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A FINITE ELEMENT BOUNDARY INTEGRAL FORMULATION FOR RADIATION AND SCATTERING BY CAVITY ANTENNAS USING TETRAHEDRAL ELEMENTS

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ABSTRACT

A hybrid finite element-boundary integral formulation is developed using tetrahedral/triangular elements for discretizing the cavity/aperture of microstrip antennas/arrays. The tetrahedral elements with edge based linear expansion functions are chosen for modeling the volume region and triangular elements are used for discretizing the aperture. The edge-based expansion functions are divergenceless thus removing the requirement to introduce a penalty term and the tetrahedral elements permit greater geometrical adaptability than the rectangular bricks. The underlying theory and resulting expressions are discussed in detail together with some numerical scattering examples for comparison and demonstration.

Introduction

Recently, a hybrid finite element formulation was developed for a characterization of the scattering and radiation properties associated with microstrip patch antennas and arrays residing in a cavity that is recessed in a ground plane [1], [2], [3]. The technique employs the finite element method (FEM) to model the substrate in the cavity region and the mesh was terminated at the aperture of the cavity via the boundary integral method. By virtue of the FEM, the analysis is applicable to patch antennas, slots and arrays which reside on or are embedded in the layered dielectric substrate. Various feed structures and impedance loads can also be modeled within the context of the FEM without difficulty. As demonstrated in [3], this hybrid version of the finite element method proved very successful and accurate in treating complex antenna configurations and large arrays. The last is owed to the sparsity of the finite element matrix and although the boundary integral resulted in a partially full matrix it did not burden the memory requirements because it was Toeplitz in form. Specifically, when the system is solved via the biconjugate gradient method in conjunction with the fast Fourier transform (FFT) [4, 5], the required memory is only $6.25N_t + 10.5N_s$, where N_t and N_s denote, respectively, the total number of unknowns in the entire cavity and the unknowns or edge elements at the aperture of the cavity.

So far the implementation of the proposed finite element method has only been carried out by subdividing the cavity volume using rectangular bricks (rectangular hexahedra). Obviously, this limits the utility of the method to those geometries and antennas which fit in a rectangular uniform grid. Consequently, circular patches, non-rectangular slots or irregular cavities and feeding lines cannot be modeled with these discretization elements. A more adaptable volume element is the tetrahedron (see figure 1(a)) which leads to a discretization of the cavity surface in terms of triangles as illustrated in figures 1(b) and 1(c) In the following, we describe the implementation of the proposed hybrid FE-BI method using tetrahedra and triangular facets for volume and surface elements, respectively. The analysis for generating the required system is outlined and some preliminary results are presented which validate the formulation.

Basic Equations

Consider the geometry given in figure 2, that of a cavity in a ground plane.
A source is placed in the cavity and we are interested in computing the radiated field in the region above S_{cav} (i.e. in region I). Using the procedure discussed by Jin and Volakis [1–3], this type of problem can be readily handled by subdividing the computational domain into two regions to be referred to as regions I and II. In region II, which encompasses the volume enclosed by the cavity walls and S_{cav}, the finite element method will be employed to formulate the fields. The primary reason for using the finite element method is its adaptability in modeling a variety of cavities and radiating elements. The fields in region I (exterior region) will be computed via the boundary integral method. This amounts to introducing equivalent sources over

the cavity's aperture which are then integrated to obtain the radiated fields. The exterior and interior fields are then coupled by imposing the continuity condition across the aperture.

1. Interior Region Formulation

From Maxwell's equations we obtain the vector wave equation

$$\nabla \times \left(\frac{1}{\mu_r}\nabla \times \mathbf{E}\right) - k_0^2\epsilon_r\mathbf{E} = -jk_0Z_0\mathbf{J}^i + \nabla \times \left(\frac{1}{\mu_r}\mathbf{M}^i\right) \tag{1}$$

The solution of this equation is equivalent to minimizing the variational function

$$\begin{aligned} F(\mathbf{E}) &= \frac{1}{2}\iiint_{V_{cav}} \left\{\frac{1}{\mu_r}(\nabla \times \mathbf{E})\cdot(\nabla \times \mathbf{E}) - k_0^2\epsilon_r\mathbf{E}\cdot\mathbf{E}\right\} dv \\ &\quad + \iiint_{V_{cav}} \mathbf{E}\cdot\left[jk_0Z_0\mathbf{J}^i - \nabla \times \left(\frac{1}{\mu_r}\mathbf{M}^i\right)\right] dv \\ &\quad + jk_0Z_0\iint_S \mathbf{E}\cdot(\mathbf{H}\times\hat{z})\, ds \end{aligned} \tag{2}$$

Before taking the variation of F with respect to $\mathbf{E}$ we must first discretize it for numerical implementation. To this end, we subdivide V_{cav} into N_e small tetrahedral elements of volume V_e. Within each volume element (say the eth element) we expand the field as

$$\mathbf{E} = \sum_{i=1}^{N_v} E_i^e \mathbf{V}_i^e \tag{3}$$

where $\mathbf{V}_i^e$ are the basis elements for the eth element and E_i^e are the unknown coefficients of the expansion. Referring to figure 3, $\mathbf{V}_i^e$ are given by

$$\mathbf{V}_{7-i}^e(\mathbf{r}) = \begin{cases} \mathbf{f}_{7-i} + \mathbf{g}_{7-i}\times\mathbf{r} & \mathbf{r}\in V_e \\ 0 & \text{outside element} \end{cases} \tag{4}$$

$$\mathbf{f}_{7-i} = \frac{b_{7-i}}{6V_e}\mathbf{r}_{i_1}\times\mathbf{r}_{i_2} \qquad \mathbf{r}_{i_1},\mathbf{r}_{i_2}: \text{ position vectors of vertices } i_1 \text{ and } i_2 \text{ (see Table 1)} \tag{5}$$

$$\mathbf{g}_{7-i} = \frac{b_i b_{7-i}}{6V_e}\mathbf{e}_i \tag{6}$$

$$\mathbf{e}_i = \frac{(\mathbf{r}_{i_2} - \mathbf{r}_{i_1})}{b_i} \tag{7}$$

$$b_i = |\mathbf{r}_{i_2} - \mathbf{r}_{i_1}| = \text{length of the } i\text{th edge (see Table 1)}$$

$$V_e = \text{element's volume}$$

and

$$i = 1,2,3,\ldots,6.$$

To understand the physical meaning of the expansion (3) it is necessary to examine the properties of the expansion functions $\mathbf{V}_i$. We observe that

$$\nabla\cdot\mathbf{V}_i^e = 0 \tag{8}$$

$$\nabla \times \mathbf{V}_i^e = 2\mathbf{g}_i \tag{9}$$

and

$$\mathbf{V}_i^e(\mathbf{r}^j) \cdot \mathbf{e}_j = \delta_{ij} \tag{10}$$

where $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ and $\mathbf{r}^j$ has its tip on the jth edge of the tetrahedron. The last property of $\mathbf{V}_i^e$ points out that E_i^e in the expansion (3) is simply the fields at the ith edge of the tetrahedron. The basis functions $\mathbf{V}_i^e$ then provide a linear transition of the fields from one of the edges to the other and have the important property of being divergenceless. This is quite essential because such a condition is equivalent to stating that $\nabla \cdot \mathbf{E} = 0$ within the element, one of Maxwell's equations which was implied in deriving the wave equation (1).

Substituting (3) into (2) yields

$$F = F_{1V} + F_{2V} + F_S \tag{11}$$

where

$$\begin{aligned}
F_{1V} &= \sum_{e=1}^{N_e} F_{1V}^e \\
F_{2V} &= \sum_{e=1}^{N_e} F_{2V}^e \\
F_{1V}^e &= \frac{1}{2}\sum_{j=1}^{N_V}\sum_{i=1}^{N_V} E_j^e E_i^e \iiint_{V_e} \left\{ \frac{1}{\mu_r}(\nabla \times \mathbf{V}_i^e) \cdot (\nabla \times \mathbf{V}_j^e) - k_0^2 \epsilon_r \mathbf{V}_i^e \cdot \mathbf{V}_j^e \right\} dv \\
F_{2V}^e &= \sum_{i=1}^{N_V} E_i^e \iiint_{V_e} \mathbf{V}_i^e \cdot \left[jk_0 Z_0 \mathbf{J}^i - \nabla \times \left(\frac{1}{\mu_r} \mathbf{M}^i \right) \right] dv \\
N_e &= \text{number volume elements} \\
N_v &= 6 = \text{number of tetrahedron edges}
\end{aligned}$$

and F_s will be considered later. Taking the variation of $F(E)$ and setting it to zero gives

$$\begin{aligned}
\frac{\partial F}{\partial E_i} &= \sum_{e=1}^{N_e} \frac{\partial F^e(E_i^e)}{\partial E_i^e} \\
&= \sum_{e=1}^{N_e} \left\{ \sum_{j=1}^{N_v} E_j^e \iiint_{V_e} \left(\frac{1}{\mu_r} \nabla \times \mathbf{V}_i^e \cdot \nabla \times \mathbf{V}_j^e - k_0^2 \epsilon_r \mathbf{V}_i^e \cdot \mathbf{V}_j^e \right) dv \right. \\
&\quad \left. + \iiint_{V_e} \cdot \left[jk_0 Z_0 \mathbf{J}^i - \nabla \times \left(\frac{1}{\mu_r} \mathbf{M}^i \right) \right] dv \right\} + \frac{\partial F_s}{\partial E_i} = 0
\end{aligned}$$

This can also be rewritten as

$$\frac{\partial F}{\partial E_i} = \sum_{e=1}^{N_e} \frac{\partial F^e}{\partial E_i^e} = \sum_{e=1}^{N_e} \left\{ [A_{ij}^e]\{E_j^e\} + \{K_i^e\} \right\} + \frac{\partial F_s}{\partial E_i} = 0 \tag{12}$$

where $[A^e_{ij}]$ is the volume element matrix whose elements are given by

$$A^e_{ij} = \iiint_{V_e} \left\{ \frac{1}{\mu_r} (\nabla \times \mathbf{V}^e_i) \cdot (\nabla \times \mathbf{V}^e_j) - k_0^2 \epsilon_r \mathbf{V}_i \cdot \mathbf{V}^e_j \right\} dv \tag{13}$$

The elements of the excitation vector $\{K^e_i\}$ are given by

$$K^e_j = \iiint_{V_e} \mathbf{V}^e_j \left[jk_0 Z_0 \mathbf{J}^i - \nabla \times \left(\frac{1}{\mu_r} \mathbf{M}^i \right) \right] dv \tag{14}$$

Explicit values for A^e_{ij} in terms of the geometrical parameters of V_e are worked out in the Appendix A.

Boundary Integral Equation

To solve (12), it is necessary to specify the discrete form of the boundary integral F_s. This can only be accomplished by replacing $\mathbf{H}$ with a functional of $\mathbf{E}$ or alternatively by imposing a condition on S_V which relates $\mathbf{E}$ and $\mathbf{H}$. Such a condition or equation is supplied from the exterior region boundary equation. In particular we have

$$\begin{aligned} \mathbf{H} &= \mathbf{H}^i + \mathbf{H}^r - j2k_0 Y_0 \iint_{S'_{cav}} \left(\bar{\mathbf{I}} + \frac{1}{k_0^2} \nabla\nabla \right) G_0(\mathbf{r}, \mathbf{r}') \cdot \mathbf{M}(\mathbf{r}')\, ds' \\ &= \mathbf{H}^i + \mathbf{H}^r - j2k_0 Y_0 \iint_{S_{cav}} \left(\bar{\mathbf{I}} + \frac{1}{k_0^2} \nabla\nabla \right) G_0(\mathbf{r}, \mathbf{r}') \cdot (\mathbf{E}(\mathbf{r}') \times \hat{z})\, ds' \end{aligned} \tag{15}$$

in which $\mathbf{H}^i$ is the incident field, if any, from the exterior region, $\mathbf{H}^r$ is the reflected field due to $\mathbf{H}^i$ if the aperture is closed, $\bar{\mathbf{I}}$ is the unit dyad and

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{16}$$

is the free space Green's function with $\mathbf{r}$ and $\mathbf{r}'$ representing the observation and integration points. The factor of two in front of the integral in (15) is due to image theory.

Substituting the above expression (15) into the last integral of (2) yields

$$\begin{aligned} F_s &= jk_0 Z_0 \Bigg\{ \iint_{S_{cav}} \mathbf{E} \cdot [(\mathbf{H}^i + \mathbf{H}^r) \times \hat{z}]\, ds \\ &\quad - 2k_0^2 \iint_{S_{cav}} (\mathbf{E}(\mathbf{r}) \times \hat{z}) \cdot \left\{ \iint_{S_{cav}} \left(\bar{\mathbf{I}} + \frac{1}{k_0^2} \nabla\nabla \right) G_0(\mathbf{r}, \mathbf{r}') \cdot (\mathbf{E}(\mathbf{r}') \times \hat{z})\, ds' \right\} ds \Bigg\} \\ &= jk_0 Z_0 \Bigg\{ \iint_{S_{cav}} \mathbf{E} \cdot [(\mathbf{H}^i + \mathbf{H}^r) \times \hat{z}]\, ds \\ &\quad + 2k_0^2 \Bigg[\iint_{S_{cav}} (\hat{z} \times \mathbf{E}(\mathbf{r})) \cdot \left\{ \iint_{S_{cav}} (E(\mathbf{r}') \times \hat{z}) G_0(\mathbf{r}, \mathbf{r}')\, ds' \right\} ds \\ &\quad + 2 \Bigg[\iint_{S_{cav}} \nabla \cdot (\mathbf{E}(\mathbf{r}) \times \hat{z}) \cdot \left\{ \iint_{S_{cav}} \nabla' \cdot (E(\mathbf{r}') \times \hat{z}) G_0(\mathbf{r}, \mathbf{r}')\, ds' \right\} ds \Bigg\} \end{aligned} \tag{17}$$

To discretize these surface integrals we must again expand $\mathbf{E}$ and in this case it is necessary to discretize the surface using triangular elements which coincide with one of the faces associated with the chosen volume elements. To this end we introduce the expansion

$$\mathbf{E} = \sum_{i=1}^{N_{se}} E_i \mathbf{S}_i = \sum_{i,e \in S_{\mathrm{cav}}} E_i^e \mathbf{S}_i^e \tag{18}$$

$$\mathbf{S}_i(\mathbf{r}) = \begin{cases} \dfrac{l_i}{2A_i^+} \hat{n}_i^+ \times (\mathbf{r} - \mathbf{r}_i^+) & \mathbf{r} \in T_i^+ \\ \\ -\dfrac{l_i}{2A_i^-} \hat{n}_i^- \times (\mathbf{r} - \mathbf{r}_i^-) & \mathbf{r} \in T_i^- \end{cases} \tag{19}$$

where $A_i^{\pm}$ are the areas of the T_i^+ and T_i^- triangles (see Figure 3) and the last sum in (18) is in terms of the global cavity volume element indices. The main properties of S_i are

$$\nabla_S \cdot \mathbf{S}_i(\mathbf{r}) = 0 \qquad \mathbf{r} \in T_i = T_i^+ + T_i^- \tag{20}$$

$$\nabla_S \times \mathbf{S}_i(\mathbf{r}) = \pm \frac{l_i}{A_i^{\pm}} \hat{n}_i^{\pm} \qquad \mathbf{r} \in T_i^{\pm} \tag{21}$$

and

$$\mathbf{S}_i(\mathbf{r}^j) \cdot \mathbf{e}_j = \delta_{ij} \tag{22}$$

where $\mathbf{r}^j$ denotes the vector on the jth edge, and for this application $\hat{n}_i^{\pm} = \hat{z}$. Consequently, E_i in (18) is simply the field at the ith edge shared by the triangle pair, and N_{se} then denotes the number of edges generated in the discretization process of the surface S_{cav}. To solve the system resulting from (12), it is necessary that these edges belong to one of the volume elements which border the aperture of the cavity. Of course, the field at an edge which is located on a perfectly conducting portion of the surface or at the periphery of the aperture must be set to zero a priori and the same must also be done for those edges of the volume elements which border a conductor. It should be clear from the above representation that the field in each surface triangle is given by the linear sum of three basis functions unless that element borders a conductor, in which case one or more of the three coefficients may be zero.

Substituting (18) into (17) and differentiating with respect to E_i^e (this differentiation does not apply to the $\mathbf{E}$ field which is extracted from the integral representation of $\mathbf{H}$) gives

$$\begin{aligned} \frac{\partial F^S}{\partial E_i^e} &= 2jk_0 Z_0 \iint_{T_i} \mathbf{S}_i^e \cdot (\mathbf{H}^i \times \hat{z})\, ds \\ &+ 2k_0^2 \sum_{j,e \in S_{\mathrm{cav}}} E_j^e \iint_{T_i} (\hat{z} \times \mathbf{S}_i^e(\mathbf{r})) \cdot \left\{ \iint_{T_j} (\mathbf{S}_j^e(\mathbf{r}') \times \hat{z}) G_0(\mathbf{r}, \mathbf{r}')\, ds' \right\} ds \\ &+ 2 \sum_{j,e \in S_{\mathrm{cav}}} E_j^e \iint_{T_i} \nabla_S \cdot (\mathbf{S}_i^e(\mathbf{r}) \times \hat{z}) \cdot \left\{ \iint_{T_j} \nabla_S' \cdot (\mathbf{S}_j^e(\mathbf{r}) \times \hat{z}) G_0(\mathbf{r}, \mathbf{r}')\, ds' \right\} ds \end{aligned} \tag{23}$$

where the presence of $T_i = T_i^+ + T_i^-$ denotes integration over the entire triangle pair. Thus, the complete form of the system (12) is

$$\sum_{e=1}^{N_e} \left\{ [A_{ij}^e]\{E_j^e\} + \{K_i^e\} \right\} + \sum_{i,j,e \in S_{cav}} \left\{ [B_{ij}^e]\{E_j^e\} + \{L_i^e\} \right\} = 0 \tag{24}$$

with

$$\begin{aligned} B_{ij}^e &= +2k_0^2 \iint_{T_i} \hat{z} \times \mathbf{S}_i^e(\mathbf{r}) \cdot \left\{ \iint_{T_j} (\mathbf{S}_j^e(\mathbf{r}') \times \hat{z}) G_0(\mathbf{r},\mathbf{r}')\, ds' \right\} ds \\ &\quad + 2 \iint_{T_i} \nabla_S \cdot (\mathbf{S}_i^e(\mathbf{r}) \times \hat{z}) \cdot \left\{ \iint_{T_j} \nabla'_S \cdot (\mathbf{S}_j(\mathbf{r}') \times \hat{z}) G_0(\mathbf{r},\mathbf{r}')\, ds' \right\} ds \end{aligned} \tag{25}$$

and

$$L_i^e = 2jk_0 Z_0 \iint_{T_i} \mathbf{S}_i^e \cdot (\mathbf{H}^i \times \hat{z})\, ds \tag{26}$$

are the excitation elements which are non-zero for scattering computation. Note that the matrix $[A_{ij}^e]$ is sparse and banded, but $[B_{ij}^e]$ is full. However, because it only accounts for the interaction among the surface elements, it is a relatively small matrix. It should not therefore appreciably impact the memory requirements to any degree.

The computation of the surface matrix elements B_{ij}^e must be done carefully because $G_0(\mathbf{r},\mathbf{r}')$ is singular when $\mathbf{r} = \mathbf{r}'$ and this occurs when $i = j$. To facilitate its evaluation we rewrite it as

$$B_{ij} = B_{ij}^{++} + B_{ij}^{+-} + B_{ij}^{-+} + B_{ij}^{--} \tag{27}$$

where

$$\begin{aligned} B_{ij}^{pq} &= -\frac{k_0^2 l_i l_j}{8\pi A_i^p A_j^q} \iint_{T_i^p} \iint_{T_i^q} \bar{\rho}_i^p(\mathbf{r}) \cdot \bar{\rho}_j^q(\mathbf{r}') \frac{e^{-jkR}}{R}\, ds'\, ds \\ &\quad + \frac{l_i l_j}{2\pi A_i^p A_j^q} \epsilon_{pq} \iint_{T_i^p} \iint_{T_i^q} \frac{e^{-jkR}}{R}\, ds'\, ds \end{aligned} \tag{28}$$

in which $R = |\mathbf{r} - \mathbf{r}'|$ and the superscripts p and q denote either $+$ or $-$. Also,

$$\epsilon_{pq} = \begin{cases} 1 & p = q \\ -1 & p \neq q \end{cases}$$

and

$$\bar{\rho}_i^{\pm}(\mathbf{r}) = \pm(\mathbf{r} - \mathbf{r}_i^{\pm}) \tag{29}$$

where $\mathbf{r}_i^{\pm}$ is the position vector of the $T_i^{\pm}$ triangle vertex opposite to the ith (shared) edge between the two triangles T_i^+ and T_i^-. To evaluate B_{ij}^{pq}, it is convenient to introduce local coordinate variables. The area coordinates have been found useful for this purpose. Referring to the triangle in Figure 4 we denote its vertices by $\mathbf{r}_n$ ($n = 1$, 2 or 3), its edge vectors by $\mathbf{l}_n$ and $\bar{\rho}_n = \mathbf{r} - \mathbf{r}_n$ coinciding with the definition (29) if this is a + triangle. The vectors $\bar{\rho}_n$ drawn from the triangle vertices to a position $\mathbf{r}$ within the triangle separate that triangle in three smaller ones whose area is denoted by A_n. It is then readily seen that $\bar{\rho}_n$ can be written as

$$\bar{\rho}_n = \frac{A_{n+1}}{A}\mathbf{l}_{n-1} - \frac{A_{n-1}}{A}\mathbf{l}_{n+1} = \xi_{n+1}\mathbf{l}_{n-1} - \xi_{n-1}\mathbf{l}_{n+1} \tag{30}$$

where $A = \sum_{n=1}^{3} A_n$ and $\xi_{n\pm1}$ are referred to as the area coordinates. By varying these from 0 to unity, we can generate all possible positions of ρ_n within the triangle's surface. In terms of the area coordinates we also find that the global position vector can be written as

$$\mathbf{r} = \xi_{n-1}\mathbf{r}_{n-1} + \xi_{n+1}\mathbf{r}_{n+1} + \xi_n\mathbf{r}_n \tag{31}$$

with $\sum_{n=1}^{3}\xi_n = 1$. Alternatively, since $\mathbf{r}_{n\pm1} - \mathbf{r}_{n\mp1} = \mp\mathbf{l}_n$ and $\mathbf{r}_{n\mp1} - \mathbf{r}_n = \mp\mathbf{l}_{n\pm1}$, we can express $\mathbf{r}$ as

$$\mathbf{r} = \mp(\xi_{n\pm1}\mathbf{l}_n - \xi_n\mathbf{l}_{n\pm1}) + \mathbf{r}_{n\mp1} \tag{32}$$

Also, we can show that the differential area in terms of these area coordinates is

$$\begin{aligned} ds = \left|\left(\frac{\partial\mathbf{r}}{\partial\xi_{n\pm1}} \times \frac{\partial\mathbf{r}}{\partial\xi_n}\right)\right| d\xi_{n\pm1}\, d\xi_n &= |\mathbf{l}_n \times \mathbf{l}_{n\pm1}|\, d\xi_{n\pm1}\, d\xi_n \\ &= 2A\, d\xi_{n\pm1}\, d\xi_n \end{aligned} \tag{33}$$

The integrals in (28) can now be readily rewritten in terms of the new coordinates ξ_n. Before doing so, though, we first simplify it by employing midpoint integration for the T_i^{pq} integrals. This gives

$$\begin{aligned} B_{ij}^{pq} &= -k_0^2 \frac{l_i l_j}{8\pi A_j^q}\bar{\rho}_i^p(\mathbf{r}_c) \cdot \iint_{T_j^q} \bar{\rho}_j^q(\mathbf{r}')\frac{e^{-jkR_c}}{R_c}\, ds' \\ &\quad + \frac{l_i l_j}{2\pi A_j^q}\epsilon_{pq} \iint_{T_j^q} \frac{e^{-jkR_c}}{R_c}\, ds' \end{aligned} \tag{34}$$

in which $R_c = |\mathbf{r}_c - \mathbf{r}'|$ and $\mathbf{r}_c$ is the position vector whose tip is at the centroid of the T_i^p triangles. The integrals in (34) can now be evaluated numerically and analytically as described in (6) and (7).

