

INVERSE PROBLEMS AND OPTIMAL EXPERIMENT DESIGN
IN UNSTEADY HEAT TRANSFER PROCESSES IDENTIFICATION

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ABSTRACT

Experimental-computational methods for estimating characteristics of unsteady heat transfer processes are analysed. The methods are based on principles of distributed parameter system identification. Theoretical basis of such methods is numerical solutions of nonlinear ill-posed inverse heat transfer problems and optimal experiment design problems. Numerical techniques for solving problems pointed out are briefly reviewed. The results of practical application of identification methods are demonstrated when estimating effective thermo-physical characteristics of composite materials and thermal contact resistance in two-layer systems.

INTRODUCTION

In creating different thermally stressed structures and systems, of wide importance are mathematical modelling and simulation of heat transfer processes occurring inside them. The use of mathematical simulation allows to predict a thermal state of the dynamical system under consideration in wide range of its operational conditions and to estimate the effect of different factors on the system behaviour. Accurate enough thermal state simulation for the system is one of the main procedure, when optimizing thermal conditions and design parameters.

The thermal mathematical model of a system or a process analysed is formed basing on the heat and mass exchange theory (see, e.g. [1]) and it contains a set of characteristics. Characteristics are usually determined by experimental way. By this, most of them can be determined only by means of indirect measurements. In this case a mathematical model is used which is of the given structure and usually contains unknown constant parameters.

It should be emphasized that in determining characteristics, methods of carrying out experiments as well as methods of data processing should consider peculiarities of mathematical models used to simulate thermal conditions. But this factor is not taken into account in overwhelming majority of traditional methods for determining characteristics. Simple mathematical models and severely controlled heating conditions for specimen are used in these methods. Traditional methods for determining of thermophysical characteristics can serve as an example [2]. As the result, a desired accuracy of determining characteristics is not provided. In this case mathematical simulation of thermal conditions is also realized with the low accuracy.

Shortcomings of traditional methods for determining characteristics are displayed when analysing a wide enough range of thermal processes. In particular, one can refer to such processes heat transfer in composite heat shield and thermo-insulating materials, contact heat transfer in high-temperature power plants, heat and mass exchange when materials and structures interact with high-enthalpy homogeneous and heterogeneous flows and

many others. That is why one must develop and implement new methods of study, providing trustworthy information on different characteristics of thermal processes analysed.

Since characteristics should correspond to the mathematical model used, their determination should be considered as a part of mathematical model building by using experimental data. This procedure is called identification problem [3]. When determining characteristics, the mathematical model structure is supposed to be known. In this case one can speak about parametric identification problem [4] or parameter estimation problem [5]. Unsteady thermal processes are referred, as a rule, to the category of dynamical distributed parameter systems. This allows to use experimental-computational methods for determining characteristics based on the main principles and approaches of distributed parameter system identification [6].

IDENTIFICATION OF HEAT TRANSFER CHARACTERISTICS

In identifying heat transfer processes, problems of determining characteristics in mathematical models with given structure are formulated as coefficient-type inverse heat transfer problems [7]. Methods and algorithms for solving these problems are the effective means for determining characteristics of different thermal processes and systems [8,9]. In spite of achievements available, the inculcation of methods based on solving coefficient inverse heat transfer problems was not very active, till recently, because of the following. The fact is that the solution of such problems strongly depends on the used scheme of temperature measurements [4,10,11]. It means that quite different results can be obtained for the same heating conditions of the system analysed but for different number of temperature sensors and their locations. That is why almost every experimental-computational study is followed by labour-intensive analysis of trustworthiness of the results obtained on the basis of numerous parametric computations (See, e.g., [12]). Preliminary optimal design of temperature measurements and other experiment conditions allows to reduce considerably the volume of work. The combination of methods and algorithms for solving inverse problems and experiment design problems is the methodological foundation of identification procedure. This combination forms the new approach increasing essentially an efficiency of thermal studies and determination of heat transfer characteristics.

The voluminous literature is devoted to methods and algorithms for solving inverse heat transfer problems. One can point out, in particular, monographs [4,5,13-20] and bibliography inside them. Most of works available deal with the solution of boundary inverse heat conduction problems, in which thermal boundary conditions are determined by using unsteady temperature measurements inside the body analysed. The considerably lesser number of publications is devoted to solving coefficient inverse problem (see, e.g., bibliography inside [4,14,16,18,19]).

