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A NEW METHOD OF OPTIMAL DESIGN FOR A TWO–DIMENSIONAL DIFFUSER BY USING DYNAMIC PROGRAMMING

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ABSTRACT

A new method for predicting the optimal velocity distribution on the wall of a two–dimensional diffuser is presented in the paper. The method by Principle of Dynamic Programming solves the optimal control problem with inequality constraints of state variables. The physical model of optimization is to protect the separation of the boundary layer while getting to be maximum pressure ratio in a diffuser of a specified length (or getting to the shortest length in a specified pressure ratio). The calculation results are fairly in agreement with the experimental ones. It shows that optimal velocity distribution on a diffuser wall should be as: the flow decelerates first quickly and then smoothly, while the flow is near separation but always protects from it. The optimal velocity distribution can directly be used to design the contour of the diffuser.

INTRODUCTION

A diffuser is an important part of compressors, fans and other air ducts. More and more attentions have been paid to its design. In the past dozens of years, the popularization and development of the optimization technique make it possible to design a diffuser with optimal velocity distribution

The index of optimizing a diffuser is to obtain the highest pressure ratio under the condition of a minimum constructional length. Generally speaking, in order to get an optimal shape of a diffuser, it is necessary to know an optimal velocity distribution on its wall. With the distribution, the boundary layer can be avoided seperation and a maximum pressure ratio (or pressure recovery) can be obtained in a specified length.

Nowadays, most designs of diffusers, which are two–dimensional or axial–symmetrical, are still based on experience. Designers often use the criteria of the diffusing angle or the equivalent diffusing angle and one–dimensional calculational method to design it. Obviously, it is too simple to reach the index of the optimal design.

Stratford(1959) proposed that the loss in a diffuser with the minimum length is the minimum while the boundary layer inside it is close to but just before occurrence of separation, then the velocity distribution is the best and the shape of the diffuser is optimal. Some researchers, such as H. liebeck, H. Fernboly, have used this principle to make some optimal designs.

Many authors also investigated the flow field in a diffuser and study how to control the flow

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separaction.

Some authors attempt to use the optimal control theory to solve the optimal design of a diffuser, because the governing equations of the flow in it are differential ones.

Gu and Ji (1987) proposed an optimal design problem of a diffuser, using the optimal control theory and the boundary layer theory. The optimal velocity distribution on its wall was obtained by using Pontryangin's maximum principle.

In order to meet the demands of engineering application, the optimal problem has to satisfy some constraints in both aerodynamics and strength which can be divided into two parts: one is called as constraint of state variable and another constraint of control variable. Those constraints are often inequality and make it very difficult to solve the problem in mathematical treatment.

It is well known that Pontryagin's principle can only solve the optimal control problem with constraints of control variables. To overcome the difficulty, many authors have done some research work and modifications such as continuous transfer techique (Jocobson, 1969; Gu, 1987) and expanded penalty function method (Gu and Miao, 1987).

However, for a problem with more inequality constraints of state variables the treatment is not efficient which have been stated by Gu and Miao(1987). That is to say, The more the constraints, the more the difficulties. On the contrary, the principle of dynamic programming is quite good at treating of state and control constraints. The more the constrains, the faster the calculation, because the numer of considered states and decisions decreases in seeking optimal decision.

In the present work, a physical model and a mathematical expression for dynamic programming are established and calculated. The result yielded by the method is quite in agreement with not only the experimental ones but also the result by Pontryagin's maximum principle.

ESTABLISHMENT OF AN OPTIMAL DESIGN PROBLEM OF A DIFFUSER

It is well known that the flow losses in a diffuser mainly consist of separation loss and friction one. Obviously, the former is greater than the latter. The friction loss is always inevitable. However, it doesn't vary greatly because the friction coefficient is approximately a constant in the fully–developed turbulent flow. The total friction loss can be considered as increasing proportionally with the axial length of a diffuser. So the key to designing an efficient diffuser is to avoid the separation of boundary layer. Considering the two factors mentioned above, Stratford. (1959) proposed that the properties of a diffuser with the minimum length and without boundary layer separation is optimal.