Code Implementation and Validation

Based on the presented formulation, a computer code was written which is listed in Appendix C. The code relies on a preprocessor to supply all required information pertaining to the geometry, mesh discretization and material parameters of the cavity-backed antenna. In particular the following tables of data must be supplied to the code (see Appendix B):

<u>Table 1</u>

Volume element No., Global edge No.,Pair of global node numbers forming the edge, Element dielectric constants(six line entries are required per volume element)

<u>Table 2</u>

Global edge No., Unnormalized vector coordinates joining the nodes forming the edge.

Table 3
Volume element No., Global node Nos forming the element, element's dielectric constant.
Table 4
Global node No.,(x, y, z) coordinates of the node.
Table 5
List of edge Nos on the perfectly conducting walls/cavity surface.
Table 6
Volume elements bordering open surface (global Nos.), Nodes of triangle coinciding with surface, corresponding edge Nos.
Table 7
List of edge Nos. at the boundary line joining the PEC and open/dielectric surface of the cavity.

As listed above, some redundancy exists among the information provided in the Tables. This is only done to simplify the processor and in the future we shall consider a more concise input data list. Regardless of this, it should be clear that the user of the code must generate these tables on his/her own from the universal tables outputted by the employed commercial mesh generation package.

From the supplied generated data the analysis portion of the code (processor) which we have written computes the elements A_{ij}, K_i, B_{ij} and L_i. As noted earlier, the $[A]$ matrix is large but because it is very sparse and banded, its storage and fill time is very small. This is actually the main advantage of the methodology along with the geometrical adaptability of the tetrahedral elements. The boundary matrix $[B]$ is unfortunately full and its computation is cumbersome, consuming a large portion of the code. For an efficient solution of the resulting system, it is necessary to force this matrix to be Toeplitz requiring that all surface triangles be identical. However, such requirement cannot be imposed (externally) on most commercial mesh generation packages. A simple approach is to allow the preprocessor to generate only those volume elements up to one cell below S_{cav}. The last layer of volume elements, bordering S_{cav} is then appended externally.

At the moment, the solution of the system is done without any provisions to force the $[B]$ matrix to be Toeplitz in form. This was done as a first step in the development of the final code since our initial and foremost goal was to test the validity of the code. The scattering patterns given in figures 5 and 6 were generated with this version of the code. The comparison with other numerical data clearly demonstrates the validity of the formulation.

In the next few months we shall concentrate on the development of a more efficient code as described above. More importantly, we shall consider the modeling of practical antenna geometries and arrays. The new code will also allow input impedance computations and modeling of feeds, lumped loads and distributed/resistive loads.

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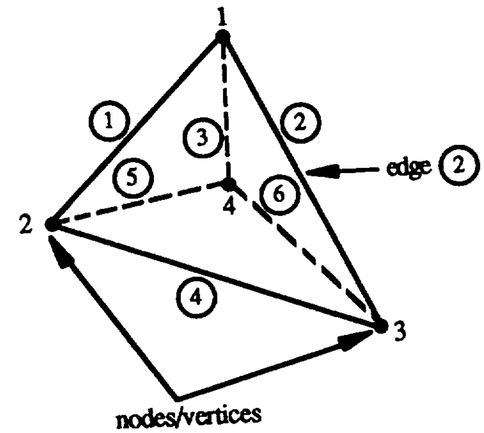
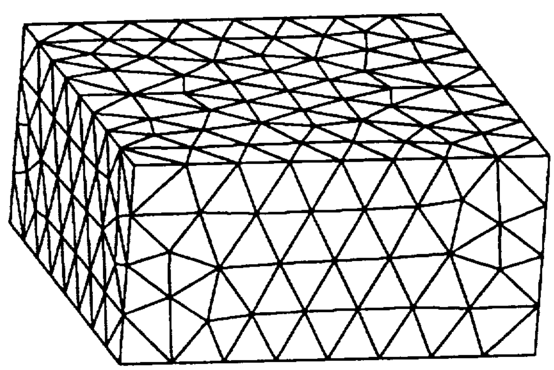
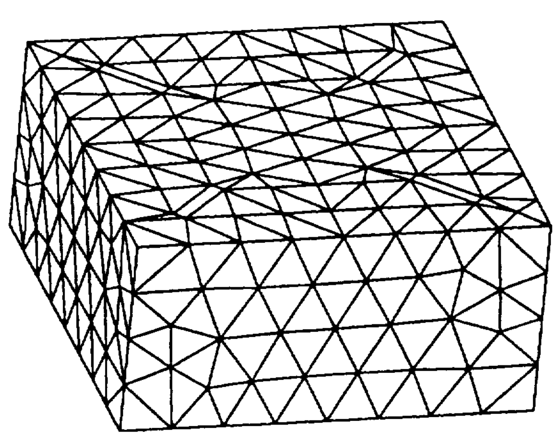


(a).



(b).



(c).

Figure 1. (a).Tetrahedron Geometry (b). Mesh with Circular Patch at aperture (c). Mesh with four slots at aperture.

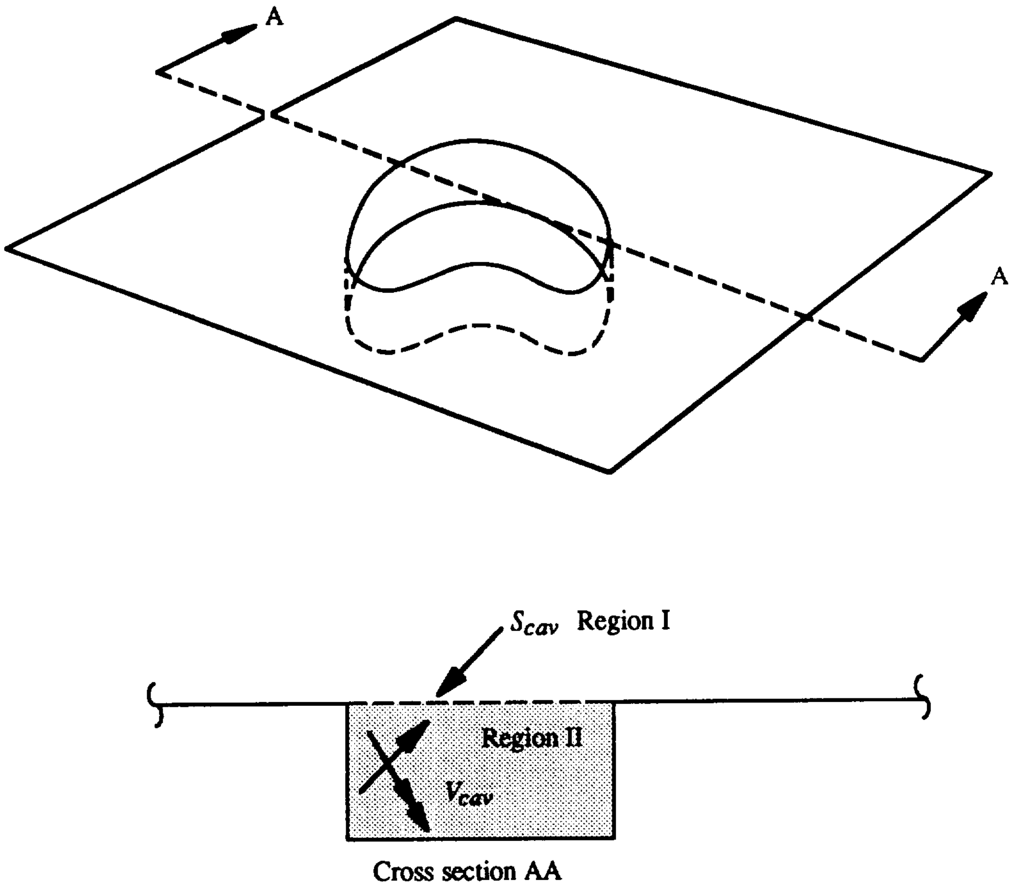


Figure 2. Cavity Geometry Recessed in a Ground Plane

TABLE 1

Edge Number	Vertex Number i_1	Vertex Number i_2
1	1	2
2	1	3
3	1	4
4	2	3
5	4	2
6	3	4

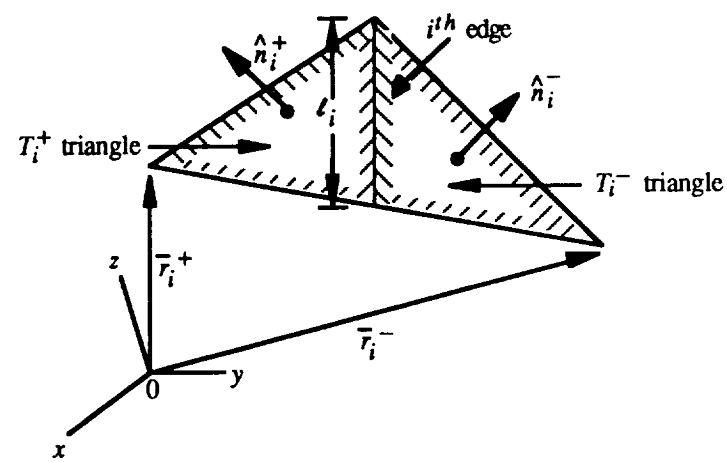


Figure 3. Pair of triangles sharing the i^{th} edge (i^{th} pair)

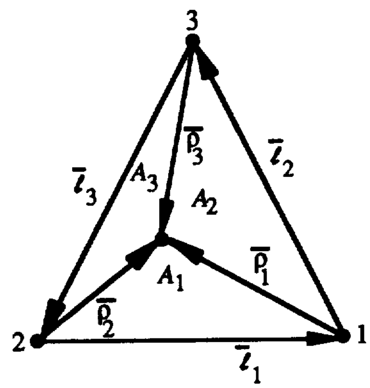


Figure 4. Illustration of the local vectors for a triangle on S_{cav}

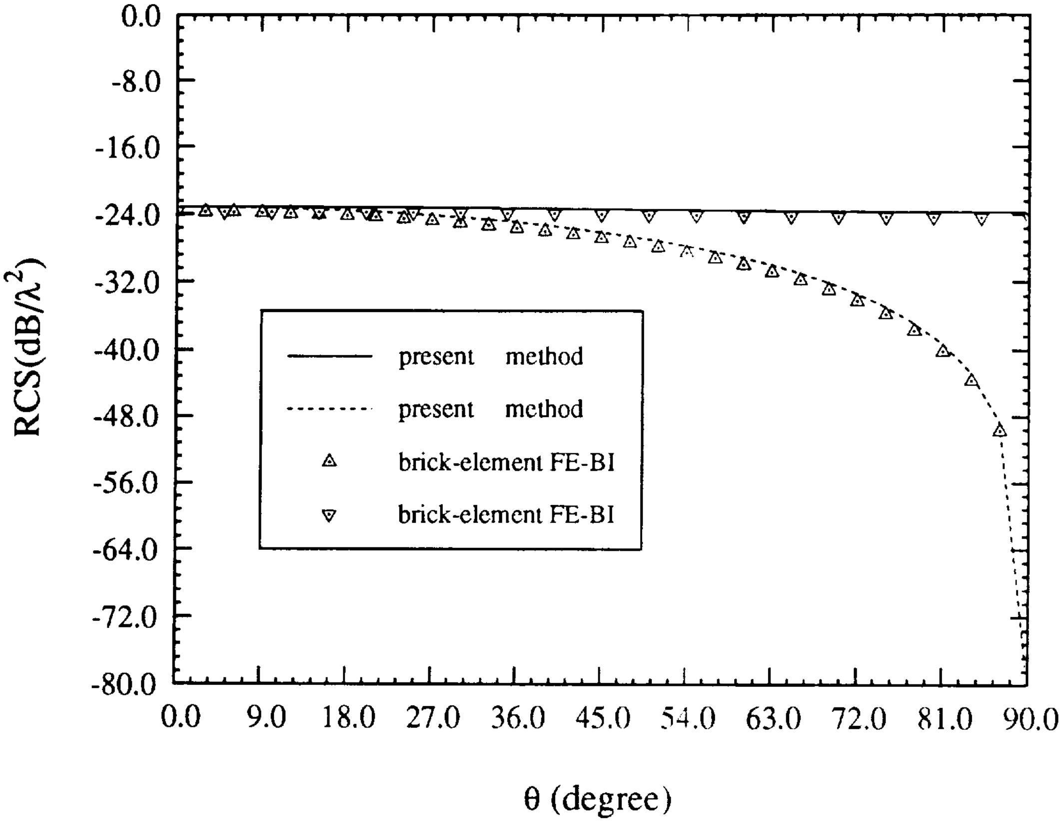


Figure 5. Comparison of bistatic RCS of the fields scattered by $0.2\lambda \times 0.2\lambda \times 0.1\lambda$ rectangular cavity of empty filling, with the incident field of $E_z{}^i$-pol and $\theta^i = 45°$.

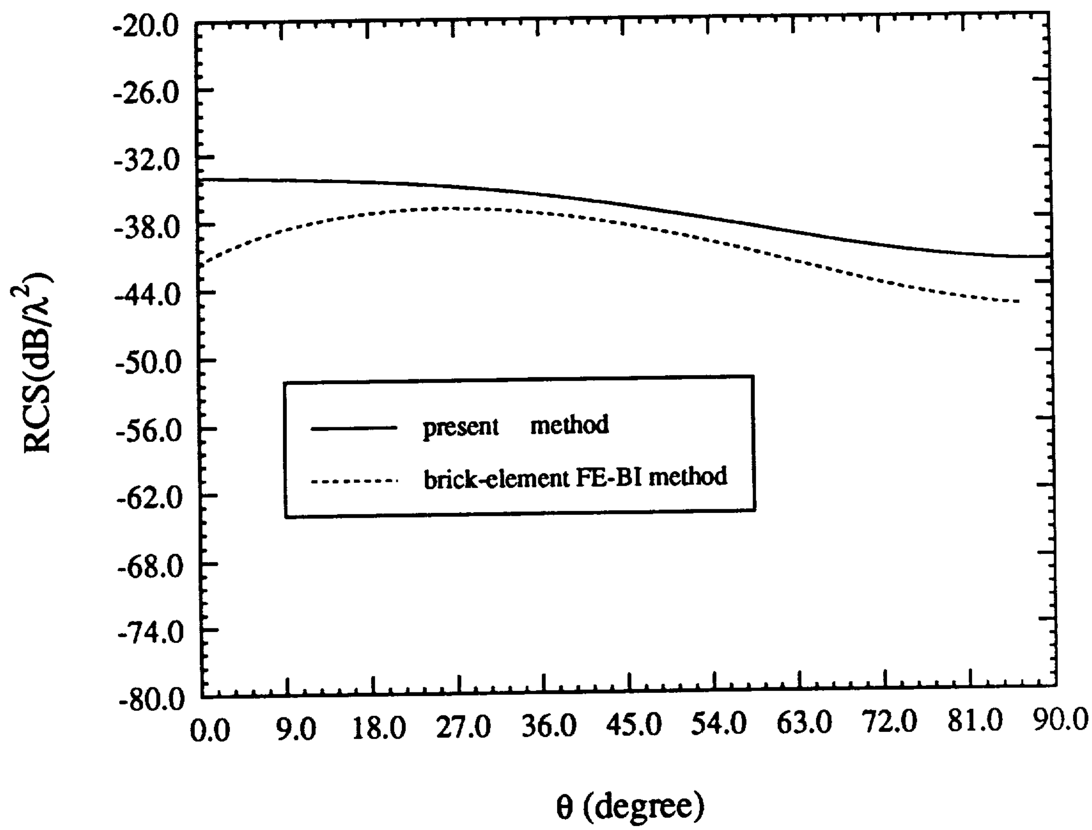


Figure 6. Demonstration of the scattering by two different patch geometries at the aperture center of $0.8\lambda \times 0.8\lambda \times 0.02\lambda$ cavity with the same incidence as described in figure 5. The dashed curve represents the bistatic RCS pattern by a rectangular patch of an area $0.32\lambda^2$, calculated from the brick-element FE-BI code [3]. The upper curve denotes for demonstration the scattering by a circular patch of an area $0.28\lambda^2$.

Appendix A

Computation of Matrix Elements A_{ij}

Appendix A

The derivation of the $[A]$ matrix elements in (12) amounts to evaluating the integral in (13). We have

$$\int_{V_e} \frac{1}{\mu_r}(\nabla \times \mathbf{V}_i^e) \cdot (\nabla \times \mathbf{V}_j^e) = \frac{4}{\mu_r}\mathbf{g}_i.\mathbf{g}_j V_e \tag{1}$$

since from (9) $\nabla \times \mathbf{V}_i^e = 2\mathbf{g}_i$. Also, from (4) we have

$$\begin{aligned} \epsilon_r \int_{V_e} \mathbf{V}_i^e.\mathbf{V}_j^e dv &= \epsilon_r \int_{V_e} \{(\mathbf{f}_i.\mathbf{f}_j) + (\mathbf{r}.\mathbf{D}) + (\mathbf{g}_i \times \mathbf{r}).(\mathbf{g}_j \times \mathbf{r})\}\, dv \quad (2) \\ &= \epsilon_r (I_1 + I_2 + I_3) \end{aligned}$$

where

$$\mathbf{D} = (\mathbf{f}_i \times \mathbf{g}_j) + (\mathbf{f}_j \times \mathbf{g}_i)$$

and

$$I_1 = \int_{V_e} \mathbf{f}_i.\mathbf{f}_j dv \tag{3}$$

$$I_2 = \int_{V_e} \mathbf{r}.\mathbf{D} dv \tag{4}$$

$$I_3 = \int_{V_e} (\mathbf{g}_i \times \mathbf{r}).(\mathbf{g}_j \times \mathbf{r}) dv \tag{5}$$

Since $\mathbf{f}$ is a constant vector, I_1 reduces to

$$I_1 = \mathbf{f}_i.\mathbf{f}_j\, V_e \tag{6}$$

Since

$$x = \sum_{i=1}^{4} L_i x_i$$

$$y = \sum_{i=1}^{4} L_i y_i$$

$$z = \sum_{i=1}^{4} L_i z_i$$

where L_i are the shape functions for the tetrahedral element and $x_i, y_i, z_i (i = 1, \cdots, 4)$ denote the x, y and z co-ordinates of the vertices of the tetrahedral element. Using the standard formula for volume integration within a tetrahedral element and simplifying, we have

$$I_2 = \frac{V_e}{4}\left[D_x \sum_{i=1}^{4} x_i + D_y \sum_{i=1}^{4} y_i + D_z \sum_{i=1}^{4} z_i\right] \tag{7}$$

where D_m is the mth component of $\mathbf{D}$. The evaluation of I_3 can be simplified by the use of basic vector identities. Therefore,

$$\begin{aligned}
I_3 &= \mathbf{g}_i.\mathbf{g}_j \int_{V_e} |\mathbf{r}|^2\, dv - \int_{V_e} (\mathbf{g}_i.\mathbf{r})(\mathbf{g}_j.\mathbf{r})\, dv \\
&= (g_{iy}g_{jy} + g_{iz}g_{jz}) \int_{V_e} x^2 dv + (g_{ix}g_{jx} + g_{iz}g_{jz}) \int_{V_e} y^2 dv + (g_{ix}g_{jx} + g_{iy}g_{jy}) \int_{V_e} z^2 dv \\
&\quad - (g_{ix}g_{jy} + g_{jx}g_{iy}) \int_{V_e} xy dv - (g_{ix}g_{jz} + g_{jx}g_{iz}) \int_{V_e} zx dv - (g_{iy}g_{jz} + g_{jy}g_{iz}) \int_{V_e} yz dv
\end{aligned} \tag{8}$$

where g_{im} represents the mth component of the vector $\mathbf{g}_i$. Each of the integrals above can be easily evaluated analytically and the result expressed in the following general form:

$$\int_{V_e} a_l a_m dv = \frac{V_e}{20}\left[\sum_{i=1}^{4} a_{li}a_{mi} + \sum_{i=1}^{4} a_{li} \sum_{i=1}^{4} a_{mi}\right] \tag{9}$$

where $l, m = 1, \cdots, 3$ and a_1 represents the variable x, a_2 stands for the variable y and a_3 denotes the variable z.

Appendix B

Example Input Data Files/Tables

17 53 10 17 1
18 11 9 11 1
18 33 9 10 1
18 12 4 9 1
18 52 10 11 1
18 15 4 11 1
18 51 4 10 1
19 24 11 18 1
19 32 10 18 1
19 6 9 18 1
19 52 10 11 1
19 11 9 11 1
19 33 9 10 1
20 38 11 17 1
20 53 10 17 1
20 50 17 18 1
20 52 10 11 1
20 24 11 18 1
20 32 10 18 1
21 45 6 14 1
21 49 14 18 1
21 54 13 14 1
21 17 6 18 1
21 18 6 13 1
21 21 13 18 1
22 55 8 16 1
22 56 16 17 1
22 57 15 16 1
22 58 8 17 1
22 30 8 15 1
22 59 15 17 1
23 60 3 20 1
23 7 3 15 1
23 61 3 16 1
23 62 15 20 1
23 63 16 20 1
23 57 15 16 1
24 63 16 20 1
24 64 10 16 1
24 61 3 16 1
24 65 10 20 1
24 60 3 20 1
24 31 3 10 1
25 65 10 20 1
25 64 10 16 1
25 53 10 17 1
25 63 16 20 1
25 66 17 20 1
25 56 16 17 1
26 60 3 20 1
26 2 3 18 1
26 7 3 15 1
26 67 18 20 1
26 62 15 20 1
26 9 15 18 1
27 60 3 20 1
27 31 3 10 1
27 2 3 18 1
27 65 10 20 1

```
27 67 18 20 1
27 32 10 18 1
28 65 10 20 1
28 53 10 17 1
28 32 10 18 1
28 66 17 20 1
28 67 18 20 1
28 50 17 18 1
29 62 15 20 1
29 9 15 18 1
29 59 15 17 1
29 [illegible] 18 20 1
2[illegible] [illegible]7 20 1
29 [illegible]7 18 1
30 [illegible] 16 20 1
30 57 15 16 1
30 56 16 17 1
30 62 15 20 1
30 66 17 20 1
30 59 15 17 1
31 30 8 15 1
31 68 8 14 1
31 58 8 17 1
31 69 14 15 1
31 59 15 17 1
31 46 14 17 1
32 26 13 15 1
32 54 13 14 1
32 27 8 13 1
32 69 14 15 1
32 30 8 15 1
32 68 8 14 1
33 9 15 18 1
33 49 14 18 1
33 21 13 18 1
33 69 14 15 1
33 26 13 15 1
33 54 13 14 1
34 59 15 17 1
34 46 14 17 1
34 50 17 18 1
34 69 14 15 1
34 9 15 18 1
34 49 14 18 1
```

*** TABLE 2 ***

a:edge #
b:(x2-x1)
c:(y2-y1)
d:(z2-z1)

a	b	c	d
1	0.0000000	0.0000000	-0.1000000
2	0.1000000	0.1000000	0.1000000
3	0.0000000	0.1000000	0.1000000

ORIGINAL PAGE IS
OF POOR QUALITY

63 -3.3333339E-02 5.0000001E-02 3.3333331E-02
64 0.1000000 -0.1000000 0.0000000
65 6.6666663E-02 -5.0000001E-02 3.3333331E-02
66 -3.3333339E-02 -5.0000001E-02 3.3333331E-02
67 -3.3333339E-02 -5.0000001E-02 -6.6666670E-02
68 0.0000000 0.1000000 0.0000000
69 -0.1000000 -0.1000000 0.1000000

*** TABLE 3 ***

a:element #
b:node of the element
c:node of the element
d:node of the element
e:node of the element
f:dielectric constant of the element

a	b	c	d	e	f
1	3	1	18	9	1
2	3	1	15	18	1
3	9	2	11	4	1
4	6	5	18	13	1
5	6	5	11	18	1
6	13	7	15	8	1
7	10	3	18	9	1
8	12	4	17	11	1
9	6	19	11	12	1
10	12	19	14	6	1
11	14	19	12	17	1
12	6	19	18	11	1
13	6	19	14	18	1
14	18	19	14	17	1
15	11	19	18	17	1
16	12	19	11	17	1
17	4	11	10	17	1
18	9	11	10	4	1
19	18	11	10	9	1
20	17	11	10	18	1
21	14	6	18	13	1
22	16	8	17	15	1
23	3	20	15	16	1
24	16	20	10	3	1
25	10	20	16	17	1
26	3	20	18	15	1
27	3	20	10	18	1
28	10	20	17	18	1
29	15	20	18	17	1
30	16	20	15	17	1
31	8	15	14	17	1
32	13	15	14	8	1
33	18	15	14	13	1
34	17	15	14	18	1

