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FINAL REPORT

DETERMINATION OF THE CORONAL MAGNETIC FIELD FROM VECTOR MAGNETOGRAPH DATA

Science Applications International Corporation

An Employee-Owned Company

Final Report on work performed under contract
NASW-4571
with NASA (Solar Physics Supporting Research and Technology)

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1. INTRODUCTION

This document represents the Final Report on work performed under Contract NASW-4571 between NASA and Science Applications International Corporation. Under this contract, SAIC has conducted research into the determination of coronal magnetic fields from vector magnetograms, including the development and application of an algorithm to determine force-free coronal fields above NOAA active region AR5747 on 20 October, 1989. A vector magnetogram for this region was obtained at the Mees Solar Observatory of the University of Hawaii using the Haleakala Stokes Polarimeter, and was used in the present investigation.

The principal results from the investigation are the following:

1. The "evolutionary algorithm" which was proposed to find the force–free coronal field has a sound theoretical basis, and is directly applicable to the practical determination of coronal fields above active regions.

2. The algorithm has been applied to a vector magnetogram of NOAA AR5747 of 20 October, 1989, and has yielded detailed quantitative estimates of the coronal magnetic field above a solar active region. This exercise has demonstrated that the solution to the nonlinear boundary value problem (with non-constant α) can be found, and, in particular, that it has been developed to the point where it can be applied practically to observational data.

Section 2 of this report lists the attendance of SAIC staff at scientific meetings and publications in preparation. Section 3 includes a detailed account of the research performed under this contract. Section 4 contains suggestions for further research. Section 5 contains a summary of the results obtained.

2. PRESENTATIONS AT SCIENTIFIC MEETINGS AND PUBLICATIONS

During the contract period, progress has been presented by SAIC researchers Dr. Zoran Mikić and Dr. Dalton D. Schnack at the scientific meetings listed below:

1. Dr. Zoran Mikić attended the meeting of the American Astronomical Society (Solar Physics Division) in Huntsville, Alabama, April 9–11, 1991, at which he gave an oral presentation entitled "Calculation of Force-Free Coronal Magnetic Fields from Vector Magnetograms." Appendix A contains a copy of the presentation.

2. Drs. Zoran Mikić and Dalton Schnack attended the Gordon Conference on *Solar Plasma and MHD Processes*, in Plymouth, New Hampshire, August 5–9, 1991. Dr. Mikić was invited as a discussant in the session on "Reconnection and Resistive MHD." This conference had a workshop format which prohibits publication of proceedings by charter.

3. Dr. Zoran Mikić attended the meeting of the American Physical Society (Division of Plasma Physics) in Tampa, Florida, November 4–8, 1991, at which he presented the paper "Calculation of Force-Free Coronal Magnetic Fields above Active Regions."

The research conducted under this contract has contributed to two publications which are in preparation. We intend to publish these two papers in *The Astrophysical Journal.*

1. Mikić, Z., and Barnes, D. C. 1991, "Determination of Force-Free Coronal Fields from Vector Magnetograms: I. Theoretical Basis," in preparation.

2. Mikić, Z. 1991, "Determination of Force-Free Coronal Fields from Vector Magnetograms: II. Application to AR 5747 of 20 October, 1989," in preparation.

3. DETERMINATION OF CORONAL FIELDS

3.1 Availability of Vector Magnetograms

High resolution vector magnetograms of active regions are currently being taken on a routine basis. The present availability of such vector magnetograms, and the advent of improved instruments with higher resolution in the near future, has encouraged us to develop a method to determine the coronal magnetic field from photospheric measurements. A reliable technique for deducing the detailed spatial structure of the coronal magnetic field will significantly advance our understanding and characterization of solar phenomena, including solar flares. The limited quantitative knowledge of the coronal field has been a major obstacle to the theoretical advancement of solar flare theory.

3.2 Evolutionary Algorithm for Determination of Force-Free Fields

We have developed a new technique for the determination of the coronal magnetic field using the force-free approximation. The method can generate nonlinear force-free fields (with non-constant-α) which match vector magnetograms. The nonlinear boundary-value problem for the magnetic field is solved using an iterative scheme which we call the *evolutionary technique.* We present an overview of the method here; a detailed discussion of the technique is in preparation for publication (Mikić and Barnes 1991).

The basis of our coronal force-free field solution relies on an ingenious formulation of the associated boundary-value problem. In our algorithm, the force-free solution is found using an *evolutionary* approach. Instead of directly solving the equilibrium equations, we solve a related time-dependent problem whose steady-state solution satisfies the force-free equations and the boundary data. The method finds solutions to the boundary-value problem by dynamically adjusting the electric current in the corona to match the observed boundary data. The computation proceeds in a fictitious time, allowing the field to relax towards a steady state subject to given boundary conditions on the magnetic flux and transverse electric field. In the asymptotic state the field satisfies the force-free equations as well as the appropriate boundary data. An adaptive external circuit is used to drive the plasma toward a state in which the normal electric current density at the photosphere matches the observed current density.

The coronal magnetic field has been modeled at various levels of sophistication. The crudest approximation to the coronal magnetic field is a potential (i.e., current-free) field. This technique has been used to provide estimates of the structure of the coronal magnetic field (Schmidt 1964;

Teuber *et al.* 1977). The assumption of vanishing coronal electric current is clearly unrealistic, especially for the analysis of active regions, in which the magnetic field is significantly nonpotential. (The release of magnetic energy which is believed to power a flare is stored in the magnetic field due to coronal electric currents, since the potential field corresponds to a minimum-energy state which has no "free" magnetic energy.) Nevertheless, potential field solutions are useful in characterizing the non-potential features of observed active-region fields (Gary *et al.* 1987). At the next level of sophistication, the coronal field has been assumed to obey the constant-α force-free equations (Nakagawa and Raadu 1972; Chiu and Hilton 1977; Seehafer 1978; Alissandrakis 1981; Hannakam, Gary, and Teuber 1984). The assumption of constant α reduces the magnetic field equation to a linear one, which can be solved directly. A comparison of such solutions with observations has shown that solar magnetic fields are not described well by constant-α solutions (Levine 1976; Sakurai 1979). Constant-α solutions have enjoyed popularity because the force-free equation becomes a linear equation for $\mathbf{B}$, and can be solved easily. Even though constant-α solutions exhibit closer agreement with observations of coronal features than those obtained from potential field solutions, both observations (Levine 1976) and model problems (Sakurai 1979) indicate that typical coronal fields are not described well by constant-α solutions. In addition, constant-α fields are unphysical because they have a infinite energy content (Seehafer 1978).

Most generally, there have been several attempts at a full solution to the nonlinear force-free equations (with non-constant α). Sakurai (1981) presented a scheme for the iterative solution of the nonlinear force-free field equations. The technique is based on an idea of Grad (1958) which is relies on magnetic field line tracing. The convergence of the scheme is not robust for highly stressed fields. The method has not been applied widely to data from vector magnetographs. Pridmore-Brown (1981) has presented a data-fitting technique, but it too has not been used widely, and it is not known how well the method works in practice. Recently, Wu *et al.* (1990) developed a method of "extrapolation" which integrates the force-free equations (starting from the boundary data) upward into the corona using a Taylor series expansion.

The estimated coronal magnetic field can be checked against features inferred from in microwave, EUV, and X-ray coronal observations, as well as from Hα observations (Levine 1975). Coordinated observing campaigns of active regions, such as the CoMStOC series, can significantly improve our understanding of the nature of coronal magnetic fields.

3.3 Data for NOAA AR5747 on 20 October, 1989

NOAA active region AR5747 was observed at the Mees Solar Observatory (MSO) of the University of Hawaii during its transit across the solar disk during

October 1989. Coordinated measurements of this active region were performed at the Big Bear Solar Observatory of the California Institute of Technology. Daily vector magnetograms of this active region were taken at MSO for each of the five days during the period 18 October through 22 October, 1989, using the Haleakala Stokes Polarimeter. This active region showed highly nonpotential photospheric vector magnetic field structure and produced many solar flares, three of which were observed at MSO (Canfield *et al.* 1990).

This active region is very interesting from the point of view of solar flare theory and coronal magnetic field modeling. The availability of these vector magnetograms afforded us an opportunity to test the *evolutionary algorithm* on observational data. We thus decided to take up the the challenge which this data set afforded, and applied our technique to determine the coronal field above this active region. Prior to this effort, we had only applied the method to (necessarily idealized) models of coronal fields in order to assess its properties and performance on problems with known solutions. The results, as presented below, are very encouraging.

The data for the third of the five days (20 October, 1989) proved to be the most suitable for analysis, mainly because most of the flux of the active region was captured by the magnetogram. This data set has been analyzed in detail by the University of Hawaii group (Canfield *et al.* 1990), including Richard Canfield, Don Mickey, Alexander McClymont, Yuhong Fan, K. D. Leka, and Tom Metcalf. Dr. Alexander McClymont kindly provided us with a data set of the vector magnetogram of 20 October, 1989 for analysis. The data consisted of the three components of the magnetic field in the photosphere over the magnetogram area.

The data undergoes an extensive amount of processing between measurement and the estimation of the photospheric magnetic fields. This stage of the analysis has been developed and is carried out at the University of Hawaii. First, the Stokes parameters I, U, V, and Q are determined at each pixel as a function of frequency from the measured signals using the calibration matrix. The Stokes profiles are then processed using the Lites-Skumanich code to determine the line-of-sight field, the transverse field magnitude, the transverse field azimuth, and the filling factor. This scheme uses the Unno method and a nonlinear least-squares fitting procedure. The Stokes profiles are also processed using an "integral method" to determine the magnetic fields in weak-field regions (where the Unno method fails). The fields estimated by these two methods are merged. The next step is to resolve the 180° ambiguity in the transverse field direction. This involves comparison of the transverse field direction with the potential magnetic field, minimization of electric currents, and minimization of the divergence of $\mathbf{B}$. Finally, the magnetic fields are transformed to a local Cartesian coordinate system in heliospheric coordinates, and are written to a text data file.

The magnetic field components thus produced (B_x, B_y, and B_z) in the photospheric plane x–y are used to determine the coronal magnetic field corresponding to the photospheric observations. This step is carried out at SAIC, and is described below.

3.4 Coronal Magnetic Fields for NOAA AR5747 on 20 October, 1989

The next step is to interpolate and extrapolate the data over the magnetogram into a rectangular region, as required by the evolutionary algorithm. The vector magnetogram (VM) was taken on a mesh of 30×30 pixels. Even though the telescope aperture was rectangular, when projected onto the solar disk (off disk-center) the magnetogram area corresponds to a "diamond-shaped" region. (Even though this area does not lie strictly in a plane, the curvature associated with the Sun's spherical surface is small for an area the size of an active region. The magnetogram in this case covers an area of roughly 150,000 km $\times$ 150,000 km.) Figure 1 shows the positions of the magnetogram pixels on the solar disk.

The magnetogram points are shifted and rotated to best fit a rectangular region. For the present data, the points are rotated by $-27°$ about (25,25) pixels. The resulting points are placed in a square region of dimension 50×50 pixels (*i.e.*, a region of 207,000 km $\times$ 207,000 km, since each pixel measures 5.7" $\times$ 5.7"). The 900 data points are surrounded by a region of extrapolation points at which the magnetic fields are smoothly merged into a zero-field region which surrounds the magnetogram data points. Figure 2 shows the rotated magnetogram points and the extrapolation points.