Algorithms suggested at present for solving coefficient inverse problems are based, in overwhelming majority, on minimizing the residual functional. The minimization procedure is built by using an exhaustive method [14,21], matching method [18], method of optimal dynamical filtration [19] and gradient methods. To compute a gradient of the residual functional, the following techniques are used: finite difference method [22], sensitivity functions [23] and a solution of boundary-value problems for conjugate variables, which are written down for linearized direct problems [24-26], as well as for finite difference analogues of direct problems [27]. Efficiency of these techniques is mainly analysed in application to coefficient inverse heat conduction problems to determine thermo-physical characteristics depending on

temperature. The analysis of recent publications [28] says that the most "popular" techniques for solving coefficient inverse problems are based on iterative regularization principle [4].

In contrast to the number of publications on inverse heat transfer problems, works on optimal design of thermal experiments are not very numerous. One can see the bibliography on this topic in works [4,13].

Methods for solving experiment design problems are based on a finite-dimensional approximation of unknown functions. In this case the inverse problem is reduced to determining the vector of unknown parameters. Then, properties of Fisher's information matrix are analysed, elements of which are computed by using sensitivity functions. The elements depend on experiment conditions (see, e.g. [4]).

The determinant of the information matrix or the square root from the minimum eigenvalue of this matrix are used as the criterion of an experiment quality. Experiment conditions are chosen by exhaustion of a given set of possible conditions [29], by the parametric accuracy analysis of the inverse heat transfer problem solution [25] or by solving optimization problem [30,31].

At last, only several publications are available on analysing the complex procedure of heat transfer processes identification and on simultaneous usage of techniques for solving inverse problems and experiment design problems. There exist isolated works devoted to design, carrying out and data processing of real experiments [32,33].

The main goal of this lecture is to demonstrate the efficiency of parametric identification methods through the examples of experimental-computational investigations of heat transfer processes.

INVERSE HEAT TRANSFER PROBLEMS

Many different particular inverse heat transfer problem statements are considered in practice. To describe general features of methods and algorithms for solving ill-posed inverse problems and to avoid details it is convenient to use the general inverse problem formulation in the operator form.

Let us consider an unsteady heat transfer process or thermal system, state model of which has the form of a boundary-value problem

$$L \left(x, \tau, T, \frac{\partial T}{\partial \tau}, \frac{\partial T}{\partial x}, \frac{\partial^2 T}{\partial x^2}, u, v \right) = 0, \quad x \in Q, \quad \tau \in [0, \tau_m] \quad (1)$$

$$T(x, 0) = T_0(x), \quad x \in \bar{Q} = Q + \Gamma, \quad (2)$$

$$B \left(x, \tau, T, \frac{\partial T}{\partial x}, u \right) = v(\tau), \quad x \in \Gamma \quad (3)$$

where $L(\cdot)$ is a non-linear operator; $B(\cdot)$ is an operator of boundary conditions; T is the state variable (temperature); τ is time; x is space; u is vector of characteristics of the system analysed; $v(\tau)$ is an external action.

In the model (1)-(3) the state variable T can be a scalar or vector function of space and time.

The model (1)-(3) is a direct problem and under given values for external action $v(\tau)$ and vector of characteristics u permits to predict the system's thermal state. If vector u is unknown or given with low accuracy, but there is some additional information about the solution of the problem (1)-(3), then an inverse problem appears for determining vector of characteristics u . The additional information is formed on the basis of measuring the state variable in a subdomain of Q . When the state variable is temperature, measurements are usually carried out in some number N of separate points of the domain Q

$$T_{\text{meas}}(X_i, \tau) = f_i(\tau), \quad i = \overline{1, N} \quad (4)$$

where $X_i, i = \overline{1, N}$ are coordinates of temperature sensor locations. The inverse problem is to determine u from conditions (1)-(4). By this, the form of operator B as well as the number of sensors N are chosen so that we can provide uniqueness of the inverse problem solution [4].

