in 1981 [6]. The main difficulty is the treatment of unknown boundary(blade surface) and was successfully overcome by introducing a nonorthogonal curvilinear(streamline) coordinate system (von-Mises coordinates) defined by (Fig. 2, where η should be replaced by ψ)

$$\left.\begin{array}{l} \xi = \ell \quad [or: \ \xi = f(\ell)] \\ \psi = \psi(\ell, \varphi) \end{array}\right\} \tag{1}$$

where the stream function ψ is defined by

$$\frac{\partial \psi}{\partial \ell} = -\tau \rho \Lambda_\varphi , \qquad \frac{\partial \psi}{\partial \varphi} = \tau r \rho \Lambda_\ell \qquad (2)$$

It is expedient to regard Eq.(1) as a mapping, which transforms the original irregular periodic flow domain with some unknown boundary(AB in H_A-problem, Fig.2a) on the physical stream surface into a simple rectangular one with fully known boundary in the image plane (Fig.2b). Moreover, the following four alternative formulations were derived in Ref. [5,6]:

    i) first-order partial differential equation (PDE) system

    ii) second-order PDE

    iii) integro-differential equation

    iv) integral equation system,

of which only the integro-differential formulation for homentropic flow is given here for reference [6]:

$$\oint_{(\delta C)} \frac{1}{\tau \rho} \left\{ \left[1 + \left(r \frac{\partial \varphi}{\partial \xi} \right)^2 \right] \left(r \frac{\partial \varphi}{\partial \psi} \right)^{-1} d\xi + r \frac{\partial \varphi}{\partial \xi} d\psi \right\} = L_I \qquad (3)$$

where

$$\frac{\partial \varphi}{\partial \xi} = \Lambda_\varphi / (r \Lambda_\ell) , \qquad \frac{\partial \varphi}{\partial \psi} = 1 / \tau r \rho \Lambda_\ell \qquad (4)$$

$$\rho = p^{1/\varkappa} = \left\{ 1 - \frac{1}{2m} (\Lambda^2 - \Lambda_u^2) \right\}^m \qquad (5)$$

$$L_I = \frac{-2\omega B}{a_o} \iint_{(\delta A)} r \cdot \sin \sigma \frac{\partial \varphi}{\partial \psi} d\xi d\psi . $$

Here $\Lambda (= W/a_o)$ and $\Lambda_u (= \omega r/a_o)$ are dimensionless relative velocity and blade speed respectively; $m = (\varkappa - 1)^{-1}$; a_o---reference speed of sound; ω---rotor angular speed; δC---the contour abcda of a finite area δA (Fig.2b).

A H_A-and an inverse problems of a cascade on a general streamsurface of revolution have been solved by this method by Chen et al. in Ref.[7].

This method was then improved considerably by Liu [8] via introducing a new moment function Ω defined by

$$\frac{\partial \Omega}{\partial \xi} = \tau r p / \varkappa , \qquad \frac{\partial \Omega}{\partial \psi} = -(\Lambda_\varphi + \Lambda_u) r \qquad (6)$$

The moment function has some special features, for instance: (i) its increment around any closed contour enclosing an airfoil $\Delta \Omega$ is just equal to the aerodynamic moment M_z exerted on airfoil:

$$\Delta \Omega = M_z = \Gamma \cdot \Delta \psi \qquad (7)$$

This is just a generalization of the well-known Kutta's lift theorem for 2-D flow, showing that M_z is proportional to both absolute circulation around airfoil Γ and flow rate through an interblade channel $\Delta \psi$. (ii) the pressure p can be computed directly from Eq.(5), resulting in two advantages: first, no density ambiguity[80] appears; second, for the inverse problem the B.C.(i.e. the distribution of Ω) on the airfoil contour is of the Dirichlet's type and hence easy to deal with. Also in this case the four alternative formulations mentioned above are possible, of which only the second-order PDE formulation is given below:

$$A \frac{\partial^2 \Omega}{\partial \xi^2} + 2B \frac{\partial^2 \Omega}{\partial \xi \partial \psi} + C \frac{\partial^2 \Omega}{\partial \psi^2} + D \frac{\partial \Omega}{\partial \xi} + E \left(\frac{\partial \Omega}{\partial \psi}\right)^2 = F. \tag{8}$$

where A, B, C, D, E are functions of Mach number, ρ, τ and δ (Fig.2); F is a function of the gradients of entropy and rothalpy. In Ref.[8] an inverse problem of a cascade on a conical streamsurface taken from Ref.[78] was solved and the result is given in Fig.3.

Later, a number of methods using this image plane $\xi \psi$ have been also published in Refs[9-12], differing from one another, however, by different choice between the four above-mentioned formulations and by different iterative strategies. Thus, in contrast to Refs.[5,6], Shen & Ma [9] solved the H_A -and H_C -problems of 2-D transonic cascade flow by employing the 1st-order PDE formulation and Jameson's rotated difference scheme, while Chen & Zhang[10], using the second-order PDE formulation for the dependent variable $\varphi(\xi,\psi)$, presented a numerical method for solving direct, inverse and hybrid H_A -problems along with three numerical inverse problem examples, of which the one for a tandem cascade is given in Fig.4. Some difference between the calculated and original profiles might be attributed to the use of the measured velocity distribution as input for the calculation. This method has been modified by Sun et al. in Ref.[11] by using a boundary-fitted coordinate η (see Eq.(9)) instead of φ. The numerical result of a supercritical cascade together with its modified design is shown in Figs.5 & 6. In Ref. [12] a method similar to Ref.[10] for H_C - & I-problems was presented for rotational flow, and a method for removing the density ambiguity is also given. In addition, a rational cascade design procedure consisting of successive use of H_C -and I-problems is proposed.

1-2. Methods based on image-plane $\xi \eta$.