Since the MHD code which is used in the determination of the coronal force-free field is periodic in the two transverse directions (x and y), it is necessary to fit the magnetogram data to periodic functions in the x–y plane. We have developed a linear least-squares fitting procedure for this task. The fitting code uses *sin* and *cos* functions (*i.e.*, finite Fourier series) to fit the magnetic fields at the magnetogram data points. To create a smooth transition to zero fields in the extrapolation region, the fields are fit to zero and the derivatives of the fields are minimized in the extrapolation region. This fitting of the magnetic fields is also used to filter the short-wavelength components in the data which are due to random measurements error (noise). For the VM data, the peak longitudinal field B_z is 2800 G, and the peak transverse field is 1800 G. The expected errors in B_z are an the order of tens of Gauss, while those in the transverse fields are several hundred Gauss. These error estimates are used to set the criteria for

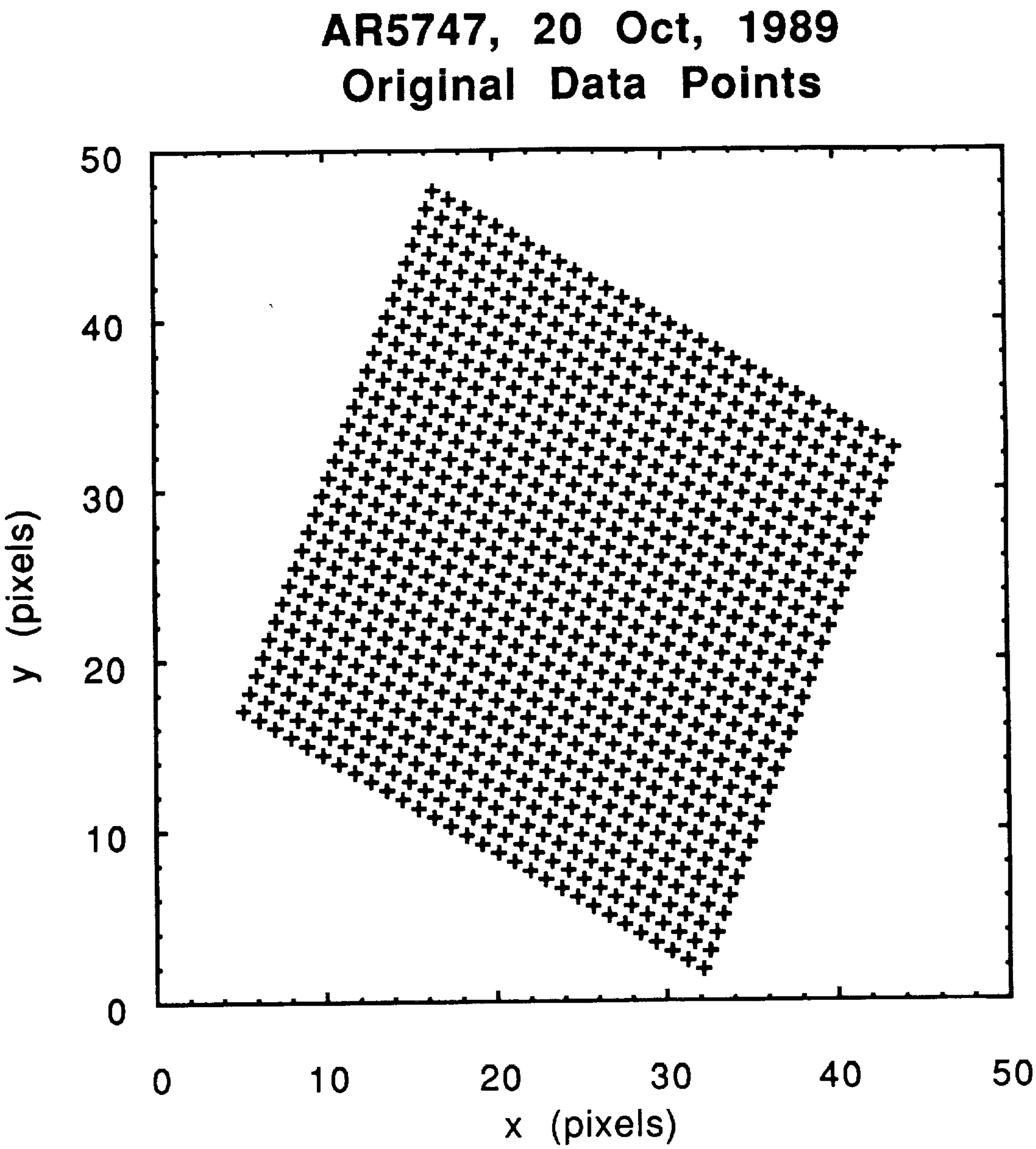

Figure 1. Locations of the 900 vector magnetogram data points as projected on the solar disk for NOAA AR5747 on 20 October, 1989. The data was taken using the Haleakala Stokes Polarimeter at the Mees Solar Observatory of the University of Hawaii.

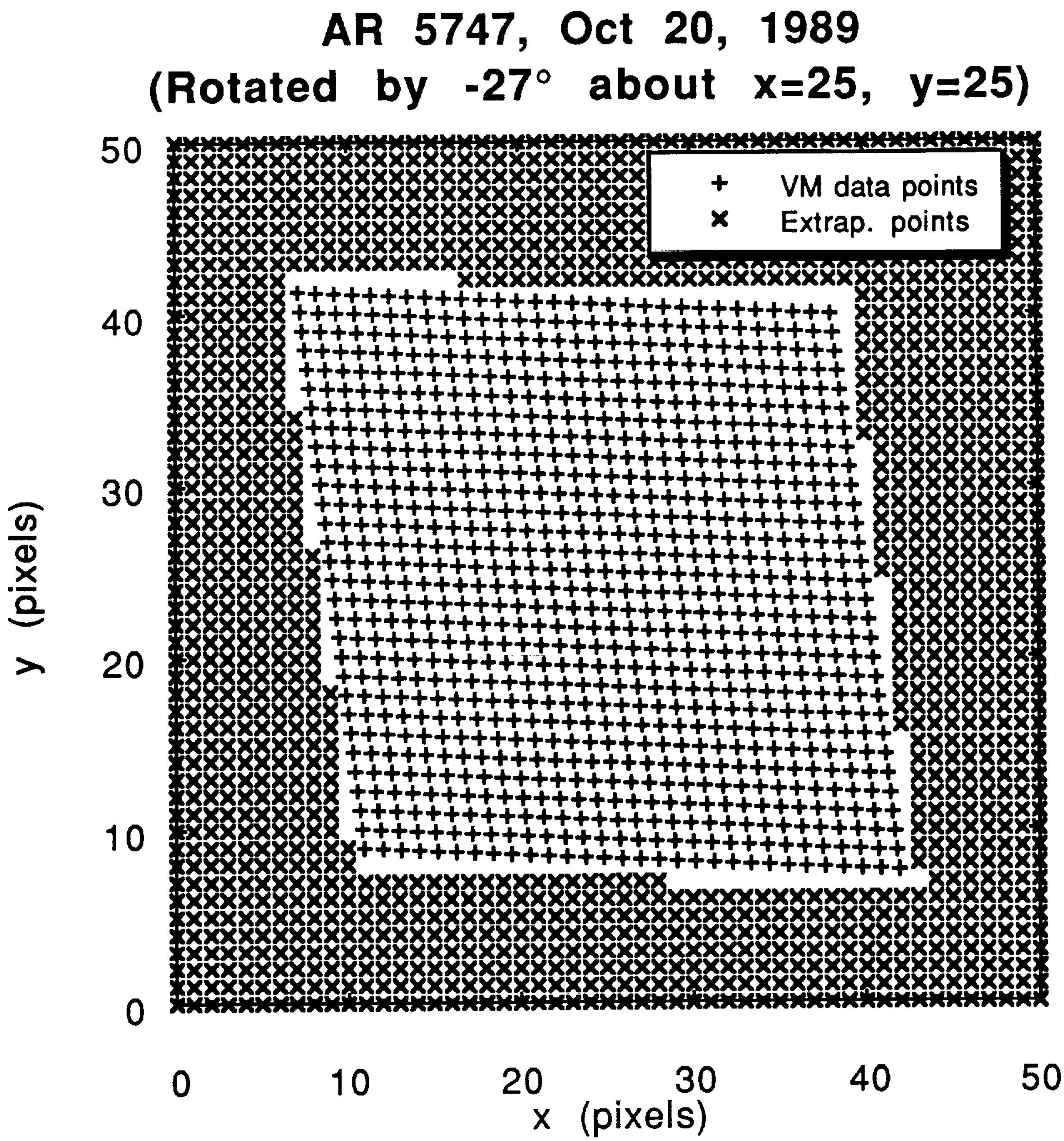

Figure 2. Locations of the 900 vector magnetogram data points after rotation of the data, and the points used in extrapolation to a zero field in the surrounding region. The square region has a dimension of 207,000 km. The vector magnetogram covered the diamond-shaped area.

accuracy in the fitted fields. The vertical current density J_z is computed from the fitted transverse fields B_x and B_y.

Figures 3 and 4 show the fitted vertical magnetic field B_z and the vertical current density J_z corresponding to the vector magnetogram of NOAA AR5747 on 20 October, 1989, which are needed in the evolutionary algorithm as boundary conditions in order to determine the coronal magnetic field.

The evolutionary algorithm was applied using these boundary conditions. A mesh of $63 \times 64 \times 64$ mesh points in 3D Cartesian coordinates was used. The code ran on the Cray-2 computer. Many runs were made to explore the effect of various parameters.

When our algorithm was applied to this data set, the method converged to a steady-state solution in a time on the order of a hundred Alfvén times. The steady-state solution did not converge to a completely force-free field. In steady state the coronal field had $\int |\mathbf{J} \times \mathbf{B}| dV$ equal to 23% of $\int |\mathbf{J} \cdot \mathbf{B}| dV$. In the code, this large remnant $\mathbf{J} \times \mathbf{B}$ force is supported by the (unphysically large) viscous friction corresponding to a relatively large flow. Attempts to reduce this remnant force by reducing the viscosity and resistivity were unsuccessful. Our current interpretation is that the applied boundary conditions are not consistent with a force-free coronal field. Our "force-free" solution should presently be considered as an *estimate* of the coronal field in which the current is almost aligned with the magnetic field. Improvements to this method in the presence of noisy data are clearly indicated.

Upon analysis of the remnant $\mathbf{J} \times \mathbf{B}$ force, we found that the largest forces were concentrated on the neutral line. This led us to the discovery that the data violated an important constraint which a force-free field is required to satisfy. Namely, for a force-free field, on the neutral line, $B_z = 0$, the vertical current J_z ought to vanish also (since $\mathbf{J}$ and $\mathbf{B}$ have to be parallel everywhere). An analysis of the VM data shows that the data does not satisfy this constraint (J_z is finite on the neutral line; in fact it has a value which is a sizable fraction of the maximum), so that if the raw data is used, a non-zero $\mathbf{J} \times \mathbf{B}$ force would be required at the neutral line. This is consistent with our observation that the largest forces occur at the neutral line. In future research we intend to take heed of this constraint. It is not clear at present if the finite J_z on the neutral line is due to thermal forces in the photosphere (at which the data is taken) due to the finite plasma beta, or whether it is due to measurement errors in the determination of the magnetic fields.

The energy of the potential magnetic field was 1.44×10^{33} ergs, while the estimated coronal field had an energy of 2.23×10^{33} ergs, indicating that the

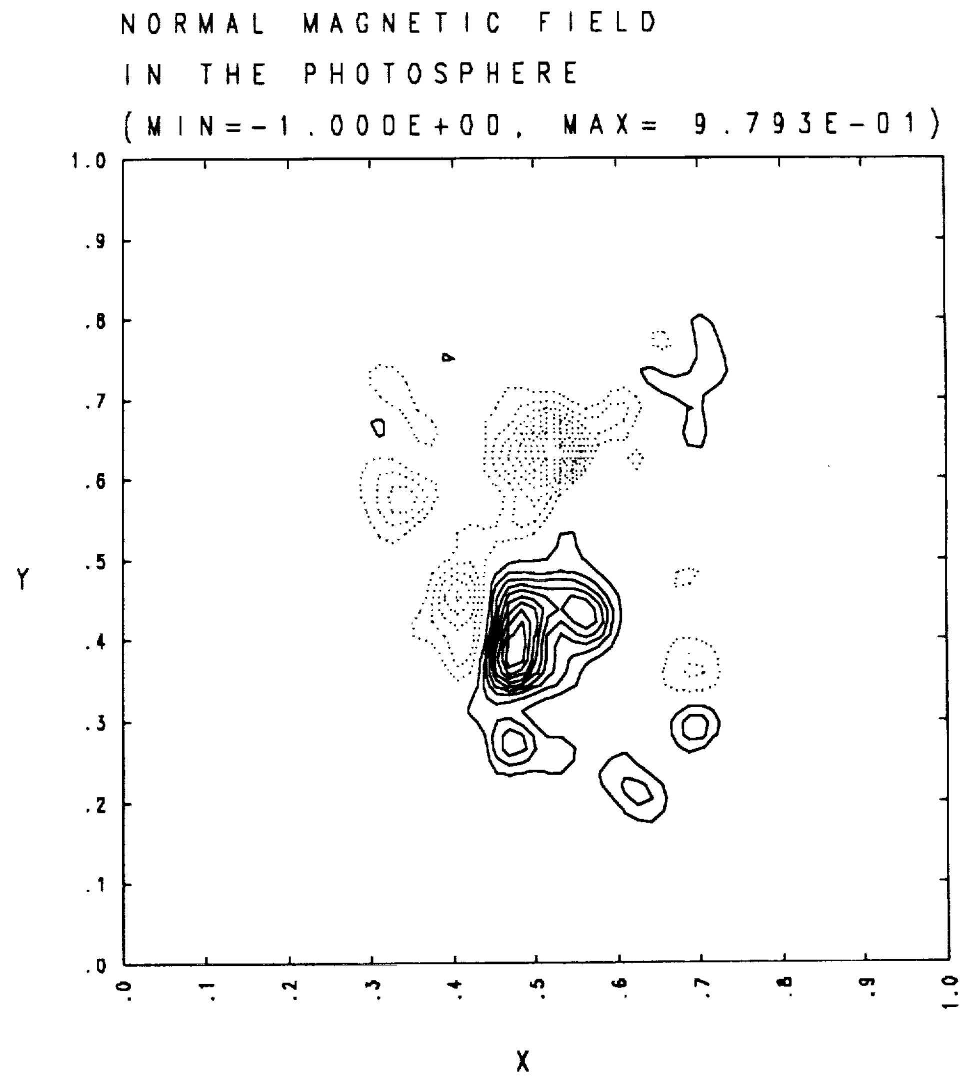

Figure 3. Contours of the normal photospheric magnetic field B_z as obtained from the vector magnetogram, after fitting and extrapolation to a zero-field region.