Generally, the turbulence degree at the inlet of a diffuser in engineering is so high that we can assume for convenience that the boundary layer has become a turbulent one at the inlet edge. And incompressible flow is only considered in present work.

The typical expressions of a optimal problem for a diffuser are as follows:

A) Pressure rise coefficient is maximum (i.e. the discharge velocity is minimum) under the condition of a provided constructional length and without separation of the boundary layer.

B) The length of a diffuser is minimum under the condition of a provided discharge velocity (i.e. the pressure rise coefficient is known) and without separation of the boudnary layer.

It can be proved that the expressions A) and B) are correlative.

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The separation of the boundary layer is a very complex problem. According to the change of velocity in main flow, the separation of boundary layer can be predicted by some experimental formulas to some extent. The following equations are adopted as the basis of solving the optimal problem, (Ref. 1,3,4,5,6,7,8,9,10).

The velocity shape factor of boundary layer is introduced as follows:

$$\Gamma = \frac{\delta_2 du}{U dx} Re_{\delta_2}^{m} \qquad \begin{cases} m = 1/4 & \text{turbulent flow} \\ m = 1 & \text{laminar flow} \end{cases} \tag{1}$$

where

$$\delta_2 = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \quad Re_{\delta_2} = \frac{U\delta_2}{\nu}$$

U is the velocity of main flow.

$\Gamma > 0$ denotes acceleration flow, and $\Gamma < 0$ deceleration one. So Γ can be used to judge whether the separation happens or not.

For a deceleration flow, the relationship between $\delta_2$and U is as follows (Ref.1) :

$$\frac{d}{dx}\left[\delta_2\left(\frac{U\delta_2}{\nu}\right)^{1/4}\right] = 0.0175 - 4.15\frac{\delta_2}{U}\frac{dU}{dx}\left(\frac{U\delta_2}{\nu}\right)^{1/4} \tag{2}$$

For convenience, the length of a diffuser, L, is used as a characteristic length; the velocity at the inlet, C, as a characteristic velocity. Then Eq. (2) can be rewritten in a non-dimensional form. The non-dimensional length of a diffuser contour is $S = X/L$, the non-dimensional velocity $V = U/C$, the non-dimensional momentum thickness $\theta = \delta_2/L$. So the Re δ_2 is

$$Re_{\delta_2} = U \cdot \delta_2 / \nu = V \cdot \theta \cdot Re_0$$

where $Re_0 = C \cdot L/\nu$ at the inlet of the diffuser.

Then Eq. (2) becomes as:

$$\frac{d}{ds}\left[\theta(Re_0 \cdot V \cdot \theta)^{1/4}\right] = 0.0175 - 4.15 \cdot \frac{\theta}{V}\frac{dV}{ds}(Re_0 \cdot V \cdot \theta)^{1/4} \tag{3}$$

Substituting the non-dimensional form of Eq. (1) into Eq. (3), we yield

$$\frac{d\theta}{ds} = (0.014 - 3.52 \cdot \Gamma)(Re_o \cdot V \cdot \theta)^{-1/4} \tag{4}$$

According to the result of Nikuradse's experiments, Buri proposed that the boundary layer will separate when Γ is not greater than -0.06. As stated by Gu and Ji(1987) to ensure the flow in a diffuser to be far from separation, we utilize the limit of Γ as :

$$-0.04 \leqslant \Gamma \leqslant 0$$

MATHEMATICAL EXPRESSION OF THE OPTIMAL PROBLEM

The mathematical expression of the index of optimization A) is as follows:

Index function:

$$J(*) = V(*) \rightarrow min \tag{5}$$

s.t.