***TABLE 4 ***

4 0.1000000 0.1000000 0.0000000
5 0.0000000 0.1000000 0.0000000
6 0.1000000 0.0000000 0.0000000
7 0.1000000 0.0000000 0.1000000
8 0.1000000 0.0000000 0.0000000
9 0.0000000 0.1000000 0.0000000
10 0.0000000 -0.1000000 0.0000000
11 0.1000000 0.1000000 0.0000000
12 0.0000000 -0.1000000 0.1000000
13 0.1000000 0.0000000 0.0000000
14 0.0000000 0.0000000 -0.1000000
15 0.1000000 0.0000000 0.1000000
16 0.0000000 0.0000000 -0.1000000
17 -0.1000000 -0.1000000 0.1000000
18 0.0000000 -0.1000000 0.1000000
19 -0.1000000 -0.1000000 0.0000000
20 0.0000000 -0.1000000 0.0000000
21 -0.1000000 0.0000000 0.0000000
22 -0.1000000 0.0000000 0.1000000
23 -0.1000000 0.0000000 0.0000000
24 0.0000000 -0.1000000 0.0000000
25 0.0000000 0.1000000 0.0000000
26 -0.1000000 -0.1000000 0.0000000
27 0.0000000 0.1000000 0.1000000
28 -0.1000000 0.0000000 0.0000000
29 0.0000000 0.0000000 -0.1000000
30 -0.1000000 0.0000000 0.1000000
31 0.0000000 0.1000000 0.0000000
32 0.1000000 0.0000000 0.1000000
33 0.0000000 0.0000000 -0.1000000
34 0.1000000 0.0000000 0.0000000
35 0.0000000 -0.1000000 0.0000000
36 0.0000000 0.0000000 -0.1000000
37 0.1000000 -0.1000000 0.0000000
38 0.0000000 -0.1000000 -0.1000000
39 -6.6666663E-02 -5.0000001E-02 3.3333331E-02
40 -0.1000000 0.0000000 0.0000000
41 3.3333339E-02 -5.0000001E-02 -6.6666670E-02
42 3.3333339E-02 -5.0000001E-02 3.3333331E-02
43 0.1000000 -0.1000000 0.0000000
44 -6.6666663E-02 5.0000001E-02 3.3333331E-02
45 0.0000000 -0.1000000 0.0000000
46 -0.1000000 0.0000000 0.0000000
47 3.3333339E-02 5.0000001E-02 3.3333331E-02
48 3.3333339E-02 5.0000001E-02 -6.6666670E-02
49 -0.1000000 0.0000000 0.1000000
50 0.0000000 0.0000000 0.1000000
51 0.0000000 -0.1000000 0.0000000
52 0.1000000 0.1000000 0.1000000
53 0.1000000 0.0000000 0.0000000
54 0.0000000 0.0000000 -0.1000000
55 -0.1000000 0.0000000 0.0000000
56 0.0000000 0.1000000 0.0000000
57 0.0000000 0.0000000 -0.1000000
58 -0.1000000 0.1000000 0.0000000
59 0.0000000 0.1000000 -0.1000000
60 6.6666663E-02 5.0000001E-02 3.3333331E-02
61 0.1000000 0.0000000 0.0000000
62 -3.3333339E-02 5.0000001E-02 -6.6666670E-02

:node #
:x coordinate of the node
:y coordinate of the node
:z coordinate of the node

a	x	y	z
1	-1.000000E-01	-1.000000E-01	0.000000E+00
2	-1.000000E-01	1.000000E-01	0.000000E+00
3	-1.000000E-01	-1.000000E-01	-1.000000E-01
4	-1.000000E-01	1.000000E-01	-1.000000E-01
5	1.000000E-01	1.000000E-01	0.000000E+00
6	1.000000E-01	1.000000E-01	-1.000000E-01
7	1.000000E-01	-1.000000E-01	0.000000E+00
8	1.000000E-01	-1.000000E-01	-1.000000E-01
9	-1.000000E-01	0.000000E+00	0.000000E+00
10	-1.000000E-01	0.000000E+00	-1.000000E-01
11	0.000000E+00	1.000000E-01	0.000000E+00
12	0.000000E+00	1.000000E-01	-1.000000E-01
13	1.000000E-01	0.000000E+00	0.000000E+00
14	1.000000E-01	0.000000E+00	-1.000000E-01
15	0.000000E+00	-1.000000E-01	0.000000E+00
16	0.000000E+00	-1.000000E-01	-1.000000E-01
17	0.000000E+00	0.000000E+00	-1.000000E-01
18	0.000000E+00	0.000000E+00	0.000000E+00
19	3.333334E-02	5.000000E-02	-6.666667E-02
20	-3.333334E-02	-5.000000E-02	-6.666667E-02

** TABLE 5 ***

on-pec edge #

1
5
14
10
3
31
12
51
33
13
15
34
16
23
22
40
36
20
18
45
29
25
27
68
54
8

7
61
28
30
55
57
37
58
64
53
43
35
46
56

*** TABLE 6 ***

a:element #
b1,b2,b3:edge # of the element
c1,c2:node # of the edge b1
d1,d2:node # of the edge b2
e1,e2:node # of the edge b3
(in c1,c2,d1,d2,e1 and e2, there are just three
different numbers corresponding to three node #).

a	b1	c1	c2	b2	d1	d2	b3	e1	e2
3	10	2	9	13	2	11	11	9	11
6	25	7	13	28	7	15	26	13	15
1	5	1	9	4	1	18	6	9	18
2	8	1	15	4	1	18	9	15	18
4	20	5	13	19	5	18	21	13	18
5	23	5	11	19	5	18	24	11	18
19	11	9	11	6	9	18	24	11	18
33	26	13	15	21	13	18	9	15	18