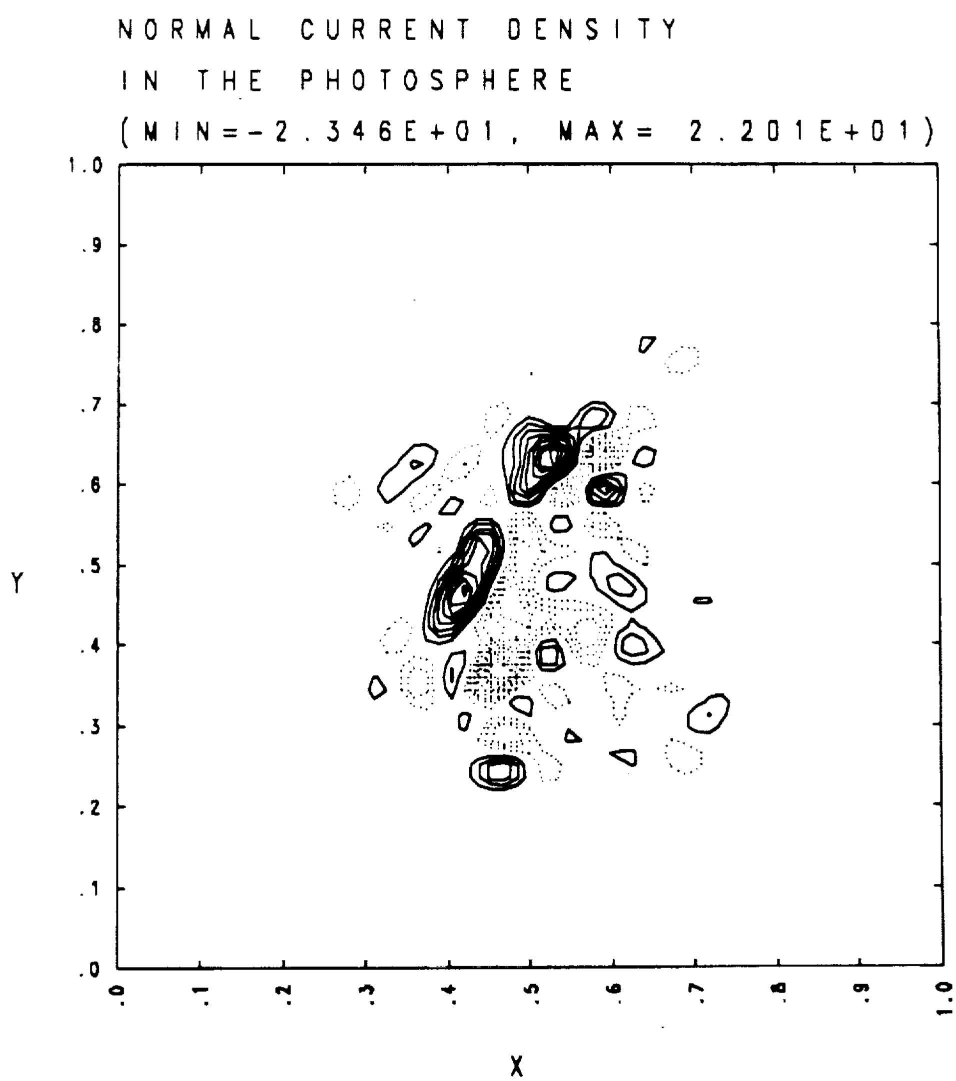

Figure 4. Contours of the normal photospheric electric current density J_z as deduced from the vector magnetogram.

coronal field is significantly nonpotential. Figure 5 shows the field lines in the potential field corresponding to the observed flux distribution, while Figure 6 shows the field lines for the estimated coronal field, indicating that the field is twisted substantially. Figure 7 shows the observed (measured vector magnetograph) transverse field, while Figure 8 shows the transverse field from the force-free solution in the photosphere. Note that the observed and computed transverse fields are qualitatively and quantitatively similar, lending credence to the conclusion that the computed coronal field is a good estimate of the coronal field above AR 5747 on 20 October, 1989.

3.5 Improved Data for NOAA AR5747 on 20 October, 1989

The original vector magnetogram for this active region (corresponding to the data shown in Figures 3 and 4 was obtained from the University of Hawaii in November 1990. Since then the researchers at the University of Hawaii have improved the calibration and fitting of the Stokes profiles. Recently (November 1991) we obtained an improved data set for the same region. This data is expected to give a better estimate of the magnetic fields in the active region. We have begun to analyze this improved data set. We have refitted the data in the manner described above, and we have also rerun the evolutionary algorithm for these boundary conditions. Figures 9 and 10 show B_z and J_z in the photosphere for the improved data set. Note that we have fit the data to a smaller region in the transverse plane (which now measures 40×40 pixels), but with the same number of mesh points, in order to improve the spatial resolution of the data. However, we have sacrificed the extent of the separation of the data from its periodic neighbors. The errors associated with this are not expected to be too severe.

The results for this improved data set are presently being evaluated. The detailed results for the determination of the coronal field above NOAA active region AR5747 on 20 October, 1989 using our evolutionary algorithm are in preparation for submission to *The Astrophysical Journal* (Mikić 1991).

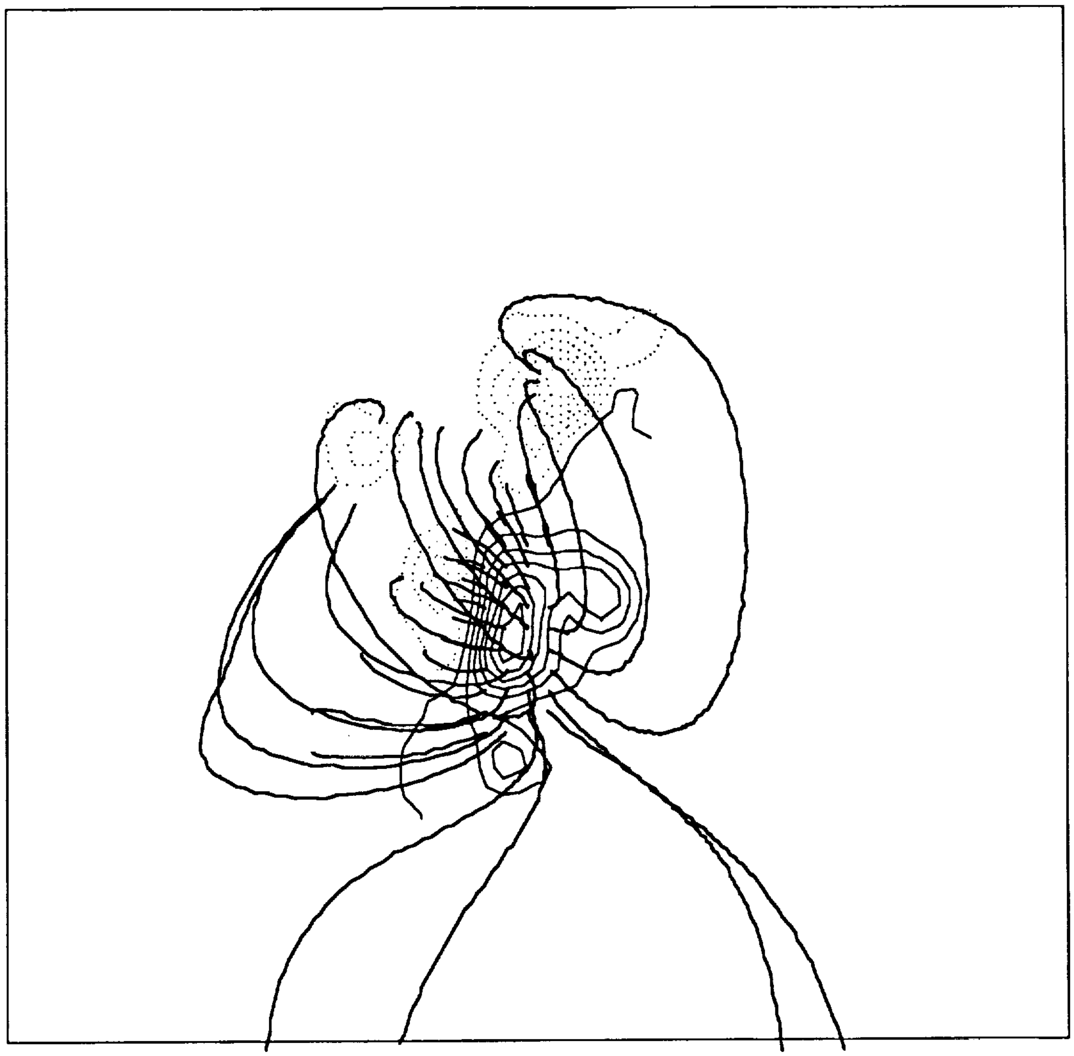

Figure 5. Top view of representative field lines superimposed on contours of the normal photospheric magnetic field for the potential field.

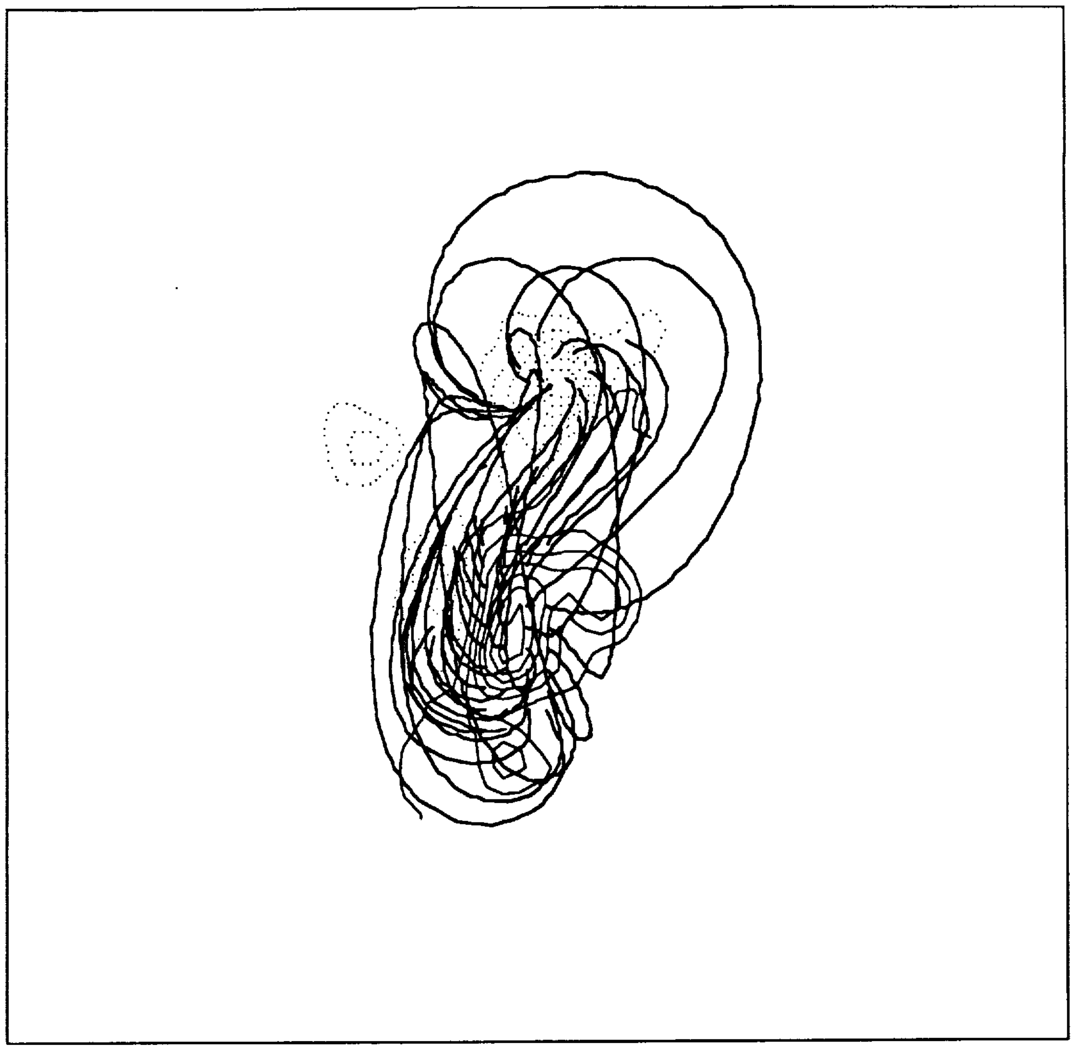

Figure 6. Top view of representative field lines superimposed on contours of the normal photospheric magnetic field for the estimated force-free coronal field. Note the significant twist in the force-free field compared to the potential field (Figure 5).