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$$\frac{dV}{ds} = \Gamma \cdot V^{3/4} \cdot \theta^{-5/4} \cdot Re_o^{-1/4} \tag{6-1}$$

$$\frac{d\theta}{ds} = (0.014 - 3.52 \cdot \Gamma)(Re_o \cdot V \cdot \theta)^{-1/4} \tag{6-2}$$

$$0 > \Gamma \geqslant -0.04 \tag{7}$$

$$V(o) = 1, \qquad \theta(o) = \theta_o \tag{8}$$

$$V(*), \qquad \theta(*), free \tag{9}$$

So the mathematical expression for dynamic programming solution is as follows:

Index function:

$$J(1) = \int_o^1 \Gamma V^{3/4} \theta^{-5/4} Re_o^{-1/4} ds \rightarrow min \tag{10}$$

s.t.

$$dV/ds = \Gamma V^{3/4} \theta^{-5/4} Re_o^{-1/4} \tag{6-1}$$

$$d\theta/ds \quad = (0.014 - 3.52\Gamma)(Re_o V\theta)^{-1/4} \tag{6-2}$$

initial condition: $\quad V(o) = 1 \qquad \theta(o) = \theta_o$ (11)

control constraint: $\quad -0.04 \leqslant \Gamma < 0$ (12)

state constraint: $\quad 0 < V < 1 \qquad \theta_o \leqslant \theta$ (13)

Quantizing the equations listed above, we yield:

$$J = \min_{\Gamma(k)} \left\{ \sum_{k=0}^{N} \Delta S * \left[\Gamma(k) V(k)^{3/4} \theta(k)^{-5/4} Re_o^{-1/4} \right] \right\} \tag{14}$$

$$V(k+1) = V(k) + \Delta S * \left[\Gamma(k) V(\bar{k})^{3/4} \theta(k)^{-5/4} Re_o^{-1/4} \right] \tag{15}$$

$$\theta(k+1) = \theta(k) + \Delta S * \left\{ (0.014 - 3.52\Gamma(k)[Re_o V(k)\theta(k)]^{-1/4} \right\} \tag{16}$$

$$-0.04 \leqslant \Gamma(k) < 0 \tag{17}$$

$$0 < V(k) \leqslant 1 \tag{18}$$

$$\theta_o \leqslant \theta(k) \tag{19}$$

and the iterative relation becomes

$$\begin{cases} J(V,\theta,k) = \min_{\Gamma(k)} \left\{ \Delta S * [\Gamma(k) V(k)^{3/4} \theta(k)^{-5/4} Re_o^{-1/4}] + J(V,\theta,k+1) \right\} \\ J(V,\theta,N) = 0 \end{cases} \tag{20}$$

CALCULATION RESULT AND ANALYSIS

In calculation, Reo and N are taken as 10^6 and 10 respectively. The state variables and control variables are quantized respectively. The sets of admissible state variables are as follows:

V= {0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.0}

θ= {0.00226, 0.00508, 0.00781, 0.01058, 0.01336, 0.01890, 0.02168, 0.02445, 0.02722, 0.03}

Then the allowed quantized states are:

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$\{(0.4, 0.00226), (0.4, 0.00508), (0.4, 0.0078), \cdots,\cdots,\cdots,\cdots,\cdots,(1, 0.03)\}$