The state model (1)-(3) can be treated as the transformation Au of a space of characteristics into a space of the state variable in measurement points. As the result of measurements a vector-function $f = \{f_1(\tau), f_2(\tau), \dots, f_N(\tau)\}$ is formed. The inverse problem is to determine u so that computed state variable in measurement points is equal to measured values. In this case the inverse problem (1)-(4) can be presented as a non-linear operator equation of the first kind

$$Au = f, \quad u \in U, \quad f \in F, \quad A: U \Rightarrow F \quad (5)$$

where operator A is constructed on the basis of the model (1)-(3); U is the solution space; F is the space of vector-functions being measured.

The main distinction of inverse problems is ill-posedness. The inverse operator A^{-1} can be unlimited and small errors in the right part can lead to large deviation in the solution. So, to solve inverse problems it is necessary to use special, regulating methods [34].

It should be noted that the solution space U in the inverse problem (5) is constructed by taking into account constraints arising from physical point of view. For example unknown characteristics must be positive in many cases.

To solve coefficient inverse problems the iterative regularization method has displayed quite a high efficiency. This method is based on minimizing, by means of gradient methods of the first order, residual functional

$$J(u) = \|Au - f\|_F^2 \quad (6)$$

The regularization parameter is the number of the last iteration, which is determined in the process of problem solving from a regularizing condition

$$J(u) \cong \delta^2 \quad (7)$$

where δ^2 is the error of input data, calculated in space F.

It should be noted that for linear ill-posed problems the iterative regularization method received severe mathematical substantiation. In a non-linear case such substantiation is not available. However, extensive computational experiments confirm high efficiency of this method for solving non-linear problems as well (see, e.g. [4,16]).

In constructing algorithms for solving coefficient inverse heat transfer problems, when the unknown characteristics depend on the state variable, a common approach is the parametrization of functions sought for, in particular by means of cubic B-splines [35]. The solution is sought for as

$$z(T) = \sum_{k=1}^m p_k \varphi_k(T) \quad (8)$$

where $z(T)$ is an unknown characteristic; p_k , $k = \overline{1, m}$ are constant parameters; $\varphi_k(T)$, $k = \overline{1, m}$ is the given system of basis functions. The inverse problem is to determine a vector of parameters $u = [p_1, p_2, \dots, p_m]^T$, the composition of which includes coefficients of approximation of all functions sought for. An iterative procedure of minimizing the residual functional (6) by using the method of conjugate gradient projection is built via formulas

$$p_k^{r+1} = P_v \left[p_k^r + \gamma_k g_k^r \right], \quad r = 0, 1, \dots, R, \quad (9)$$

$$g_k^r = J_k^{(r)} + \beta_r g_k^{r-1},$$

$$\beta_0 = 0, \quad \beta_r = \frac{\sum_{k=1}^m (J_k^{(r)} - J_k^{(r-1)}) J_k^{(r)}}{\sum_{k=1}^m (J_k^{(r-1)})^2},$$

where P_v is the operator of projecting on the multitude W of admissible solutions; R is the number of the last iteration. The calculation of gradient components J_k , $k = \overline{1, m}$ is accomplished through the solution of a boundary-value problem for conjugate variable [9]. An approximate method is used to realize the projection operation [38].

A descent parameter is determined from the condition

$$\gamma_r = \text{Arg min}_{\gamma > 0} J \left[P_v (u^r + \gamma g^r) \right] \quad (10)$$

where $g = [g_1, g_2, \dots, g_m]^T$. If one characteristic is unknown in the

Inverse problem, then to solve a problem of minimization (10) we can make use of known methods, such as a "golden section" method [36]. For multiparameter inverse problems, of much greater computational efficiency is the technique based on representation of a descent parameter γ as a vector value [37]. Various modifications of this technique are described in [4]. Parameter γ_r is determined for unconstraint minimization procedure and then projection operations are successively realized for all unknown functions [38]. It should be emphasized that in many cases good results have been obtained by using unconstraint methods for minimizing the residual functional. The high capacity for work of such iterative algorithms for solving coefficient inverse problems is demonstrated, for example, in [4,10,24,39,40].