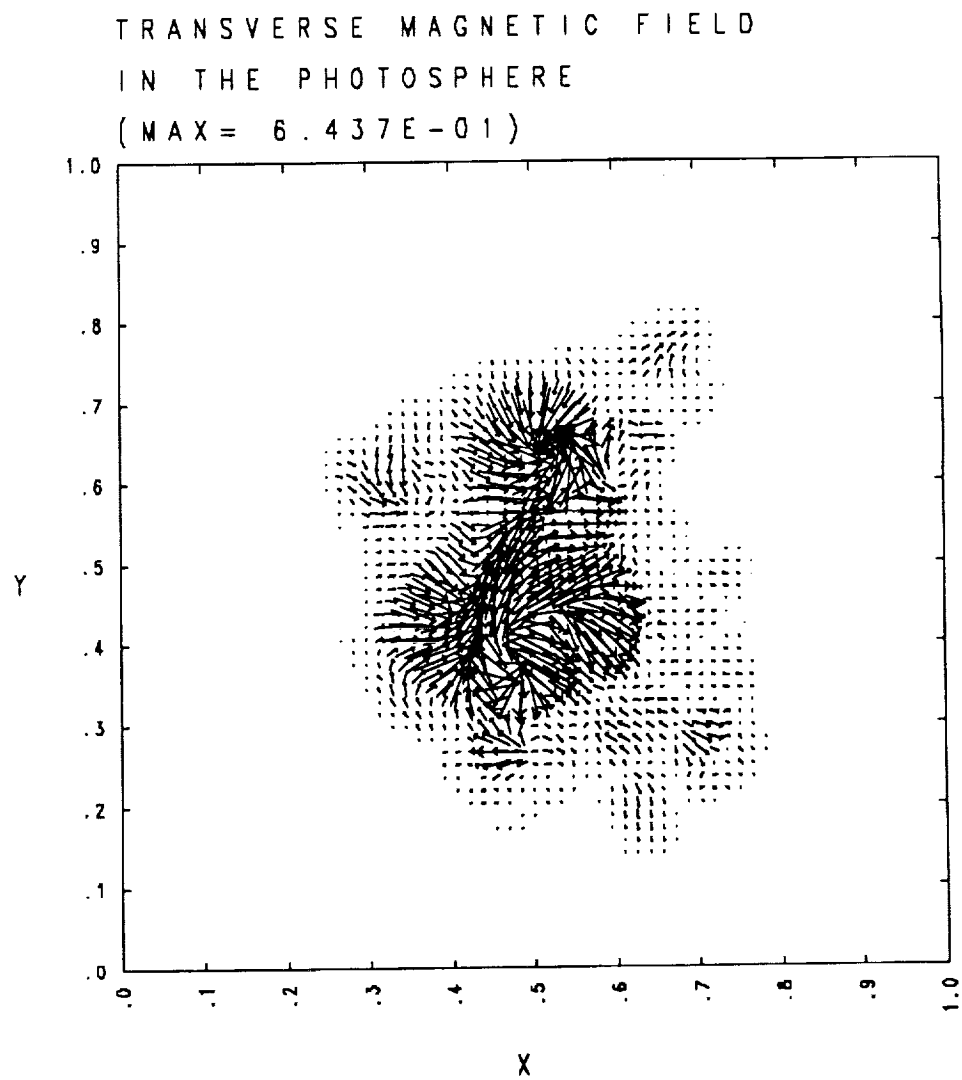

Figure 7. Transverse photospheric magnetic field as obtained from the vector magnetogram.

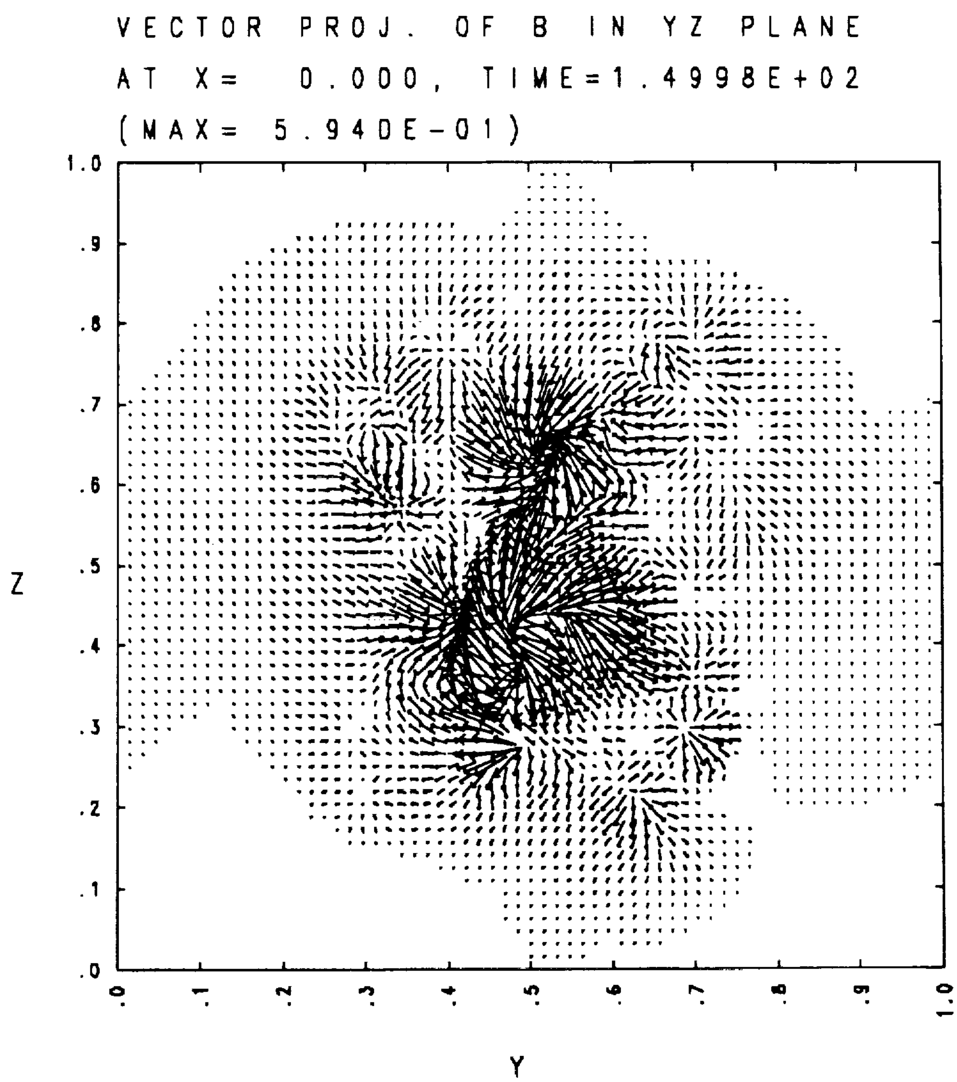

Figure 8. Transverse photospheric magnetic field from the force-free solution. Note that the computed force-free field is qualitatively and quantitatively similar to the measured photospheric transverse field (Figure 7).

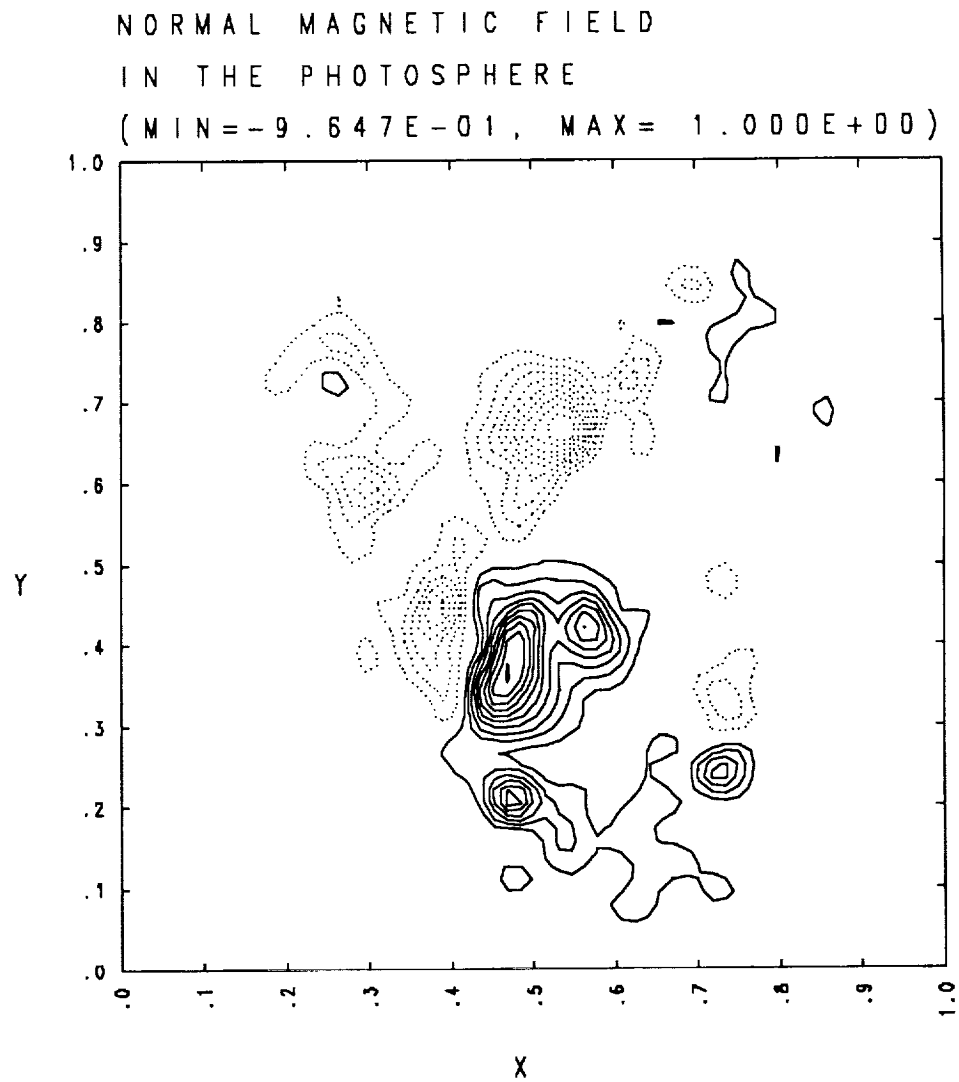

Figure 9. Contours of the normal photospheric magnetic field B_Z as obtained from the improved vector magnetogram (November 1991 version), after fitting and extrapolation to a zero-field region. Note that the fitting region is 40×40 pixels, compared to 50×50 pixels for Figure 3.