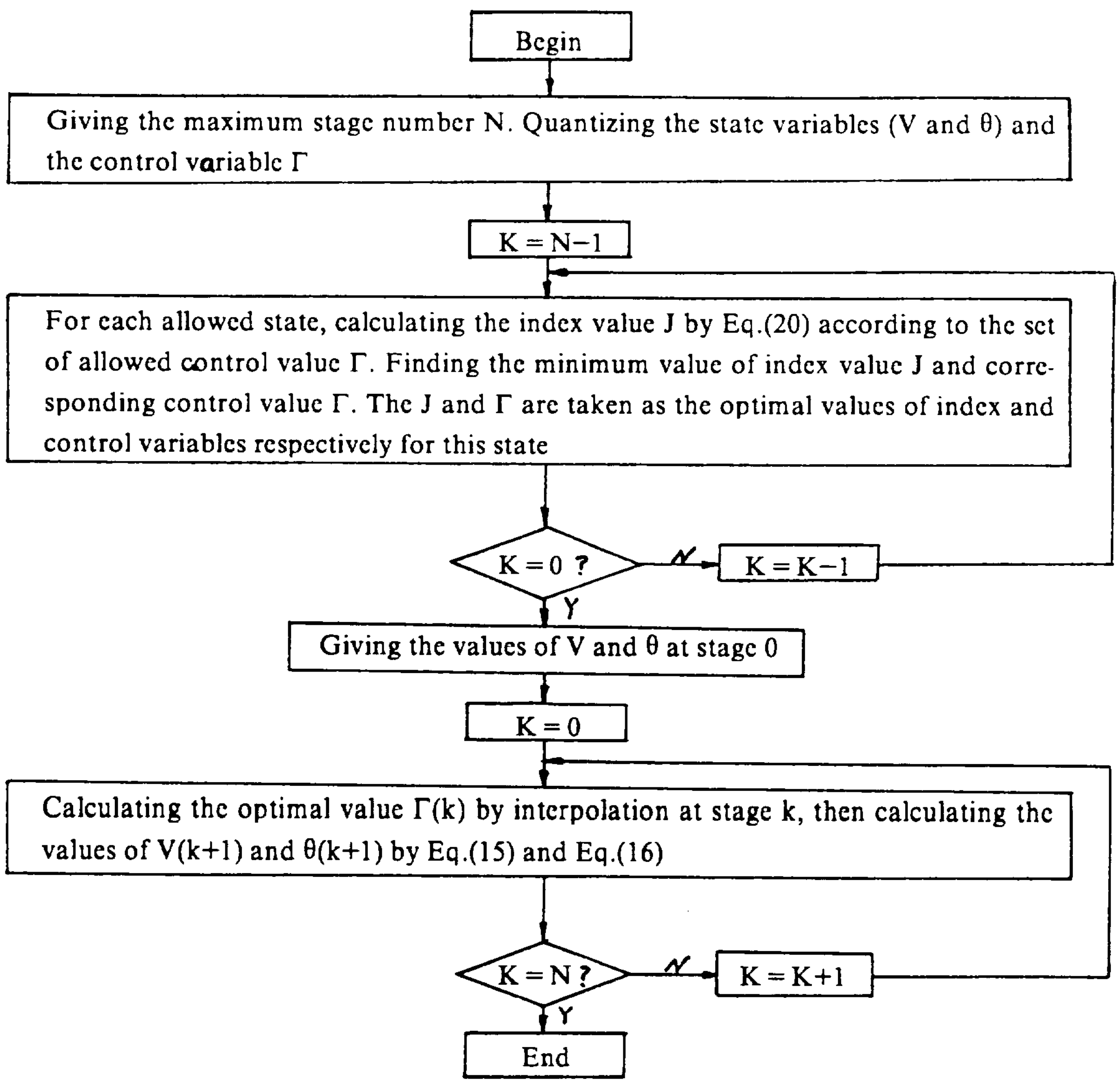
And the set of admissible control variable is as follows:

$\Gamma = \{-0.04, -0.035, -0.03, -0.025, -0.02, -0.015, -0.01, -0.005\}$

The two-dimensional dynamic Programming Computatinal procedure is used because there are two state variables, the calculation procedure is presented in the computer flow chart. The initial value of V and θ, that is , the values at stage o, are taken as 1 and 0.00226 respectively. The calculation results at stage 5 are presented in Table 1. There is only a part of all results because the results are too many to list them all. At each stage, a similar table can also be listed.

In the table 1, a grid point stands for a allowed quantized state, the value put to the right-up of a grid point is the optimal value of index J at this state, and the one put to the right-down of the grid point is the corresponding optimal value of control variable Γ.