*** TABLE 7 ***

node numbre on outer boundary of die-electric surface
1
2
5
7
9
11
13
15

Appendix C

Computer Code Listing

```fortran
C===============================================================================
C   PROCESSE A UNIVERSAL FILE OBTAINED FROM IDEAS AND CONVERTS THE NODAL INFO
C   TO EDGE INFO NEEDED FOR CONSTRUCTING AN EDGE-BASED THREE-DIMENSIONAL FINITE
C   ELEMENT MESH USING TETRAHEDRAL ELEMENTS.
C(1)STORES THE NODE NUMBERS AND RESPECTIVE NODAL COORDINATES IN 'NODDY'
C(2)STORES THE ELEMENT NUMBERS AND CORRESPONDING NODES IN 'ELNO'
C(3)PROCESSES 'NODDY' AND 'ELNO' AND STORES THE EDGE NOS. AND NODAL CONNECTIONS
C   IN 'GLOB' AND EDGE NOS. WITH CORRESPONDING EDGE VECTOR IN 'EDGY'
C   NOTE: EDGE OVERLAPS ARE TAKEN CARE OF.
C   STORAGE LIMIT: 800 NODES, 3000 ELEMENTS, 4300 EDGES
C===============================================================================
        PROGRAM UNV_FILE_PROCESSOR
        CHARACTER STRING*80, YASTRN*40, UNV*20
        INTEGER N1(3000,4),TAB(4300,2),NN(100),TR(3000,10),NUN(3000)
        INTEGER BC(3000,3),EDGV(18000,3),MAT(3000),ZE(10)
        INTEGER NC(10), PEDGE(1000)
        REAL X(800),Y(800),Z(800)
        COMMON /BANK/N1,X,Y,Z,NE,MAT
        COMMON /DBASE/EDGV
        COMMON /LOCAL/NCOUNT
        COMMON /PECEDGE/PEDGE,NSURF
        DO I=1,8
        ZE(I)=0
        ENDDO
        OPEN(8,FILE='ESURFC')
        OPEN(9,FILE='ESURFD')
        WRITE(6,*)'NAME OF UNIVERSAL FILE ?'
        READ(5,'(A)')UNV
        WRITE(6,*)'INPUT # OF UNCONNECTED PEC SECTIONS(<10)'
        READ(5,*)NPC
        OPEN(1,FILE=UNV)
        OPEN(2,FILE='ENODDY')
        OPEN(3,FILE='ELNO')
        OPEN(4,FILE='EGLOB')
        OPEN(7,FILE='EDGY')
        OPEN(10,FILE='OUTB')

C....PROCESSING UNIVERSAL FILE FOR INFO ON VARIOUS PARAMETERS
        IT=1
15      READ(1,'(A)')STRING
        L=0
        IF (STRING(4:6).EQ.'151') THEN
          WRITE(6,*)'ENCOUNTERED HEADER'
          GO TO 15
        ELSEIF (STRING(4:6).EQ.'747') THEN
2         READ(1,'(A)')STRING
          IF (STRING(5:6).NE.'-1') THEN
            IF (STRING(9:9).NE.' ') THEN
              WRITE(4,*)STRING(2:13)
              DO IK=1,4
                READ(1,'(A)')STRING
              ENDDO
            ENDIF
            GO TO 2
          ENDIF
C....PROCESSING NODAL INFO; DATA STORED IN 'NODDY'
        ELSEIF (STRING(4:6).EQ.' 15') THEN
```

```
5          READ(1,'(A)')STRING
           IF (STRING(5:6).NE.'-1') THEN
             WRITE(2,*)STRING(7:10),' ',STRING(41:53),' ',STRING(54:66),' '
     1                  ,STRING(67:79)
             L=L+1
             GO TO 5
           ENDIF
           L1=L
           WRITE(6,*)'THERE ARE ',L,' NODES.'
           WRITE(4,*)L
C....PROCESSING ELEMENT INFO; DATA STORED IN 'ELNO'
         ELSEIF (STRING(4:6).EQ.' 71') THEN
10         READ(1,'(A)')STRING
           IF (STRING(5:6).NE.'-1') THEN
             READ(1,'(A)')YASTRN
             WRITE(3,*)STRING(7:10),' ',YASTRN(7:10),' ',YASTRN(17:20),
     1       ' ',YASTRN(27:30),' ',YASTRN(37:40),'   ',STRING(49:50)
             L=L+1
             GO TO 10
           ELSE
             WRITE(6,*)'THERE ARE ',L,' ELEMENTS.'
             WRITE(4,*)L
             L2=L
           ENDIF
         ELSEIF (STRING(4:6).EQ.'752') THEN
30         READ(1,'(A)')STRING
           IF (STRING(19:20).EQ.' 0') THEN
             READ(STRING(9:10),'(I2)')NG
             READ(STRING(57:60),'(I4)')NUMB
                 IF(NG.EQ.IT.AND.NG.LE.NPC)THEN
                 NC(IT)=NUMB
         WRITE(6,*)'THERE ARE',NC(IT),'NODES ON',IT,'TH PEC SURFACE'
                 IT=IT+1
                 L=0
                 ELSEIF(NG.EQ.(NPC+1))THEN
                 ND=NUMB
             WRITE(6,*)'THERE ARE ',ND,' NODES ON DIE SURFACE'
                 L=0
                 ELSEIF(NG.EQ.(NPC+2))THEN
                 NB=NUMB
             PRINT*,'THERE ARE ',NB,' NODES AT DIE SURFACE OUTERS'
                 L=0
                 ENDIF
           ELSEIF (STRING(5:6).EQ.'-1') THEN
             GO TO 20
           ELSEIF (STRING(1:1).EQ.' ') THEN
             DO I=1,4
               IF (STRING((20*I-3):20*I).NE.'    ') THEN
                 L=L+1
                 READ(STRING((20*I-3):20*I),'(I4)')TR(L,NG)
               ENDIF
             ENDDO
           ENDIF
           GO TO 30
         ENDIF
         GO TO 15
20       CLOSE(1)

C-------
         DO LL=1,NB
         WRITE(10,*)TR(LL,NPC+2)
         ENDDO

C....DATA FROM 'NODDY' AND 'ELNO' STORED IN
C    N1    - TABLE OF NODES MAKING UP THE CORRESPONDING ELEMENT
C    X,Y,Z - NODAL COORDINATE TABLE
```

```
      REWIND 2
      REWIND 3
      DO I=1,L1
        READ(2,*)NA,X(I),Y(I),Z(I)
      ENDDO
      DO I=1,L2
        READ(3,*)NK,N1(I,1),N1(I,2),N1(I,3),N1(I,4),MAT(I)
      ENDDO
C....CLOSE 'NODDY' AND 'ELNO' FOR GOOD
      WRITE(2,*)ZE
      WRITE(3,*)ZE
      CLOSE(2)
      CLOSE(3)
      NCOUNT=0
      WRITE(6,*)'BE PATIENT #*!?/#@*!!!'
      DO NE=1,L2
C....STORE EDGE INFO IN AN INTEGER ARRAY 'TAB' AFTER CHECKING FOR
C    OVERLAP. THE SUBROUTINE 'COMPARE' IS THE HEART OF THE PROGRAM.
        CALL COMPARE(TAB)
C....COMMENTING OUT THE FOLLOWING STATEMENT CAUSES A SPEEDUP OF 250%
C       WRITE(6,*)'PROCESSED ELEMENT NO. ',NE,' :EDGE COUNT= ',NCOUNT
      ENDDO
      WRITE(6,*)'EDGE COUNT = ',NCOUNT
      REWIND(4)
C     WRITE(6,*)'NO. OF DIELECTRIC LAYERS?'
C     READ(5,*)ITEMP
C     READ(4,*)(XTMP,I=1,ITEMP)
      READ(4,*)XTMP
      READ(4,*)(NJ,I=1,2)
      DO I=1,(6*L2)
        READ(4,*)NK,EDGV(I,1),EDGV(I,2),EDGV(I,3),NJ
      ENDDO
      WRITE(4,*)ZE
      WRITE(7,*)ZE
      CLOSE(4)
      CLOSE(7)
      NSURF=0
C----
      DO 79 IT=1,NPC
      LSURF=0
C     WRITE(8,*)'ON THE',IT,'TH PEC SURFACE FOR ON-SURFACE EDGES'
       WRITE(6,*)'PROCESSING THE',IT,'TH PEC SURFACE.'
          DO J=1,NC(IT)
            DO K=1,NC(IT)
              DO L=1,NCOUNT
                IF (TAB(L,1).NE.0) THEN
       IF ((TR(J,IT).EQ.TAB(L,1)).AND.(TR(K,IT).EQ.TAB(L,2))) THEN
                    WRITE(8,*)L
                    TAB(L,1)=0
                    NSURF=NSURF+1
                    PEDGE(NSURF)=L
                    LSURF=LSURF+1
                    GO TO 40
                  ENDIF
                ENDIF
              ENDDO
40          ENDDO
          ENDDO
       PRINT*,'THERE ARE',LSURF,'ON-SURFACE EDGES ON',IT,'TH PEC SURFACE'
79     CONTINUE
       WRITE(8,*)'0'
         WRITE(6,*)'TOTAL NO. OF PEC ON-SURFACE EDGES= ',NSURF
         WRITE(6,*)'FEM MATRIX TO BE SOLVED IS OF ORDER ',(NCOUNT-NSURF)
C-----------
         WRITE(6,*)
         WRITE(6,*)'PROCESSING DIE SURFACE FOR ON-SURFACE EDGES.'
```

```
         NCNT=0
           DO IK=1,L2
             NUN(IK)=0
           ENDDO
           DO I=1,ND
             DO J=1,L2
               DO K=1,4
                 IF (TR(I,(NPC+1)).EQ.N1(J,K)) THEN
                   NUN(J)=NUN(J)+1
                   BC(J,NUN(J))=N1(J,K)
                   IF (NUN(J).EQ.3) THEN
                     CALL EDGE(J,BC(J,1),BC(J,2),BC(J,3),NFLAG)
                       IF(NFLAG.EQ.0)GOTO 80
                     NCNT=NCNT+1
                     GO TO 80
                   ENDIF
                 ENDIF
               ENDDO
80           ENDDO
           ENDDO
         WRITE(6,*)'THERE ARE ',NCNT,' DIE ON-SURFACE TRIANGLES.'
       CLOSE(8)
       WRITE(9,*)ZE
       WRITE(10,*)ZE
       CLOSE(9)
       CLOSE(10)
       STOP
       END
C=============================================================================
C   'COMPARE' CHECKS FOR ALL POSSIBLE EDGES AVOIDING OVERLAP AND STORES THE
C    INFO IN AN ARRAY 'TAB'. THE FILE 'EDGY' CONTAINS THE ELEMENT NO.
C    AND THE SIX EDGE VECTORS CORRESPONDING TO THAT ELEMENT.
C=============================================================================
       SUBROUTINE COMPARE(TAB)
       INTEGER N1(3000,4),MAT(3000)
       REAL X(800),Y(800),Z(800)
       INTEGER NT(6,2),TAB(4300,2)
       COMMON /BANK/N1,X,Y,Z,NE,MAT
       COMMON /LOCAL/NCOUNT
       NT(1,1)=N1(NE,1)
       NT(1,2)=N1(NE,2)
       NT(2,1)=N1(NE,1)
       NT(2,2)=N1(NE,3)
       NT(3,1)=N1(NE,1)
       NT(3,2)=N1(NE,4)
       NT(4,1)=N1(NE,2)
       NT(4,2)=N1(NE,3)
       NT(5,1)=N1(NE,2)
       NT(5,2)=N1(NE,4)
       NT(6,1)=N1(NE,3)
       NT(6,2)=N1(NE,4)
C....ARRANGING NODE COUPLETS IN ASCENDING ORDER; A PERSONAL CHOICE
       DO I=1,6
         IF (NT(I,1).GT.NT(I,2))THEN
           NTMP=NT(I,1)
           NT(I,1)=NT(I,2)
           NT(I,2)=NTMP
         ENDIF
       ENDDO
C....A BRUTE FORCE SEARCH FOR OVERLAPPING EDGES
       L=NCOUNT
       DO II=1,6
         IF (NE.EQ.1) GO TO 32
         DO I=1,NCOUNT
           IF ((TAB(I,1).EQ.NT(II,1)).AND.(TAB(I,2).EQ.NT(II,2))) THEN
             WRITE(4,*)NE,I,NT(II,1),NT(II,2),MAT(NE)
```

```
              GO TO 35
            ENDIF
          ENDDO
32        L=L+1
          TAB(L,1)=NT(II,1)
          TAB(L,2)=NT(II,2)
          WRITE(4,*)NE,L,NT(II,1),NT(II,2),MAT(NE)
          WRITE(7,*)L,X(NT(II,2))-X(NT(II,1)),Y(NT(II,2))-Y(NT(II,1)),
     1               Z(NT(II,2))-Z(NT(II,1))
35      ENDDO
        NCOUNT=L
        RETURN
        END
C===========================================================================
C   DETERMINES EDGES CORRESPONDING TO THE SURFACE ELEMENT
C===========================================================================
        SUBROUTINE EDGE(M,J1,J2,J3,NFLAG)
        INTEGER EDGV(18000,3),TMP(3),PEDGE(1000),NC(10)
        COMMON /DBASE/EDGV
        COMMON /PECEDGE/PEDGE,NSURF
        DO IJ=(6*M)-5,(6*M)
          IF (J1.EQ.EDGV(IJ,2)) THEN
            IF (J2.EQ.EDGV(IJ,3)) THEN
              TMP(1)=EDGV(IJ,1)
              L1=IJ
            ELSEIF (J3.EQ.EDGV(IJ,3)) THEN
              TMP(2)=EDGV(IJ,1)
              L2=IJ
            ENDIF
          ELSEIF (J2.EQ.EDGV(IJ,2)) THEN
            IF (J3.EQ.EDGV(IJ,3)) THEN
              TMP(3)=EDGV(IJ,1)
              L3=IJ
            ENDIF
          ENDIF
        ENDDO
C       CHECK OVERLAPPING WITH PEC EDGES
        ML=0
        DO LL=1,3
        DO K=1,NSURF
        IF(PEDGE(K).EQ.TMP(LL))THEN
        ML=ML+1
        END IF
        END DO
        END DO
        IF(ML.GT.3)THEN
        PRINT*,'PEDGE(K)',PEDGE(K)
        ENDIF
        IF (ML.EQ.3)THEN
        NFLAG=0
        ELSE
        NFLAG=1
        WRITE(9,*)M,TMP(1),EDGV(L1,2),EDGV(L1,3),TMP(2),EDGV(L2,2),
     1          EDGV(L2,3),TMP(3),EDGV(L3,2),EDGV(L3,3)
C       WRITE(9,*)M,J1,J2,J3,TMP
        ENDIF
101     CONTINUE
        RETURN
        END

C  ********************************************************************
C PROGRAM TO PRECESS DATA FROM PREP1.FTN. IT OUTPUT THE DATA FILES
C 1. PLTAE0.DAT (FOR BI);2. TABI (FOR ON-SURFACE NEW/OLD NUMBERING
C SYSTEM CONTRAST TABLES.) TAB1:FOR TRIANGLES; TAB2 FOR NODES;
C TAB3: FOR EDGES. (BE CARE OF THE MAXIMUM DIMENSIONS DEFINED
C IN THE "DIMENSION" PARAMETER STATEMENT, BEFORE RUNNING!
```

```
C  *********************************************************************
      PARAMETER (MAXA=1500,MAX=1000)
      INTEGER ITAB1(MAX), ITAB2(MAX), ITAB3(MAX), ITAB5(MAX,2)
      INTEGER IA(3), IB(6),IC(MAX), ID(3,2),IP,ITAB6(MAX,3)
      INTEGER ITAB6A(MAX,3),ITAB3A(MAX),ITAB3B(MAX)
      INTEGER ITAB5A(MAX,2),ITAB5B(MAX,2)
      REAL    XYZ(MAXA,3), TAB4(MAX,3)
      OPEN(2,FILE='enoddy')
 100  READ (2,*)I1,XYZ(I1,1),XYZ(I1,2),XYZ(I1,3)
      IF(I1.EQ.0)GO TO 200
      GO TO 100
 200  CLOSE(2)
      OPEN(3,FILE='esurfc')
      J1=1
 300  READ(3,*)IC(J1)
      IF(IC(J1).EQ.0)GO TO 400
      J1=J1+1
      GO TO 300
 400  CLOSE(3)
      LL=J1-1
      OPEN(4,FILE='esurfd')
      I=0
      J=1
      L1=1
      L2=1
 500  READ(4,*)IP,IA(1),IB(1),IB(2),IA(2),IB(3),IB(4),IA(3),IB(5),IB(6)
      IF(IP.EQ.0)GO TO 600
      K1=0
      DO 10 I1=1,3
      DO 5  J1=1,2
      ID(I1,J1)=IB(J1+K1)
  5   CONTINUE
      K1=K1+2
 10   CONTINUE
      I=I+1
      ITAB1(I)=IP
      DO 20 I1=1,3
      ITAB6A(I,I1)=IA(I1)
 20   CONTINUE
      DO 30 I1=1,6
      DO 25 J1=1,J
      IF(IB(I1).EQ.ITAB2(J1))GO TO 30
 25   CONTINUE
      ITAB2(J)=IB(I1)
      J=J+1
 30   CONTINUE
      DO 40 I1=1,3
      DO 32 J1=1,LL
      IF(IA(I1).EQ.IC(J1))THEN
         DO 31 K1=1,L2
         IF(IA(I1).EQ.ITAB3B(K1))GO TO 40
 31      CONTINUE
         ITAB3B(L2)=IA(I1)
         ITAB5B(L2,1)=ID(I1,1)
         ITAB5B(L2,2)=ID(I1,2)
         L2=L2+1
         GO TO 40
      ELSE
      END IF
 32   CONTINUE
         DO 35 K1=1,L1
         IF(IA(I1).EQ.ITAB3A(K1)) GO TO 40
 35      CONTINUE
         ITAB3A(L1)=IA(I1)
         ITAB5A(L1,1)=ID(I1,1)
         ITAB5A(L1,2)=ID(I1,2)
```

```
          L1=L1+1
 40     CONTINUE
        GO TO 500
600     CLOSE(4)
        II=I
        JJ=J-1
        INEDS=L1-1
        IEXEDS=L2-1
        KK=(L1-1)+(L2-1)
        DO 50 I1=1,JJ
        DO 50 J1=1,3
        M=ITAB2(I1)
        TAB4(I1,J1)=XYZ(M,J1)
50      CONTINUE
        DO 55 I1=1,L1-1
        ITAB3(I1)=ITAB3A(I1)
        DO 55 J1=1,2
        DO 51 K1=1,JJ
        M1=ITAB5A(I1,J1)
        IF(M1.EQ.ITAB2(K1))THEN
        ITAB5(I1,J1)=K1
        ELSE
        END IF
51      CONTINUE
55      CONTINUE
        M=L1-1
        DO 60 I1=1,L2-1
        ITAB3(M+I1)=ITAB3B(I1)
        DO 60 J1=1,2
        DO 59 K1=1,JJ
        M1=ITAB5B(I1,J1)
        IF(M1.EQ.ITAB2(K1))THEN
        ITAB5(M+I1,J1)=K1
        ELSE
        END IF
 59     CONTINUE
 60     CONTINUE
        DO 70 I1=1,II
        DO 65 J1=1,3
        M=ITAB6A(I1,J1)
        DO 63 K1=1,KK
        IF(M.EQ.ITAB3(K1))THEN
        ITAB6(I1,J1)=K1
        GO TO 65
        ELSE
        END IF
 63     CONTINUE
 65     CONTINUE
 70     CONTINUE
        OPEN(14,FILE='PLATE0.DAT')
        WRITE(14,*)'FLAG1'
        WRITE(14,*)'NODCRDS'
        DO I1=1,JJ
        WRITE(14,*)I1,(TAB4(I1,J1),J1=1,3)
        END DO
        WRITE(14,*)'-1, 0, 0, 0'
        WRITE(14,*)'FLAG2'
        WRITE(14,*)'NUMBER OF INTERIOR EDGES'
        WRITE(14,*)INEDS
        WRITE(14,*)'NUMBER OF EXTERIOR EDGES'
        WRITE(14,*)IEXEDS
        WRITE(14,*)'EDGNODS'
        DO I1=1,KK
        WRITE(14,*)I1,(ITAB5(I1,J1),J1=1,2)
        END DO
        WRITE(14,*)'FLAG3'
```

```
WRITE(14,*)'NUMBER OF TRIANGLES'
WRITE(14,*)II
WRITE(14,*)'TRIEDGS'
DO I1=1,II
WRITE(14,*)I1,(ITAB6(I1,J1),J1=1,3)
END DO
CLOSE(14)
OPEN(11,FILE='TAB1')
OPEN(21,FILE='TAB2')
OPEN(31,FILE='TAB3')
DO I1=1,II
WRITE(11,*)I1,ITAB1(I1)
END DO
DO I1=1,JJ
WRITE(21,*)I1,ITAB2(I1)
END DO
DO I1=1,KK
WRITE(31,*)I1,ITAB3(I1)
END DO
CLOSE(11)
CLOSE(21)
CLOSE(31)
END
```

```
From jxgum Thu Dec 26 14:34:33 1991
Received: by jaguar.engin.umich.edu (5.64/1.35)
        id 5600a3122.000bdb7; Thu, 26 Dec 91 14:30:09 -0500
Date: Thu, 26 Dec 91 14:30:09 -0500
From: Jian Gong <jxgum>
Message-Id: <5600a3122.000bdb7@jaguar.engin.umich.edu>
To: volakis
Status: R

      PROGR.. FEM_MOM
C*********************************************************************
C*      THE PROGRAM INCLUDES TWO SECTIONS: ONE IS FEM TO SOLVE THE    *
C* THE INTERIOR REGION; THE OTHER IS MOM (OR BI) TO DEAL WITH THE     *
C* THE EXTERIOR REGION. ALL DATA FILES NEEDED BY THE TWO METHODS HAVE *
C* BEEN CREATED BY "PREP1.FTN" AND "PREP2.FTN". USER ONLY NEED TO KEY *
C* IN SOME INFO REQUESTED BY THIS CODE, WHICH INDICATES WHERE THE INFO *
C* MAY BE OBTAINED.                                                   *
C*      THE OUTPUT MAY BE THE SURFACE FIELD (MAGNITUDE/PHASE) OR      *
C* SCATTERED/RADIATED FIELDS (IN TERMS OF RCS)                        *
C*      IN THE FIRST STAGE, THE PROGRAM FEM-MOM SOLVES THE PROBLEM OF *
C* SCATTERING FROM A CAVITY ON AN INFINITE GROUNG PLANE (PEC). THE    *
C* ARBITRARILY SHAPED TOP SURFACE MODELED WITH TRIANGULAR PATCHES.THE *
C* INTERIOR IS DIRCRETIZED INTO TETRAHEDRA ELEMENTS.                  *
C*********************************************************************

      INCLUDE 'CONST.INC'
      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3),EDGLEN(MAXEDG),R(MAXTRI),PI,VRCS,DEG,
     *     RCOND,FANGO,ANG1O,STEPANG,ALPHA,THETAI,PHII,THETAO,PHIO,
     *     AE(MAXA,MAXA)
      INTEGER NINTEDG,EDGNODS(MAXEDG,2),NTRIS,TRIEDGS(MAXTRI,3),CT
      INTEGER TRISIGN(MAXTRI,3),IPVT(MAXA),K,PTYPE,FIX,NUMPTS,STYPE
      INTEGER EDST(MAXA,2),EDGSGN(MAXZ),TAB3(MAXEDG)
      COMPLEX A(MAXA,MAXA), VC(MAXA),EN(MAXA),DUM(MAXA)
      COMPLEX     EZ(MAXZ,MAXZ),V(MAXZ),ES(MAXZ),PJ
      CHARACTER*40 MESH_FILE,RES_FILE,OUT_FILE

    2 FORMAT(' ',F15.10,'    ',F17.10)

      PI=3.14159265359
      PJ=(0.,1.)

      PRINT*,'INPUT TOTAL # OF EDGES'
      READ*,NED
      PRINT*,'INPUT TOTAL # OF PEC EDGES'
      READ*,NES
      PRINT*,'INPUT # OF ON-DIE-SURFACE NODES'
      READ*,NNOD

      CALL USER(MESH_FILE,STYPE,RES_FILE,OUT_FILE,PTYPE,ALPHA,
     *             THETAI,PHII,FIX,FANGO,ANG1O,NUMPTS,STEPANG)

      CALL MESHDATA(MESH_FILE,NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS)

      IF (STYPE.EQ.RESIST) THEN
      CALL GET_R(RES_FILE,NTRIS,R)
      ENDIF

      CALL ACREATOR(NED,NES,EDST,A)
      NET=NED-NES

      CALL ZCREATOR(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,STYPE,R,
     *           EDGLEN,TRISIGN,EZ)

      PRINT*,'FINISH Z CREATING'
```

```
      CALL COMB1(MESH_FILE,TRISIGN,NNOD,TAB3,EDGSGN)

      PRINT*,'FINISH COMB1'

      CALL COMB2(NINTEDG,EDST,TAB3,EDGSGN,EZ,A)
      DO I=1,NET
      DO J=1,NET
      IF(CABS(A(I,J)).NE.0.)THEN
      A(I,J)=CZERO*A(I,J)
      ENDIF
      ENDDO
      ENDDO

      PRINT*,'FINISH COMB2'

      CALL CGECO(A,MAXA,NET,IPVT,RCOND,DUM)

      OPEN(9,FILE='RCS.DAT')
      OPEN(8,FILE='RCS1.DAT')

      IF (PTYPE.EQ.BISTAT) THEN

      CALL EXCIT(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,EDGLEN,
     *           TRISIGN,ALPHA,THETAI,PHII,V)


      DO II=1,NINTEDG
      LL=EDST(TAB3(II),1)
      VC(LL)=CZERO*V(II)
      ENDDO

      CALL BACK_SUBST(A,VC,NET,IPVT,EN)

      WRITE(212,*)'ES'
      DO II=1,NINTEDG
      LL=EDST(TAB3(II),1)
      ES(II)=EN(LL)
      WRITE(212,*)ES(II)
      ENDDO

      WRITE(9,*) 'E FIELD ON SURFACE (ES)'
      WRITE(9,*) 'MAGNITUDE'
      DO 5 CT=1,NINTEDG
      ESR=REAL(ES(CT))
      ESI=(ES(CT)-CONJG(ES(CT)))/2./PJ
      ESMAG=SQRT(ESR**2+ESI**2)
      WRITE(9,*) CT,ESMAG
    5 CONTINUE
      WRITE(9,*)'PHASE'
      DO 8 CT=1,NINTEDG
      ESR=REAL(ES(CT))
      ESI=(ES(CT)-CONJG(ES(CT)))/2./PJ
      ESPH=ATAN(ESI/ESR)
      ESPH=180.*ESPH/PI
      WRITE(9,*)CT, ESPH
    8 CONTINUE
      WRITE(9,*) ' '
      END IF

      IF (FIX.EQ.ANGPHI) THEN
      PHIO=FANGO
      THETAO=ANG1O
      ELSE
      THETAO=FANGO
      PHIO=ANG1O
```

```
      END IF

      DO 10 K=1,NUMPTS
      IF (PTYPE.EQ.BACKSCAT) THEN
      THETAI=THETAO
      PHII=PHIO
      CALL EXCIT(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,EDGLEN,
     *            TRISIGN,ALPHA,THETAI,PHII,V)

      DO II=1,NINTEDG
      LL=EDST(TAB3(II),1)
      VC(LL)=CZERO*V(II)
      END DO

      CALL BACK_SUBST(A,VC,NET,IPVT,EN)
      END IF

      WRITE(212,*)'ES'
      DO II=1,NINTEDG
      LL=EDST(TAB3(II),1)
      ES(II)=EN(LL)
      WRITE(212,*)ES(II)
      ENDDO


      CALL  RCS(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,EDGLEN,
     *           TRISIGN,ES,THETAO,PHIO,RCSTH,RCSPHI)

      IF (FIX.EQ.ANGPHI) THEN
      DEG=THETAO*180./PI
      THETAO=THETAO+STEPANG
      ELSE
      DEG=PHIO*180./PI
      PHIO=PHIO+STEPANG
      END IF

      IF (K.EQ.1) THEN
      WRITE(9,*) 'INPUT DATA'
      WRITE(8,*) 'INPUT DATA'
      END IF
      WRITE(9,2) DEG,RCSTH
      WRITE(8,2) DEG,RCSPHI

   10 CONTINUE
      CLOSE(212)
      CLOSE(9)
      CLOSE(8)
      STOP
      END




      SUBROUTINE MESHDATA(MESH_FILE,NODCRDS,NINTEDG,EDGNODS,NTRIS,
     *                    TRIEDGS)
C***********************************************************************
C*   SR GET_MESH READS IN MESH DATA FROM FILE SPECIFIED BY             *
C*              USER.                                                  *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3)
