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A MINIMUM PROPELLANT SOLUTION TO AN ORBIT-TO-ORBIT TRANSFER USING A LOW THRUST PROPULSION SYSTEM

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Introduction

The Space Exploration Initiative is considering use of low thrust (nuclear electric, solar electric) and intermediate thrust (nuclear thermal) propulsion systems for transfer to Mars and back. Due to the duration of such a mission, a low thrust minimum-fuel solution is of interest; a savings of fuel can be substantial if one allows the propulsion system to be turned off and back on. This switching of the propulsion system helps distinguish the minimum-fuel problem from the well-known minimum-time problem.

Optimal orbit transfers are also of interest to the development of a guidance system for orbital maneuvering vehicles which will be needed, for example, to deliver cargoes to Space Station Freedom.

The problem of optimizing trajectories for an orbit-to-orbit transfer with minimum-fuel expenditure using a low thrust propulsion system was discussed in (1), but the code, SECKSPOT (2), which was used is incapable of handling a general minimum propellant problem.

Analysis

To avoid the singularities that may occur when using the classical elements a, e, i, ω, Ω, (semi-major axis, eccentricity, angle of inclination, argument of perigee, longitude of the ascending node) the equinoctial elements a, h, k, p, and q are used to define the spacecraft's state. The sixth state element is the eccentric longitude F, which represents the angular position of the satellite in its orbit. Thus, the spacecraft's state, $\mathbf{z}$, is

$\mathbf{z} = (a, h, k, p, q)^T$, or in classical elements,

$$\mathbf{z} = (a,\ e\sin(\omega + \Omega),\ e\cos(\omega + \Omega),\ \tan(i/2)\sin(\Omega), \tan(i/2)\cos(\Omega))^T,$$

and [1]

$$F = E + \omega + \Omega.$$

The convention of denoting vectors in bold print and unit vectors with the circumflex $\wedge$ is used.

It follows that the equations of motion in terms of these equinoctial elements are

$$\dot{\mathbf{z}} = 2P\ M\hat{\mathbf{u}}\ /\ mc,$$

[2]

$$\dot{m} = -2P\ /\ c^2,$$

where P (≥ 0) = power due to thrusters, m (> 0) = mass of the spacecraft,

c (> 0) = jet exhaust speed, M = 5 × 3 matrix representing the partial derivative of $\mathbf{z}$ with respect to the velocity vector $\mathbf{r}$, and the unit vector in the direction of thrust is $\hat{\mathbf{u}}$, which is also the control for this optimization problem.

Applying the maximum principle to the optimal pair $(\hat{\mathbf{u}}, \mathbf{z})$, it is clear that one should maximize the Hamiltonian function, H, with respect to the thrust direction at each point along the optimal trajectory of the transfer to obtain the minimum-fuel solution. The Hamiltonian function is defined in terms of the costate (adjoint) variables $\lambda_{\mathbf{z}}$ and λ_m so that

$$\dot{\mathbf{z}} = \partial H / \partial \lambda_{\mathbf{z}}, \quad \dot{m} = \partial H / \partial \lambda_m,$$

$$\dot{\boldsymbol{\lambda}}_{\mathbf{z}} = - \partial H / \partial \mathbf{z}, \quad \dot{\lambda}_m = - \partial H / \partial \lambda_m, \qquad [3]$$

where

$$H(\mathbf{z}, m, \boldsymbol{\lambda}_{\mathbf{z}}, \lambda_m, \hat{\mathbf{u}}) = 2P / mc \, \lambda_{\mathbf{z}}^T M \hat{\mathbf{u}} - 2P / c^2 \lambda_m. \qquad [4]$$

Hence, the Hamiltonian H in [4] is maximized by aligning $\hat{\mathbf{u}}$ in the direction of the primer vector, $\lambda_{\mathbf{z}}^T M$. Let λ denote the magnitude of this primer vector. Then define the switching function, σ, by $\sigma = \lambda - m\lambda_m / c$. With this definition, H can be rewritten as $H = (2P / mc)\sigma$.

At any point along the trajectory for which σ, and hence H, is negative, H can be maximized by taking $P = 0$, which amounts to turning the propulsion system off; while if σ, and hence H, is positive, then allow the propulsion system to be used at full power in the optimal direction of thrust $\hat{\mathbf{u}}$. So $\sigma < 0$ determines a "coast phase", while $\sigma > 0$ yields a "thrust phase". The points at which $\sigma = 0$ are called switch points.

Around each orbit the equinoctial elements vary "slowly" and can be held constant; thus only the "fast" variable F remains. However, in this situation the actual position in an orbit of the spacecraft is not of interest, so the dependence on this variable is removed by integration. Using Kepler's equation which relates the time t to the mean anomaly and hence the eccentric anomaly, one can perform averaging over an orbit of some specified period by changing the variable of integration from time t to the eccentric longitude F. Thus the Hamiltonian and hence the equations of motion can be averaged; note that since the Hamiltonian is zero for coast phases, one needs only integrate over thrust phases. In order to perform this integration using a Gaussian quadrature method, it is necessary to predetermine the switch points along the trajectory and integrate only over thrust periods, which will begin and end with a switch point. These averaged derivatives are then solved using a Runge-Kutta method.

To find the solution it is necessary to determine the initial values for the costate variables λ_z such that the boundary conditions are satisfied; that is, the desired final state is reached. The transversality conditions at the final time (which is unspecified) t_f, are $\lambda_z(t_f) = \alpha$, where α is a vector of parameters and $\lambda_m(t_f) = 1$.

Numerical Algorithm

The minimum-fuel solution is found by an iterative method which determines the initial costate values $\lambda z(t_0)$ and $\lambda_m(t_0)$ such the boundary conditions $a - a_f = 0$, $h - h_f = 0$, $k - k_f = 0$, $p - p_f = 0$, $q - q_f = 0$ and $\lambda_{mf} - 1 = 0$ (and $f - f_f$ if not averaging) are satisfied. This is a system of nonlinear equations in terms of $\lambda_z(t_0)$, whose solution can be found by a secant method. For an initial guess, an optimal trajectory is found in the following manner: call the Runge-Kutta integrator,which calls RKFCT which calculates the derivatives . RKFCT calls SWCHPTS to find the thrust periods over and orbit, by finding the switchpoits and calls INTEG to prepare the integrands for quadrature if AVGTST determines that the variables are changing slowly enough, and if not averaging the partial derivatives of the equations of motion are calculated; INTEG calls QUAD4 for integration. QUAD4 calls FCT, which calculates the Hamiltonian, and calls EVALMP and PRIMER to find the thrust direction. From this Runge-Kutta integrator, a new state and costate is found. The stopping criterion used is the size of the semi-major axis.

Conclusions

It appears that one of the difficulties involved with this problem is convergence of the method considering the sensitivity to slight perturbations in the initial costate values. Another problem is the accurate determination of the switch points for the case of averaging and also not averaging.

References

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(2) Sackett, L.L., Malchow, H.L, Edelbaum, T.N., Solar Electric Geocentric Transfer with Attitude Constraints: Analysis. Technical Report NASACR-134927,The Charles Stark Draper Laboratory, Inc. August 1975.