OPTIMAL EXPERIMENT DESIGN

The input data for solving the inverse problem are formed basing on information obtained in the result of corresponding experiments and measurements. Under formation are two groups of values. The first group includes values displayed in model (1)–(3), determining the conditions of an experiment: a dimension of a specimen \bar{Q} in study, duration of an experiment τ_m , initial distribution of a state variable $T_0(x)$, external action $v(\tau)$. Combine these conditions to vector

$$w = \{ \bar{Q}, \tau_m, T_0(x), v(\tau) \} \quad (11)$$

The second group of values characterizes the conditions for measuring a state variable and in the case under consideration includes the sensor number N and vector of their space positioning in the specimen $X = [X_1, X_2, \dots, X_N]^T$. These values make up a scheme or a plan of measurements

$$\xi = \{ N, X \} \quad (12)$$

In total, vectors w and ξ determine a plan of the experiment

$$\pi = \{ w, \xi \} \quad (13)$$

The inverse problem (5) can be solved, generally speaking, with different plans of the experiment Π . But the results of studies have shown that quite an arbitrary selection of elements of the experiment plan (13) can lead to large errors in the inverse problem solution [4,10,31,41]. Hence, a problem arises on optimization or optimal design of experiments in identifying thermal processes with the aim of providing maximum accuracy for the unknown characteristics determination in the assumed mathematical model [42]. A search for optimal plans of experiments leads to the necessity of solving extreme problems

$$\pi_0 = \text{Arg max } \Psi(\pi), \quad \pi \in \Pi \quad (14)$$

where $\Psi(\pi)$ is the quality criterion of the experiment, characterizing the accuracy of solution of the inverse problem under analysis; Π is a set of admissible plans.

The accuracy of solution of the inverse problem (5) is determined by properties of the Freshe derivative A' of operator A , reflecting the nature of error transformation of the right part f into errors of solution u [43]. It is possible to show [4] that the above properties of A' are characterized by eigenvalues μ_k , $k = \overline{1, m}$ of matrix

$$M = \{ \Phi_{j,k}, \quad j, k = \overline{1, m} \} \quad (15)$$

$$\text{where } \Phi_{j,k} = \frac{1}{N} \sum_{i=1}^N \int_0^{\tau_m} x_i(\tau) \theta_j(x_i, \tau) \theta_k(x_i, \tau) d\tau$$

$x_i(\tau)$, $i = \overline{1, N}$ are weight functions, giving a possibility to consider the presupposed errors in the measurements of a state variable, $\theta_k(x, \tau) = \partial T(x, \tau) / \partial p_k$, $k = \overline{1, m}$ are sensitivity functions. For the inverse problem (5) matrix (10) coincides with the Fisher's normalized information matrix, widely used in the theory of experiment design [44]. The following values can be used in particular as an optimization criterion: a square root from the minimum eigenvalue $\sqrt{\mu_{\min}}$ and a determinant $\det M = \prod_{k=1}^m \mu_k$. The computational experiments carried out showed high capacity for work of the given criteria [4, 32, 33, 41]. A set of admissible plans is formed with regards for the conditions of uniqueness of solution of the inverse problem and with constraints, characterizing the capacity of the experiment equipment used and that of measurements [4].

To determine the elements of matrix (15) it is necessary to calculate sensitivity functions $\theta_k(x, \tau)$, $k = \overline{1, m}$. These functions are calculated using a boundary-value problem obtained as the result of differentiation of relations (1)-(3) through parameters p_k , $k = \overline{1, m}$. Here, due to non-linearity of operators L and B , sensitivity functions depend on the vector of unknown parameters u . Hence, it is possible to construct only approximate, locally optimal plans of experiments involving apriori information about vector u [44]. The studies carried out show that apriori information, usually available, gives a possibility to get local-optimal plans close enough to exact plans [4, 41, 46].

Using described methods for solving problems of optimal design of thermal experiments there have been developed corresponding computational algorithms based on the scanning method [45] and on the optimal control theory [31]. Their high efficiency is shown, for example, in [4, 32].

IDENTIFICATION OF EFFECTIVE THERMOPHYSICAL
CHARACTERISTICS OF COMPOSITE MATERIALS

The latest two decades witness constant increase of publications devoted to methods and algorithms for solving coefficient inverse heat conduction problems and to their application. In fact, the first results of investigations in this field were published in 1963 [47,48]. The main part of subsequent works dealt with suggesting algorithms and analysing their computational efficiency. Experimental-computational studies are considered in considerably smaller part of publications.