```

```fortran
      INTEGER I,NODE,NINTEDG,NEXTEDG,NEDG,J,EDGNODS(MAXEDG,2),NTRIS,
     *        TRIEDGS(MAXTRI,3),K,DUM
      CHARACTER*120 LINE
      CHARACTER*40 MESH_FILE


      OPEN (UNIT=7,FILE=MESH_FILE)
  100 CONTINUE
      READ(7,'(A)',END=1000) LINE
      IF (LINE(2:6).EQ.'FLAG1') THEN
      READ(7,'(A)') LINE
      I=1
  200 READ(7,*) NODE,NODCRDS(I,1),NODCRDS(I,2),NODCRDS(I,3)
      IF (NODE.EQ.-1) THEN
      GOTO 300
      END IF
      I=I+1
      GOTO 200
  300 CONTINUE

      ELSE IF (LINE(2:6).EQ.'FLAG2') THEN
      READ(7,'(A)') LINE
      READ(7,*) NINTEDG
      READ(7,'(A)') LINE
      READ(7,*) NEXTEDG
      READ(7,'(A)') LINE
      NEDG=NINTEDG+NEXTEDG
      DO 400 J=1,NEDG
      READ(7,*) DUM,EDGNODS(J,1),EDGNODS(J,2)
  400 CONTINUE

      ELSE IF (LINE(2:6).EQ.'FLAG3') THEN
      READ(7,'(A)') LINE
      READ(7,*) NTRIS
      READ(7,'(A)') LINE
      DO 500 K=1,NTRIS
      READ(7,*) DUM,TRIEDGS(K,1),TRIEDGS(K,2),TRIEDGS(K,3)
  500 CONTINUE

      END IF
      GOTO 100
 1000 CONTINUE
      CLOSE(7)
      RETURN
      END


      SUBROUTINE ZCREATOR(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,
     *           STYPE,R,EDGLEN,TRISIGN,Z)
C***********************************************************************
C* SR FILLZ CALCULATES ELEMENTS OF IMPEDANCE MATRIX Z.                 *
C***********************************************************************

      INCLUDE 'CONST.INC'
      INCLUDE 'DIM.INC'
      REAL NODCRDS(MAXNOD,3),RC,R(MAXTRI),CTRD,LONGEST,EDGLEN(MAXEDG),
     *     OSIGN,SSIGN,PI,ETA,SAREA,SRCAREA
      INTEGER NEDG,NINTEDG,EDGNODS(MAXEDG,2),NTRIS,TRIEDGS(MAXTRI,3),
     *        TRISIGN(MAXTRI,3),EDGUSE(MAXEDG),P,Q,NUMTRIS,M(3),N(3),
     *        SINGFLAG,SINGPT,INM,OUTM,V,W,K,SRC,OBS,I1,I2,I3,I4,I5,
     *        STYPE,RESPT
      COMPLEX J,FSG,PFSG,A,PHI,Z(MAXZ,MAXZ),UNIFNC,LINFNC,PHISING,ASING,
     *        RESFNC,VINT,ZRES
      EXTERNAL FSG,PFSG,UNIFNC,LINFNC,RESFNC

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
```

```
      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN
      REAL NEAR
      COMMON /ADJ/NEAR

      DO 3 I1=1,MAXEDG
      EDGUSE(I1)=0
    3 CONTINUE

      DO 5 I2=1,MAXTRI
      DO 7 I3=1,3
      TRISIGN(I2,I3)=0
    7 CONTINUE
    5 CONTINUE
      DO 8 I4=1,NINTEDG
      DO 9 I5=1,NINTEDG
      Z(I4,I5)=(0.0,0.0)
    9 CONTINUE
    8 CONTINUE
      J=(0.0,1.0)
      PI=3.1415926536
      ETA=376.7343091821
      SRC=1
      OBS=0

      DO 10 P=1,NTRIS

      DO 25 I1=1,3
      M(I1)=TRIEDGS(P,I1)
   25 CONTINUE

      CALL GET_VERTS(M,OBS,NODCRDS,EDGNODS)

      DO 20 Q=1,NTRIS

      DO 30 I2=1,3
      N(I2)=TRIEDGS(Q,I2)
   30 CONTINUE


      CALL GET_VERTS(N,SRC,NODCRDS,EDGNODS)

      RC=CTRD()

      CALL EDGE_LEN(M,N,NODCRDS,EDGNODS,EDGUSE,EDGLEN,LONGEST)


      IF (P.EQ.Q) THEN
      SINGFLAG=1
      SINGPT=1
      NEAR=1.0
      INM=7
      OUTM=1
      RESPT=7
      ELSE IF (RC.LE.LONGEST) THEN
      SINGFLAG=1
      SINGPT=1
      NEAR=1.0
      INM=7
      OUTM=1

      ELSE
      SINGFLAG=0
      NEAR=0.0
      INM=7
```

```
      OUTM=1
      END IF

      CALL AC_INT(FSG,OUTM,INM,PHI)

      IF (SINGFLAG.EQ.1) THEN
      CALL SINT(SINGPT,UNIFNC,PHISING)
      SAREA=SRCAREA()
      PHISING=PHISING/SAREA
      PHI=PHI+PHISING
      END IF

      DO 40 V=1,3

      IF (M(V).LE.NINTEDG) THEN
      DO 50 W=1,3

      IF (N(W).LE.NINTEDG) THEN

      DO 60 K=1,3
      RM(K)=OBSV(V,K)
      RN(K)=SRCV(W,K)
   60 CONTINUE

      CALL AC_INT(PFSG,OUTM,INM,A)

      IF (SINGFLAG.EQ.1) THEN
      CALL SINT(SINGPT,LINFNC,ASING)
      ASING=ASING/SAREA
      A=A+ASING
      END IF


      IF ((P.EQ.Q).AND.(STYPE.EQ.RESIST)) THEN
      CALL SINT(RESPT,RESFNC,VINT)
      ZRES=ETA/4.*EDGLEN(M(V))*EDGLEN(N(W))*OSIGN*SSIGN*
     *      R(Q)*VINT/SAREA
      ELSE
      ZRES=(0.0,0.0)
      END IF

      Z(M(V),N(W))=Z(M(V),N(W))-0.5*PI*EDGLEN(M(V))
     *               *EDGLEN(N(W))*OSIGN*SSIGN*(A-PHI/(PI*PI))

      ELSE
      EDGUSE(N(W))=1
      END IF

   50 CONTINUE

      ELSE
      EDGUSE(M(V))=1
      END IF

   40 CONTINUE

   20 CONTINUE

   10 CONTINUE

      RETURN
      END


      SUBROUTINE GET_VERTS(EDG,FLAG,NODCRDS,EDGNODS)
C***********************************************************************
```

```
C*   SR GET_VERTS RETURNS THE VERTICES OF A TRIANGLE THRU              *
C*                COMMON BLOCK "TRICONST" GIVEN THE NUMBERS OF THE     *
C*                TRIANGLE EDGES.  THE CALLING ROUTINE SPECIFIES IF THE *
C*                TRIANGLE IS SOURCE OR OBSERVATION.  THE TRIANGLE     *
C*                EDGE NUMBERS ARE RECEIVED ARRAY EDG.  THE VERTICES   *
C*                (POSITION VECTORS) ARE THEN ASSIGNED SUCH THAT THE   *
C*                VERTEX IN OBSV/SRCV ARRAY POSITION (I,-) IS OPPOSITE *
C*                THE EDGE IN EDG ARRAY POSITION (I).                  *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3)
      INTEGER EDG(3),NODE(3),ORDNODE(3),EDGNODS(MAXEDG,2),
     *        I,J1,J2,K1,K2,SRC,OBS,FLAG


      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON TRICONST/OBSV,SRCV

      SRC=1
      OBS=0

      NODE(1)=EDGNODS(EDG(1),1)
      NODE(2)=EDGNODS(EDG(1),2)
      IF ( (EDGNODS(EDG(1),1).NE.EDGNODS(EDG(2),1)).AND.
     *     (EDGNODS(EDG(1),2).NE.EDGNODS(EDG(2),1)) ) THEN
      NODE(3)=EDGNODS(EDG(2),1)
      ORDNODE(1)=EDGNODS(EDG(2),1)
      ELSE
      NODE(3)=EDGNODS(EDG(2),2)
      ORDNODE(1)=EDGNODS(EDG(2),2)
      END IF

      DO 10 I=2,3
      IF ( (EDGNODS(EDG(I),1).NE.NODE(1)).AND.
     *   (EDGNODS(EDG(I),2).NE.NODE(1)) ) THEN
         ORDNODE(I)=NODE(1)
      ELSE IF ( (EDGNODS(EDG(I),1).NE.NODE(2)).AND.
     *        (EDGNODS(EDG(I),2).NE.NODE(2)) ) THEN
      ORDNODE(I)=NODE(2)
      ELSE
      ORDNODE(I)=NODE(3)
      END IF
   10 CONTINUE


      IF ( [illegible] OBS) THEN
      DO 2[illegible]
      DO 30 K1=1,3
      OBSV(J1,K1)=NODCRDS(ORDNODE(J1),K1)
   30 CONTINUE
   20 CONTINUE
      END IF

      IF (FLAG.EQ.SRC) THEN
      DO 50 J2=1,3
      DO 40 K2=1,3
      SRCV(J2,K2)=NODCRDS(ORDNODE(J2),K2)
   40 CONTINUE
   50 CONTINUE
      END IF

      RETURN
      END
```

```
      SUBROUTINE EDGE_LEN(M,N,NODCRDS,EDGNODS,EDGUSE,EDGLEN,LONGEST)
C*********************************************************************
C*   SR EDGE_LEN UPDATES THE EDGE LENGTH TABLE GIVEN THE            *
C*               EDGE NUMBERS OF THE SOURCE AND OBSERVATION TRIANGLES. *
C*               IT ALSO RETURNS THE LENGTH OF THE LONGEST EDGE OF THE *
C*               SOURCE AND OBSERVATION TRIANGLES.                   *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3),LONGEST,VECT1(3),VECT2(3),DIFF(3),VDOT,
     *     EDGLEN(MAXEDG)
      INTEGER EDGNODS(MAXEDG,2),EDGUSE(MAXEDG),M(3),N(3),I,J,K,P,
     *        NODE1,NODE2,S

      LONGEST=0.0

      DO 10 I=1,3

      IF (EDGUSE(M(I)).EQ.0) THEN
      NODE1=EDGNODS(M(I),1)
      NODE2=EDGNODS(M(I),2)
      DO 20 J=1,3
      VECT1(J)=NODCRDS(NODE1,J)
      VECT2(J)=NODCRDS(NODE2,J)
   20 CONTINUE
      CALL VECT_DIFF(1.0,VECT1,1.0,VECT2,DIFF)
      EDGLEN(M(I))=SQRT(VDOT(DIFF,DIFF))
      END IF
      IF (EDGLEN(M(I)).GT.LONGEST) THEN
      LONGEST=EDGLEN(M(I))
      END IF
   10 CONTINUE

      DO 30 K=1,3

      IF (EDGUSE(N(K)).EQ.0) THEN

      IF ((N(K).NE.M(1)).AND.(N(K).NE.M(2)).AND.(N(K).NE.M(3))) THEN
      NODE1=EDGNODS(N(K),1)
      NODE2=EDGNODS(N(K),2)
      DO 40 P=1,3
      VECT1(P)=NODCRDS(NODE1,P)
      VECT2(P)=NODCRDS(NODE2,P)
   40 CONTINUE
      CALL VECT_DIFF(1.0,VECT1,1.0,VECT2,DIFF)
      EDGLEN(N(K))=SQRT(VDOT(DIFF,DIFF))
      END IF
      END IF
      IF (EDGLEN(N(K)).GT.LONGEST) THEN
      LONGEST=EDGLEN(N(K))
      END IF
   30 CONTINUE
      RETURN
      END

      SUBROUTINE UPDATE_SIGNS(M,N,V,W,P,Q,TRISIGN,EDGUSE,OSIGN,SSIGN)
C*********************************************************************
C*  SR UPDATE_SIGNS UPDATES THE TRIANGLE SIGN AND EDGE              *
C*               USE TABLES FOR AN INPUT OBSERVATION INTERIOR EDGE AND *
C*               AN INPUT SOURCE INTERIOR EDGE.                      *
C*                                                                   *
C*               THE TRIANGLE SIGN TABLE IS AN ARRAY OF TRIANGLE     *
C*               NUMBERS VS. TRIANGLE EDGE CURRENT DIRECTIONS.  THE  *
```

```
C*               SIGN IN POSITION (K,L) CORRESPONDS TO EDGE N IN        *
C*               TRIANGLE EDGE TABLE (TRIEDGS) POSITION (K,L).  THE     *
C*               SIGN DESIGNATES THE CURRENT DIRECTION ACROSS THE NTH   *
C*               EDGE IN THE KTH TRIANGLE.                              *
C*               CASES:  -1-CURRENT DIRECTED ACROSS INTERIOR EDGE N     *
C*                          INTO TRIANGLE K                             *
C*                       +1-CURRENT DIRECTED ACROSS INTERIOR EDGE N     *
C*                          OUT OF TRIANGLE K                           *
C*                        0-EXTERIOR EDGE, NO CURRENT                   *
C*               SAY TRIANGLES A AND B SHARE EDGE N. ONE TRIANGLE MUST  *
C*               BE DESIGNATED AS + (SAY A) AND THE OTHER AS - (B).     *
C*               CURRENT IS DIRECTED ACROSS EDGE N FROM + -> -          *
C*               (A -> B).                                              *
C*                                                                      *
C*               CURRENT DIRECTIONS ARE ASSIGNED ON THE BASIS OF THE    *
C*               EDGE USE TABLE.  THE EDGE USE TABLE RECORDS THE        *
C*               NUMBER OF TIMES AN EDGE HAS BEEN USED BY DIFFERENT     *
C*               TRIANGLES.                                             *
C*               CASES:  0-EDGE NEVER USED                              *
C*                       1-EDGE USED BY SOME TRIANGLE I                 *
C*                       2-EDGE USED BY TRIANGLE I AND TRIANGLE J,      *
C*                         WHERE I DOES NOT EQUAL J                     *
C*                                                                      *
C*               THIS ROUTINE CHECKS THE EDGE USE TABLE FOR INPUT       *
C*               INTERIOR EDGE N, AND UPDATES THE TRIANGLE SIGN TABLE   *
C*               ACCORDING TO...                                        *
C*                 EDGE N NEVER USED BEFORE                     -> +1   *
C*                 EDGE N USED PREVIOUSLY BY DIFFERENT TRIANGLE -> -1   *
C*               THE ROUTINE THEN UPDATES THE EDGE USE TABLE, AND       *
C*               RETURNS THE TRIANGLE EDGE SIGNS.                       *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL OSIGN,SSIGN
      INTEGER TRISIGN(MAXTRI,3),EDGUSE(MAXEDG),M(3),N(3),V,W,P,Q

      IF (EDGUSE(M(V)).EQ.0) THEN
      EDGUSE(M(V))=1
      TRISIGN(P,V)=1
      OSIGN=1.0

      ELSE IF (TRISIGN(P,V).EQ.0) THEN
      EDGUSE(M(V))=2
      TRISIGN(P,V)=-1
      OSIGN=-1.0

      ELSE
      IF (TRISIGN(P,V).EQ.1) THEN
      OSIGN=1.0
      ELSE
      OSIGN=-1.0
      END IF
      END IF


      IF (EDGUSE(N(W)).EQ.0) THEN
      EDGUSE(N(W))=1
      TRISIGN(Q,W)=1
      SSIGN=1.0
      ELSE IF (TRISIGN(Q,W).EQ.0) THEN
      EDGUSE(N(W))=2
      TRISIGN(Q,W)=-1
      SSIGN=-1.0
      ELSE
      IF (TRISIGN(Q,W).EQ.1) THEN
```

```
      SSIGN=1.0
      ELSE
      SSIGN=-1.0
      END IF
      END IF

      RETURN
      END


      REAL FUNCTION CTRD()
C*********************************************************************
C* FNC CTRD COMPUTES THE DIFFERENCE BETWEEN OBSERVATION              *
C*              AND SOURCE TRIANGLE CENTROIDS.                       *
C*********************************************************************

      INCLUDE 'DIM.INC'
      REAL VECT1(3),VECT2(3),COEFF,DIFF(3)
      INTEGER I

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV

      DO 10 I=1,3
      VECT1(I)=OBSV(1,I)+OBSV(2,I)+OBSV(3,I)
      VECT2(I)=SRCV(1,I)+SRCV(2,I)+SRCV(3,I)
   10 CONTINUE
      COEFF=1./3.
      CALL VECT_DIFF(COEFF,VECT1,COEFF,VECT2,DIFF)
      CTRD=SQRT(VDOT(DIFF,DIFF))
      RETURN
      END


      COMPLEX FUNCTION FSG(LP1,LP2,LP3,L1,L2,L3)
C*********************************************************************
C*   FNC FSG RETURNS THE VALUE OF THE...                             *
C*             1. FREE SPACE GREEN'S FNC, EXP(-JKR)/R, IF NEAR=0.0   *
C*             2. FREE SPACE GREEN'S FUNCTION WITH SINGULARITY       *
C*                REMOVED,  [EXP(-JKR)-1]/R, IF NEAR=1.0             *
C*             THE ARGUMENTS OF FSG ARE THE LOCAL AREA COORDINATES   *
C*             OF OBSERVATION/SOURCE POINTS WITHIN GIVEN             *
C*             OBSERVATION/SOURCE TRIANGLES.                         *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL LP1,LP2,LP3,L1,L2,L3,COEFF(NMAX),R,PI,VECTS(NMAX,NMAX),
     *     VSUM(3),VDOT
      INTEGER I,K,M
      COMPLEX J

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL NEAR
      COMMON /ADJ/NEAR

      J=(0.0,1.0)
      PI=3.1415926536
      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      COEFF(4)=-1.0*LP1
      COEFF(5)=-1.0*LP2
      COEFF(6)=-1.0*LP3
      DO 10 I=1,3
```

```
      DO 20 K=1,3
      VECTS(I,K)=OBSV(I,K)
      VECTS(I+3,K)=SRCV(I,K)
   20 CONTINUE
   10 CONTINUE

      CALL SUM_VECTS(6,VECTS,COEFF,VSUM)
      R=SQRT(VDOT(VSUM,VSUM))
      IF (R.LE.1E-6) THEN
      FSG=(0.0,-6.283185307)
      ELSE
      FSG=(CEXP(-1*J*2.0*PI*R)-NEAR)/R
      ENDIF
      RETURN
      END



      COMPLEX FUNCTION PFSG(LP1,LP2,LP3,L1,L2,L3)
C***********************************************************************
C*   FNC PFSG RETURNS THE VALUE OF...                                  *
C*                 1. (R-RI).(R'-RJ)*EXP(-JKR)/R, IF NEAR=0.0          *
C*                 2. (R-RI).(R'-RJ)*[EXP(-JKR)-1]/R IF NEAR=1.0       *
C*                    (NOTE REMOVED SINGULARITY)                       *
C*                 WHERE RI/RJ IS THE POSITION VECTOR TO THE ITH/JTH   *
C*                 VERTEX OF THE OBSERVATION/SOURCE TRIANGLE.          *
C*                 THE ARGUMENTS OF PFSG ARE THE LOCAL AREA COORDINATES *
C*                 OF OBSERVATION/SOURCE POINTS WITHIN GIVEN           *
C*                 OBSERVATION/SOURCE TRIANGLES.                       *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL LP1,LP2,LP3,L1,L2,L3,COEFF(NMAX),R,ARG,PI,VECTS(NMAX,NMAX),
     *     RO(3),RP(3),RMINRP(3),RMINRI(3),RPMINRJ(3),IMAG,VDOT
      INTEGER I,K
      COMPLEX J

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL NEAR
      COMMON /ADJ/NEAR
      REAL RI(3),RJ(3)
      COMMON /BASES/RI,RJ

      J=(0.0,1.0)
      PI=3.1415926536

      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      DO 10 I=1,3
      DO 20 K=1,3
      VECTS(I,K)=OBSV(I,K)
   20 CONTINUE
   10 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,RO)

      COEFF(1)=LP1
      COEFF(2)=LP2
      COEFF(3)=LP3
      DO 30 I=1,3
      DO 40 K=1,3
      VECTS(I,K)=SRCV(I,K)
```

```
   40 CONTINUE
   30 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,RP)


      CALL VECT_DIFF(1.0,RO,1.0,RP,RMINRP)
      CALL VECT_DIFF(1.0,RO,1.0,RI,RMINRI)
      CALL VECT_DIFF(1.0,RP,1.0,RJ,RPMINRJ)
      R=SQRT(VDOT(RMINRP,RMINRP))
      ARG=VDOT(RMINRI,RPMINRJ)
      IF (R.LE.1E-6) THEN
      IMAG=-2*PI*ARG
      PFSG=CMPLX(0.0,IMAG)
      ELSE
      PFSG=ARG*(CEXP(-1*J*2.0*PI*R)-NEAR)/R
      ENDIF
      RETURN
      END


      COMPLEX FUNCTION UNIFNC(L1,L2,L3)
C*********************************************************************
C*   FNC UNIFNC RETURNS THE VALUE OF THE INTEGRATION                 *
C*               OVER THE SOURCE TRIANGLE OF THE REMOVED SINGULARITY *
C*               PHI:  1/R.  THE ARGUMENTS OF UNIFNC ARE THE LOCAL   *
C*               AREA COORDINATES OF THE OBSERVATION PT. WITHIN THE  *
C*               OBSERVATION TRIANGLE.                               *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL L1,L2,L3,COEFF(NMAX),VECTS(NMAX,NMAX),R(3),UNIPOT,
     *     LINPOT(3),NHAT(3),P(3),VERTS(4,3)
      INTEGER I,J,UNIFLAG,LINFLAG,CT1,CT2
      COMPLEX DUM

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV

      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      DO 10 I=1,3
      DO 20 J=1,3
      VECTS(I,J)=OBSV(I,J)
   20 CONTINUE
   10 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,R)

      UNIFLAG=1
      LINFLAG=0

      DO 30 CT1=1,3
      DO 40 CT2=1,3
      VERTS(CT1,CT2)=SRCV(CT1,CT2)
   40 CONTINUE
   30 CONTINUE
      CALL POT_INTS_SS(3,R,VERTS,UNIFLAG,UNIPOT,LINFLAG,LINPOT,NHAT,P)
      DUM=(0.0,0.0)
      UNIFNC=DUM+UNIPOT
      RETURN
      END

      COMPLEX FUNCTION LINFNC(L1,L2,L3)
C*********************************************************************
C*   FNC LINFNC RETURNS THE VALUE OF THE INTEGRATION                 *
```

```
C*                 OVER THE SOURCE TRIANGLE OF THE REMOVED SINGULARITY   *
C*                 A:  (R-RI).(P'-P)/R + (R-RI).(P-PJ)/R                  *
C*                 WHERE P'/P/PJ IS THE PROJECTION OF POSITION VECTOR    *
C*                 R'/R/RJ ONTO THE PLANE OF THE SOURCE TRIANGLE, AND    *
C*                 RI/RJ IS THE POSITION VECTOR TO THE ITH/JTH VERTEX OF *
C*                 THE OBSERVATION/SOURCE TRIANGLE.  THE ARGUMENTS OF    *
C*                 LI~~NC ARE THE LOCAL AREA COORDINATES OF THE          *
C*                 OE   VATION PT. WITHIN THE OBSERVATION TRIANGLE.      *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL L1,L2,L3,COEFF(NMAX),VECTS(NMAX,NMAX),R(3),UNIPOT,
     *     LINPOT(3),NHAT(3),P(3),RMINRI(3),C2,PJ(3),PMINPJ(3),VDOT,
     *     VERTS(4,3)
      INTEGER MPT,INDX,I,J,UNIFLAG,LINFLAG,CT1,CT2
      COMPLEX DUM

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL RI(3),RJ(3)
      COMMON /BASES/RI,RJ

      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      DO 10 I=1,3
        DO 20 J=1,3
          VECTS(I,J)=OBSV(I,J)
   20   CONTINUE
   10 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,R)

      UNIFLAG=1
      LINFLAG=1

      DO 30 CT1=1,3
        DO 40 CT2=1,3
          VERTS(CT1,CT2)=SRCV(CT1,CT2)
   40 CONTINUE
   30 CONTINUE
      CALL POT_INTS_SS(3,R,VERTS,UNIFLAG,UNIPOT,LINFLAG,LINPOT,NHAT,P)

      CALL VECT_DIFF(1.0,R,1.0,RI,RMINRI)

      C2=VDOT(NHAT,RJ)
      CALL VECT_DIFF(1.0,RJ,C2,NHAT,PJ)

      CALL VECT_DIFF(1.0,P,1.0,PJ,PMINPJ)
      DUM=(0.0,0.0)
      LINFNC=DUM+VDOT(RMINRI,LINPOT)+VDOT(RMINRI,PMINPJ)*UNIPOT
      RETURN
      END


      SUBROUTINE SUM_VECTS(NVECTS,VECTS,COEFF,VSUM)
C***********************************************************************
C*   SR SUM_VECTS SUMS N VECTORS EACH MULTIPLIED BY A                   *
C*                 COEFFICIENT, I.E. VSUM = C1*V1 + C2*V2 + ... CN*VN.  *
C***********************************************************************

      INCLUDE 'DIM.INC'

      REAL VECTS(NMAX,NMAX),COEFF(NMAX),VSUM(3)
      INTEGER NVECTS,I,J,K
```

```
      DO 10 I=1,3
      VSUM(I)=0.0
   10 CONTINUE

      DO 20 J=1,NVECTS
      DO 30 K=1,3
      VSUM(K)=VSUM(K)+COEFF(J)*VECTS(J,K)
   30 CONTINUE
   20 CONTINUE
      RETURN
      END


      BLOCK DATA
C***********************************************************************
C*   THIS BLOCK DATA SEGMENT INITIALIZES COMMON BLOCK                  *
C*               INTCONST WHICH CONTAINS THE LOCAL AREA COORDINATE     *
C*               INTEGRATION POINTS AND WEIGHTS USED IN ALL            *
C*               INTEGRATION ROUTINES (SRS AC_INT, SINT, VECTINT).     *
C*               THE 1, 4, AND 7-POINT FORMULAS WERE DEVELOPED         *
C*               ESPECIALLY FOR TRIANGULAR REGIONS.                    *
C***********************************************************************

      REAL W(12),ZETA1(12),ZETA2(12),ZETA3(12)
      COMMON /INTCONST/ZETA1,ZETA2,ZETA3,W

      DATA ZETA1(1),ZETA2(1),ZETA3(1),W(1)