All methods using image-plane $\xi \psi$ suffer from the shortcoming that singularities appear in the vicinity of blunt leading and trailing edges due to local multivaluedness of the mapping Eq.(1). To circumvent this difficulty, another method for hybrid problems was suggested by Liu [13], where a new image-plane $\xi \eta$ defined by (Fig.2)

$$\left. \begin{array}{l} \xi = \ell \quad [or: \quad \xi = f(\ell)] \\[2mm] \eta = \dfrac{\eta_p - \eta_s}{\varphi_p - \varphi_s}(\varphi - \varphi_s) + \eta_s \end{array} \right\} \tag{9}$$

was introduced, where η_p and η_s (the η-values on suction & pressure sides) are given constants. Also in this case four alternative formulations can be derived, but only the integro-differential formulation is given here for reference (Fig.2b):

$$\oint_{(\delta c)} \frac{1}{\tau \rho}\left\{ \left(tg\,\gamma \frac{\partial \psi}{\partial \xi} - \frac{\sec^2 \gamma}{H_\eta}\frac{\partial \psi}{\partial \eta}\right)d\xi + \left(H_\eta \frac{\partial \psi}{\partial \xi} - tg\,\gamma \frac{\partial \psi}{\partial \eta}\right)d\eta \right\}$$

$$= 2\frac{\Lambda_u}{r}\Delta\eta \int_{(ab)} H_\eta \cdot \sin\delta \cdot d\xi. \tag{10}$$

$$\frac{\partial \psi}{\partial \xi} = \tau\rho(\Lambda_\ell \cdot tg\,\gamma - \Lambda_\varphi), \qquad \frac{\partial \psi}{\partial \eta} = \tau\rho H_\eta \Lambda_\ell \tag{11}$$

$$\rho = \left[1 - \frac{1}{2m}(\Lambda^2 - \Lambda_u^2)\right]^m. \tag{12}$$

where $tg\,\gamma$ is the slope of the ξ-coordinate line; $H_\eta = r\,\delta\varphi/(\eta_p - \eta_s)$ is the scale factor of the coordinate η; $\delta\varphi = \varphi_p - \varphi_s$ is the angular width of the blade channel. We can see that this new image-plane method is particularly advantageous for solving those hybrid prob-

lems with given airfoil thickness (and hence H_η is also known).

Other methods based on $\xi\eta$-image plane, using, however, the following second-order PDE formulation, have been presented by Chen et al. [14,15] and Ge[16]:

$$A_1 \frac{\partial^2 \psi}{\partial \xi^2} + A_2 \frac{\partial^2 \psi}{\partial \xi \partial \eta} + A_3 \frac{\partial^2 \psi}{\partial \eta^2} + A_4 \frac{\partial \psi}{\partial \xi} + A_5 \frac{\partial \psi}{\partial \eta} = B_1 \qquad (13)$$

where A_1, A_2, A_3, A_4, A_5 are functions of ρ and the metric tensor g_{22}, g_{23}; B_1 depends on the gradients of entropy and rothalpy, velocity, ω, σ and viscous forces. In Ref. [14] the H_A-problem of potential flow is solved and one of the numerical examples is given in Fig.7. The figures 8 & 9 taken from the viscous inverse problem solutions of Refs. [15] & [16] respectively show that for the same inlet and outlet flow angles the airfoil in viscous flow is more strongly curved than that in inviscid flow.

2) Unified Approach Based on Variational Principles

Basically there have been developed two completely different variational approaches, following a systematic way suggested by Liu[17].

2-1. Approach based on VPs in the image plane $\xi\psi$(Fig.2)

In Ref. [18] Liu established two families of VPs and generalized VPs in terms of the moment function Ω and angular function respectively for the H_A- and H_B-problems in the image plane $\xi\psi$, which were modified by Liu & Yao to give the VPs for the H_C-problem in Ref. [19]. Only one of these VPs is given below:

$$J_1(\Omega) = \iint_{(A)} \left\{ \Lambda_u^2 + 2m\left[1 - \left(\frac{\kappa}{\tau r}\frac{\partial\Omega}{\partial\xi}\right)^{\frac{1}{\kappa m}}\right] - \left(\frac{1}{r}\frac{\partial\Omega}{\partial\psi} + \Lambda_u\right)^2 \right\} d\xi \cdot d\psi + L_\Omega \qquad (14)$$

where the boundary integral term L_ψ takes different form for different problem types. Based on these VPs involving Ω, some finite element (FE) solutions to H_A-and H_B-problems have been presented in Ref. [20] by Yao et al., from which Figs.10 & 11 for a cascade on a conical stream surface [78] are taken.

2-2. Approach based on VPs with variable domain.

Making use of the functional variation with variable domain, Liu was able to establish three families of VPs and generalized VPs for H_A-, H_B-and H_C-problems in terms of the potential and stream functions Φ & ψ for potential and rotational flows in Refs. [21,22] and extended them to transonic flow with shocks in Ref. [24]. Moreover, variable-domain VPs using Clebsch variables have been also developed for 2-D transonic rotational channel flow by Liu[25].

Numerical solutions to H_A-and H_C-problems based on VPs of Refs. [21,22] have been obtained by Yan & Liu[22,23] by means of a new finite element with self-adjusting nodes for numerical realization of the functional variation with variable domain(Figs.12 & 13).

Perhaps a very attractive merit of this variable-domain approach is that it can be straightforwardly extended to fully 3-D flow.

3) The Mean-Streamline Method (MSLM).

This method originally suggested by Wu & Brown[30] was improved in many aspects in China. A survey of this development before 1984 has been presented by Cai[2]. Recent research includes Cai's paper [3] and Wang's paper [4].

4) Miscellaneous Approaches.

Several known approaches to inverse design of cascades were improved or modified in China.

4-1. Iterative method based on direct problem solver.

Such a method is suggested by Wang in Ref. [26] to solve inverse and various hybrid problems(including H_A & H_C) and extended to viscous flow in Ref. [27] by incorporating a boundary layer solver of integral type. Based on this method, Wang et al. proposed a

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quasi-3-D design procedure for impellers[28,29].

4-2. Time-dependent method.