Computer flow chart.



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As shown in Fig. 1, curve ABC is an optimal velocity distribution of the diffuser. The results are is quite in agreement with the experimental data and the results calculated by Pontryagin's maximum principle by Gu and Ji,(1987).

The optimal velocity distribution ABC is also called as an optimal deceleration curve for a diffuser. In the range above the curve, including the curve, there is no separation while Γ is not less than -0.04; on the contrary, in the range under the curve, separation will happen. So the curve ABC is a critical line ($\Gamma = -0.04$) between separation and non-separation.

Drawing a deceleration curve AB'C' in the nonseparation range, we can find that for a diffuser with a specified length, the discharge velocity V^* is always less then V'. On the other hand, for a specified discharge velocity V' (which menas a specified pressue rise coefficient) the corresponding optimal length S^* is always less than S'. That means reducing velocity along the optimal deceleration curve ABC will yield the maximum pressure rise coefficient in a specified length or the minimum length of a diffuser in a specified pressure rise. In this case, the length is minimum and the loss is nearly minimum because there is no separation.

From the physical explanation of the optimal deceleration curve in Fig.1 It is also proved that the two expressions of index functions A) and B) of the optinal control problem are correlative. That is, the optimal velocity distribution obtained by one index function can satisfy another automatically.

THE CALCULATION OF THE DIFFUSER CONTOUR

The contour of the diffuser is calculated by means of the optimal velocity distribution on the surface, so that it is also called as optimal design problem or inverse problem. It is well known that solving the problem directly in X−Y plane will involve non−identifed of calculated region. So coordinate transformation is neccessary. It is the easiest way to transfer the X−Y plane to $\Phi-\Psi$ plane.

The governing Equations. in $\Phi-\Psi$ plane has been deduced strictly in the paper as:

Taking an element in X−Y plane and considering incompressible, potential flow, the continuity equation and non−rotation equation are as follows:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \tag{21}$$

$$\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 0 \tag{22}$$

Velocity vector is

$$\bar{V} = |V| \cdot (\bar{i} \cdot \cos\beta + \bar{j} \cdot \sin\beta) \tag{23}$$

Where |V| is the amplitude of $\bar{V}$. β is the angle between $\bar{V}$ and coordinate line X.

The transform relation between X−Y plane and $\Phi-\Psi$ plane is:

$$\left.\begin{aligned} f_x &= (Y_\Psi \cdot f_\Phi - Y_\Phi \cdot f_\Psi)/J \\ f_y &= (-X_\Psi f_\Phi + X_\Phi \cdot f_\Psi)/J \end{aligned}\right\} \tag{24}$$

Where J is Jacobi matrix.

Substituting Eq. (23) and Eq. (24) into Eq.(21), because of $J \neq 0$, then we yield:

$$\frac{\partial V}{\partial \Phi}(Y_\Psi \cos\beta - X_\Psi \sin\beta \quad) + \frac{\partial V}{\partial \Psi}(-Y_\Phi \cos\beta + X_\Phi \sin\beta \quad)$$

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$$+ V \cdot (- X_{\Psi}\frac{\partial \sin\beta}{\partial \Phi} + Y_{\Psi}\frac{\partial \cos\beta}{\partial \Phi}) + V(X_{\Phi}\frac{\partial \sin\beta}{\partial \Psi} - Y_{\Phi}\frac{\partial \cos\beta}{\partial \Psi}) = 0 \tag{25}$$