One of the main goal of investigations is to create a reliable non-stationary method for determining effective thermophysical characteristics of composite thermal protective materials at high temperatures [39,49]. It is clear today that such methods should be built by using identification approaches including the solution of coefficient inverse heat conduction problems and experiment design.

To illustrate the practical application of identification methods let us consider determination of the effective thermal conductivity for glass-reinforced plastic on silicone binder, heated by a high-enthalpy gas flow [32].

To realize a complex procedure of identification there has been conducted a number of experiments with a one-side gas-dynamic heating of flat specimens of the material of 20 mm thickness. Temperature measurements at different depth from a heated surface were taken by means of thermocouples. The nominal heating conditions and duration of the experiments were given beforehand. For control of its reproduction in the experiments and for formation of a boundary condition of the first kind, measurements have been used by the thermocouple nearest to a heated surface. The location of these thermocouples in specimens was further considered as the origin of a solid axis x . The indications of thermocouples located at the biggest distance from a heated surface served as the second boundary condition of the first kind. The location of thermocouples was determined by means of X-ray radioscopy. All subsequent experiments showed approximately the same results, heating conditions of all specimens being similar to each other.

The mathematical model of heat transfer process in the material looked like a boundary-value problem for the non-linear heat conduction equation

$$C(T) \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x} \left(\lambda(T) \frac{\partial T}{\partial x} \right), \quad 0 < x < L, \quad 0 < \tau \leq \tau_m \quad (16)$$

$$T(x, 0) = T_0(x), \quad 0 \leq x \leq L \quad (17)$$

$$T(0, \tau) = v_1(\tau) \quad (18)$$

$$T(L, \tau) = v_2(\tau) \quad (19)$$

The results of temperature measurements of (4) type in the internal points of interval $[0, L]$ served as input data for solving the inverse problem on determination of function $\lambda(T)$. The inverse problem analyzed has a

unique solution at $N \geq 1$ [4].

Optimal planning of an experiment was made first. Since its heating conditions and time were given, the control over the experiment quality was carried out only by means of selecting an optimal plan of measurements (12). For this purpose an extreme problem of measurement planning has been solved

$$\xi_0 = \text{Arg max } \Psi [M(\xi, \lambda(T))], \quad \xi \in \Xi \quad (20)$$

where $\Xi = \{(n, X): N \geq 1, 0 < X_i < 1, i=1, N\}$

To solve problem (20) there has been used a procedure of work [45]. The boundary conditions of the first kind (15), (16) are shown in Figure 1. The dependence $\lambda(T)$ obtained by traditional method served as a priori information about the unknown function. Function $\lambda(T)$ was approximated by a cubic B-spline of (8) type with "natural" boundary conditions [35] with the parameter number $m = 4$. So, vector $p = [p_k, k=1,4]$ was unknown. Sensitivity functions $\theta_k(x, \tau)$, $k = \overline{1, m}$ were determined from a solution of boundary-value problems obtained by differentiating relations (16)-(19) through parameters p_k , $k = \overline{1, 4}$.

The results of solving of a problem on selecting an optimal location of one/ and two thermocouples are given in Figure. 2, where a change of the experiment quality criterion is illustrated $\Psi [M(\xi, \lambda(T))] = \sqrt{\mu_{\min}}$ depending on the sensors setting coordinates. For two sensors there are shown surface sections $\Psi(X_1, X_2)$ by planes drawn through the point of maximum value of criterion parallel to coordinate planes.

The results obtained show that in the analyzed experiment to provide high accuracy of solution of an inverse problem one sensor should be set in the narrow enough domain close to the origin of coordinates. Besides, in this experiment two sensors will be sufficient since at $N > 2$ the location coincidence of the second and successive sensors seems most optimal. The conclusions made are fully confirmed by data of computational experiments [32].

A solution of the inverse problem followed then using the procedure of work [24]. A thermogram of the corresponding experiment is shown in Figure. 1. To verify validity of the measurement plan and to estimate the authenticity of the inverse problem solution analysis was made of the effect of the initial guess about the unknown function on the solution [32]. The results of such an analysis are given in Figure. 3.