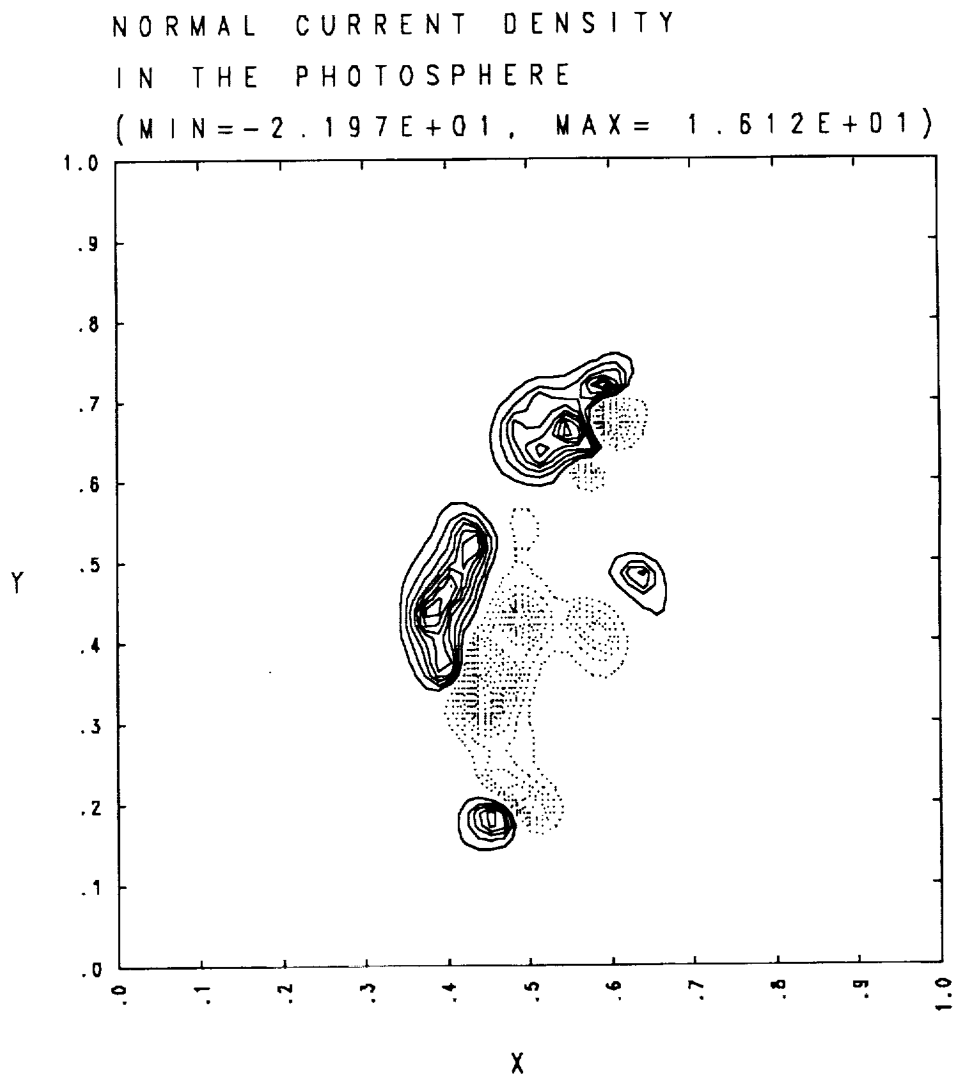

Figure 10. Contours of the normal photospheric electric current density J_z as deduced from the improved vector magnetogram (November 1991 version).

4. SUGGESTIONS FOR FURTHER RESEARCH

Based on the promising results to date it is prudent to continue the development and application of the evolutionary algorithm for the determination of the force-free magnetic fields above active regions.

In the short term, the continued development of the technique, including a careful examination of its limitations and the accuracy of solutions, is indicated. Its further application to additional active-region observations, such as those taken using the vector magnetogram at the NASA Marshall Space Flight Center, the new Imaging Vector Magnetograph at the University of Hawaii, and the Applied Physics Laboratory Vector Magnetograph, is very worthwhile.

In the longer term, it would be interesting to study the dynamic response of computed force-free fields to applied shear in order to understand the mechanism which causes solar flares. This is possible since the computer code which is used to compute the force-free field has been designed to obtain dynamical solutions of the MHD equations [and has been used previously to study the structure and dynamics of coronal fields (Mikić, Barnes, and Schnack 1988; Mikić, Schnack, and Van Hoven 1989, 1990)]. The automation of the application of the technique so that the method can be applied *routinely* to determine force-free fields from vector magnetograms should also be considered as a practical improvement. Furthermore, the development of more sophisticated boundary conditions at the outer edges of the computational domain (especially in generalizing to a non-periodic representation of the transverse coordinates) would be an important advancement in the realism with which the coronal field can be modeled.

5. SUMMARY

During the present contract with NASA we have developed, tested, and applied a new algorithm to determine coronal magnetic fields above solar active regions. We have successfully estimated the coronal field above NOAA active region AR5747 on 20 October, 1989 from data taken at the Mees Solar Observatory of the University of Hawaii. We have shown that observational data can be used to obtain realistic estimates of coronal magnetic fields. Our model has significantly extended the realism with which the coronal magnetic field can be inferred from observations.

The understanding of coronal phenomena will be greatly advanced by a reliable technique, such as the one presented here, for deducing the detailed spatial structure of the coronal field. The payoff from major current and proposed NASA observational efforts is heavily dependent on the success with which the coronal field can be inferred from vector magnetograms. In particular, the present inability to reliably obtain the coronal field has been a major obstacle to the theoretical advancement of solar flare theory and prediction.

The results presented here have shown that the evolutionary algorithm can be used to estimate coronal magnetic fields. Several improvements and refinements have been indicated in Section 5. Their investigation and implementation will further improve the accuracy with which coronal magnetic fields can be estimated.

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APPENDIX A:

PAPER PRESENTED AT THE AMERICAN ASTRONOMICAL SOCIETY
(SOLAR PHYSICS DIVISION)
MEETING, APRIL 9-11, 1991, HUNTSVILLE, ALABAMA

Calculation of Force-Free Coronal Magnetic Fields from Vector Magnetograms*

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SAIC, San Diego

*Supported by NASA contract NASW-4571

INTRODUCTION

- Find force-free fields ($\mathbf{J} \times \mathbf{B} = 0$) in the corona using an "evolutionary algorithm"
- Nonlinear force-free solution (i.e., non-constant α)
- Boundary value problem formulation
- Force-free field is the asymptotic state of a related time-dependent problem
- Has worked successfully on a 3D model of a sunspot (simulated VM data)
- Applied to vector magnetogram data from the Stokes Polarimeter, IfA, U. Hawaii, for NOAA AR5747 of Oct. 20, 1989

FORCE-FREE APPROXIMATION

- Assume that the coronal plasma pressure is small ($\nabla p = 0$). The coronal plasma is thus force-free:

$$\mathbf{J} \times \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

This gives:

$$\frac{c}{4\pi}\nabla \times \mathbf{B} = \alpha \mathbf{B}$$

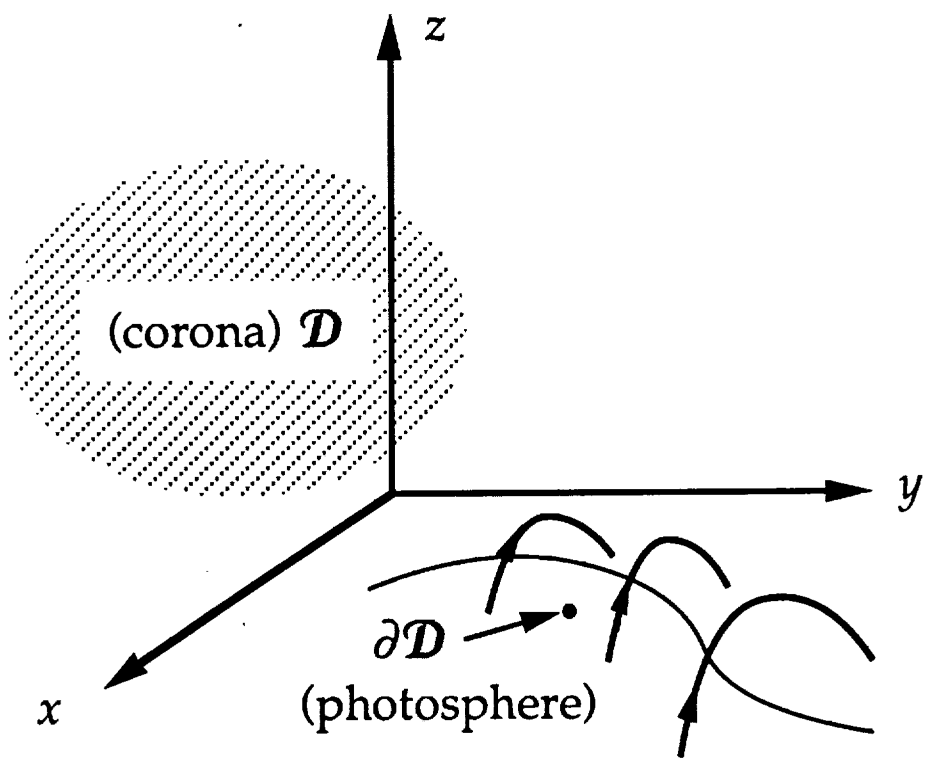
Topological constraint:

$$\mathbf{B} \cdot \nabla \alpha = 0$$

FORCE-FREE CORONAL FIELDS

Let $\mathcal{D} = \{z > 0\}$ (Planar approx.)

$$\boxed{\begin{array}{c}\text{Find } \mathbf{B} \text{ in } \mathcal{D} \text{ satisfying} \\ \nabla \times \mathbf{B} = \alpha \mathbf{B} \\ \text{given } \mathbf{B} \text{ on } \partial\mathcal{D}.\end{array}}$$



EXISTENCE AND UNIQUENESS OF THE SOLUTION

- The existence and uniqueness of the nonlinear FFF solution is not known under all conditions
- The potential field is unique for a given B_n
- The constant-α field is unique for a given B_n and specified angle of $\mathbf{B}_t$ (Chiu and Hilton 1977)
- Schmidt (1968) conjectured that a solution exists when B_n and α are specified on $\partial\mathcal{D}$
- Given B_n on $\partial\mathcal{D}$ and α on $\partial\mathcal{D}^+$, the solution exists for small α (Bineau 1972), and does not exist for large α (Bineau 1972; Aly 1984, 1988, 1989)

- The FFF solution is stable with respect to changes in boundary conditions to the same extent that the potential field is (Molodensky 1974)

- We have confirmed that a solution can be found with these boundary conditions (for a 3D model of a sunspot)

- Sakurai (1981) proposed an iterative solution technique which works for very small α (based on Grad 1958)

Description of Method

- Apply boundary conditions on B_n and J_n (derived from VM data)
- Construct an adaptive external circuit to drive the required current through the photosphere into the corona
- Use the resistive MHD equations to evolve the plasma
- In steady state, plasma approaches a force-free state which matches the VM boundary data

RESISTIVE MHD EQUATIONS

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \frac{1}{c} \mathbf{J} \times \mathbf{B} + \nu \rho \nabla^2 \mathbf{v}$$

STEADY STATE SOLUTION

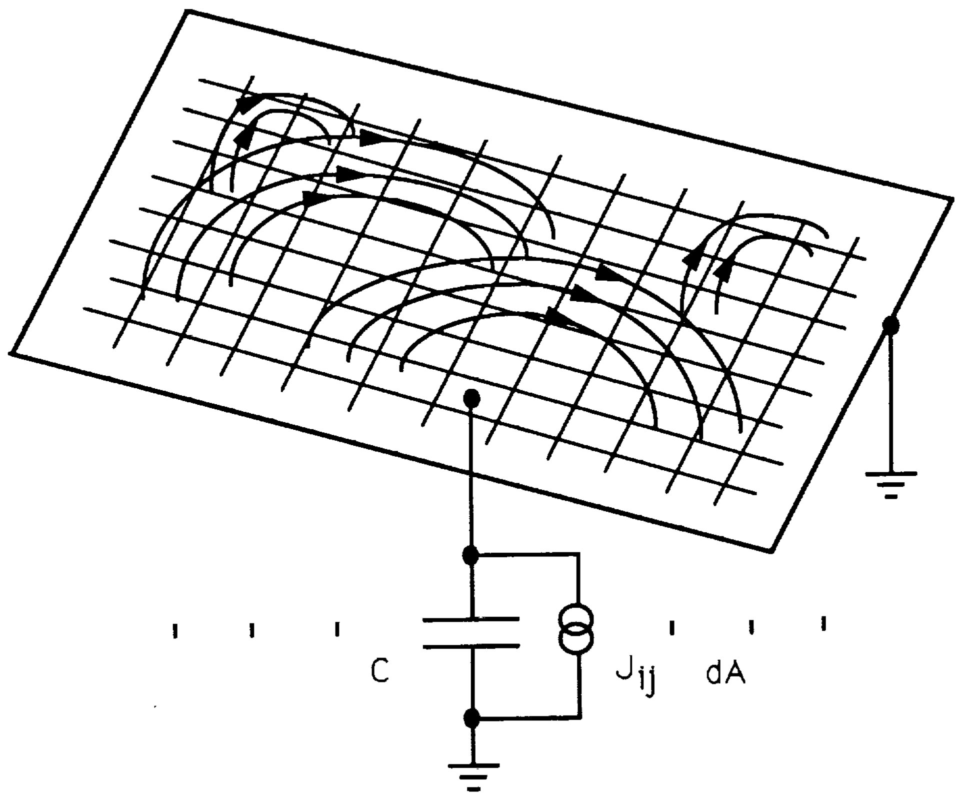
$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

$$\mathbf{J} \times \mathbf{B} = -c\rho\left(\nu\nabla^2\mathbf{v} - \mathbf{v}\cdot\nabla\mathbf{v}\right)$$

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \eta\,\mathbf{J}$$

$$\nabla \times \mathbf{E} = 0$$

- Driven resistive MHD equilibrium with flow
- For small η and ν (and hence $\mathbf{v}$), the solution approaches a force-free state


C
J_{ij}
dA

SOLUTION TECHNIQUE

1. For the given B_n on $\partial\mathcal{D}$, find the potential field $\mathbf{B}_{\text{pot}}$ in $\mathcal{D}$

2. Apply a potential $V(x,y,t)$ on $\partial\mathcal{D}$ using an adaptive external circuit to force $J_n = J_n^{\,o}$ (the measured normal current), while keeping B_n fixed

- If a steady-state is reached, it will satisfy $\mathbf{J} \times \mathbf{B} \approx 0$ in $\mathcal{D}$, and will have the required B_n and J_n on $\partial\mathcal{D}$.