The four items on the left side of Eq. (25) are as follows respectively,

$$\frac{\partial V}{\partial \Phi} \cdot (Y_{\Psi} \cdot \cos\beta - X_{\Psi}\sin\beta) = \frac{\partial V}{\partial \Phi} \cdot \sqrt{X_{\Psi}^2 + Y_{\Psi}^2}$$

$$\frac{\partial V}{\partial \Psi} \cdot (- Y_{\Phi} \cdot \cos\beta + X_{\Phi}\sin\beta = 0$$

$$V \cdot \left(- X_{\Psi}\frac{\partial \sin\beta}{\partial \Phi} + Y_{\Psi}\frac{\partial \cos\beta}{\partial \Phi}\right) = 0$$

$$V \cdot (X_{\Phi}\frac{\partial \sin\beta}{\partial \Psi} - Y_{\Phi}\frac{\partial \cos\beta}{\partial \Psi}) = V \cdot \frac{\partial \beta}{\partial \Psi} \cdot \sqrt{X_{\Phi}^2 + Y_{\Phi}^2}$$

In deducing the definations of normal and tangent unit vectors of equal Φ and equal Ψ lines, Eq. (25) can be rewritten as,

$$\frac{\partial V}{\partial \Phi} \cdot \sqrt{X_{\Psi}^2 + Y_{\Psi}^2} + V \cdot \frac{\partial \beta}{\partial \Psi}\sqrt{X_{\Phi}^2 + Y_{\Phi}^2} = 0 \tag{26}$$

It can be easily proved that:

$$\sqrt{X_{\Psi}^2 + Y_{\Psi}^2} = \sqrt{X_{\Phi}^2 + Y_{\Phi}^2} = V / J$$

And substituting it into Eq.(26), finally, the continuity equation in $\Phi-\Psi$ plane can be given as:

$$\frac{\partial \ln V}{\partial \Phi} + \frac{\partial \beta}{\partial \Psi} = 0 \tag{27}$$

In the similar way, the non-rotation equation in $\Phi-\Psi$ plane is:

$$\frac{\partial \ln V}{\partial \Psi} - \frac{\partial \beta}{\partial \Phi} = 0 \tag{28}$$

Two Laplace's Eqs. can be obtained from Eq. (27) and Eq.(28):

$$\frac{\partial^2 \ln V}{\partial^2 \Psi} + \frac{\partial^2 \ln V}{\partial \Phi^2} = 0 \tag{29}$$

$$\frac{\partial \beta^2}{\partial \Phi^2} + \frac{\partial \beta^2}{\partial \Psi^2} = 0 \tag{30}$$

The velocity distribution within the diffuser can be obtained by solving Eq. (29) with ADI method. Then from Eq.(28) the values of β on the top line ($\Psi = 0$ or $\Psi = 1$) of the potential flow region can also be yielded, so the shape of potential flow region can be defined. The diffuser contour can be modified by adding thickness of boundary layer. The calculation result is shown in Fig.4, the line of $Y / L = 0$ is the central line of diffuser. The shape is quite similar to the real size of B. S. Stratford's diffuser. The equivalent diffusing angle of the diffuser is 19 ° (integral angle), and is much greater than ordinary recommended angle.

CONCLUSIONS

The optimal deceleration curve (i.e. optimal velocity distribution) on the wall of a diffuser is first obtained by using the principle of dynamic programming. In solving optimal control problem of fluid mechnics with inequality constraints of state and control variables, the dynamic programming method has many advantages over others. The physical model of optimization for a diffuser is to avoid the

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separation of boundary layer while getting to the maximum pressure rise in a diffuser of a specified length (or getting to the shortest length in a specified pressure rise). The calculation results are fairly in agreement with the experimental ones and the results calcaulated by Pontrayagin's maximum principle.

The optimal velocity distribution on a diffuser wall should be as: the flow decelerates first quickly and then smoothly, and the flow is near separation but always protects from it. The optimal velocity distribution can also be expanded to design an unsymmetric diffuser.