It is seen that the solution of the inverse problem does not depend on values of initial guess, thus proving high authenticity of identification results obtained. For comparison on Fig. 3 there is also given a temperature dependence of thermal conductivity obtained by the method of monotonic heating. It is seen that in the high-temperature region there is a considerable difference of this dependence from that obtained from the solution of the inverse problem. Here, the dependence $\lambda(T)$ obtained as a result of identification provides much better temperature correspondence, calculated from (16)-(19), with values measured experimentally, this confirming high authenticity of results as well.

It should be noted as a conclusion of this section that effective thermophysical characteristics of high-temperature composite materials can strongly depend on heating conditions. It is caused by thermal destruction of a binder [49], this process depending on heating rate [50]. The analogous dependence takes place for semi-transparent materials [51]. To avoid this factor, methods based on solution of inverse heat transfer problems have been developed for determining heat transfer characteristics in more complicated mathematical models taking into account effects of thermal decomposition [25,39,52,53] and heat transfer by radiation [40,54].

IDENTIFICATION OF CONTACT THERMAL RESISTANCES IN MULTILAYER STRUCTURES

Contact heat transfer is important in different technical systems. The main characteristic of this process is contact thermal resistance. At present stationary methods are widely used to determine contact thermal resistances in different joints [55,56]. Non-stationary methods based on solving inverse heat transfer problems are more effective (see, e.g. [7,17]) but in spite of the fact that the first works devoted to such methods were published about twenty years ago [57-59], only isolated investigations are known in this field especially experimental-computational studies. Works devoted to optimization of experiments for identifying contact thermal resistances are also isolated.

The application of identification method is considered in this section to determine thermal contact resistances between fuel and shell in fuel rods of a nuclear-power reactor. Transient processes between successive stationary states are analyzed when the reactor is started up for the first time. The results of experimental-computational studies presented in works [33,60-63] are briefly discussed.

The mathematical model of a non-stationary heat transfer process in a fuel rod is given by the following boundary-value problem

$$C_1(T) \frac{\partial T_1}{\partial \tau} = \frac{1}{x} \frac{\partial}{\partial x} \left(\lambda_1(T) \frac{\partial T_1}{\partial x} \right) + q_v(x, \tau), \quad L_0 < x < L_1, \quad 0 < \tau \leq \tau_m \quad (21)$$

$$C_2(T) \frac{\partial T_2}{\partial \tau} = \frac{1}{x} \frac{\partial}{\partial x} \left(\lambda_2(T) \frac{\partial T_2}{\partial x} \right), \quad L_1 < x < L_2, \quad 0 < \tau \leq \tau_m \quad (22)$$

$$T_1(x, 0) = T_{0,1}(0), \quad L_0 \leq x \leq L_1 \quad (23)$$

$$T_2(x, 0) = T_{0,2}(0), \quad L_1 \leq x \leq L_2 \quad (24)$$

$$\frac{\partial T_1(L_0, \tau)}{\partial x} = 0 \quad (25)$$

$$\lambda_1(T_1(L_1, \tau)) \frac{\partial T_1(L_1, \tau)}{\partial x} = \lambda_2(T_2(L_1, \tau)) \frac{\partial T_2(L_1, \tau)}{\partial x} \quad (26)$$

$$-R \lambda_1 (T_1(L_1, \tau)) \frac{\partial T_1(L_1, \tau)}{\partial x} = T_1(L_1, \tau) - T_2(L_1, \tau) \quad (27)$$

$$\lambda_2 (T_2(L_2, \tau)) \frac{\partial T_2(L_2, \tau)}{\partial x} = \alpha(\tau) [T_2(L_2, \tau) - T_0(L_2, \tau)] \quad (28)$$

In the model (21)-(28) initial temperature distributions $T_{o,1}(x)$ and $T_{o,2}(x)$ are computed by solving the corresponding stationary problem. Energy release in fuel $q_f(x, \tau)$ was computed taking into account radial nonuniformity $\bar{q}(x)$ and integral heat release $q_t(\tau)$ measured by neutron detectors

$$q_f(x, \tau) = q(\tau) \bar{q}(x) \quad (29)$$

where $q(\tau) = \frac{q_t(\tau)}{2\pi \int_{L_0}^{L_1} \bar{q}(x) dx}$

The contact thermal resistance R is unknown but temperatures are available measured in some points of the structure analysed

$$T_{meas}(X_{j,i}, \tau) = f_{j,i}(\tau), \quad i = \overline{1, N_j}, \quad j = 1, 2 \quad (30)$$

The inverse problem is to determine R from conditions (21)-(30). During each transient regime contact thermal resistance R was considered as a constant. The main goal of the investigation was to determine experimentally R depending on integral heat release q_t .