Starting from the integral form of aerodynamic equations, a finite-volume method for inverse cascade problem is given by Zhou & Zhu[31].

4-3. Hodograph method.

It was improved in the transonic region by incorporating some analytical nozzle solutions and generalized to cascade flow along general streamsheet of revolution independently by Chen[32] and Yao[33].

IV. INVERSE & HYBRID PROBLEMS OF S_2-FLOW IN TURBOMACHINES

Similarly to S_1-flow, the image-plane approach and the VP-based approach mentioned above can be applied to S_2-flow as well.

1) Unified VP-Based Approach.

Starting from the basic equations of Wu's S_2-flow model[1,79], first complete VPs and generalized VPs for the semi-inverse problem were established by Liu[34] and the corresponding FE solutions were obtained by Qin et al.[35]. Inverse and hybrid problems of S_2-flow were formulated in a unified manner by VPs with variable domain by Liu[36] and by VPs in an image-plane $\xi\psi$ by Cai & Liu[37], which have been generalized to flow of pure substance by Xu[38]. In Ref.[39] VPs for hybrid problems of axisymmetric channel flow were derived by Tao & Liu.

2) Universial Image-Plane Approach.

Using an image plane $\xi\psi$ and given a distribution of circulation $V_\varphi \Gamma$ on S_2-surface, Ge presented a method for solving the complete inverse problem and a hybrid problem(with unknown hub(or casing) wall, see Table II), thereby a second-order PDE for $r(\xi,\psi)$ was derived and solved [40].

V. INVERSE & HYBRID PROBLEMS OF FULLY 3-D ROTOR-FLOW

For these problems three approaches have been developed in China.

1) Method of Mean-Stream Surface.

It was originally suggested by Wu in 1952[1] by a Taylor-serise expansion of flow parameters in the azimuthal direction as an extension of MSLM[30, 2]. It was improved, numerically elaborated and applied to design by Zhao et al. in Refs.[41, 42], where an annular constraint condition is set up, which must be satisfied to ensure that the hub/casing walls are axisymmetric.

2) Universal Image-Space Approach.

In Ref.[43] Liu developed a universal image-space theory of hybrid problems for fully 3-D potential flow, which is a generalization of the image-plane approach of Ref.[13]. Applying tensor calculus and Stokes theorem, the basic flow equations are transformed into the following integro-differential equation system for the stream functions ψ_1 and ψ_2 in the image space $\xi^1\xi^2\xi^3$ (Fig.14b):

$$\oint_{(c)} \left(g_{ij}V^j + \Lambda_u r \frac{\partial \varphi}{\partial \xi^i} \right) d\xi^i = 0 \tag{15}$$

$$\sqrt{g}\, \rho \{V^1, V^2, V^3\} = \left\{ \frac{\partial(\psi_1,\psi_2)}{\partial(\xi^2,\xi^3)},\ \frac{\partial(\psi_1,\psi_2)}{\partial(\xi^3,\xi^1)},\ \frac{\partial(\psi_1,\psi_2)}{\partial(\xi^1,\xi^2)} \right\} \tag{16}$$

and Eq.(5), where V^i are the contravariant components of the velocity $\vec{A}$ in a body-fitted

nonorthogonal curvilinear coordinate system ξ^i(Fig.16a). Similarly, a corresponding potential function formulation by integro-differential equations of this theory has been also presented by Liu et al.[44].

A similar method for solving 3-D hybrid problems was put forth by Chen et al.[45], using, however, a second-order PDE formulation:

$$\frac{\partial}{\partial \xi^i}\left(\sqrt{g}\,\rho g^{ij}\frac{\partial \Phi}{\partial \xi^j}\right) - \frac{\partial}{\partial \xi^2}\left(\sqrt{g/g^{22}}\,\rho\Lambda_u\right) = 0 \tag{17}$$

This equation was solved numerically by the method AF2, the multigrid technique and the artificial density in the transonic region. To greatly simplify the numerical solution, the inverse problem is modified in such a way that the physical contravariant velocity components Λ^i(assuming that ξ^i-coordinate is the streamlike line) rather than the fully velocity Λ is prescribed on the blade surface. An axial compressor rotor was redesigned and improved by this method as shown in Figs.15 & 16.

3) Unified Variable-Domain Variational Approach

The variable-domain variational approach[21, 22] has been extended by Liu to hybrid problems for fully 3-D incompressible[46], compressible[47] and transonic[48] flows in rotors. Only one of the VPs is given below for reference:

$$J_2(\Phi, A_b^{**}, A_{2d}) = \frac{1}{K}\iiint\limits_{(V)}\left\{1 - \frac{1}{2m}\left[(\nabla\Phi)^2 - \frac{2\Lambda u}{r}\frac{\partial \Phi}{\partial \varphi}\right]\right\}^{Xm}\cdot dV + L. \tag{18}$$

where the boundary integral term L takes different form for different problem type. Note that the variable-domain variation of J_2 should be taken at the unknown boundaries A_b^{**} (blade surface)and A_{2d} (free trailing vortex sheet). Corresponding numerical solutions to incompressible(H_AxD)-problem of Mizuki's centrifugal compressor[51] and to compressible ([Hc+D]XD)-problem of an axial turbine stator have been obtained by a novel FE with self-adjusting nodes in Refs.[49] and [50] respectively and are shown partly in Figs.17 & 18.
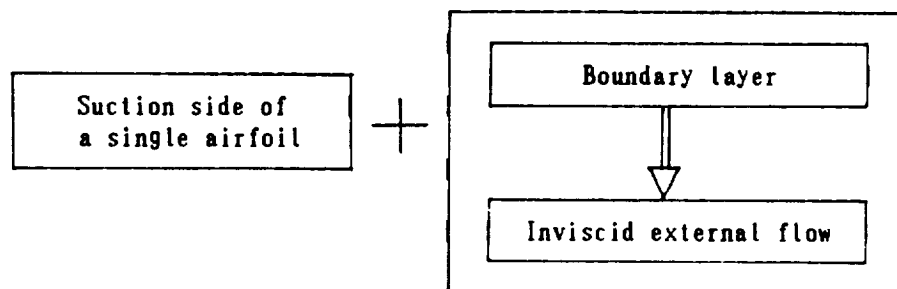
VI. OPTIMIZATION OF AERODYNAMIC DESIGN OF BLADING.