BOUNDARY ALGORITHM

- Impedance Z:

$$Z = \frac{\eta l}{d A}$$

- Choose a capacitance C:

$$C = \upsilon_o Z \Delta t$$

with $\upsilon_o > 1$; υ_o controls the time scale of the external circuit

- Evolve V according to:

$$\frac{\partial V}{\partial t} = \frac{d A}{C}(J_n^o - J_n)$$

- This gives $\mathbf{E}_t$ on the boundary:

$$\mathbf{E}_t = -\nabla_t V$$

- Note that B_n remains fixed on the boundary (since $\nabla_t \times \mathbf{E}_t = 0$)

APPLICATION TO VM DATA

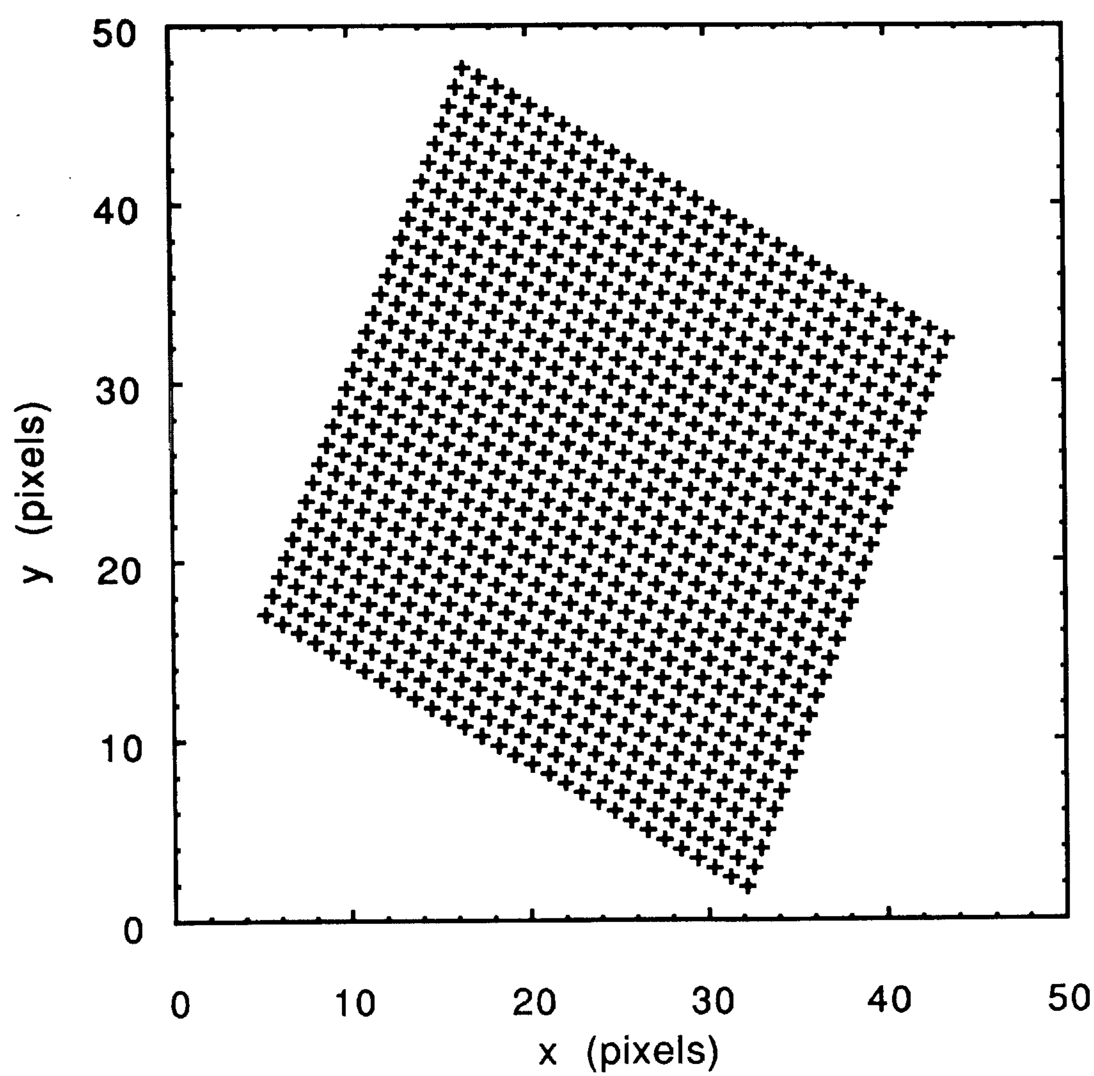
IfA, U. Hawaii (McClymont *et al.*):

- Use Lites-Skumanich code to fit Stokes profiles, and deduce **B**
- Resolve 180° ambiguity in transverse **B**

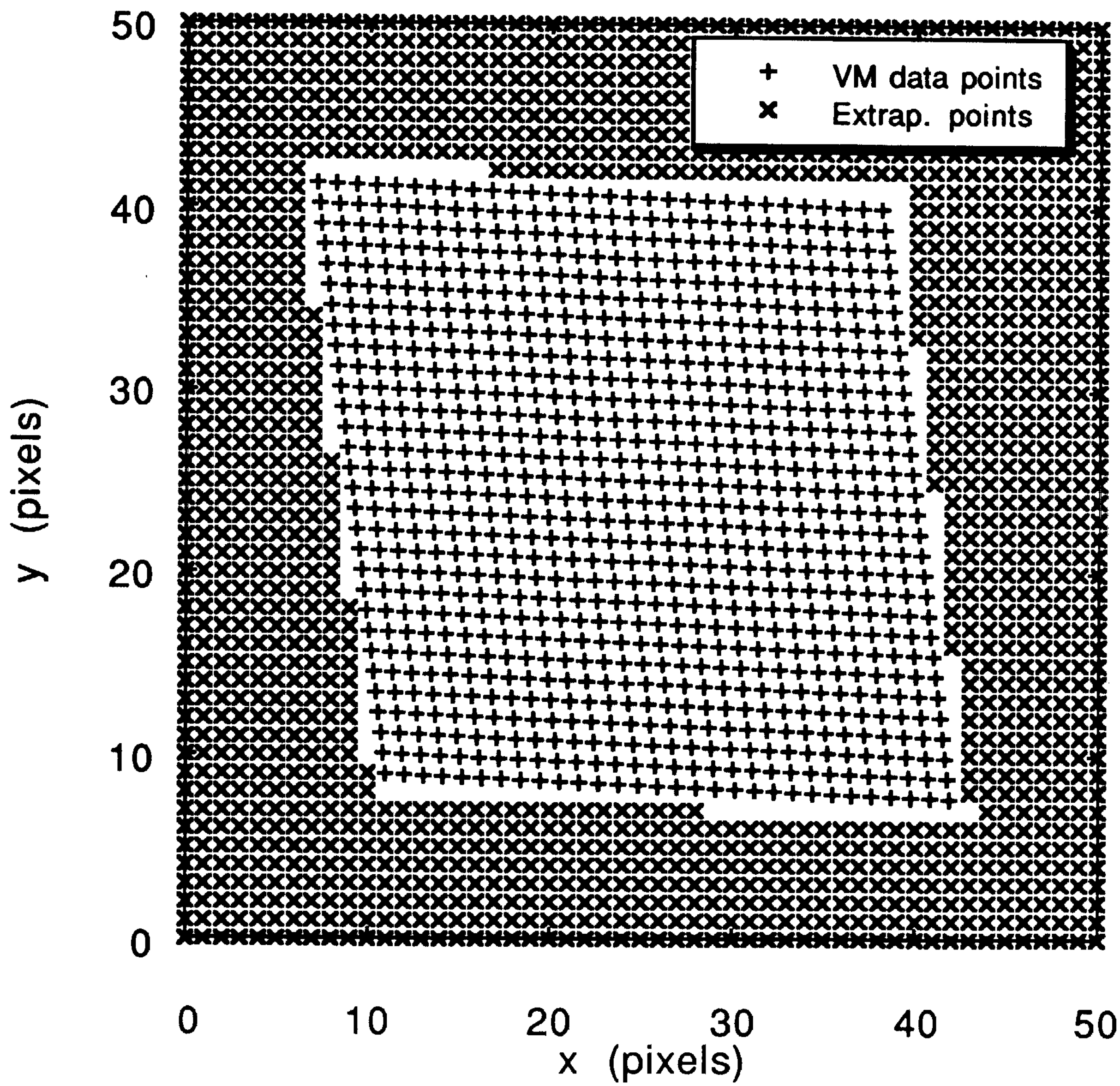
SAIC, San Diego:

- Rotate and translate data to best fit a rectangular region [for AR5747, rotate by -27° about (25,25) pix.]
- Interpolation/Extrapolation of data. Least squares fit to Fourier series, with derivative constraints [for AR5747, use 32x32 modes]
- Run resistive MHD code [used a 63x64x64 mesh on Cray-2]
- Converges in $O(100\tau_A)$

AR5747, 20 Oct, 1989
Original Data Points



AR 5747, Oct 20, 1989
(Rotated by -27° about x=25, y=25)



- Data is not consistent with a force-free solution:

$$\int |\mathbf{J} \times \mathbf{B}|\, dV \approx 23\% \text{ of } \int |\mathbf{J} \cdot \mathbf{B}|\, dV$$

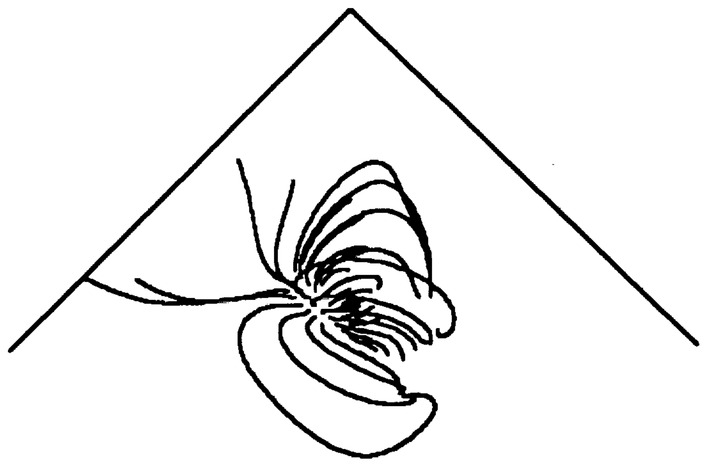
- This has been noted by U. Hawaii researchers (Canfield *et al.* 1990) as an imbalance of positive and negative flux between arbitrary intervals of α in the magnetogram

- Source of error: measurement errors; fitting is only good to ~ 15%; thermal forces?