Iterative numerical algorithms were developed for solving inverse problems analysed [60,62], the residual functional being written down in the form

$$J = \sum_{j=1}^2 \sum_{i=1}^{N_j} \left(\int_0^{\tau_m} (T_{j,i}(X_{j,i}, \tau) - f_{j,i}(\tau))^2 d\tau + (T_{j,i}(X_{j,i}, 0) - f_{j,i}(0))^2 \right) \quad (31)$$

Algorithms for solving temperature measurements design were also developed. These algorithms were used in carrying out experimental-computational studies of contact heat transfer processes in fuel rods. Some results are briefly discussed below.

General sequence of stages was similar to that for thermophysical

characteristics identification. At the first stage parametric analysis of the accuracy of inverse problem solution as well as optimal temperature measurement design were made. Input data for solving these problems one can see in work [61].

The results of measurement design for one thermocouple are shown in Figure 4. One can see that the thermocouple installation into the fuel is much more effective. The conclusions made are fully confirmed by data of parametric accuracy analysis of the inverse problem solution (61). Computations show that it is quite enough to use one thermocouple. Basing on the results obtained, thermocouples were installed on the internal surface of fuel tablets which had the shape of hollow cylinders.

The results of experimental data processing are illustrated in Figure 5. One can see that a decrease takes place when q_1 is approximately equal to 210W/cm. It testifies to the fact that the fuel gets in touch with the shell. For comparison the dependence analysed is shown here, which was obtained by using the method of work [64]. The last one does not predict the moment of touch and so gives much more optimistic results of safety analysis.

CONCLUSION

The results presented demonstrate high efficiency of methods for thermal studies based on distributed parameter system identification. Such methods facilitate to obtain trustworthy data for heat transfer characteristics and increase the accuracy of mathematical simulation of thermal conditions.

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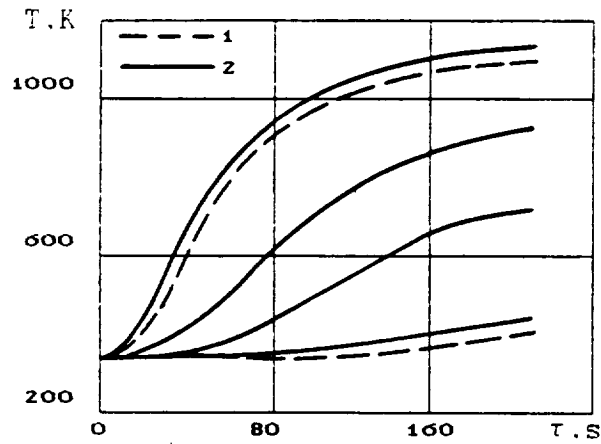


Fig. 1 Results of temperature measurements:
1 - for solving the design problems;
2 - for solving the inverse problem.

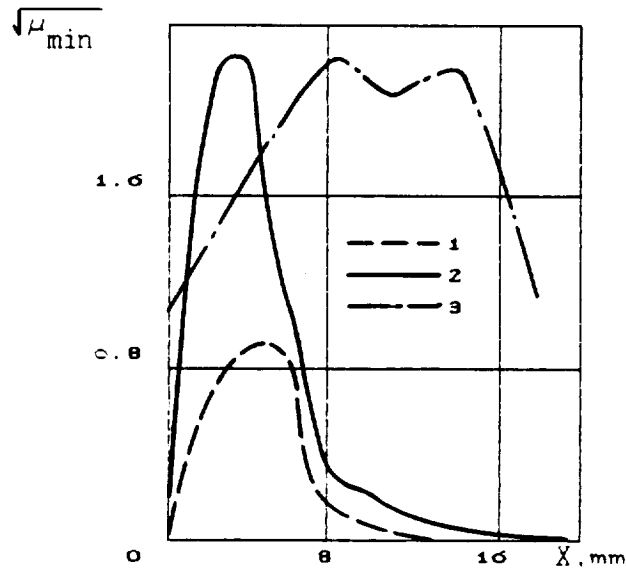


Fig. 2 Dependence of the efficiency criterion on the coordinates of thermosensor locations:
1 - N=1; 2 - N=2, section 1;
3 - N=2, section 2.