1) Optimization of S_2-Flow.

A basic and very important problem in this context is the optimal flow type(i.e. optimal radial distribution of flow) in bladings. This problem was first studided by a variational method by Liu[52] and later by Xue[53] and Lu[54]. Recently, this problem was treated by an optimal control method by Gu & Miao[55], so that various inequality design constraints can be accounted for.
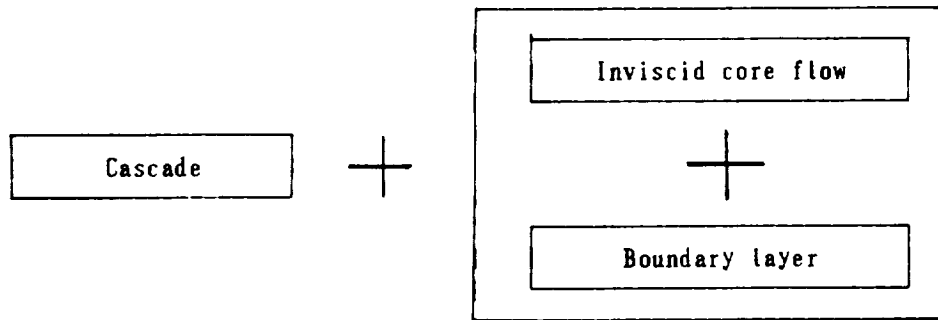
2) Optimization of 2-D Cascades.

Theory of optimization of cascade profile shape can be founded on the basis of one of the following flow models.
i) Simplified model(LeFoll, Citavy)

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ii) Refined model (Liu[57-59])

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                              +-----------------------------+
                              |  +-----------------------+  |
                              |  |  Inviscid core flow   |  |
                              |  +-----------------------+  |
  +-----------------+         |             +               |
  |    Cascade      |    +    |             +               |
  +-----------------+         |  +-----------------------+  |
                              |  |    Boundary layer     |  |
                              |  +-----------------------+  |
                              +-----------------------------+
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Perhaps, a third, most accurate flow model based on Navier-Stokes equations incorporating some turbulence models should be also tried.

Two possible mathematical models(formulations) can be used here, namely: a) mathematical programing problem, b) optimal control problem.

2-1. Optimization based on the simplified flow model.

The problem of determining the optimal velocity distribution along the suction side was formulated as an optimal control problem with some inequality design constraints (e.g. separation-free, maximum or minimum velocity limit, etc) and solved by a heuristic grapho-analytical method by Liu[56,63]. This approach is followed by Wang[60] in the design of an axial ventilator. Having calculated the optimal velocity distribution along suction side and specified a reasonable airfoil thickness distribution, a Hc-problem was solved by the image-plane method given in Refs. [5,6] to yield the optimal airfoil shape. A similar method, with some modifications, for optimizing 2-D compressor cascade was presented by Hua & Chen[61], where a method for estimating the airfoil circulation was given and the optimal airfoil shape was obtained by MSLM[20,30].

The optimal velocity distribution along the suction side on a general streamsheet of revolution was obtained by Zou[62], using Mager's transformation of turbulent boundary layer.

In Ref. [63] some generalizations of the LeFoll's optimization theory of blades were given by Liu & Wu to accommodate different objective functionals with more general constraints.

2-2. Optimization based on refined model.

In Refs. [57,59] Liu suggested a new theory of optimal 2-D cascades based on the above-mentioned refined flow model, in which this problem has been formulated as an optimal control problem with multiple inequality design constraints on control-and phase-spaces. Two typical optimal control problems were considered: cascade with minimal losses and cascade with maximal loading (circulation), and a duality theorm between them has been proved theoretically, so that it is sufficient to study only the cascade with minimal losses. This theory has been generalized to 2-D compressible flow and to a 3-D axial-flow rotor by Liu[58]. The essential feature of this theory lies in its capability of handling a wide variety of practical design constraints(from stress, vibrational, cooling and technological considerations) in a unified manner so as to make the optimal solution surely feasible and suitable for use in practice.

2-3. Local optimization of transonic cascades

Jiang et al. suggested a numerical method for weakening shocks in transonic cascades by local optimization of airfoil shape[64]. The airfoil contour segment near the shock is represented by a cubic parabola with free coefficients a_1, a_2, a_3, a_4. Then the Mach number just before the shock Ms is minimized with respect to a_i.

3) Optimal Design of Diffusers.

The optimal design of 2-D diffusers was considered by Gu & Ji[66] using optimal

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control for searching optimal wall shape that maximizes the pressure recovery of diffuser.
A more general optimization problem of diffuser was put forth by Liu et al. in Ref.[65],
where not only the wall shape but also the wall suction distribution that maximize the
pressure recovery without boundary layer separation are sought by optimal control method.

An 1-D optimal design method for turbine annular axial-radial exhaust diffuser was
presented by Ling & Jin[67] based on an approximate loss model. The pressure recovery
coefficient of an optimal diffuser designed in this way has been shown higher than the
conventional one by 7% by expriment.

4) Other Optimization Problems.

Making use of the Parson's number and the concept of optimal reaction degree, Yao
presented a method for optimizing aerothermodynamic parameters in one-and multi-stage
steam turbine design. Some guidelines for optimal design of long twisted blades are given.

In Ref.[69], based on the diffusion factor and equivalent diffusion ratio, the
optimal solidity problem of 2-D compressor cascades is formulated by Liu as nonlinear
programing problems, whose analytical solution in form of simple formulae is very
convenient for practical use.

A simple approximate method for determining the optimal relative azimuthal position
of two blade rows in tandem cascades is suggested by Wu & Feng[70] using a simple total
pressure loss model.

