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FINAL REPORT

Background

The purpose of this research effort was to see if displacement or mixed version hp finite elements in time could be used to efficiently solve the rotor trim problem. At the end of the first year of effort it appeared the funding would end. Therefore, we prepared a "Final Report" that summarized the first year's effort. The effort included all of the linear results for flap and flap-lag with both displacement and mixed elements.

Second-Year Effort

However, it turned out that second-year funding did become available. Thus, the "Final Report" became a Semi-Annual Report. Therefore, the report given here summarizes the second year's effort which is the solution of the nonlinear trim problem. As it turns out, this was also successful. The conclusion of our work, therefore, states that finite elements in time are a good way to solve rotor trim problems for a moderate number of degrees of freedom. The technical content of this second-year effort follows.

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(NASA-CR-[illegible]) PERIODIC TRIM SOLUTIONS WITH hp-VERSION FINITE ELEMENTS IN TIME Final Report, Jul. [illegible] - Dec. [illegible] (Georgia Inst. of Tech.) [illegible] p CSCL 01C
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INTRODUCTION

Finite Element in Time has been proven to be a powerful alternative strategy for rotor craft trim problem. Recently, Finite Element Method in Time has been well developed in various versions like time-marching framework, Galerkin framework, Rayleigh-Ritz framework and mixed formulation. Dr. Peters and Mr. Hou applied this method on the rotorcraft trim problem to obtain linearized solutions and obtained a very good result. This project is aimed to expand the application in rotorcraft trim problem from linearized solution to nonlinear solution.

The rotorcraft trim problem is to find a period solution for period-coefficient, differential equations subject to side constraints that certain force and momentum balance equations have to be zero. There are certain free (or trim) parameters that must be chosen to meet these side constraints. A lot of successful works have been done by Dr. Peters and Mr. Hou in this area. They linearized the flap-lag equation and compared the results of Fourier series analysis, displace method and mixed method for accuracy and efficiency. Finite Element in Time presented a strong potential to outperform the conventional Fourier series analysis. Therefore, it is interesting to find out if Finite Element in Time can further provide a good flap-lag nonlinear solution. In this project, we use the displacement method for the hovering flight.

NONLINEAR EQUATION

The model which we use is the same as in Ref [1] [2]. The schematical model is shown in Fig 1. The reverse flow effect is neglected. We also assume that the drag coefficient C_{d_0} is small with respect to the lift curve slope α.

The euations of motion of this model are [2]:

$$\ddot{\beta} + \sin\beta \; \cos\,(1+\dot{\zeta})^2 + (P-1)(\beta - \beta_{pc}) + Z\,\zeta = \int_0^1 \overline{F}_\beta \,\bar{r}\, d\,\bar{r} \tag{1}$$

$$\cos^2\beta\,\ddot{\zeta} - 2\sin\beta\cos\beta\,(1+\dot{\zeta})\,\dot{\beta} + W\,\dot{\zeta} + Z\,(\beta - \beta_{pc}) = \cos\beta \int_0^1 \overline{F}_\zeta \,\bar{r}\, d\,\bar{r}$$

For hovering flight

$$\overline{F}_\beta = \frac{\gamma}{2}\left[\,\bar{u}_t^2 \sin\theta - \bar{u}_t\,\bar{u}_p\,(\cos\theta + \frac{C_{do}}{a})\right]$$

$$\overline{F}_\zeta = \frac{\gamma}{2}\left[\,\bar{u}_p^2\,(\cos\theta - \frac{1}{2}\frac{C_{do}}{a}) - \bar{u}_p\,\bar{u}_t \sin\theta - \bar{u}_t^2\,\frac{C_{do}}{a}\right]$$

$$\bar{u}_t = (1+\dot{\zeta})\,\bar{r}\cos\beta$$

$$\bar{u}_p = \bar{r}\,\dot{\beta} + \lambda\cos\beta$$

$$P = 1 + \frac{1}{\Delta}\left[\omega_\beta^2 + R\,(\omega_\zeta^2 - \omega_\beta^2)\sin^2\theta\right]$$

$$W = \frac{1}{\Delta}\left[\omega_\zeta^2 - R\,(\omega_\zeta^2 - \omega_\beta^2)\sin^2\theta\right]$$

$$Z = \frac{R}{2\Delta}\,(\omega_\zeta^2 - \omega_\beta^2)\sin 2\theta$$

$$\Delta = 1 + R'(1-R)\sin^2\theta\;(\omega_\zeta^2 - \omega_\beta^2)^2 /\; \omega_\zeta^2\,\omega_\beta^2$$

$$R = \frac{1/K_{\beta B} - 1/K_{\zeta B}}{(1/K_{\beta B} + 1/K_{\beta H}) - (1/K_{\zeta B} + 1/K_{\zeta H})}$$

$$R = \frac{1/K_{\beta B} - 1/K_{\zeta B}}{(1/K_{\beta B} + 1/K_{\beta H}) - (1/K_{\zeta B} + 1/K_{\zeta H})}$$

When $\theta = 0$ or $R = 0$, there is no elastic coupling between flap and inplane. Then

$$P = 1 + \omega_\beta^2 = p \qquad \omega = \omega_\zeta^2 \qquad Z = 0$$

Expanding θ as

$$\theta = \bar{\theta} + \theta_\beta(\beta - \beta_{pc}) + \theta_\zeta \zeta = (\bar{\theta} - \theta_\beta \beta_{pc}) + \theta_\beta \beta + \theta_\zeta \zeta$$

$$\bar{\theta} = \theta_0 + \theta_s \sin \Psi + \theta_c \cos \Psi$$

By small angle assumption

$$\cos \beta \approx 1 - \frac{1}{2}\beta^2 \qquad \cos^2\beta \approx 1 - \beta^2 \qquad \cos^3\beta \approx 1 - \frac{3}{2}\beta^2$$

$$\sin \beta \approx \beta \qquad \sin 2\beta \approx 2\beta$$

$$\cos \theta \approx 1 - \frac{1}{2}\theta^2 \qquad \sin \theta \approx \theta$$