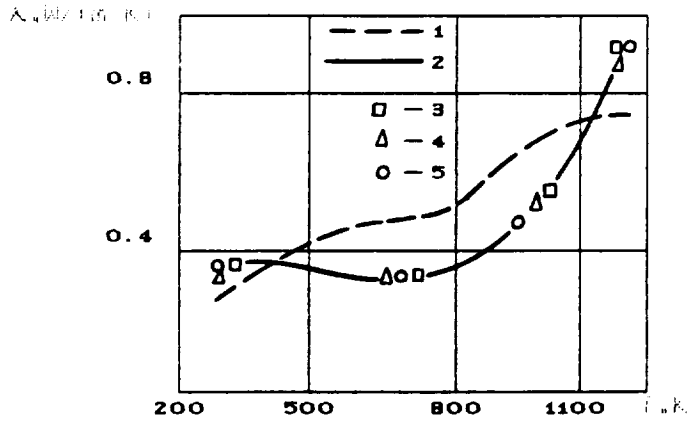


Fig. 3 Dependence of thermal conductivity on temperature: 1 - traditional method; 2 - inverse problem solution; 3 - $\lambda=0.2$ W/(m·K); 4 - $\lambda=0.4$; 5 - $\lambda=0$.

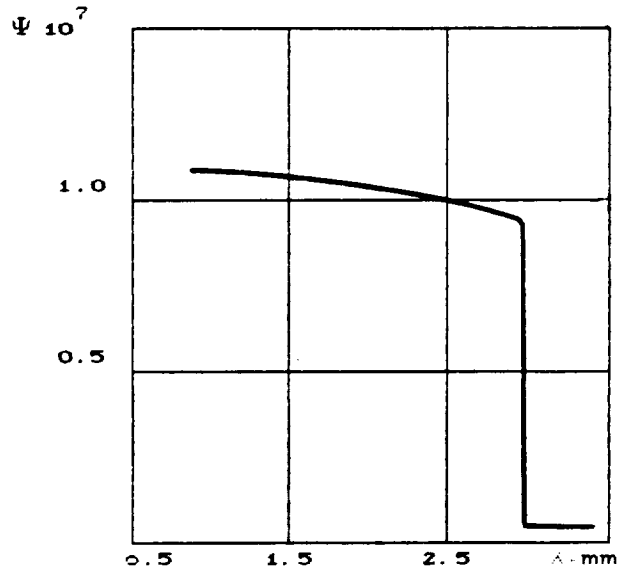


Fig. 4 Change of design criteria Ψ depending on sensor coordinate λ .

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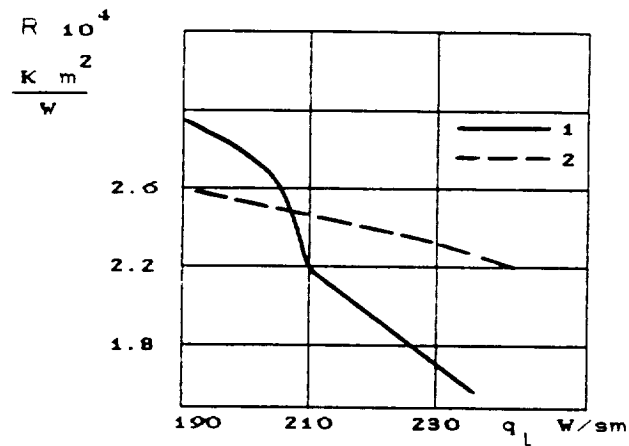


Fig.5 Dependence of contact thermal resistance on integral heat release:
1 - inverse problem solution;
2 - prediction.