VII. MISCELLANEOUS INVERSE, HYBRID & OPTIMIZATION PROBLEMS IN ENGINEERING SCIENCES

In Ref.[71] the finite element method is generalized by Liu & Zhao via variable-
domain variations in such a way that the nodes are movable. It allows both optimal grid
and flow field to be cogenrated simultaneously and naturally using directly the VPs of
aerodynamic problems.

The inverse problem of heat conduction with unknown boundary was handled by Liu &
Zhang[72] using Ritz's and FEM based on Variable-domain VPs. An alternative method for
solving this problem was suggested by Liu[73] by introducing an image plane $T\psi'$ (T and
ψ' stand for tranformed temperature and heat stream function respectively). An interest-
ing invariance property of the nonlinear inverse problem solution with respect to varia-
ble conductivity is pointed out. An example is solved by FEM based on a pair of comple-
mentary extremum principles.

The inverse and hybrid problems of free surface flow under gravity over a dam are
posed and handled by Liu via VPs in an image-plane $\zeta\psi$ [74] and VPs with variable domain
in the physical plane [75,76].

In Ref.[77] Liu suggested a novel problem in gear theory—-optimal meshing(i.e.
optimal tooth profile) of spur gears and its variational theory. An analytical solution
to the optimal meshing with minimal friction losses has been obtained and it has been
revealed that the cycloidal gearing with radial tooth profile on the lower half tooth
height used widely in watches and clocks can be regarded approximately as a practical
gearing with maximal efficiency.

VIII. CONCLUDING REMARKS

Research on inverse, hybrid and optimization problems is of great theoretical as
well as practical importance in engineering sciences. To our experiences, the three new
approaches(image-space approach and VP-based approach, especially its variable-domain
variational variat, for inverse and hybrid problems; optimization approach based on
optimal control) suggested and intensively developed in China in the last two decades
have proved to be efficient tools for inverse design and optimization not only in turbo-
machinery aerodynamics in particular but also in engineering sciences in general and
deserve further development and application to practice. Design engineers and industry
will surely benefit a lot from them, if a complete set of computer codes based on these

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approaches can be finished and organized into a computerized automated interactive
design system(something like that of Ref. [81].

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$[\, I \times H_A \,]$-Problem $[(H_c+D)\times H_A]$-Problem
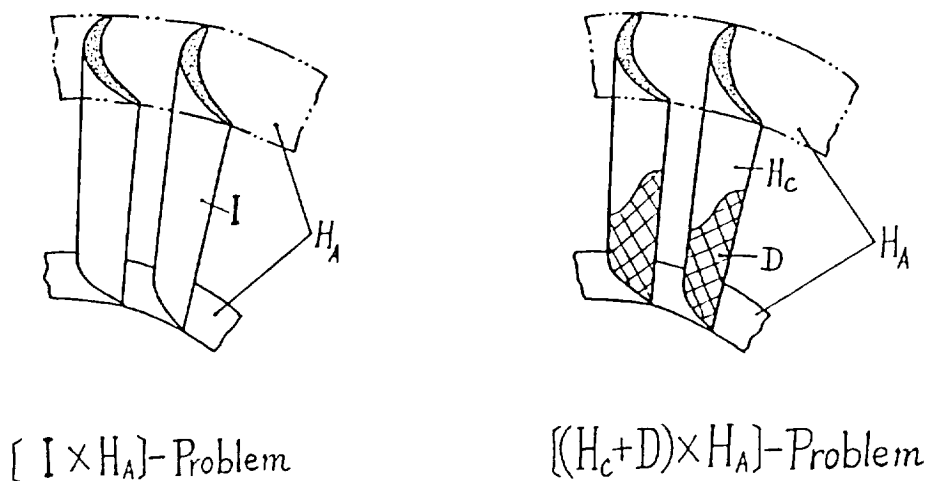
Fig.1 Definition of Symbols for fully 3-D hybrid problems

a) physical stream surface   b) image-plane $\xi\psi$ (or $\xi\eta$)
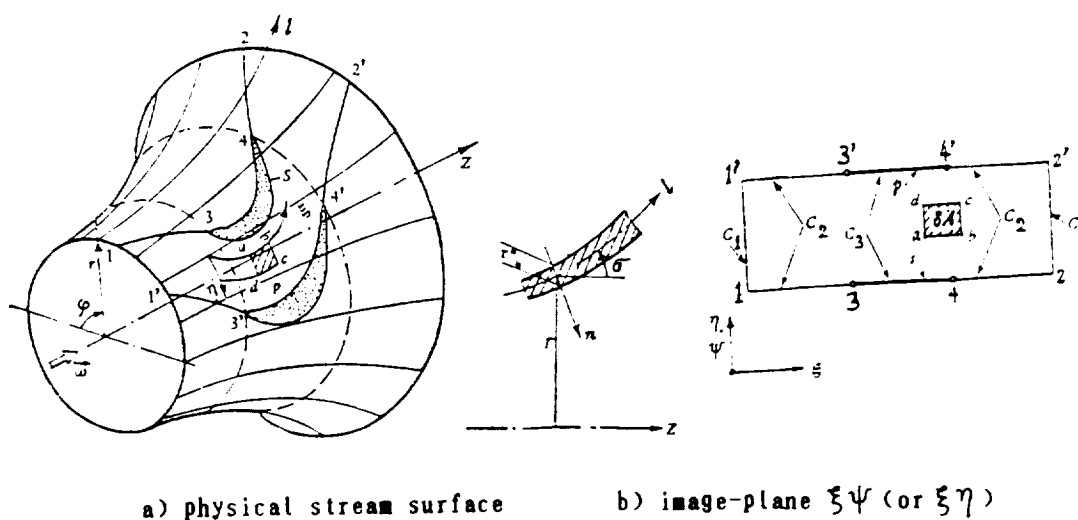
Fig.2 Cascade flow and its image plane