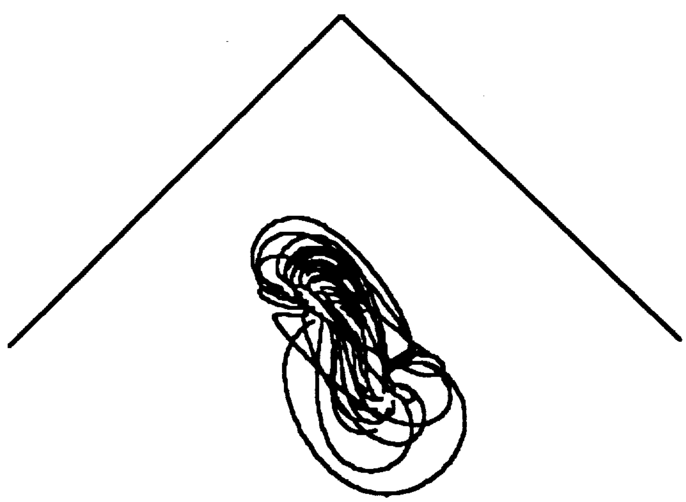
DETAILS OF SOLUTION

- Coronal field is sheared significantly
- $W_{pot} = 1.44 \times 10^{33}$ ergs
- $W_{ff} = 1.55 W_{pot} = 2.23 \times 10^{33}$ ergs
- Free energy = 8×10^{32} ergs
- The transverse magnetic field from the solution is similar to that from the VM data

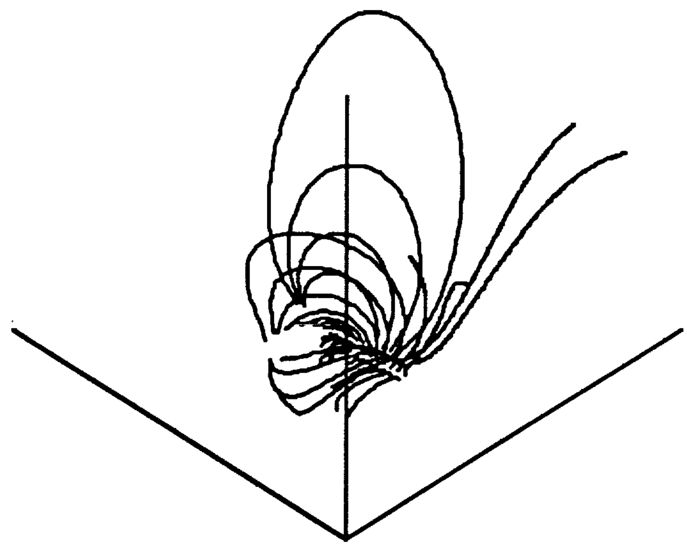
Potential Field



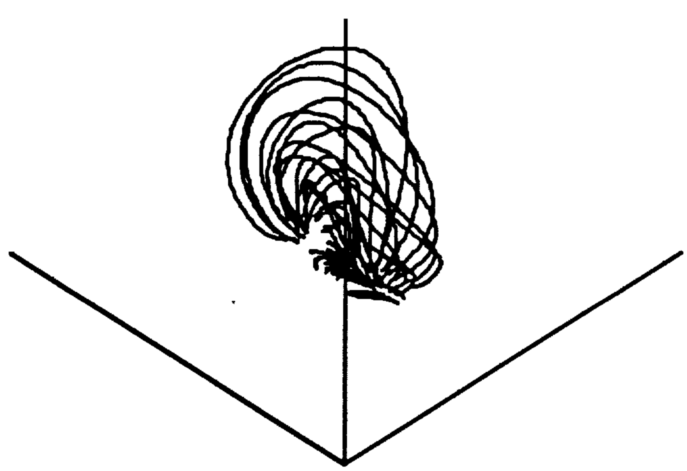
Force-Free Field



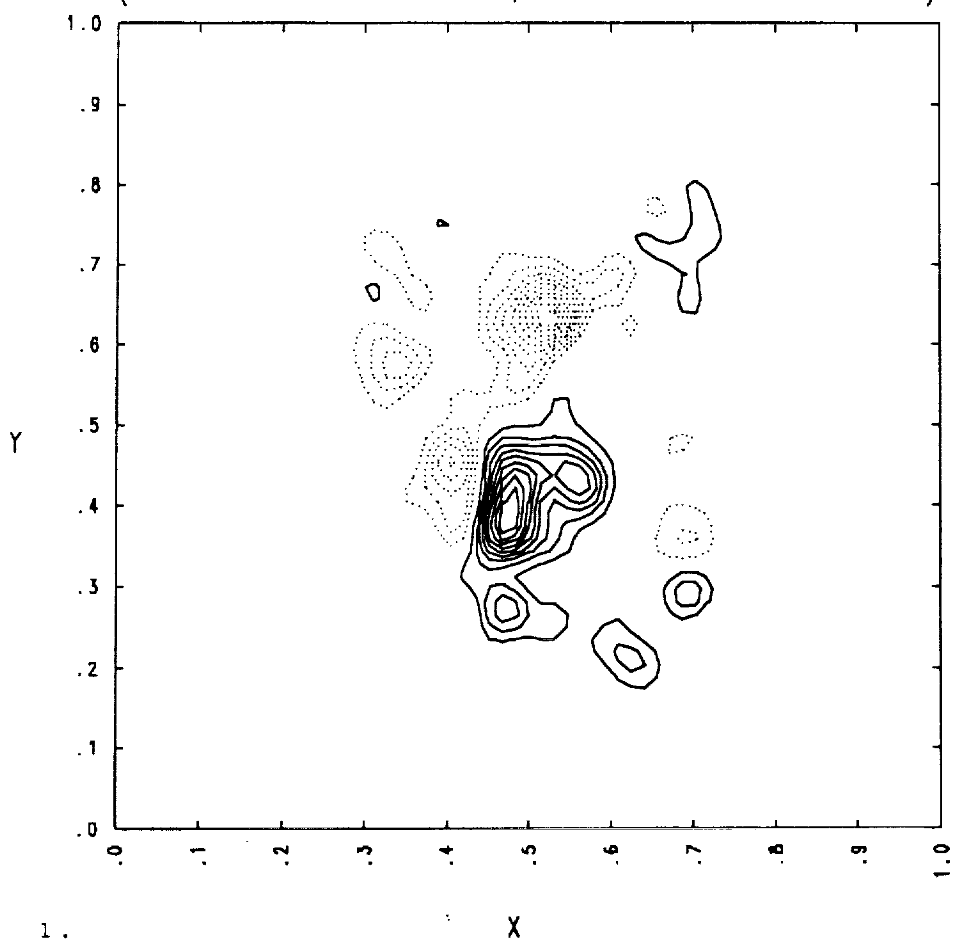
Potential Field



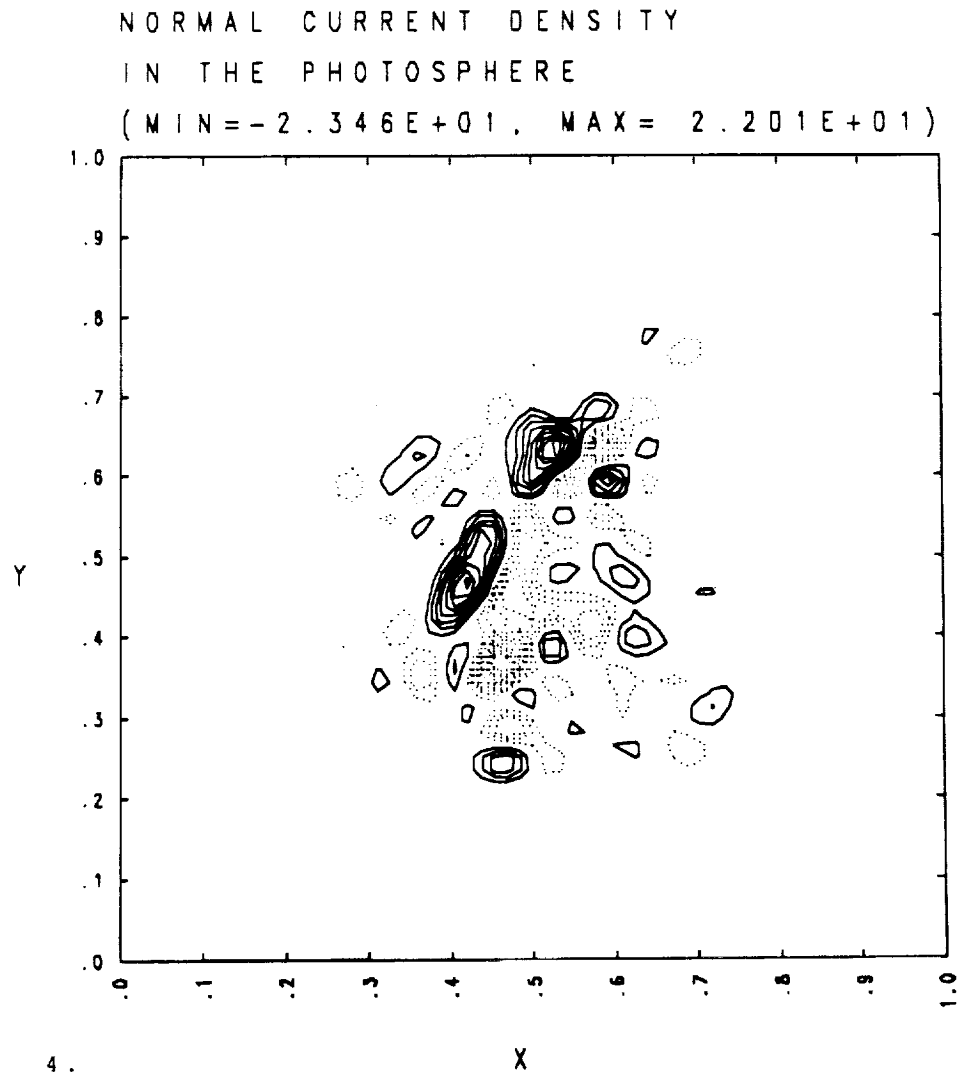
Force-Free Field



NORMAL MAGNETIC FIELD
IN THE PHOTOSPHERE
(MIN=-1.000E+00, MAX= 9.793E-01)

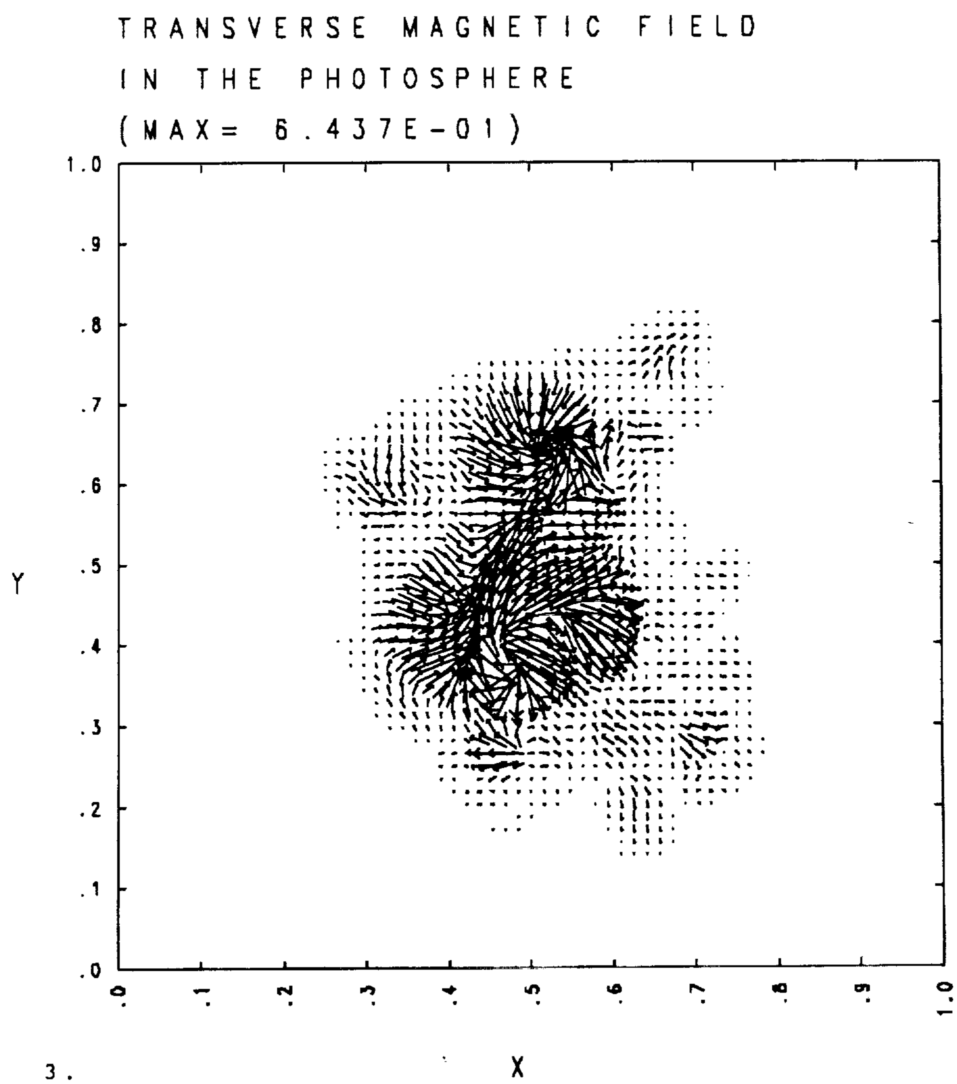


frame # 1.