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	V=0.85	V=0.80	V=0.75	V=0.70	V=0.65	V=0.60	V=0.55	V=0.50	V=0.45
θ=.00226	0.577	0.601	0.626	0.655	0.684	0.710	0.732	0.750	0.780
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.00503	0.750	0.763	0.776	0.789	0.803	0.818	0.842	0.870	0.930
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.00781	0.822	0.831	0.840	0.849	0.858	0.867	0.879	0.907	0.993
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.01058	0.863	0.870	0.877	0.884	0.891	0.897	0.906	0.926	1.017
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.01336	0.890	0.895	0.901	0.906	0.912	0.917	0.924	0.938	1.028
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.01613	0.909	0.913	0.917	0.922	0.926	0.931	0.936	0.947	1.031
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.01890	0.923	0.926	0.930	0.934	0.938	0.941	0.946	0.954	1.030
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040
θ=.02168	0.935	0.938	0.941	0.944	0.948	0.951	0.954	0.961	1.028
	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040	-.040

Table 1

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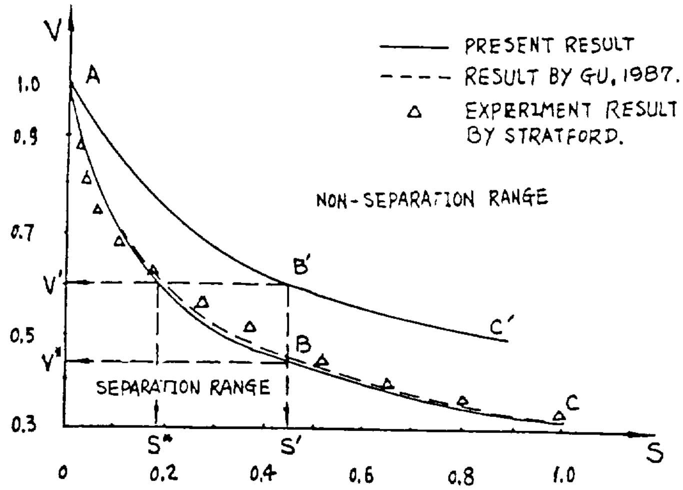

Fig.1 Optimal velocity distribution

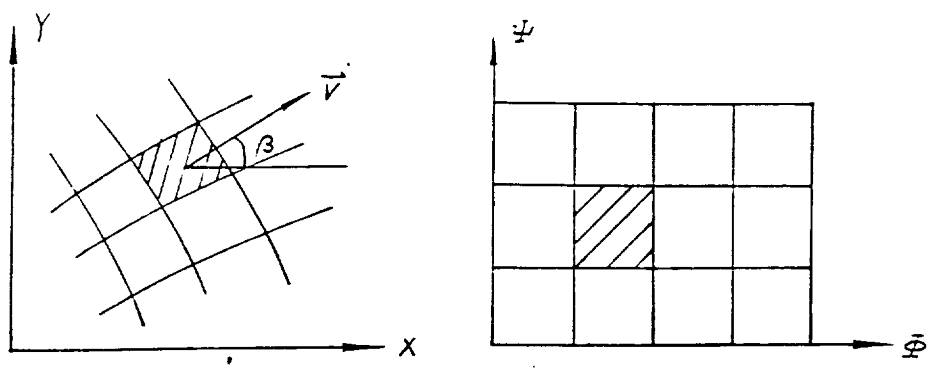

Fig.2. Coordinate tranformation

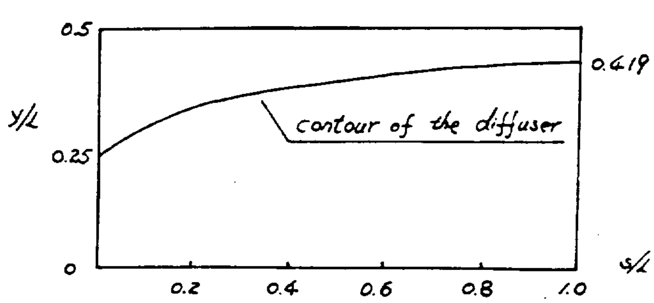

Fig.3 contour of the optimal diffuser