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$M_1 = 0.457$
$\beta_1 = 10.0$
$\beta_2 = -55.1$
••• Present
—— Ref.[78]
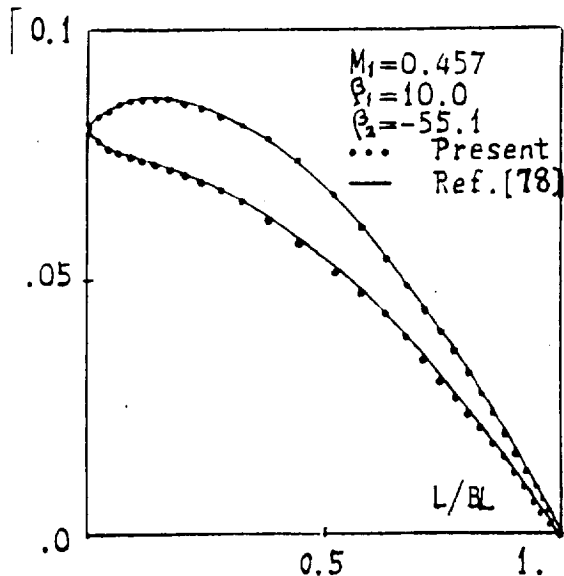
Fig.3 Image-plane solution by

moment function
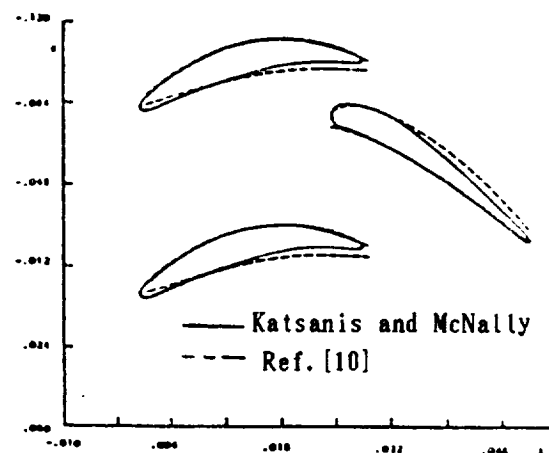
Fig.4 Tandem cascade profiles by an

inverse solution

—— Katsanis and McNally

- - - Ref. [10]

Original
- - - Modified
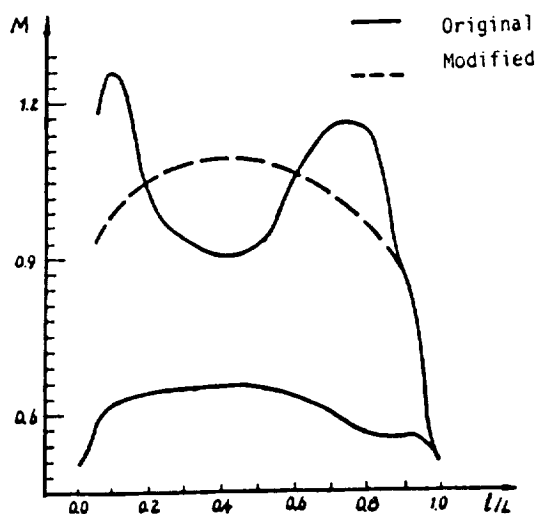
Fig.5 Modified distribution of
Mach number
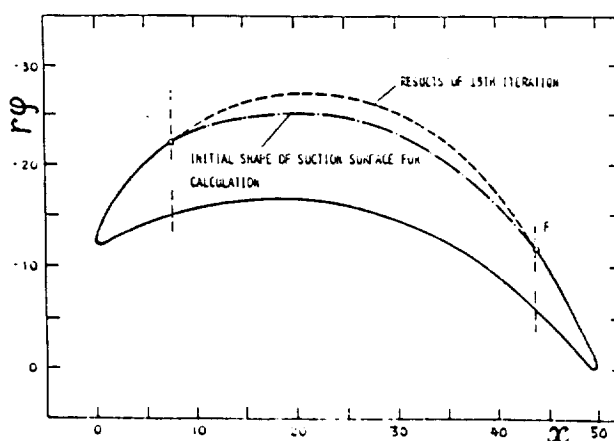
Fig.7 Cascade profile obtained by H_A-

problem solution

—— Profile from NACA L55 F07
- - - Present method

--- 有粘结果 Visous
—— 无粘结果 inviscid
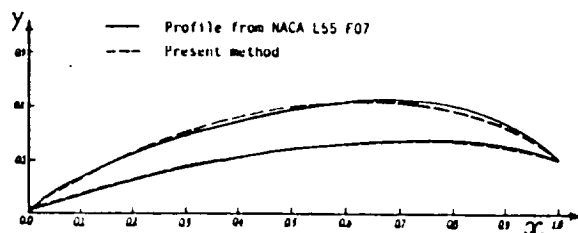
Fig.6 Comparison of profiles corresponding

to Fig.5
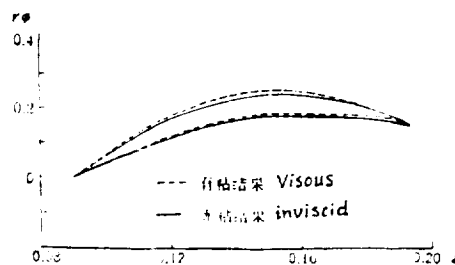
Fig.8 Viscous effect on airfoil shape
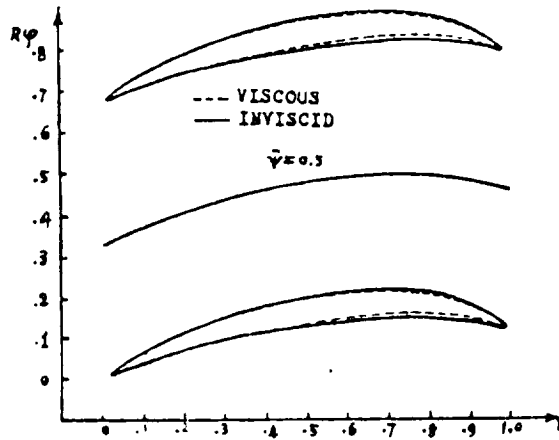
Fig.9 Viscous effect on airfoil shape
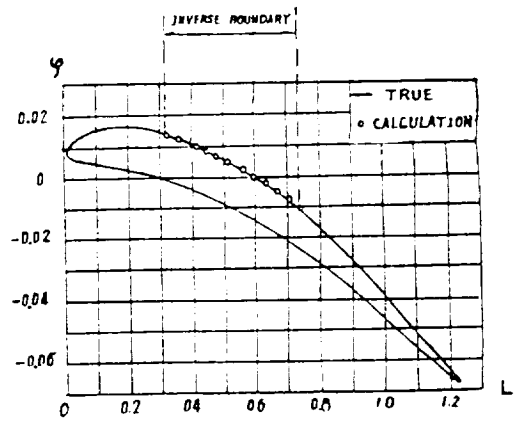
Fig.10 FE solution of H_A-problem