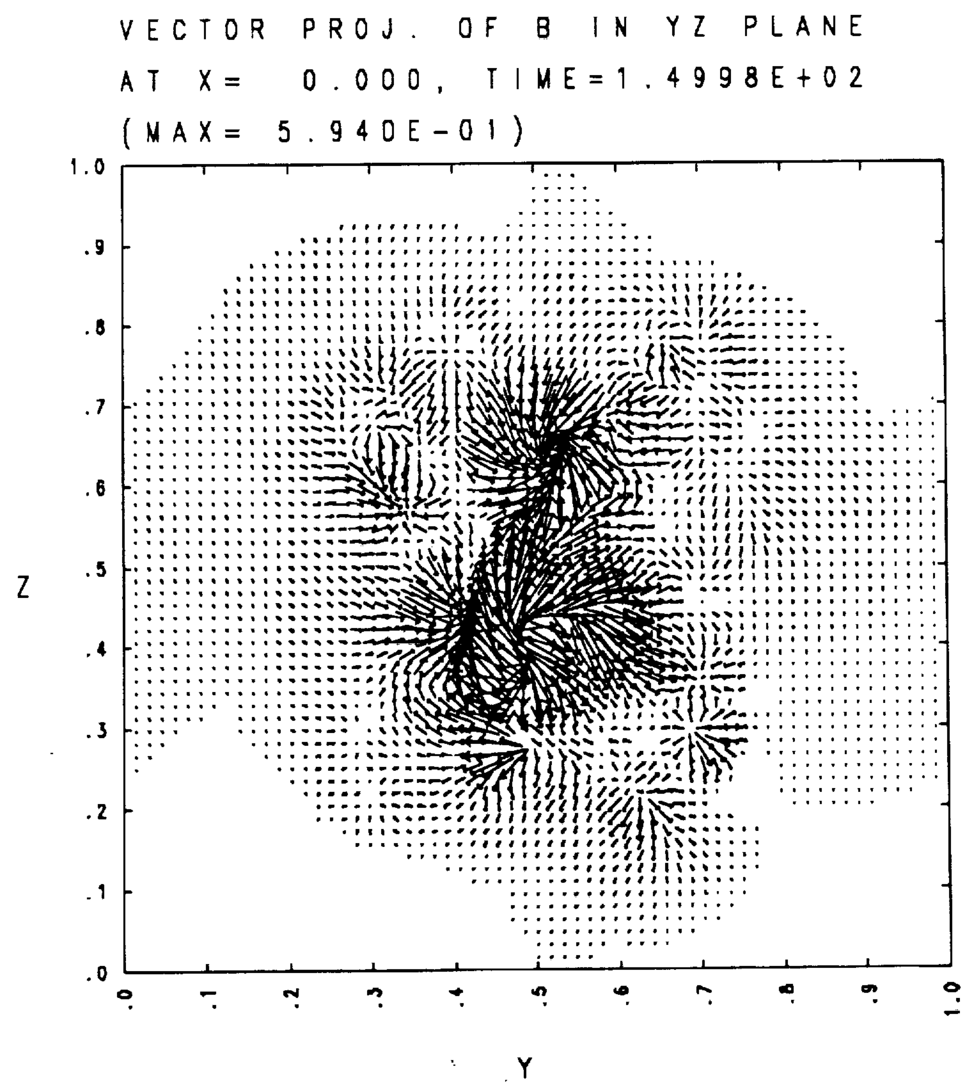


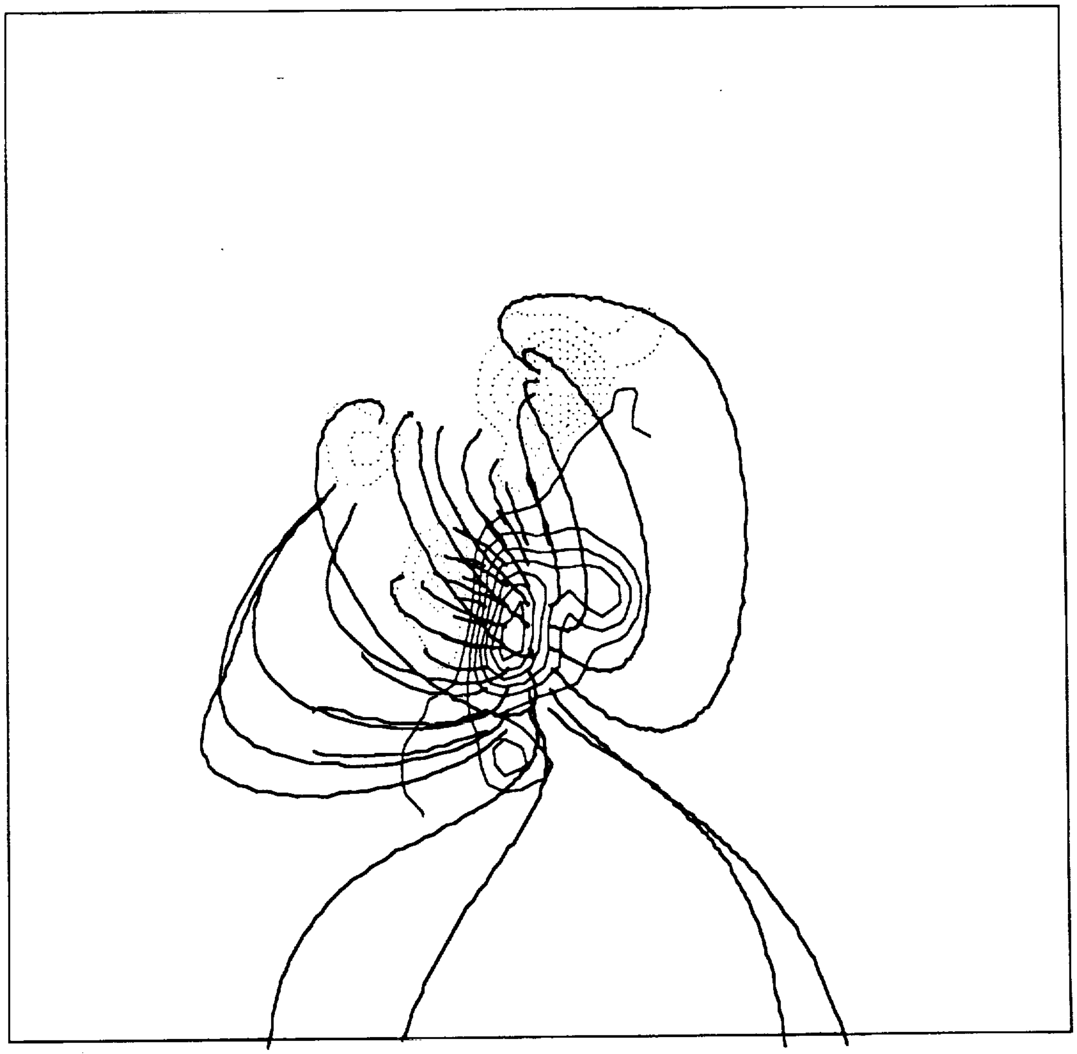
NORMAL CURRENT DENSITY
IN THE PHOTOSPHERE
(MIN=-2.346E+01, MAX= 2.201E+01)
1.0
.9
.8
.7
.6
.5
Y
.4
.3
.2
.1
.0
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
X

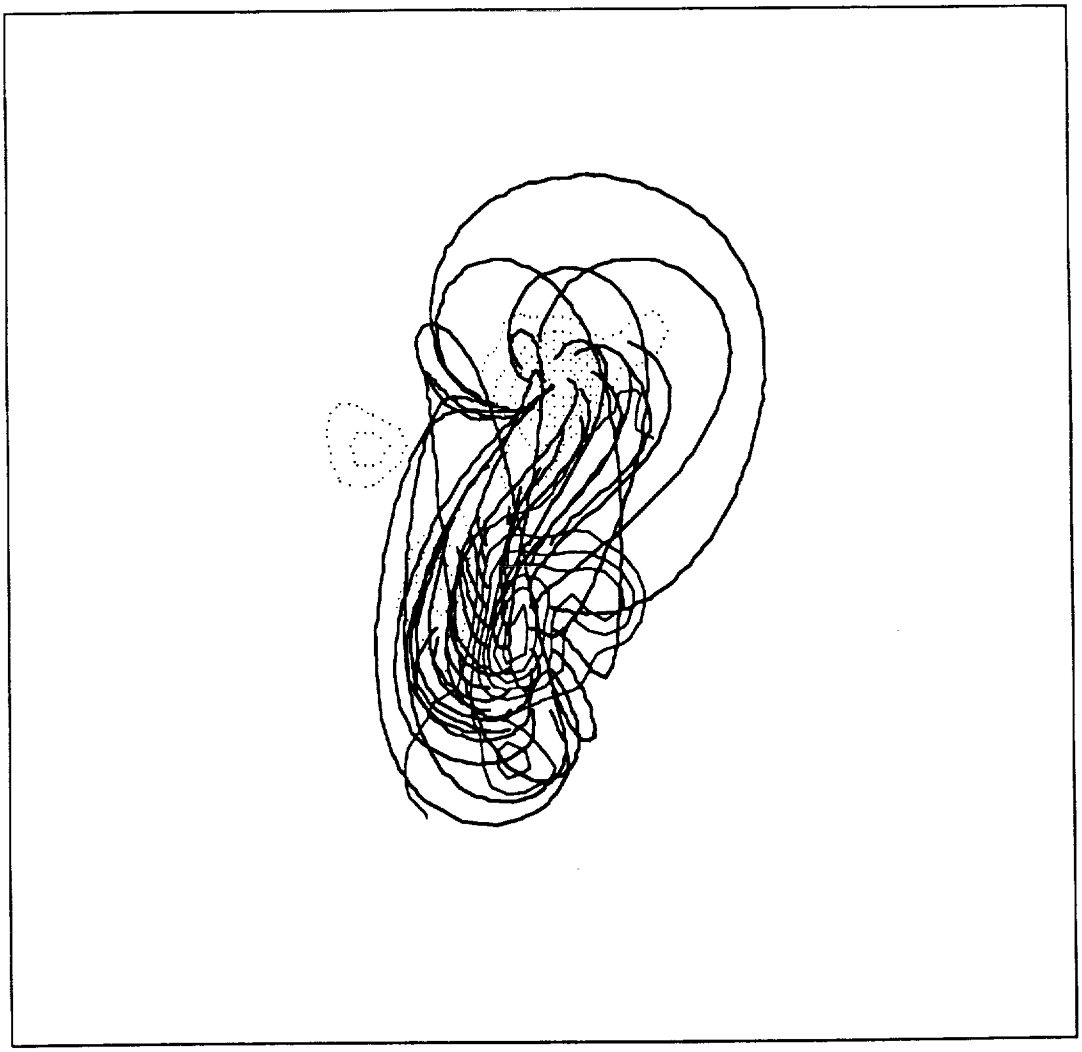
frame # 4.


TRANSVERSE MAGNETIC FIELD
IN THE PHOTOSPHERE
(MAX= 6.437E-01)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
Y
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
X

frame # 3.


VECTOR PROJ. OF B IN YZ PLANE
AT X= 0.000, TIME=1.4998E+02
(MAX= 5.940E-01)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
Z
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
Y





FEATURES

- Evolve **B** toward a force-free state using resistive MHD
- Resistivity is essential to allow the topology to change during the evolution
- The dynamical equations allow ideal and resistive instabilities to occur (giving a minimum-energy final state subject to given boundary conditions?)
- Adaptive external circuit automatically forces the right current to flow in the corona

CONCLUSION

- The "Evolutionary Method" has a sound theoretical basis
- It has worked on model force-free fields with simulated VM data; more importantly, it also works on actual VM data, as demonstrated here
- This technique offers exciting new possibilities for detailed analysis of coronal magnetic fields

<u>Future work:</u>

- Assess measurement errors
- Optimally extract the required boundary conditions (force-free VM problem is overdetermined)
- Resolve 180° ambiguity in transverse **B**

NASA National Aeronautics and Space Administration

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15. Supplementary Notes

16. Abstract

This document represents the Final Report on work performed under Contract NASW-4571 between NASA and Science Applications International Corporation. Under this contract, SAIC has conducted research into the determination of coronal magnetic fields from vector magnetograms, including the development and application of an algorithm to determine force-free coronal fields above a solar active region. The "evolutionary algorithm" which was proposed to find the force-free coronal field has a sound theoretical basis, and is directly applicable to the practical determination of coronal fields above active regions. The algorithm has been applied to a vector magnetogram of NOAA AR5747 of 20 October, 1989, and has yielded detailed quantitative estimates of the coronal magnetic field above a solar active region. This exercise has demonstrated that the solution to the nonlinear boundary value problem (with non-constant α) can be found, and, in particular, that it has been developed to the point where it can be applied practically to observational data.

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SECTION J., ATTACHMENT (3)