     */0.3333333333,0.3333333333,0.3333333333,1.0/

      DATA ZETA1(2),ZETA2(2),ZETA3(2),W(2),
     *     ZETA1(3),ZETA2(3),ZETA3(3),W(3),
     *     ZETA1(4),ZETA2(4),ZETA3(4),W(4),
     *     ZETA1(5),ZETA2(5),ZETA3(5),W(5)
     */0.3333333333,0.3333333333,0.3333333333,-0.5625,
     * 0.6,0.2,0.2,0.5208333333,
     * 0.2,0.6,0.2,0.5208333333,
     * 0.2,0.2,0.6,0.5208333333/

      DATA ZETA1(6),ZETA2(6),ZETA3(6),W(6),
     *     ZETA1(7),ZETA2(7),ZETA3(7),W(7),
     *     ZETA1(8),ZETA2(8),ZETA3(8),W(8),
     *     ZETA1(9),ZETA2(9),ZETA3(9),W(9),
     *     ZETA1(10),ZETA2(10),ZETA3(10),W(10),
     *     ZETA1(11),ZETA2(11),ZETA3(11),W(11),
     *     ZETA1(12),ZETA2(12),ZETA3(12),W(12)
     */0.3333333333,0.3333333333,0.3333333333,0.225,
     * 0.0597158717,0.4701420641,0.4701420641,0.1323941527,
     * 0.4701420641,0.0597158717,0.4701420641,0.1323941527,
     * 0.4701420641,0.4701420641,0.0597158717,0.1323941527,
     * 0.7974269853,0.1012865073,0.1012865073,0.1259391805,
     * 0.1012865073,0.7974269853,0.1012865073,0.1259391805,
     * 0.1012865073,0.1012865073,0.7974269853,0.1259391805/

       END

      SUBROUTINE AC_INT(FNC,OUTM,INM,VOUT)
C***********************************************************************
C*  SR AC_INT PERFORMS A DOUBLE SURFACE INTEGRATION OVER               *
C*              TWO TRIANGLES OF A FUNCTION IN LOCAL AREA COORDINATES.*
C*              THE USER CAN CHOOSE 1,4, OR 7 POINTS FOR EACH SURFACE *
C*              INTEGRATION.  NOTE: THE RESULTING VALUE IS NORMALIZED *
C*              TO THE AREAS OF THE TRIANGLES.                         *
C***********************************************************************
```

```
      INTEGER OUTM,INM,OUTINDX,ININDX,I,J,K,L,M,N,P
      COMPLEX FNC,VOUT,VIN

      REAL W(12),ZETA1(12),ZETA2(12),ZETA3(12)
      COMMON /INTCONST/ZETA1,ZETA2,ZETA3,W

      IF (OUTM.EQ.1) THEN
      OUTINDX=1
      ELSE IF (OUTM.EQ.4) THEN
      OUTINDX=2
      ELSE
      OUTINDX=6
      END IF

      IF (INM.EQ.1) THEN
      ININDX=1
      ELSE IF (INM.EQ.4) THEN
      ININDX=2
      ELSE
      ININDX=6
      END IF

      VOUT=(0.0,0.0)

      DO 10 I=OUTINDX,OUTINDX+OUTM-1
      VIN=(0.0,0.0)
      DO 20 J=ININDX,ININDX+INM-1
      VIN=VIN+W(J)*FNC(ZETA1(J),ZETA2(J),ZETA3(J),
     *     ZETA1(I),ZETA2(I),ZETA3(I))
   20 CONTINUE
      VOUT=VOUT+W(I)*VIN
   10 CONTINUE
      RETURN
      END

      SUBROUTINE SINT(MPT,FN,VINT)
C***********************************************************************
C*   SR SINT PERFORMS A SINGLE SURFACE INTEGRATION OVER                *
C*                A TRIANGLE OF A FUNCTION IN LOCAL AREA COORDINATES.  *
C*                THE USER CAN CHOOSE 1,4, OR 7 POINTS FOR THE SURFACE *
C*                INTEGRATION.  NOTE: THE RESULTING VALUE IS NORMALIZED *
C*                TO THE AREA OF THE TRIANGLE.                         *
C***********************************************************************

      INTEGER MPT,INDX,I
      COMPLEX FN,VINT

      REAL ZETA1(12),ZETA2(12),ZETA3(12),W(12)
      COMMON /INTCONST/ZETA1,ZETA2,ZETA3,W

      IF (MPT.EQ.1) THEN
      INDX=1
      ELSE IF (MPT.EQ.4) THEN
      INDX=2
      ELSE
      INDX=6
      END IF

      VINT=(0.0,0.0)

      DO 10 I=INDX,INDX+MPT-1
      VINT=VINT+W(I)*FN(ZETA1(I),ZETA2(I),ZETA3(I))
   10 CONTINUE

      RETURN
      END
```

```
      SUBROUTINE POT_INTS_SS(NUMSIDES,R,VERTS,UNIFLAG,UNIPOT,LINFLAG,
     *                       LINPOT,NHAT,P)
C*********************************************************************
C*  SR POT_INTS_SS COMPUTES THE POTENTIAL INTEGRALS FOR              *
C*             UNIFORM AND LINEARLY VARYING SURFACE SOURCES          *
C*             DISTRIBUTED ON A PLANAR POLYGON S.  THE INTEGRALS ARE *
C*             CALCULATED USING CLOSED FORM ANALYTICAL EXPRESSIONS.  *
CC********************************************************************

      REAL R(3),VERTS(4,3),UNIPOT,LINPOT(3),V1(3),V2(3),N(3),NMAG,
     *     NHAT(3),RIPLUS(3),RIMIN(3),UIHAT(3),D,CAPRIMIN,CAPRIPLUS,
     *     LIMIN,LIPLUS,MAGPIO,PIOHAT(3),RIO,VDOT,VMAG,UNIELEM,
     *     LINELEM(3),RDUM(3),C2,RPROJ(3),P(3),DIFF(3),MARG,PARG
      INTEGER NUMSIDES,UNIFLAG,LINFLAG,I,J,K,M,Q,S,T,U,CT,INDX

      DO 10 J=1,3
      VERTS(NUMSIDES+1,J)=VERTS(1,J)
   10 CONTINUE

      DO 20 K=1,3
      V1(K)=VERTS(2,K)-VERTS(1,K)
      V2(K)=VERTS(3,K)-VERTS(1,K)
   20 CONTINUE
      CALL CROSS(V1,V2,N)
      NMAG=VMAG(N)
      DO 30 M=1,3
        NHAT(M)=N(M)/NMAG
   30 CONTINUE

      DO 35 INDX=1,3
      RDUM(INDX)=VERTS(1,INDX)
   35 CONTINUE
      CALL VECT_DIFF(1.0,R,1.0,RDUM,DIFF)
      D=VDOT(NHAT,DIFF)

      CALL VECT_DIFF(1.0,R,D,NHAT,RPROJ)
      C2=VDOT(NHAT,RPROJ)
      CALL VECT_DIFF(1.0,RPROJ,C2,NHAT,P)

      UNIPOT=0.0
      DO 40 Q=1,3
      LINPOT(Q)=0.0
   40 CONTINUE

      DO 50 I=1,NUMSIDES

      DO 60 S=1,3
      RIMIN(S)=VERTS(I,S)
      RIPLUS(S)=VERTS(I+1,S)
   60 CONTINUE


      CALL I_PARAMS(RIPLUS,RIMIN,R,NHAT,D,P,
     *      IHAT,CAPRIMIN,CAPRIPLUS,LIMIN,LIPLUS,
     *      MAGPIO,PIOHAT,RIO)

      IF (UNIFLAG.EQ.1) THEN

      IF (MAGPIO.LE.1.0E-10) THEN
      UNIELEM=0.0
      LSE
      IF (LIMIN.LT.0) THEN
      MARG=RIO**2/(ABS(LIMIN)+SQRT(RIO**2+LIMIN**2))
      ELSE
```

```
      MARG=CAPRIMIN+LIMIN
      END IF
      IF (LIPLUS.LT.0) THEN
      PARG=RIO**2/(ABS(LIPLUS)+SQRT(RIO**2+LIPLUS**2))
      ELSE
      PARG=CAPRIPLUS+LIPLUS
      END IF
      NIELEM = VDOT(PIOHAT,UIHAT) * ( MAGPIO *
     *            ALOG(PARG/MARG) -ABS(D) *
     *            (ATAN2((MAGPIO*LIPLUS),(RIO*RIO+ABS(D)*CAPRIPLUS))
     *            -ATAN2((MAGPIO*LIMIN),(RIO*RIO+ABS(D)*CAPRIMIN))))
      END IF
      UNIPOT=UNIPOT+UNIELEM
      END IF

      IF (LINFLAG.EQ.1) THEN

      IF ((RIO.LE.1.0E-10).AND.(D.LE.1.0E-10)) THEN
      DO 65 CT=1,3
      LINELEM(CT)=UIHAT(CT) * (LIPLUS*CAPRIPLUS -
     *              LIMIN*CAPRIMIN)
      LINPOT(CT)=LINPOT(CT)+LINELEM(CT)
   65 CONTINUE
      ELSE
      IF (LIMIN.LT.0) THEN
      MARG=RIO**2/(ABS(LIMIN)+SQRT(RIO**2+LIMIN**2))
      ELSE
      MARG=CAPRIMIN+LIMIN
      END IF
      IF (LIPLUS.LT.0) THEN
      PARG=RIO**2/(ABS(LIPLUS)+SQRT(RIO**2+LIPLUS**2))
      ELSE
      PARG=CAPRIPLUS+LIPLUS
      END IF
      DO 70 T=1,3
      LINELEM(T) = UIHAT(T) * ( RIO*RIO *
     *                LOG(PARG/MARG)+
     *                LIPLUS*CAPRIPLUS-LIMIN*CAPRIMIN )
      LINPOT(T)=LINPOT(T)+LINELEM(T)
   70 CONTINUE
      END IF
      END IF

   50 CONTINUE

      IF (LINFLAG.EQ.1) THEN
      DO 80 U=1,3
      LINPOT(U)=LINPOT(U)/2.0
   80 CONTINUE
      END IF


      RETURN
      END


      SUBROUTINE I_PARAMS(RIPLUS,RIMIN,R,NHAT,D,P,
     *                    UIHAT,CAPRIMIN,CAPRIPLUS,LIMIN,LIPLUS,
     *                    MAGPIO,PIOHAT,RIO)
C**********************************************************************
C*  SR I_PARAMS CALCULATES THE GEOMETRICAL PARAMETERS FOR             *
C*             POT_INTS_SS WHICH ARE DEPENDENT ON ITH POLYGON SIDE.    *
C**********************************************************************

      REAL RIPLUS(3),RIMIN(3),R(3),NHAT(3),D,P(3),UIHAT(3),CAPRIMIN,
     *     CAPRIPLUS,LIMIN,LIPLUS,MAGPIO,PIOHAT(3),RIO,PIMIN(3),
```

```
     *     PIPLUS(3),LVECT(3),LIHAT(3),DIFF(3),PLUSDIFF(3),PIO(3),C2,
     *     LMAG,VDOT,VMAG
      INTEGER J,K,M


      C2=VDOT(NHAT,RIMIN)
      CALL VECT_DIFF(1.0,RIMIN,C2,NHAT,PIMIN)

      C2=VDOT(NHAT,RIPLUS)
      CALL VECT_DIFF(1.0,RIPLUS,C2,NHAT,PIPLUS)

      CALL VECT_DIFF(1.0,RIPLUS,1.0,RIMIN,LVECT)
      LMAG=VMAG(LVECT)
      DO 10 J=1,3
      LIHAT(J)=LVECT(J)/LMAG
   10 CONTINUE

      CALL CROSS(LIHAT,NHAT,UIHAT)

      CALL VECT_DIFF(1.0,R,1.0,RIMIN,DIFF)
      CAPRIMIN=VMAG(DIFF)

      CALL VECT_DIFF(1.0,R,1.0,RIPLUS,DIFF)
      CAPRIPLUS=VMAG(DIFF)

      CALL VECT_DIFF(1.0,PIMIN,1.0,P,DIFF)
      LIMIN=VDOT(DIFF,LIHAT)

      CALL VECT_DIFF(1.0,PIPLUS,1.0,P,PLUSDIFF)
      LIPLUS=VDOT(PLUSDIFF,LIHAT)

      MAGPIO=ABS(VDOT(PLUSDIFF,UIHAT))

      IF (MAGPIO.LT.1.0E-10) THEN
      DO 20 K=1,3
      PIOHAT(K)=0.0
   20 CONTINUE
      ELSE
      CALL VECT_DIFF(1.0,PLUSDIFF,LIPLUS,LIHAT,PIO)
      DO 30 M=1,3
      PIOHAT(M)=PIO(M)/MAGPIO
   30 CONTINUE
      END IF

      RIO=SQRT(MAGPIO*MAGPIO + D*D)

      RETURN
      END



      SUBROUTINE VECT_DIFF(C1,VECT1,C2,VECT2,VDIFF)
C**********************************************************************
C*   SR VECT_DIFF TAKES THE DIFFERENCE OF TWO VECTORS,                *
C*                 EACH MULTIPLIED BY A COEFFICIENT,                  *
C*                 I.E. VDIFF = C1*VECT1 - C2*VECT2                   *
C**********************************************************************

      REAL C1,C2,VECT1(3),VECT2(3),VDIFF(3)
      INTEGER I

      DO 10 I=1,3
      VDIFF(I)=C1*VECT1(I)-C2*VECT2(I)
   10 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION VMAG(VECT)
C*********************************************************************
C* FNC VMAG COMPUTES THE MAGNITUDE OF A VECTOR.                       *
C*********************************************************************

      REAL VECT(3),VDOT

      VMAG=SQRT(VDOT(VECT,VECT))
      RETURN
      END


      REAL FUNCTION VDOT(VECT1,VECT2)
C*********************************************************************
C*  FNC VDOT COMPUTES THE DOT PRODUCT OF TWO VECTORS.                 *
C*********************************************************************

      REAL VECT1(3),VECT2(3)

      VDOT=VECT1(1)*VECT2(1)+VECT1(2)*VECT2(2)+VECT1(3)*VECT2(3)
      RETURN
      END

       SUBROUTINE CROSS(VECT1,VECT2,RCROSS)
C*********************************************************************
C*  SR CROSS COMPUTES THE CROSS PRODUCT OF TWO VECTORS,               *
C*                 I.E. VCROSS = VECT1 X VECT2                        *
C*********************************************************************

      REAL RCROSS(3),VECT1(3),VECT2(3)

      RCROSS(1)=VECT1(2)*VECT2(3)-VECT1(3)*VECT2(2)
      RCROSS(2)=VECT2(1)*VECT1(3)-VECT1(1)*VECT2(3)
      RCROSS(3)=VECT1(1)*VECT2(2)-VECT2(1)*VECT1(2)
      RETURN
      END



      REAL FUNCTION SRCAREA()
C*********************************************************************
C* FNC SRCAREA COMPUTES THE AREA OF THE SOURCE TRIANGLE               *
C*                 GIVEN THE POSITION VECTORS TO THE VERTICES OF THE  *
C*                 SOURCE TRIANGLE.                                   *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL VECT1(3),VECT2(3),VECT3(3),DIFF(3),BASE,LIHAT(3),UIHAT(3),
     *     HEIGHT,VDOT,VMAG,V1(3),V2(3),N(3),NMAG,NHAT(3)
      INTEGER I,J,K,S

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV

      DO 10 I=1,3
      V1(I)=SRCV(2,I)-SRCV(1,I)
      V2(I)=SRCV(3,I)-SRCV(1,I)
   10 CONTINUE
      CALL CROSS(V1,V2,N)
      NMAG=VMAG(N)
      DO 20 J=1,3
      NHAT(J)=N(J)/NMAG
   20 CONTINUE
```

```
      DO 30 K=1,3
      VECT1(K)=SRCV(1,K)
      VECT2(K)=SRCV(2,K)
      VECT3(K)=SRCV(3,K)
   30 CONTINUE
      CALL VECT_DIFF(1.0,VECT2,1.0,VECT1,DIFF)
      BASE=VMAG(DIFF)
      DO 40 S=1,3
      LIHAT(S)=DIFF(S)/BASE
   40 CONTINUE
      CALL CROSS(LIHAT,NHAT,UIHAT)
      CALL VECT_DIFF(1.0,VECT3,1.0,VECT1,DIFF)
      HEIGHT=ABS(VDOT(DIFF,UIHAT))
      SRCAREA=.5*BASE*HEIGHT
      RETURN
      END

      COMPLEX FUNCTION RESFNC(L1,L2,L3)
C*********************************************************************
C*   FNC RESFNC RETURNS THE VALUE OF THE INTEGRAND OF THE            *
C*                 RESISTIVE TERM:  (R-RM).(R-RN)                    *
C*                 RM/RN ARE THE POSITION VECTORS TO THE MTH/NTH     *
C*                 VERTICES OF THE GIVEN OBSERVATION TRIANGLE.       *
C*                 THE ARGUMENTS OF RESFNC ARE THE LOCAL AREA        *
C*                 COORDINATES OF OBSERVATION POINTS WITHIN THE      *
C*                 OBSERVATION TRIANGLE.                             *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL L1,L2,L3,COEFF(NMAX),VECTS(NMAX,NMAX),R(3),PN(3),PM(3),VDOT
      INTEGER I,K
      COMPLEX DUM

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN

      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      DO 10 I=1,3
      DO 20 K=1,3
      VECTS(I,K)=OBSV(I,K)
   20 CONTINUE
   10 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,R)

      CALL VECT_DIFF(1.0,R,1.0,RN,PN)

      CALL VECT_DIFF(1.0,R,1.0,RM,PM)
      DUM=(0.0,0.0)
      RESFNC=DUM+VDOT(PN,PM)
      RETURN
      END

      SUBROUTINE USR(MESH_FILE,STYPE,RES_FILE,OUT_FILE,PTYPE,ALPHA,
     *               THETAI,PHII,FIX,FANGO,ANG1O,NUMPTS,STEPANG)
C*********************************************************************
C*   SR USR_DATA PROMPTS USER FOR, AND READS IN, DATA                *
C*                 PERTAINING TO                                     *
C*                             -FILE NAMES                           *
C*                             -INCIDENT FIELD                       *
C*                             -RCS OBSERVATION CUT                  *
C*
```

```
C*                VIA STANDARD I/O.                                   *
C**********************************************************************

      INCLUDE 'CONST.INC'

      REAL FANGO,ANG1O,ANG2O,STEPANG,ALPHA,THETAI,PHII,
     *     RNUMPTS,PI
      INTEGER PTYPE,FIX,NUMPTS,FLAG,STYPE
      CHARACTER*40 MESH_FILE,RES_FILE,OUT_FILE

   10 FORMAT(A40)
   12 FORMAT(A,F7.2,A)
      PI=3.1415926536

  100 CONTINUE

      PRINT*,'ENTER MESH FILE NAME:'
      READ(*,10) MESH_FILE
      PRINT*,'ENTER SURFACE TYPE (1-PEC, 2-RESISTIVE):'
      READ*, STYPE
      IF (STYPE.EQ.RESIST) THEN
      PRINT*,'ENTER RESISTIVITY FILE NAME:'
      READ(*,10) RES_FILE
      END IF
      PRINT*, 'ENTER OUTPUT FILE NAME:  '
      READ(*,10) OUT_FILE
      PRINT*,'ENTER PATTERN (1-BISTATIC, 2-BACKSCATTER):  '
      READ*, PTYPE
      PRINT*,'ENTER E-FIELD POLARIZATION ANGLE ALPHA (IN DEGREES):  '
      READ*, ALPHA
      IF (PTYPE.EQ.BISTAT) THEN
      PRINT*,'ENTER ANGLES OF INCIDENCE...'
      PRINT*,'PHI (IN DEGREES):  '
      READ*, PHII
      PRINT*,'THETA (IN DEGREES):  '
      READ*, THETAI
      END IF
      PRINT*,'ENTER CUT SPECIFICATIONS...'
      PRINT*,'FIX (1-PHI, 2-THETA):  '
      READ*, FIX
      IF (FIX.EQ.ANGPHI) THEN
      PRINT*,'ENTER FIXED OBSERVATION ANGLE PHI (IN DEGREES):  '
      READ*, FANGO
      PRINT*,'ENTER START OBSERVATION ANGLE THETA (IN DEGREES):  '
      READ*, ANG1O
      PRINT*,'ENTER STOP OBSERVATION ANGLE THETA (IN DEGREES):  '
      READ*, ANG2O
      ELSE
      PRINT*,'ENTER FIXED OBSERVATION ANGLE THETA (IN DEGREES):  '
      READ*, FANGO
      PRINT*,'ENTER START OBSERVATION ANGLE PHI (IN DEGREES):  '
      READ*, ANG1O
      PRINT*,'ENTER STOP OBSERVATION ANGLE PHI (IN DEGREES):  '
      READ*, ANG2O
      END IF
      PRINT*,'ENTER NUMBER OF OBSERVATION POINTS:  '
      READ*, NUMPTS

      PRINT*,' '
      PRINT*,'--------------- FILES ---------------'
      PRINT*,'             MESH:  ',MESH_FILE
      IF (STYPE.EQ.RESIST) THEN
      PRINT*,'     RESISTIVITY:  ',RES_FILE
      END IF
      PRINT*,'           OUTPUT:  ',OUT_FILE
      PRINT*,' '
```

```fortran
      PRINT*,'------------ SURFACE TYPE -----------'
      IF (STYPE.EQ.PEC) THEN
      PRINT*,'                         PEC'
      ELSE
      PRINT*,'                      RESISTIVE'
      END IF
      PRINT*,' '
      PRINT*,'------------ PATTERN TYPE -----------'
      IF (PTYPE.EQ.BISTAT) THEN
      PRINT*,'                      BISTATIC'
      ELSE
      PRINT*,'                     BACKSCATTER'
      END IF
      PRINT*,' '
      PRINT*,'---------- INCIDENT FIELD -----------'
      WRITE(*,12) '          ALPHA:  ',ALPHA,' DEG.'
      IF (PTYPE.EQ.BISTAT) THEN
      WRITE(*,12) '            PHI:  ',PHII,' DEG.'
      WRITE(*,12) '          THETA:  ',THETAI,'  DEG.'
      END IF
      PRINT*,' '
      PRINT*,'-------- OBSERVATION ANGLES ---------'
      IF (FIX.EQ.ANGPHI) THEN
      WRITE(*,12) '              PHI:  ',FANGO,' DEG.'
      WRITE(*,12) '     START THETA:  ',ANG1O,' DEG.'
      WRITE(*,12) '      STOP THETA:  ',ANG2O,' DEG.'
      ELSE
      WRITE(*,12) '            THETA:  ',FANGO,' DEG.'
      WRITE(*,12) '       START PHI:  ',ANG1O,' DEG.'
      WRITE(*,12) '        STOP PHI:  ',ANG2O,' DEG.'
      END IF
      PRINT*,' '
      PRINT*,'--- NUMBER OF OBSERVATION POINTS ----'
      PRINT*,'                        ',NUMPTS
      PRINT*,' '

      PRINT*,'ABOVE DATA O.K. (1-YES, 2-NO)?'
      READ*, FLAG
      IF (FLAG.EQ.NO) THEN
      GOTO 100
      END IF

      ALPHA=ALPHA*PI/180.
      PHII=PHII*PI/180.
      THETAI=THETAI*PI/180.
      FANGO=FANGO*PI/180.
      ANG1O=ANG1O*PI/180.
      ANG2O=ANG2O*PI/180.

      RNUMPTS=1.0*(NUMPTS-1)
      STEPANG=(ANG2O-ANG1O)/RNUMPTS
      RETURN
      END


      SUBROUTINE EXCIT(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,EDGLEN,
     *                  TRISIGN,ALPHA,THETAI,PHII,V)
C*********************************************************************
C*  SR EXCIT COMPUTES ELEMENTS OF EXCITATION VECTOR V:               *
C*             <EI,FM>.  THE INCIDENT ELECTRIC FIELD IS A PLANE WAVE *
C*             OF UNIT AMPLITUDE:                                    *
C*                          EI = EHAT EXP(J 2 PI KHAT.R)             *
C*             WHERE EHAT IS UNIT POLARIZATION VECTOR DETERMINED     *
C*             FROM ANGLE ALPHA, AND KHAT IS UNIT PROPAGATION VECTOR *
C*             DETERMINED FROM INCIDENT ANGLES THETAI AND PHII.      *
C*********************************************************************
```

```
      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3),EDGLEN(MAXEDG),ALPHA,PHII,THETAI
      INTEGER NINTEDG,EDGNODS(MAXEDG,2),NTRIS,TRIEDGS(MAXTRI,3),
     *        TRISIGN(MAXTRI,3),I,P,K,M(3),S,T
      COMPLEX V(MAXZ),ANS,PWFNC,PJ
      EXTERNAL PWFNC

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN
      REAL EHAT(3),KHAT(3),DUMY(3)
      COMMON /INCID/EHAT,KHAT

      PI=3.14159265359
      PJ=(0.0,1.0)

      EHAT(1)=COS(ALPHA)*COS(THETAI)*COS(PHII)-SIN(ALPHA)*SIN(PHII)
      EHAT(2)=COS(ALPHA)*COS(THETAI)*SIN(PHII)+SIN(ALPHA)*COS(PHII)
      EHAT(3)=-COS(ALPHA)*SIN(THETAI)

      KHAT(1)=SIN(THETAI)*COS(PHII)
      KHAT(2)=SIN(THETAI)*SIN(PHII)
      KHAT(3)=COS(THETAI)

      DO I=1,3
      DUMY(I)=EHAT(I)
      ENDDO
      EHAT(1)=KHAT(3)*DUMY(2)-DUMY(3)*KHAT(2)
      EHAT(2)=DUMY(3)*KHAT(1)-DUMY(1)*KHAT(3)
      EHAT(3)=DUMY(1)*KHAT(2)-DUMY(2)*KHAT(1)

      DO 10 I=1,MAXZ
      V(I)=(0.0,0.0)
   10 CONTINUE

      DO 20 P=1,NTRIS
      DO 30 K=1,3
      M(K)=TRIEDGS(P,K)
   30 CONTINUE
      CALL GET_VERTS(M,0,NODCRDS,EDGNODS)

      DO 40 S=1,3
      IF (M(S).LE.NINTEDG) THEN
      DO 50 T=1,3
      RM(T)=OBSV(S,T)
   50 CONTINUE
      CALL SINT(7,PWFNC,ANS)
      V(M(S))=V(M(S))+TRISIGN(P,S)*EDGLEN(M(S))*ANS
     *        *2.0*PI*PJ

      END IF
   40 CONTINUE
   20 CONTINUE

      DO I=1,3
      DUMY(I)=EHAT(I)
      ENDDO
      EHAT(1)=KHAT(2)*DUMY(3)-DUMY(2)*KHAT(3)
      EHAT(2)=DUMY(1)*KHAT(3)-DUMY(3)*KHAT(1)
      EHAT(3)=DUMY(2)*KHAT(1)-DUMY(1)*KHAT(2)


      RETURN
```

```
      END

      COMPLEX FUNCTION PWFNC(L1,L2,L3)
C**********************************************************************
C*   FNC PWFNC RETURNS THE VALUE OF THE  (R-RM).EI WHERE EI             *
C*              IS INCIDENT PLANE WAVE OF UNIT AMPLITUDE, AND RM IS     *
C*              THE POSITION VECTOR TO THE MTH VERTEX OF THE            *
C*              OBSERVATION TRIANGLE.  THE ARGUMENTS OF PWFNC ARE THE   *
C*              LOCAL AREA COORDINATES OF OBSERVATION POINTS WITHIN     *
C*              GIVEN OBSERVATION TRIANGLE.                             *
C**********************************************************************

      INCLUDE 'DIM.INC'