(a)

Fig.12 Airfoil shape by FE solution of
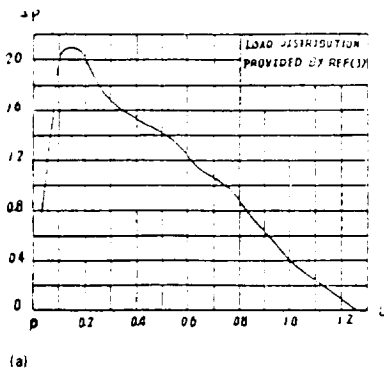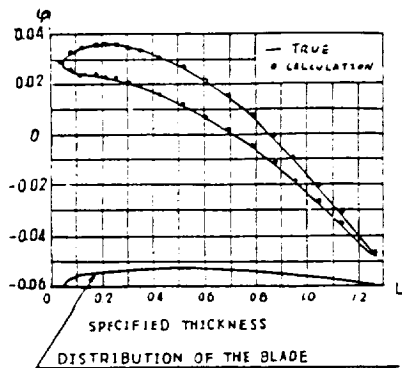
a H_C-problem

(b)

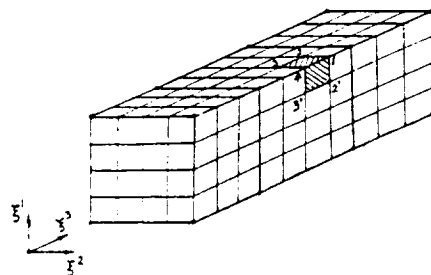**Fig.11 FE solution of H_B-problem**
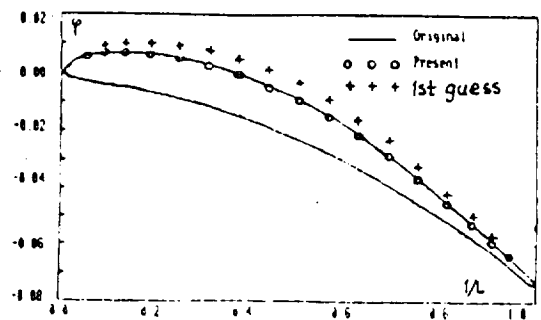
Fig.13 Airfoil shape by FE solution of

a H_A-problem
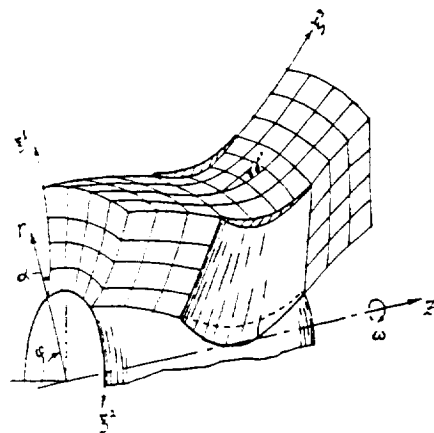
a) Physical space

b) Image space

Fig.14   3-D rotor flow and its image space

20% span
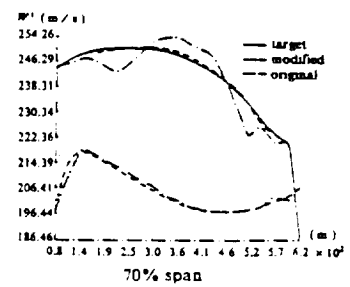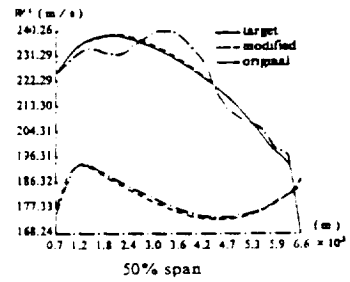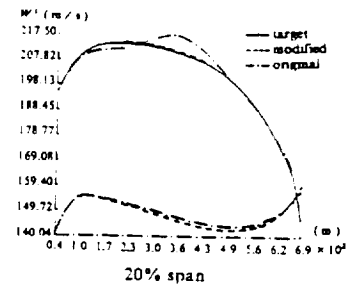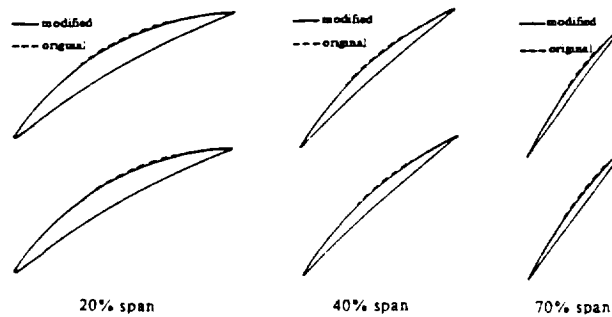
50% span