      REAL L1,L2,L3,R(3),RMINRM(3),COEFF(NMAX),VECTS(NMAX,NMAX),VDOT,PI
      INTEGER I,S
      COMPLEX J

      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN
      REAL EHAT(3),KHAT(3)
      COMMON /INCID/EHAT,KHAT

      PI=3.14159265359
      J=(0.0,1.0)

      COEFF(1)=L1
      COEFF(2)=L2
      COEFF(3)=L3
      DO 20 I=1,3
      DO 30 S=1,3
      VECTS(I,S)=OBSV(I,S)
   30 CONTINUE
   20 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,R)

      CALL VECT_DIFF(1.0,R,1.0,RM,RMINRM)
      PWFNC=VDOT(EHAT,RMINRM)*CEXP(J*2.0*PI*VDOT(KHAT,R))
      RETURN
      END

      SUBROUTINE BACK_SUBST(A,VC,NET,IPVT,EN)
C**********************************************************************
C**********************************************************************

      INCLUDE 'DIM.INC'

      REAL RCOND
      INTEGER NET,LDA,N,JOB,INDX,IPVT(MAXA)
      COMPLEX A(MAXA,MAXA),VC(MAXA),EN(MAXA),DUM(MAXA)

      LDA=MAXA
      JOB=0
      DO 10 INDX=1,NET
       EN(INDX)=VC(INDX)
   10 CONTINUE
      CALL CGESL(A,LDA,NET,IPVT,EN,JOB)
      RETURN
      END

      SUBROUTINE RCS(NODCRDS,NINTEDG,EDGNODS,NTRIS,TRIEDGS,EDGLEN,
     *                     TRISIGN,ES,THETAO,PHIO,RCSTH,RCSPHI)
C**********************************************************************
C*  FNC RCS RETURNS THE VALUE OF THE RADAR CROSS SECTION                *
C*                IN DB:                                                *
```

```
C*                   RCS = LIM[R->INFINITY] 4 PI R^2 |ES|^2/|EI|^2        *
C*                THE ARGUMENTS OF RCS ARE THE OBSERVATION ANGLES         *
C*                THETAO AND PHIO IN RADIANS.                             *
C*                                                                        *
C*                NOTE:  IN DETERMINING THE RCS, THE SPHERICAL            *
C*                FAR FIELD COEFFICIENTS SR/STH/SPHI ARE COMPUTED:        *
C*                            ECOMP=SCOMP E-JKR/KR                        *
C*                THESE VALUES MAY BE EASILY EXTRACTED.                   *
C*************************************************************************

      INCLUDE 'DIM.INC'

      REAL NODCRDS(MAXNOD,3),EDGLEN(MAXEDG),THETAO,PHIO,
     *     PI,ETA
      INTEGER NINTEDG,EDGNODS(MAXEDG,2),NTRIS,TRIEDGS(MAXTRI,3),
     *        TRISIGN(MAXTRI,3),INDX,CT,K,Q,S,T,N(3)
      COMPLEX H(3),ANS(3),ES(MAXZ),VECT(3),DUM(3),SR,STH,SPHI,J

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN
      REAL RHAT(3)
      COMMON /FFOBS/RHAT

      J=(0.0,1.0)
      PI=3.1415925953
      ETA=376.7343091821

      DO 10 INDX=1,3
      VECT(INDX)=(0.0,0.0)
   10 CONTINUE
      RHAT(1)=SIN(THETAO)*COS(PHIO)
      RHAT(2)=SIN(  ETAO)*SIN(PHIO)
      RHAT(3)=COS(   TAO)
      DO 20 Q=1,NTRIS

      DO 30 CT=1,3
      N(CT)=TRIEDGS(Q,CT)
   30 CONTINUE
      CALL GET_VERTS(N,1,NODCRDS,EDGNODS)

      DO 40 K=1,3
      IF (N(K).LE.NINTEDG) THEN
      DO 50 S=1,3
      RN(S)=SRCV(K,S)
   50 CONTINUE
      ALL VECTINT(7,ANS)
      DO 60 T=1,3
      VECT(T)=VECT(T)+ES(N(K))*EDGLEN(N(K))*TRISIGN(Q,K)*
     *        ANS(T)/2.0
   60 CONTINUE
      END IF
   40 CONTINUE
   20 CONTINUE
      CALL CPLXCROSS(RHAT,VECT,DUM)
      CALL CPLXCROSS(RHAT,DUM,H)
      STH=(H(1)*COS(THETAO)*COS(PHIO)+H(2)*COS(THETAO)*
     *    SIN(PHIO)-H(3)*SIN(THETAO))
      SPHI=(-H(1)*SIN(PHIO)+H(2)*COS(PHIO))

      RCSTH=10.*ALOG10(4.*PI*CABS(STH)**2)
      RCSPHI=10.*ALOG10(4.*PI*CABS(SPHI)**2)
      RETURN
      END
```

```
      SUBROUTINE VECTINT(MPT,VINT)
C*********************************************************************
C* SR VECTINT PERFORMS A SINGLE SURFACE INTEGRATION OVER              *
C*               A TRIANGLE OF VECTOR FUNCTION FFARG.  LOCAL AREA     *
C*               COORDINATES ARE USED.  THE CALLING ROUTINE SPECIFIES *
C*               1,4, OR 7 POINTS FOR THE SURFACE INTEGRATION.        *
C*               NOTE: THE RESULTING VALUE IS NORMALIZED TO THE AREA  *
C*               OF THE TRIANGLE.                                     *
C*********************************************************************

      INCLUDE 'DIM.INC'

      INTEGER MPT,INDX,CT,I,K
      COMPLEX VINT(3),ANS(3)

      REAL ZETA1(12),ZETA2(12),ZETA3(12),W(12)
      COMMON /INTCONST/ZETA1,ZETA2,ZETA3,W

      IF (MPT.EQ.1) THEN
      INDX=1
      ELSE IF (MPT.EQ.4) THEN
      INDX=2
      ELSE
      INDX=6
      END IF

      DO 10 CT=1,3
      VINT(CT)=(0.0,0.0)
   10 CONTINUE

      DO 20 I=INDX,INDX+MPT-1
      CALL FFARG(ZETA1(I),ZETA2(I),ZETA3(I),ANS)
      DO 30 K=1,3
      VINT(K)=VINT(K)+W(I)*ANS(K)
   30 CONTINUE
   20 CONTINUE

      RETURN
      END

      SUBROUTINE FFARG(LP1,LP2,LP3,VFUNC)
C*********************************************************************
C*  SR FFARG RETURNS THE COMPLEX VECTOR VALUE OF THE                  *
C*               E-FAR FIELD ARGUMENT:                                *
C*                     EFF ~ (R'-RN) EXP(J 2 PI R'.RHAT)              *
C*               WHERE RN IS THE POSITION VECTOR TO THE NTH VERTEX OF *
C*               THE SOURCE TRIANGLE.  THE ARGUMENTS OF FFARG ARE THE *
C*               LOCAL AREA COORDINATES OF SOURCE POINTS WITHIN THE   *
C*               GIVEN SOURCE TRIANGLE.                               *
C*********************************************************************

      INCLUDE 'DIM.INC'

      REAL LP1,LP2,LP3,PI,VECTS(NMAX,NMAX),COEFF(NMAX),RP(3),PN(3),
     *     VDOT
      INTEGER I,K,S
      COMPLEX VFUNC(3),J

      REAL OBSV(NMAX,NMAX),SRCV(NMAX,NMAX)
      COMMON /TRICONST/OBSV,SRCV
      REAL RM(3),RN(3)
      COMMON /BASES/RM,RN
      REAL RHAT(3)
      COMMON /FFOBS/RHAT
```

```
      J=(0.0,1.0)
      PI=3.1415926536

      COEFF(1)=LP1
      COEFF(2)=LP2
      COEFF(3)=LP3
      DO 10 I=1,3
      DO 20 K=1,3
      VECTS(I,K)=SRCV(I,K)
   20 CONTINUE
   10 CONTINUE
      CALL SUM_VECTS(3,VECTS,COEFF,RP)

      CALL VECT_DIFF(1.0,RP,1.0,RN,PN)
      DO 30 S=1,3
      VFUNC(S)=PN(S)*CEXP(J*2.0*PI*VDOT(RP,RHAT))
   30 CONTINUE
      RETURN
      END

      SUBROUTINE CPLXCROSS(VECT1,VECT2,VCROSS)
C**********************************************************************
C*  SR CMPLXCROSS COMPUTES THE COMPLEX CROSS PRODUCT OF A            *
C*                REAL VECTOR AND A COMPLEX VECTOR,                  *
C*                I.E. VCROSS = VECT1 X VECT2                        *
C**********************************************************************

      REAL VECT1(3)
      COMPLEX VCROSS(3),VECT2(3)

      VCROSS(1)=VECT1(2)*VECT2(3)-VECT1(3)*VECT2(2)
      VCROSS(2)=VECT2(1)*VECT1(3)-VECT1(1)*VECT2(3)
      VCROSS(3)=VECT1(1)*VECT2(2)-VECT2(1)*VECT1(2)
      RETURN
      END

C*********************************************************************
      SUBROUTINE CGECO(A,LDA,N,IPVT,RCOND,Z)
C*********************************************************************
      INTEGER LDA,N,IPVT(1)
      COMPLEX A(LDA,1),Z(1)
      REAL RCOND
C
C     CGECO FACTORS A COMPLEX MATRIX BY GAUSSIAN ELIMINATION
C     AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C     IF  RCOND  IS NOT NEEDED, CGEFA IS SLIGHTLY FASTER.
C     TO SOLVE  A*X = B , FOLLOW CGECO BY CGESL.
C     TO COMPUTE  INVERSE(A)*C , FOLLOW CGECO BY CGESL.
C     TO COMPUTE  DETERMINANT(A) , FOLLOW CGECO BY CGEDI.
C     TO COMPUTE  INVERSE(A) , FOLLOW CGECO BY CGEDI.
C
C     ON ENTRY
C
C        A       COMPLEX(LDA, N)
C                THE MATRIX TO BE FACTORED.
C
C        LDA     INTEGER
C                THE LEADING DIMENSION OF THE ARRAY  A .
C
C        N       INTEGER
C                THE ORDER OF THE MATRIX  A .
C
C     ON RETURN
C
C        A       AN UPPER TRIANGULAR MATRIX AND THE MULTIPLIERS
```

```
C                 WHICH WERE USED TO OBTAIN IT.
C                 THE FACTORIZATION CAN BE WRITTEN  A = L*U  WHERE
C                 L  IS A PRODUCT OF PERMUTATION AND UNIT LOWER
C                 TRIANGULAR MATRICES AND  U  IS UPPER TRIANGULAR.
C
C        IPVT     INTEGER(N)
C                 AN INTEGER VECTOR OF PIVOT INDICES.
C
C        RCOND    REAL
C                 AN ESTIMATE OF THE RECIPROCAL CONDITION OF  A .
C                 FOR THE SYSTEM  A*X = B , RELATIVE PERTURBATIONS
C                 IN  A  AND  B  OF SIZE  EPSILON  MAY CAUSE
C                 RELATIVE PERTURBATIONS IN  X  OF SIZE  EPSILON/RCOND .
C                 IF  RCOND  IS SO SMALL THAT THE LOGICAL EXPRESSION
C                            1.0 + RCOND .EQ. 1.0
C                 IS TRUE, THEN  A  MAY BE SINGULAR TO WORKING
C                 PRECISION.  IN PARTICULAR,  RCOND  IS ZERO  IF
C                 EXACT SINGULARITY IS DETECTED OR THE ESTIMATE
C                 UNDERFLOWS.
C
C        Z        COMPLEX(N)
C                 A WORK VECTOR WHOSE CONTENTS ARE USUALLY UNIMPORTANT.
C                 IF  A  IS CLOSE TO A SINGULAR MATRIX, THEN  Z  IS
C                 AN APPROXIMATE NULL VECTOR IN THE SENSE THAT
C                 NORM(A*Z) = RCOND*NORM(A)*NORM(Z) .
C
C     LINPACK. THIS VERSION DATED 07/14/77 .
C     CLEVE MOLER, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.
C
C     SUBROUTINES AND FUNCTIONS
C
C     LINPACK CGEFA
C     BLAS CAXPY,CDOTC,CSSCAL,SCASUM
C     FORTRAN ABS,AIMAG,AMAX1,CMPLX,CONJG,REAL
C
C     INTERNAL VARIABLES
C
      COMPLEX CDOTC,EK,T,WK,WKM
      REAL ANORM,S,SCASUM,SM,YNORM
      INTEGER INFO,J,K,KB,KP1,L
C
      COMPLEX ZDUM,ZDUM1,ZDUM2,CSIGN1
      REAL CABS1
      CABS1(ZDUM) = ABS(REAL(ZDUM)) + ABS(AIMAG(ZDUM))
      CSIGN1(ZDUM1,ZDUM2) = CABS1(ZDUM1)*(ZDUM2/CABS1(ZDUM2))
C
C     COMPUTE 1-NORM OF A
C
      ANORM = 0.0E0
      DO 10 J = 1, N
         ANORM = AMAX1(ANORM,SCASUM(N,A(1,J),1))
   10 CONTINUE
C
C     FACTOR
C
      CALL CGEFA(A,LDA,N,IPVT,INFO)
C
C     RCOND = 1/(NORM(A)*(ESTIMATE OF NORM(INVERSE(A)))) .
C     ESTIMATE = NORM(Z)/NORM(Y) WHERE  A*Z = Y  AND  CTRANS(A)*Y = E .
C     CTRANS(A)  IS THE CONJUGATE TRANSPOSE OF A .
C     THE COMPONENTS OF  E  ARE CHOSEN TO CAUSE MAXIMUM LOCAL
C     GROWTH IN THE ELEMENTS OF W  WHERE  CTRANS(U)*W = E .
C     THE VECTORS ARE FREQUENTLY RESCALED TO AVOID OVERFLOW.
C
C     SOLVE CTRANS(U)*W = E
C
```

```
      EK = CMPLX(1.0E0,0.0E0)
      DO 20 J = 1, N
         Z(J) = CMPLX(0.0E0,0.0E0)
   20 CONTINUE
      DO 100 K = 1, N
         IF (CABS1(Z(K)) .NE. 0.0E0) EK = CSIGN1(EK,-Z(K))
         IF (CABS1(EK-Z(K)) .LE. CABS1(A(K,K))) GO TO 30
            S = CABS1(A(K,K))/CABS1(EK-Z(K))
            CALL CSSCAL(N,S,Z,1)
            EK = CMPLX(S,0.0E0)*EK
   30    CONTINUE
         WK = EK - Z(K)
         WKM = -EK - Z(K)
         S = CABS1(WK)
         SM = CABS1(WKM)
         IF (CABS1(A(K,K)) .EQ. 0.0E0) GO TO 40
            WK = WK/CONJG(A(K,K))
            WKM = WKM/CONJG(A(K,K))
         GO TO 50
   40    CONTINUE
            WK = CMPLX(1.0E0,0.0E0)
            WKM = CMPLX(1.0E0,0.0E0)
   50    CONTINUE
         KP1 = K + 1
         IF (KP1 .GT. N) GO TO 90
            DO 60 J = KP1, N
               SM = SM + CABS1(Z(J)+WKM*CONJG(A(K,J)))
               Z(J) = Z(J) + WK*CONJG(A(K,J))
               S = S + CABS1(Z(J))
   60       CONTINUE
            IF (S .GE. SM) GO TO 80
               T = WKM - WK
               WK = WKM
               DO 70 J = KP1, N
                  Z(J) = Z(J) + T*CONJG(A(K,J))
   70          CONTINUE
   80       CONTINUE
   90    CONTINUE
         Z(K) = WK
  100 CONTINUE
      S = 1.0E0/SCASUM(N,Z,1)
      CALL CSSCAL(N,S,Z,1)
C
C     SOLVE CTRANS(L)*Y = V
C
      DO 120 KB = 1, N
         K = N + 1 - KB
         IF (K .LT. N) Z(K) = Z(K) + CDOTC(N-K,A(K+1,K),1,Z(K+1),1)
         IF (CABS1(Z(K)) .LE. 1.0E0) GO TO 110
            S = 1.0E0/CABS1(Z(K))
            CALL CSSCAL(N,S,Z,1)
  110    CONTINUE
         L = IPVT(K)
         T = Z(L)
         Z(L) = Z(K)
         Z(K) = T
  120 CONTINUE
      S = 1.0E0/SCASUM(N,Z,1)
      CALL CSSCAL(N,S,Z,1)
C
      YNORM = 1.0E0
C
C     SOLVE L*V = Y
C
      DO 140 K = 1, N
         L = IPVT(K)
```

```
            T = Z(L)
            Z(L) = Z(K)
            Z(K) = T
            IF (K .LT. N) CALL CAXPY(N-K,T,A(K+1,K),1,Z(K+1),1)
            IF (CABS1(Z(K)) .LE. 1.0E0) GO TO 130
               S = 1.0E0/CABS1(Z(K))
               CALL CSSCAL(N,S,Z,1)
               YNORM = S*YNORM
  130       CONTINUE
  140    CONTINUE
         S = 1.0E0/SCASUM(N,Z,1)
         CALL CSSCAL(N,S,Z,1)
         YNORM = S*YNORM
C
C        SOLVE  U*Z = V
C
         DO 160 KB = 1, N
            K = N + 1 - KB
            IF (CABS1(Z(K)) .LE. CABS1(A(K,K))) GO TO 150
               S = CABS1(A(K,K))/CABS1(Z(K))
               CALL CSSCAL(N,S,Z,1)
               YNORM = S*YNORM
  150       CONTINUE
            IF (CABS1(A(K,K)) .NE. 0.0E0) Z(K) = Z(K)/A(K,K)
            IF (CABS1(A(K,K)) .EQ. 0.0E0) Z(K) = CMPLX(1.0E0,0.0E0)
            T = -Z(K)
            CALL CAXPY(K-1,T,A(1,K),1,Z(1),1)
  160    CONTINUE
C        MAKE ZNORM = 1.0
         S = 1.0E0/SCASUM(N,Z,1)
         CALL CSSCAL(N,S,Z,1)
         YNORM = S*YNORM
C
         IF (ANORM .NE. 0.0E0) RCOND = YNORM/ANORM
         IF (ANORM .EQ. 0.0E0) RCOND = 0.0E0
         RETURN
         END

C***********************************************************************
         SUBROUTINE CGEFA(A,LDA,N,IPVT,INFO)
C***********************************************************************
         INTEGER LDA,N,IPVT(1),INFO
         COMPLEX A(LDA,1)
C
C        CGEFA FACTORS A COMPLEX MATRIX BY GAUSSIAN ELIMINATION.
C
C        CGEFA IS USUALLY CALLED BY CGECO, BUT IT CAN BE CALLED
C        DIRECTLY WITH A SAVING IN TIME IF  RCOND  IS NOT NEEDED.
C        (TIME FOR CGECO) = (1 + 9/N)*(TIME FOR CGEFA) .
C
C        ON ENTRY
C
C           A       COMPLEX(LDA, N)
C                   THE MATRIX TO BE FACTORED.
C
C           LDA     INTEGER
C                   THE LEADING DIMENSION OF THE ARRAY  A .
C
C           N       INTEGER
C                   THE ORDER OF THE MATRIX  A .
C
C        ON RETURN
C
C           A       AN UPPER TRIANGULAR MATRIX AND THE MULTIPLIERS
C                   WHICH WERE USED TO OBTAIN IT.
C                   THE FACTORIZATION CAN BE WRITTEN  A = L*U  WHERE
```

```
C                 L  IS A PRODUCT OF PERMUTATION AND UNIT LOWER
C                 TRIANGULAR MATRICES AND  U  IS UPPER TRIANGULAR.
C
C        IPVT     INTEGER(N)
C                 AN INTEGER VECTOR OF PIVOT INDICES.
C
C        INFO     INTEGER
C                 = 0  NORMAL VALUE.
C                 = K  IF  U(K,K) .EQ. 0.0 .  THIS IS NOT AN ERROR
C                      CONDITION FOR THIS SUBROUTINE, BUT IT DOES
C                      INDICATE THAT CGESL OR CGEDI WILL DIVIDE BY ZERO
C                      IF CALLED.  USE  RCOND  IN CGECO FOR A RELIABLE
C                      INDICATION OF SINGULARITY.
C
C     LINPACK. THIS VERSION DATED 07/14/77 .
C     CLEVE MOLER, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.
C
C     SUBROUTINES AND FUNCTIONS
C
C     BLAS CAXPY,CSCAL,ICAMAX
C     FORTRAN ABS,AIMAG,CMPLX,REAL
C
C     INTERNAL VARIABLES
C
      COMPLEX T
      INTEGER ICAMAX,J,K,KP1,L,NM1
C
      COMPLEX ZDUM
      REAL CABS1
      CABS1(ZDUM) = ABS(REAL(ZDUM)) + ABS(AIMAG(ZDUM))
C
C     GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
C
      INFO = 0
      NM1 = N - 1
      IF (NM1 .LT. 1) GO TO 70
      DO 60 K = 1, NM1
         KP1 = K + 1
C
C        FIND L = PIVOT INDEX
C
         L = ICAMAX(N-K+1,A(K,K),1) + K - 1
         IPVT(K) = L
C
C        ZERO PIVOT IMPLIES THIS COLUMN ALREADY TRIANGULARIZED
C
         IF (CABS1(A(L,K)) .EQ. 0.0E0) GO TO 40
C
C           INTERCHANGE IF NECESSARY
C
            IF (L .EQ. K) GO TO 10
               T = A(L,K)
               A(L,K) = A(K,K)
               A(K,K) = T
   10       CONTINUE
C
C           COMPUTE MULTIPLIERS
C
            T = -CMPLX(1.0E0,0.0E0)/A(K,K)
            CALL CSCAL(N-K,T,A(K+1,K),1)
C
C           ROW ELIMINATION WITH COLUMN INDEXING
C
            DO 30 J = KP1, N
               T = A(L,J)
               IF (L .EQ. K) GO TO 20
```

```
               A(L,J) = A(K,J)
               A(K,J) = T
   20       CONTINUE
            CALL CAXPY(N-K,T,A(K+1,K),1,A(K+1,J),1)
   30    CONTINUE
      GO TO 50
   40 CONTINUE
         INFO = K
   50 CONTINUE
   60 CONTINUE
   70 CONTINUE
      IPVT(N) = N
      IF (CABS1(A(N,N)) .EQ. 0.0E0) INFO = N
      RETURN
      END

C***********************************************************************
      SUBROUTINE CGESL(A,LDA,N,IPVT,B,JOB)
C***********************************************************************
      INTEGER LDA,N,IPVT(1),JOB
      COMPLEX A(LDA,1),B(1)
C
C     CGESL SOLVES THE COMPLEX SYSTEM
C     A * X = B  OR  CTRANS(A) * X = B
C     USING THE FACTORS COMPUTED BY CGECO OR CGEFA.
C
C     ON ENTRY
C
C        A       COMPLEX(LDA, N)
C                THE OUTPUT FROM CGECO OR CGEFA.
C
C        LDA     INTEGER
C                THE LEADING DIMENSION OF THE ARRAY  A .
C
C        N       INTEGER
C                THE ORDER OF THE MATRIX  A .
C
C        IPVT    INTEGER(N)
C                THE PIVOT VECTOR FROM CGECO OR CGEFA.
C
C        B       COMPLEX(N)
C                THE RIGHT HAND SIDE VECTOR.
C
C        JOB     INTEGER
C                = 0         TO SOLVE  A*X = B ,
C                = NONZERO   TO SOLVE  CTRANS(A)*X = B  WHERE
C                            CTRANS(A)  IS THE CONJUGATE TRANSPOSE.
C
C     ON RETURN
C
C        B       THE SOLUTION VECTOR  X .
C
C     ERROR CONDITION
C
C        A DIVISION BY ZERO WILL OCCUR IF THE INPUT FACTOR CONTAINS A
C        ZERO ON THE DIAGONAL.  TECHNICALLY THIS INDICATES SINGULARITY
C        BUT IT IS OFTEN CAUSED BY IMPROPER ARGUMENTS OR IMPROPER
C        SETTING OF LDA .  IT WILL NOT OCCUR IF THE SUBROUTINES ARE
C        CALLED CORRECTLY AND IF CGECO HAS SET RCOND .GT. 0.0
C        OR CGEFA HAS SET INFO .EQ. 0 .
C
C     TO COMPUTE  INVERSE(A) * C  WHERE  C  IS A MATRIX
C     WITH  P  COLUMNS
C           CALL CGECO(A,LDA,N,IPVT,RCOND,Z)
C           IF (RCOND IS TOO SMALL) GO TO ...
```

```
C             DO 10 J = 1, P
C                CALL CGESL(A,LDA,N,IPVT,C(1,J),0)
C          10 CONTINUE
C
C     LINPACK. THIS VERSION DATED 07/14/77 .
C     CLEVE MOLER, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.
C
C     SUBROUTINES AND FUNCTIONS
C
C     BLAS CAXPY,CDOTC
C     FORTRAN CONJG
C
C     INTERNAL VARIABLES
C
      COMPLEX CDOTC,T
      INTEGER K,KB,L,NM1
C
      NM1 = N - 1
      IF (JOB .NE. 0) GO TO 50
C
C        JOB = 0 , SOLVE  A * X = B
C        FIRST SOLVE  L*Y = B
C
         IF (NM1 .LT. 1) GO TO 30
         DO 20 K = 1, NM1
            L = IPVT(K)
            T = B(L)
            IF (L .EQ. K) GO TO 10
               B(L) = B(K)
               B(K) = T
   10       CONTINUE
            CALL CAXPY(N-K,T,A(K+1,K),1,B(K+1),1)
   20    CONTINUE
   30    CONTINUE
C
C        NOW SOLVE  U*X = Y
C
         DO 40 KB = 1, N
            K = N + 1 - KB
            B(K) = B(K)/A(K,K)
            T = -B(K)
            CALL CAXPY(K-1,T,A(1,K),1,B(1),1)
   40    CONTINUE
      GO TO 100
   50 CONTINUE
C
C        JOB = NONZERO, SOLVE  CTRANS(A) * X = B
C        FIRST SOLVE  CTRANS(U)*Y = B
C
         DO 60 K = 1, N
            T = CDOTC(K-1,A(1,K),1,B(1),1)
            B(K) = (B(K) - T)/CONJG(A(K,K))
   60    CONTINUE
C
C        NOW SOLVE CTRANS(L)*X = Y
C
         IF (NM1 .LT. 1) GO TO 90
         DO 80 KB = 1, NM1
            K = N - KB
            B(K) = B(K) + CDOTC(N-K,A(K+1,K),1,B(K+1),1)
            L = IPVT(K)
            IF (L .EQ. K) GO TO 70
               T = B(L)
               B(L) = B(K)
               B(K) = T
   70       CONTINUE
```

```
   80    CONTINUE
   90    CONTINUE
  100 CONTINUE
      RETURN
      END
C***********************************************************************
      INTEGER FUNCTION ICAMAX(N,CX,INCX)
C***********************************************************************
C
C     FINDS THE INDEX OF ELEMENT HAVING MAX. ABSOLUTE VALUE.
C     JACK DONGARRA, LINPACK, 3/11/78.
C
      COMPLEX CX(1)
      REAL SMAX
      INTEGER I,INCX,IX,N
      COMPLEX ZDUM
      REAL CABS1
      CABS1(ZDUM) = ABS(REAL(ZDUM)) + ABS(AIMAG(ZDUM))
C
      ICAMAX = 0
      IF( N .LT. 1 ) RETURN
      ICAMAX = 1
      IF(N.EQ.1)RETURN
      IF(INCX.EQ.1)GO TO 20
C
C        CODE FOR INCREMENT NOT EQUAL TO 1
C
      IX = 1
      SMAX = CABS1(CX(1))
      IX = IX + INCX
      DO 10 I = 2,N
         IF(CABS1(CX(IX)).LE.SMAX) GO TO 5
         ICAMAX = I
         SMAX = CABS1(CX(IX))
    5    IX = IX + INCX
   10 CONTINUE
      RETURN
C
C        CODE FOR INCREMENT EQUAL TO 1
C
   20 SMAX = CABS1(CX(1))
      DO 30 I = 2,N
         IF(CABS1(CX(I)).LE.SMAX) GO TO 30
         ICAMAX = I
         SMAX = CABS1(CX(I))
   30 CONTINUE
      RETURN
      END
C***********************************************************************
      REAL FUNCTION SCASUM(N,CX,INCX)
C***********************************************************************
C
C     TAKES THE SUM OF THE ABSOLUTE VALUES OF A COMPLEX VECTOR AND
C     RETURNS A SINGLE PRECISION RESULT.
C     JACK DONGARRA, LINPACK, 3/11/78.
C
      COMPLEX CX(1)
      REAL STEMP
      INTEGER I,INCX,N,NINCX
C
      SCASUM = 0.0E0
      STEMP = 0.0E0
      IF(N.LE.0)RETURN
      IF(INCX.EQ.1)GO TO 20
```

```
C
C        CODE FOR INCREMENT NOT EQUAL TO 1
C
      NINCX = N*INCX
      DO 10 I = 1,NINCX,INCX
        STEMP = STEMP + ABS(REAL(CX(I))) + ABS(AIMAG(CX(I)))
   10 CONTINUE
      SCASUM = STEMP
      RETURN
C
C        CODE FOR INCREMENT EQUAL TO 1
C
   20 DO 30 I = 1,N
        STEMP = STEMP + ABS(REAL(CX(I))) + ABS(AIMAG(CX(I)))
   30 CONTINUE
      SCASUM = STEMP
      RETURN
      END

C***********************************************************************
      SUBROUTINE CAXPY(N,CA,CX,INCX,CY,INCY)
C***********************************************************************
C
C     CONSTANT TIMES A VECTOR PLUS A VECTOR.
C     JACK DONGARRA, LINPACK, 3/11/78.
C
      COMPLEX CX(1),CY(1),CA
      INTEGER I,INCX,INCY,IX,IY,N
C
      IF(N.LE.0)RETURN
      IF (ABS(REAL(CA)) + ABS(AIMAG(CA)) .EQ. 0.0 ) RETURN
      IF(INCX.EQ.1.AND.INCY.EQ.1)GO TO 20
C
C        CODE FOR UNEQUAL INCREMENTS OR EQUAL INCREMENTS
C          NOT EQUAL TO 1
C
      IX = 1
      IY = 1
      IF(INCX.LT.0)IX = (-N+1)*INCX + 1
      IF(INCY:LT.0)IY = (-N+1)*INCY + 1
      DO 10 I = 1,N
        CY(IY) = CY(IY) + CA*CX(IX)
        IX = IX + INCX
        IY = IY + INCY
   10 CONTINUE
      RETURN
C
C        CODE FOR BOTH INCREMENTS EQUAL TO 1
C
   20 DO 30 I = 1,N
        CY(I) = CY(I) + CA*CX(I)
   30 CONTINUE
      RETURN
      END

C***********************************************************************
      COMPLEX FUNCTION CDOTC(N,CX,INCX,CY,INCY)
C***********************************************************************
C
C     FORMS THE DOT PRODUCT OF TWO VECTORS, CONJUGATING THE FIRST
C     VECTOR.
C     JACK DONGARRA, LINPACK,  3/11/78.
C
      COMPLEX CX(1),CY(1),CTEMP
      INTEGER I,INCX,INCY,IX,IY,N
C
```

```
      CTEMP = (0.0,0.0)
      CDOTC = (0.0,0.0)
      IF(N.LE.0)RETURN
      IF(INCX.EQ.1.AND.INCY.EQ.1)GO TO 20
C
C        CODE FOR UNEQUAL INCREMENTS OR EQUAL INCREMENTS
C          NOT EQUAL TO 1
C
      IX = 1
      IY = 1
      IF(INCX.LT.0)IX = (-N+1)*INCX + 1
      IF(INCY.LT.0)IY = (-N+1)*INCY + 1
      DO 10 I = 1,N
        CTEMP = CTEMP + CONJG(CX(IX))*CY(IY)
        IX = IX + INCX
        IY = IY + INCY
   10 CONTINUE
      CDOTC = CTEMP
      RETURN
C
C        CODE FOR BOTH INCREMENTS EQUAL TO 1
C
   20 DO 30 I = 1,N
        CTEMP = CTEMP + CONJG(CX(I))*CY(I)
   30 CONTINUE
      CDOTC = CTEMP
      RETURN
      END

C***********************************************************************
      SUBROUTINE  CSSCAL(N,SA,CX,INCX)
C***********************************************************************
C
C     SCALES A COMPLEX VECTOR BY A REAL CONSTANT.
C     JACK DONGARRA, LINPACK, 3/11/78.
C
      COMPLEX CX(1)
      REAL SA
      INTEGER I,INCX,N,NINCX
C
      IF(N.LE.0)RETURN
      IF(INCX.EQ.1)GO TO 20
C
C        CODE FOR INCREMENT NOT EQUAL TO 1
C
      NINCX = N*INCX
      DO 10 I = 1,NINCX,INCX
        CX(I) = CMPLX(SA*REAL(CX(I)),SA*AIMAG(CX(I)))
   10 CONTINUE
      RETURN
C
C        CODE FOR INCREMENT EQUAL TO 1
C
   20 DO 30 I = 1,N
        CX(I) = CMPLX(SA*REAL(CX(I)),SA*AIMAG(CX(I)))
   30 CONTINUE
      RETURN
      END
      SUBROUTINE  CSCAL(N,CA,CX,INCX)
C
C     SCALES A VECTOR BY A CONSTANT.
C     JACK DONGARRA, LINPACK,  3/11/78.
C
      COMPLEX CA,CX(1)
      INTEGER I,INCX,N,NINCX
C
```

```
      IF(N.LE.0)RETURN
      IF(INCX.EQ.1)GO TO 20
C
C        CODE FOR INCREMENT NOT EQUAL TO 1
C
      NINCX = N*INCX
      DO 10 I = 1,NINCX,INCX
        CX(I) = CA*CX(I)
   10 CONTINUE
      RETURN
C
C        CODE FOR INCREMENT EQ  .L TO 1
C
   20 DO 30 I = 1,N
        CX(I) = CA*CX(I)
   30 CONTINUE
      RETURN
      END

C=====================================================================
C       THIS SUBROUTINE GENERATES THE FEM MATRIX
C=====================================================================
        SUBROUTINE FEM(NED,NES,EDST,A)
        PARAMETER (MAXA=2000)
        INTEGER EDNA(15000),GNN(15000,2),TAB(6,3),SED(2000),EDST(MAXA,2)
     1            ,MAT(2500)
        CHARACTER BIT(6)*1
        REAL XYZ(3000,3),A1(6,6),X(500),Y(500),Z(500),B1(6,6),EPS(10)
        COMPLEX A(MAXA,MAXA),B(MAXA,MAXA)
        LOGICAL MATZ
        COMMON /BANK/EDNA,GNN,XYZ,EPS,MAT
        COMMON /MESS/X,Y,Z
        PI=3.14159265359
        NL=1
        OPEN(1,FILE='EGLOB')
        OPEN(2,FILE='EDGY')
        OPEN(3,FILE='ENODDY')
        OPEN(7,FILE='ESURFC')
        READ(1,*)(EPS(I),I=1,NL)
        READ(1,*)NN
        DO I=1,NN
          READ(3,*)K,X(I),Y(I),Z(I)
        ENDDO
        READ(1,*)NEL
        DO I=1,6*NEL
          ITEMP=1+(I/6.)
          READ(1,*)ELM,EDNA(I),GNN(I,1),GNN(I,2),MAT(ITEMP)
        ENDDO
        DO I=1,NED
          READ(2,*)K,XYZ(I,1),XYZ(I,2),XYZ(I,3)
        ENDDO
        READ(7,*)(SED(I),I=1,NES)
        CLOSE(1)
        CLOSE(2)
        CLOSE(3)
        CLOSE(7)
        NET=NED-NES
        DO I=1,NET
         DO J=1,NET
           A(I,J)=CMPLX(0.,0.)
           B(I,J)=CMPLX(0.,0.)
         ENDDO
        ENDDO
C       WRITE(6,*)'GENERATING FEM MATRIX'
        DO I=1,MAXA
           EDST(I,1)=0
```

```
        EDST(I,2)=0
      ENDDO
      NPTRX=1
      DO I=1,NEL
        CALL CRUX(I,A1,B1,TAB)
        DO IJ=1,6
          BIT(IJ)='0'
        ENDDO
        DO 100 J=1,6
          DO ICHK=1,NES
            IF (TAB(J,3).EQ.SED(ICHK)) THEN
              BIT(J)='1'
              GO TO 100
            ENDIF
          ENDDO
100     ENDDO
        DO J=1,6
          IF (BIT(J).EQ.'0') THEN
            IF (EDST(TAB(J,3),2).NE.TAB(J,3)) THEN
              EDST(TAB(J,3),1)=NPTRX
              EDST(TAB(J,3),2)=TAB(J,3)
              NPTRX=NPTRX+1
            ENDIF
          ENDIF
        ENDDO
        DO J=1,6
          DO K=1,6
            IF ((BIT(J).EQ.'0').AND.(BIT(K).EQ.'0')) THEN
              MMM=EDST(TAB(J,3),1)
              NNN=EDST(TAB(K,3),1)
              A(MMM,NNN)=A(MMM,NNN)+(A1(J,K)-4.*PI*PI*B1(J,K))
            ENDIF
          ENDDO
        ENDDO
      ENDDO
      WRITE(6,*)'FINISHED FEM MATRIX GENERATION'
      IF (NET.NE.(NPTRX-1)) THEN
        WRITE(6,*)'ERROR IN MATRIX ASSEMBLY'
        STOP
      ENDIF
      IR=0
       DO I=1,NPTRX-1
         DO J=1,NPTRX-1
           IF (REAL(A(I,J)-A(J,I)).GT..000001) THEN
             WRITE(6,*)A(I,J),A(J,I),I,J
             IR=1
           ENDIF
         ENDDO
       ENDDO
      IF (IR.EQ.0) THEN
        WRITE(6,*)'SYMMETRY TEST PASSED'
      ELSE
        WRITE(6,*)'SYMMETRY TEST FAILED'
      ENDIF

      END
C=================================================================
      SUBROUTINE CRUX(L,A,B,TAB)
      INTEGER EDNA(15000),GNN(15000,2),TAB(6,3),MAT(2500)
      REAL XYZ(3000,3),X(500),Y(500),Z(500),A(6,6),B(6,6),F(6,3),
     1      G(6,3),TMP(3),EPS(10)
      COMMON /BANK/EDNA,GNN,XYZ,EPS,MAT
      COMMON /MESS/X,Y,Z
      COMMON /LOCAL/SUMX,SUMY,SUMZ,XX,YY,ZZ,XY,YZ,ZX
      COMMON /FGS/F,G
      LV=6*(L-1)
```

```
      DO J=1,6
        TAB(J,1)=GNN(LV+J,1)
        TAB(J,2)=GNN(LV+J,2)
        TAB(J,3)=EDNA(LV+J)
      ENDDO
C....SORTING THE ARRAY 'TAB' ARRANGES IT ACCORDING TO LOCAL NODE NOS. SO
C    THAT THE ARRAY LOOKS LIKE THE ONE IN FILE 'INPUT'.
      CALL SORT(TAB)
C....'CALC' CALCULATES SOME DATA CORRESPONDING TO THE ELEMENT
      CALL CALC(TAB(1,1),TAB(1,2),TAB(2,2),TAB(3,2))
C....'VOLUME' COMPUTES SIX TIMES THE VOLUME OF THE TETRAHEDRAL ELEMENT
      CALL VOLUME(TAB(1,3),TAB(2,3),TAB(3,3),VOL)
      DO J=1,6
        CALL FCROSS(X(TAB(7-J,1)),Y(TAB(7-J,1)),Z(TAB(7-J,1)),X(TAB(
     1           7-J,2)),Y(TAB(7-J,2)),Z(TAB(7-J,2)),TMP)
C....'BJ' STORES THE LENGTH OF THE 'J' TH EDGE.
        BJ=SQRT((XYZ(TAB(J,3),1)**2)+(XYZ(TAB(J,3),2)**2)+(XYZ
     1     (TAB(J,3),3)**2))
        DO K=1,3
         F(J,K)=BJ*TMP(K)/VOL
         G(J,K)=BJ*XYZ(TAB(7-J,3),K)/VOL
        ENDDO
      ENDDO
      CALL FGSGN(TAB)
      VOL=VOL/6.
      DO J=1,6
        DO K=1,6
          A(J,K)=4.*DOT(J,K,G)*VOL
          B(J,K)=F1(J,K,F,G)*EPS(MAT(L))*VOL
        ENDDO
      ENDDO
      RETURN
      END
C======================================================================
      SUBROUTINE SORT(TAB)
      INTEGER TAB(6,3)
      NC=1
      DO IK=4,2,-1
        IF (NC.EQ.1) THEN
          IJ=6
        ELSE
          IJ=IK
        ENDIF
        DO II=1,IJ-1
          DO J=IJ,II+1,-1
            IF (TAB(J,NC).LT.TAB(J-1,NC)) THEN
              DO JK=1,3
                CALL EXCHG(TAB(J,JK),TAB(J-1,JK))
              ENDDO
            ENDIF
          ENDDO
        ENDDO
        NC=2
      ENDDO
      IF (TAB(5,2).LT.TAB(4,2)) THEN
         DO JK=1,3
           CALL EXCHG(TAB(5,JK),TAB(4,JK))
         ENDDO
        ENDIF
      CALL EXCHG(TAB(5,1),TAB(5,2))
      RETURN
      END
C======================================================================
      SUBROUTINE VOLUME(J1,J2,J3,V)
      INTEGER EDNA(15000),GNN(15000,2),MAT(2500)
      REAL XYZ(3000,3),B1(3),B2(3),B3(3),EPS(10)
```

```
      COMMON /BANK/EDNA,GNN,XYZ,EPS,MAT
      DO IT=1,3
        B1(IT)=XYZ(J1,IT)
        B2(IT)=XYZ(J2,IT)
        B3(IT)=XYZ(J3,IT)
      ENDDO
      V=ABS(B1(1)*((B2(2)*B3(3))-(B2(3)*B3(2)))-B1(2)*((B2(1)*B3
     1   (3))-(B2(3)*B3(1)))+B1(3)*((B2(1)*B3(2))-(B2(2)*B3(1))))
      RETURN
      END
C======================================================================
      SUBROUTINE FCROSS(X1,Y1,Z1,X2,Y2,Z2,TEMPO)
      REAL TEMPO(3)
      TEMPO(1)=Y1*Z2-Y2*Z1
      TEMPO(2)=Z1*X2-Z2*X1
      TEMPO(3)=X1*Y2-X2*Y1
      RETURN
      END
C======================================================================
      SUBROUTINE CALC(J1,J2,J3,J4)
      REAL X(500),Y(500),Z(500)
      COMMON /MESS/X,Y,Z
      COMMON /LOCAL/SUMX,SUMY,SUMZ,XX,YY,ZZ,XY,YZ,ZX
      SUMX=X(J1)+X(J2)+X(J3)+X(J4)
      SUMY=Y(J1)+Y(J2)+Y(J3)+Y(J4)
      SUMZ=Z(J1)+Z(J2)+Z(J3)+Z(J4)
      XX=SUMX*SUMX+X(J1)*X(J1)+X(J2)*X(J2)+X(J3)*X(J3)+X(J4)*X(J4)
      YY=SUMY*SUMY+Y(J1)*Y(J1)+Y(J2)*Y(J2)+Y(J3)*Y(J3)+Y(J4)*Y(J4)
      ZZ=SUMZ*SUMZ+Z(J1)*Z(J1)+Z(J2)*Z(J2)+Z(J3)*Z(J3)+Z(J4)*Z(J4)
      XY=SUMX*SUMY+X(J1)*Y(J1)+X(J2)*Y(J2)+X(J3)*Y(J3)+X(J4)*Y(J4)
      YZ=SUMY*SUMZ+Y(J1)*Z(J1)+Y(J2)*Z(J2)+Y(J3)*Z(J3)+Y(J4)*Z(J4)
      ZX=SUMZ*SUMX+Z(J1)*X(J1)+Z(J2)*X(J2)+Z(J3)*X(J3)+Z(J4)*X(J4)
      RETURN
      END
C======================================================================
      REAL FUNCTION DOT(J1,J2,VEC)
      REAL VEC(6,3)
      DOT=(VEC(J1,1)*VEC(J2,1))+(VEC(J1,2)*VEC(J2,2))+(VEC(J1,3)*VEC
     1     (J2,3))
      RETURN
      END
C======================================================================
      REAL FUNCTION F1(J1,J2,F,G)
      REAL F(6,3),G(6,3),TMP1(3),TMP2(3)
      COMMON /LOCAL/SUMX,SUMY,SUMZ,XX,YY,ZZ,XY,YZ,ZX
      CALL FCROSS(F(J1,1),F(J1,2),F(J1,3),G(J2,1),G(J2,2),G(J2,3),TMP1)
      CALL FCROSS(F(J2,1),F(J2,2),F(J2,3),G(J1,1),G(J1,2),G(J1,3),TMP2)
      TERM1=DOT(J1,J2,F)
      TERM2=((TMP1(1)+TMP2(1))*SUMX+(TMP1(2)+TMP2(2))*SUMY+
     1       (TMP1(3)+TMP2(3))*SUMZ)/4.
      TERM3=(G(J1,2)*G(J2,2)+G(J1,3)*G(J2,3))*XX+(G(J1,1)*G(J2,1)+
     1      G(J1,3)*G(J2,3))*YY+(G(J1,1)*G(J2,1)+G(J1,2)*G(J2,2))*ZZ
     2      -(G(J1,1)*G(J2,2)+G(J1,2)*G(J2,1))*XY-(G(J1,1)*G(J2,3)+
     3      G(J1,3)*G(J2,1))*ZX-(G(J1,2)*G(J2,3)+G(J1,3)*G(J2,2))*YZ
      F1=TERM1+TERM2+(TERM3/20.)
      RETURN
      END
C======================================================================
      SUBROUTINE EXCHG(J1,J2)
      NTMP=J1
      J1=J2
      J2=NTMP
      RETURN
      END
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++C
      SUBROUTINE FGSGN(TAB)
```

```
      INTEGER EDNA(15000),GNN(15000,2),TAB(6,3),MAT(2500)
      REAL XYZ(3000,3),X(500),Y(500),Z(500),A(6,6),B(6,6),F(6,3),
     1     G(6,3),TMP(3),EPS(10),V1(3),V2(3),GXE1(3),GXE2(3),EHAT(3)
     1     ,E0(3)
      COMMON /BANK/EDNA,GNN,XYZ,EPS,MAT
      COMMON /MESS/X,Y,Z
      COMMON /FGS/F,G
      DO 100 I=1,6
      CALL FCROSS(G(I,1),G(I,2),G(I,3),X(TAB(I,1)),Y(TAB(I,1)),
     1     Z(TAB(I,1)),GXE1)
      DO K=1,3
      V1(K)=F(I,K)+GXE1(K)
      ENDDO
      IF(TAB(I,2).GT.TAB(I,1))THEN
      DO K=1,3
      EHAT(K)=-XYZ(TAB(I,3),K)
      ENDDO
      ELSE
      DO K=1,3
      EHAT(K)=XYZ(TAB(I,3),K)
      ENDDO
      ENDIF
      S=0.
      AEHAT=SQRT(EHAT(1)**2+EHAT(2)**2+EHAT(3)**2)
      DO K=1,3
      S=S+V1(K)*EHAT(K)/AEHAT
      ENDDO
      IF(S.GT.0.)THEN
      GOTO 100
      ELSE
      DO K=1,3
      F(I,K)=-F(I,K)
      G(I,K)=-G(I,K)
      ENDDO
      ENDIF
 100  CONTINUE
      RETURN
      END

      SUBROUTINE COMB1(MESH_FILE,TRISIGN,NNOD,TAB3,EDGSGN)
C     THE CODE TO COMBINE THE MATRICES FROM FEM SUBROUTINE AND BI SUBROUTINE
C     SINCE THE TWO METHODS HAVE DIFFERENT NUMBERING SYSTEM, THIS CODE RELIES
C     ON THE INFORMATION PROVIDED BY PREP1.FTN AND PREP2.FTN TO CONVERT THE
C     SIGN CONVENTION OF THE FEM FOR THE ON-DIELECT-SURFACE EDGES TO MATCH
C     THE BI SIGN CONVENTION.
      PARAMETER (MAXEDGE=2000,MAXNODE=500,MAXTRI=650,MAXZ=2000)
      INTEGER P0,P1,P2,P3,EG0,EG1,EG2,EG3,EDGSGN(MAXZ),TAB2(MAXNODE)
      INTEGER TRISIGN(MAXTRI,3),EDGNODS(MAXEDGE,2),TAB3(MAXEDGE)
      INTEGER TRIEDGS(MAXTRI,3)
      REAL    NODCRDS(MAXNODE,3)
      CHARACTER*40 LINE
      CHARACTER*40 MESH_FILE

      OPEN (UNIT=4,FILE=MESH_FILE)
  100 CONTINUE
      READ(4,'(A)',END=1000) LINE
      IF (LINE(2:6).EQ.'FLAG1') THEN
        READ(4,'(A)') LINE
        I=1
  200   READ(4,*) NODE,NODCRDS(I,1),NODCRDS(I,2),NODCRDS(I,3)
        IF (NODE.EQ.-1) THEN
          GOTO 300
        END IF
        I=I+1
        GOTO 200
  300   CONTINUE
```

```
      ELSE IF (LINE(2:6).EQ.'FLAG2') THEN
        READ(4,'(A)') LINE
        READ(4,*) NINTEDG
        READ(4,'(A)') LINE
        READ(4,*) NEXTEDG
        READ(4,'(A)') LINE
        NEDG=NINTEDG+NEXTEDG
        DO 400 J=1,NEDG
          READ(4,*)NDUM,EDGNODS(J,1),EDGNODS(J,2)
 400    CONTINUE

      ELSE IF (LINE(2:6).EQ.'FLAG3') THEN
        READ(4,'(A)') LINE
        READ(4,*) NTRIS
        READ(4,'(A)') LINE
        DO 500 K=1,NTRIS
          READ(4,*)NDUM,TRIEDGS(K,1),TRIEDGS(K,2),TRIEDGS(K,3)
 500    CONTINUE

      END IF
      GOTO 100
1000  CONTINUE
      CLOSE(4)

        OPEN(4,FILE='TAB2')
        DO I=1,NNOD
        READ(4,*)NDUM,TAB2(I)
        ENDDO
        OPEN(4,FILE='TAB3')
        DO I=1,NEDG
        READ(4,*)NDUM,TAB3(I)
        ENDDO

        DO 1500 I=1,NTRIS
        EG1=TRIEDGS(I,1)
        EG2=TRIEDGS(I,2)
        EG3=TRIEDGS(I,3)
        DY1=ABS(NODCRDS(EDGNODS(EG1,2),2)-NODCRDS(EDGNODS(EG1,1),2))
        DY2=ABS(NODCRDS(EDGNODS(EG2,2),2)-NODCRDS(EDGNODS(EG2,1),2))
        DY3=ABS(NODCRDS(EDGNODS(EG3,2),2)-NODCRDS(EDGNODS(EG3,1),2))
C       FIND THE MINIMUN DY'S AND MARK THE CORRESPONDING EDGES
        IF(DY1.LE.DY2)THEN
                IF(DY1.LE.DY3)THEN
                MK=1
                ELSEIF(DY1.GT.DY3)THEN
                MK=3
                ENDIF
        ELSE
                IF(DY2.LE.DY3)THEN
                MK=2
                ELSEIF(DY2.GT.DY3)THEN
                MK=3
                ENDIF
        ENDIF
C       FIND TWO NODE #'S (P2,P3) CORRESPONDING TO THE EDGE (EG1) WITH MINIMUM 'DY'
        IF(MK.EQ.1)THEN
        P2=EDGNODS(EG1,1)
        P3=EDGNODS(EG1,2)
        ENDIF
        IF(MK.EQ.2)THEN
        P2=EDGNODS(EG2,1)
        P3=EDGNODS(EG2,2)
        EG0=EG1
        EG1=EG2
        EG2=EG0
```

```
          ENDIF
          IF(MK.EQ.3)THEN
          P2=EDGNODS(EG3,1)
          P3=EDGNODS(EG3,2)
          EG0=EG1
          EG1=EG3
          EG3=EG0
          ENDIF
C         FIND THE 3RD NODE #
          P0=EDGNODS(EG2,1)
           IF(P0.NE.P2.AND.P0.NE.P3)THEN
                  P1=P0
           ELSE
                  P1=EDGNODS(EG2,2)
           ENDIF
C         ORDER P1,P2,P3 COUNTERCLOCKWISELY
          Y1=NODCRDS(P1,2)
          Y2=NODCRDS(P2,2)
          Y3=NODCRDS(P3,2)
          X2=NODCRDS(P2,1)
          X3=NODCRDS(P3,1)
          YY=AMAX1(Y2,Y3)
          IF(Y1.GT.YY)THEN
C                 NUP=1
                  IF(X3.GT.X2)THEN
                  GOTO 1110
                  ELSE
                  P0=P2
                  P2=P3
                  P3=P0
                  ENDIF
          ELSE
C                 NUP=2
                  IF(X3.LT.X2)THEN
                  GOTO 1110
                  ELSE
                  P0=P2
                  P2=P3
                  P3=P0
                  ENDIF
          ENDIF
 1110     CONTINUE
C         FIND EG2:(P1,P3);EG3:(P1,P2)
          N21=EDGNODS(EG2,1)
          N22=EDGNODS(EG2,2)
          IF(P2.NE.N21.AND.P2.NE.N22)THEN
          GOTO 1120
          ELSE
          EG0=EG2
          EG2=EG3
          EG3=EG0
          ENDIF
 1120     CONTINUE

C         COMPARE THE SIGNS (ACCORDING TO TRISIGN FROM MOM),
C         BY CHECKING THE EDGES IN EACH TRIANGLES ONE BY ONE
          DO II=1,3
          IF(EG1.EQ.TRIEDGS(I,II))NE1=II
          IF(EG2.EQ.TRIEDGS(I,II))NE2=II
          IF(EG3.EQ.TRIEDGS(I,II))NE3=II
          ENDDO
          IF(TRISIGN(I,NE1).NE.0)THEN
            IF(TRISIGN(I,NE1).EQ.1)THEN
              IF(TAB2(P2).LT.TAB2(P3))THEN
                EDGSGN(TRIEDGS(I,NE1))=1
              ELSE
```

```
          EDGSGN(TRIEDGS(I,NE1))=-1
        ENDIF
      ELSE
        IF(TAB2(P2).GT.TAB2(P3))THEN
          EDGSGN(TRIEDGS(I,NE1))=1
        ELSE
          EDGSGN(TRIEDGS(I,NE1))=-1
        ENDIF
      ENDIF
    ENDIF
    IF(TRISIGN(I,NE2).NE.0)THEN
      IF(TRISIGN(I,NE2).EQ.1)THEN
        IF(TAB2(P3).LT.TAB2(P1))THEN
          EDGSGN(TRIEDGS(I,NE2))=1
        ELSE
          EDGSGN(TRIEDGS(I,NE2))=-1
        ENDIF
      ELSE
        IF(TAB2(P3).GT.TAB2(P1))THEN
          EDGSGN(TRIEDGS(I,NE2))=1
        ELSE
          EDGSGN(TRIEDGS(I,NE2))=-1
        ENDIF
      ENDIF
    ENDIF
    IF(TRISIGN(I,NE3).NE.0)THEN
      IF(TRISIGN(I,NE3).EQ.1)THEN
        IF(TAB2(P1).LT.TAB2(P2))THEN
          EDGSGN(TRIEDGS(I,NE3))=1
        ELSE
          EDGSGN(TRIEDGS(I,NE3))=-1
        ENDIF
      ELSE
        IF(TAB2(P1).GT.TAB2(P2))THEN
          EDGSGN(TRIEDGS(I,NE3))=1
        ELSE
          EDGSGN(TRIEDGS(I,NE3))=-1
        ENDIF
      ENDIF
    ENDIF
1500    CONTINUE
        CLOSE(4)
        END

        SUBROUTINE COMB2(NINTEDG,EDST,TAB3,EDGSGN,Z,A)
C GIVEN FEM: MATRIX A, NEW/OLD EDGE # CONTRAST TABLE EDST
C GIVEN BI:  MATRIX Z, NEW/OLD EDGE # CONTRAST TABLE TAB3
C GIVEN SIGN OF EDGES (ON DIE-SURFACE) CONTRAST TABLE EDGSGN
        PARAMETER (MAXEDGE=2000,MAXA=2000,MAXZ=2000)
        INTEGER TAB3(MAXEDGE), EDST(MAXA,2), EDGSGN(MAXZ)
        COMPLEX A(MAXA,MAXA),Z(MAXZ,MAXZ)
        DO I=1,NINTEDG
        M=TAB3(I)
        L=EDST(M,1)
        DO J=1,NINTEDG
        N=TAB3(J)
        K=EDST(N,1)
        A(L,K)=A(L,K)*EDGSGN(J)+Z(I,J)
        ENDDO
        ENDDO
        RETURN
        END
```