70% span

Fig.15   Distribution of Λ^1 over
blade surface

20% span   40% span   70% span

Fig.16   The shape of the blade after and before the
modification

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(a) hub   (b) mean   (c) shroud
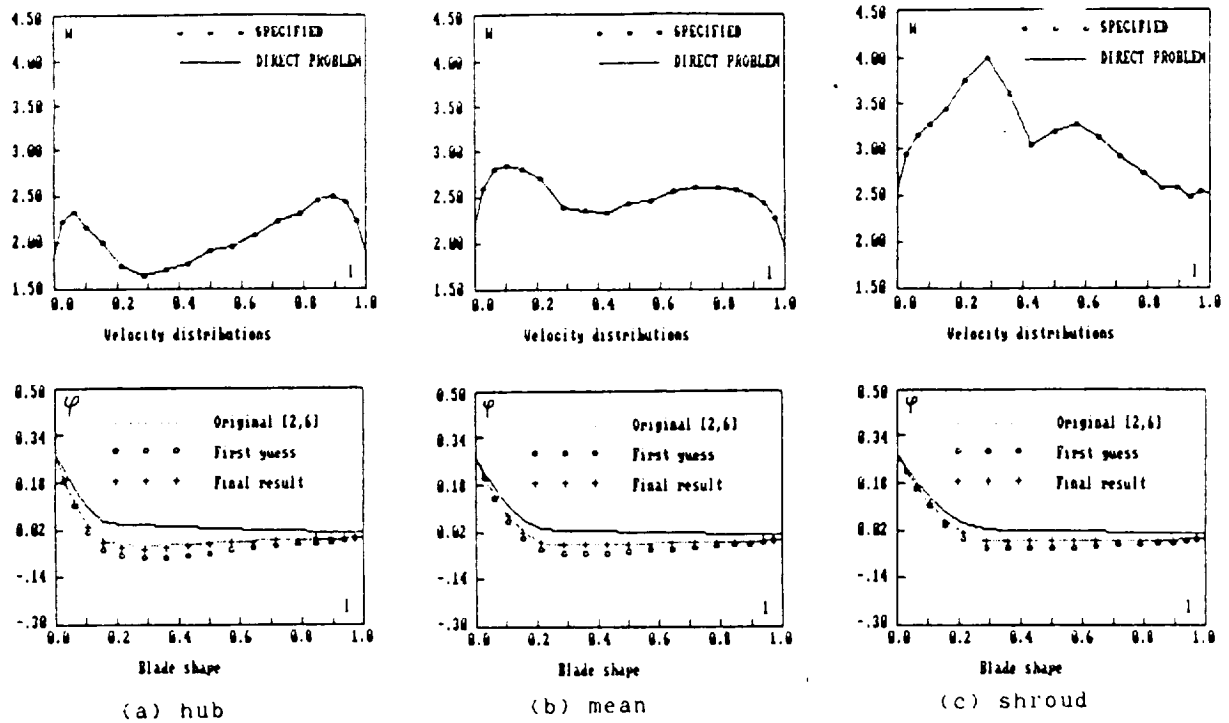
Fig. 17   FE solution of $(H_A \times D)$-problem of a centrifugal compressor

a) hub   b) mean   c)shroud
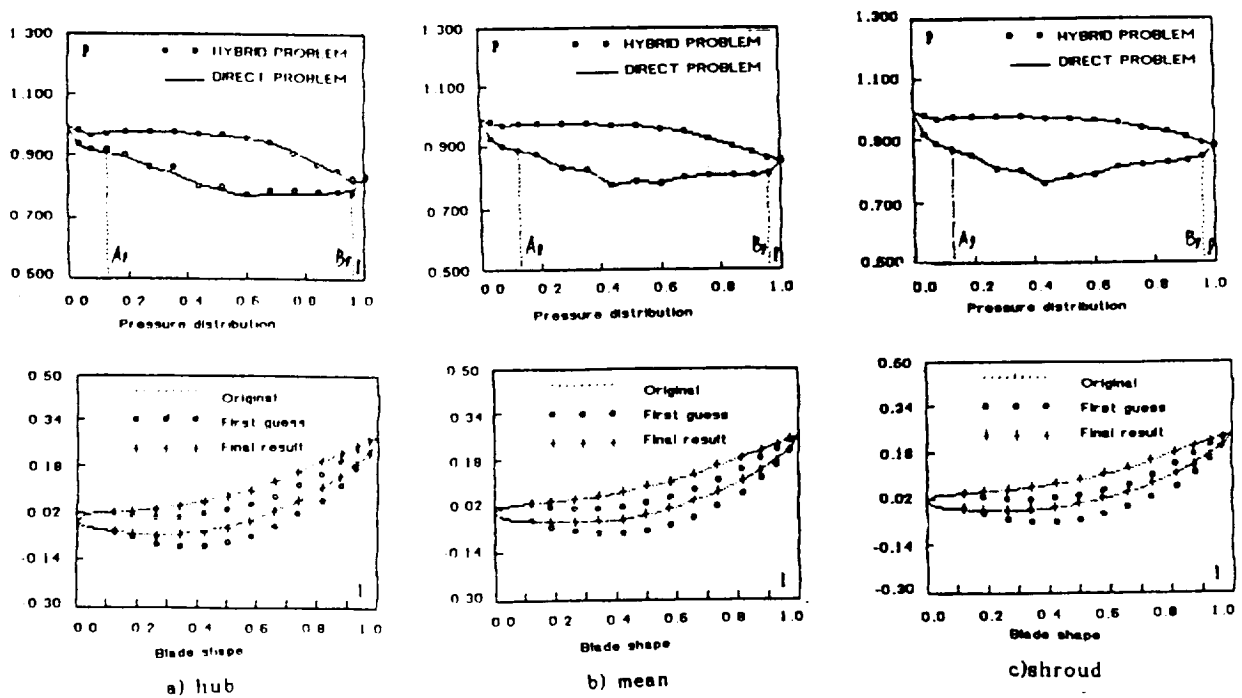
Fig. 18   FE solution of $[(H_c+D)\times D]$-problem of an axial turbine stator