Substituting all these terms into equation (1), we obtain the nonlinear equations for flap-lag forced response

$$[\mathrm{M}(\beta,\theta)]\begin{Bmatrix}\ddot{\beta}\\ \ddot{\zeta}\end{Bmatrix} + [\mathrm{C}(\dot{\beta},\dot{\zeta},\beta,\zeta,\theta)]\begin{Bmatrix}\dot{\beta}\\ \dot{\zeta}\end{Bmatrix} + [\mathrm{K}(\beta,\zeta,\theta)]\begin{Bmatrix}\beta\\ \zeta\end{Bmatrix} = \{\mathrm{F}\} \qquad (2)$$

Where:

$$[\mathrm{M}] = \begin{bmatrix} 1 & 0 \\ 0 & \cos^2\beta \end{bmatrix}$$

$$
[C] = \begin{bmatrix}
(\dot{\zeta}+2)\frac{\gamma}{16}\cos\beta(\cos\theta+\frac{C_{d_0}}{a}) & \begin{array}{c}\dot{\beta}\frac{\gamma}{16}\cos\beta(\cos\theta+\frac{C_{d_0}}{a}) \\ +(\dot{\zeta}+2)\left[\frac{1}{2}\sin 2\beta\,\frac{\gamma}{8}\cos^2\beta\sin\theta\right] \\ +\frac{\gamma}{6}\cos^2\beta\,\lambda(\cos\theta+\frac{C_{d_0}}{a})\end{array} \\
\begin{array}{c}\dot{\beta}\left[-\frac{\gamma}{8}\cos\beta(\cos\theta-\frac{1}{2}\frac{C_{d_0}}{a})\right] \\ +\dot{\zeta}(\frac{\gamma}{16}\cos^2\beta\sin\theta-\frac{1}{2}\sin 2\beta) \\ +\frac{\gamma}{8}\cos^2\beta\sin\theta \\ -\sin 2\beta\,\frac{\gamma}{3}\cos^2\beta\,\lambda(\cos\theta-\frac{C_{d_0}}{2a})\end{array} & \begin{array}{c}\dot{\beta}(\frac{\gamma}{16}\cos^2\beta\sin\theta-\frac{1}{2}\sin 2\beta) \\ +\dot{\zeta}\frac{\gamma}{8}\frac{C_{d_0}}{a}\cos^3\beta \\ +\frac{\gamma}{2}\cos^3\beta(\frac{\lambda}{3}\sin\theta+\frac{1}{2}\frac{C_{d_0}}{a})\end{array}
\end{bmatrix}
$$

$$
[K] = \begin{bmatrix}
\begin{array}{c}P-\frac{\gamma}{8}\theta_\beta \\ +\beta[\frac{\lambda}{8}\sin\theta-\frac{\gamma}{6}\lambda(\cos\theta+\frac{C_{d_0}}{a})] \\ -\frac{\gamma}{6}\lambda[\frac{1}{2}\theta_\beta^2\beta+\theta_\beta(\overline{\theta}-\theta_\beta\beta_{pc}) \\ +\frac{1}{2}\theta_\beta\theta_\zeta\zeta]\end{array} & \begin{array}{c}Z-\frac{\gamma}{8}\theta_\zeta \\ -\frac{\gamma}{6}\lambda[\frac{1}{2}\theta_\zeta^2\zeta+\theta_\zeta(\overline{\theta}-\theta_\beta\beta_{pc}) \\ +\frac{1}{2}\theta_\beta\theta_\zeta\zeta]\end{array} \\
\begin{array}{c}Z+\frac{\gamma}{6}\lambda\theta_\beta \\ +\frac{3}{2}\beta[\frac{\gamma}{4}\lambda^2(\cos\theta-\frac{1}{2}\frac{C_{d_0}}{a}) \\ -\frac{\gamma}{6}\lambda\sin\theta-\frac{\gamma}{8}\frac{C_{d_0}}{a}] \\ +\frac{\gamma}{4}\lambda^2[\frac{1}{2}\theta_\beta^2\beta+\theta_\beta(\overline{\theta}-\theta_\beta\beta_{pc}) \\ +\frac{1}{2}\theta_\beta\theta_\zeta\zeta]\end{array} & \begin{array}{c}W+\frac{\gamma}{6}\lambda\theta_\zeta \\ +\frac{\gamma}{4}\lambda^2[\frac{1}{2}\theta_\zeta^2\zeta+\theta_\zeta(\overline{\theta}-\theta_\beta\beta_{pc}) \\ +\frac{1}{2}\theta_\beta\theta_\zeta\beta]\end{array}
\end{bmatrix}
$$

$$
\{F\} = \begin{Bmatrix}
\beta_{pc}(P-1)+\frac{\gamma}{8}(\overline{\theta}-\theta_\beta\beta_{pc})-\frac{\gamma}{6}\lambda[1-\frac{1}{2}(\overline{\theta}-\theta_\beta\beta_{pc})^2+\frac{C_{d_0}}{a}] \\
\beta_{pc}Z+\frac{\gamma}{4}\lambda^2[1-\frac{1}{2}(\overline{\theta}-\theta_\beta\beta_{pc})^2-\frac{1}{2}\frac{C_{d_0}}{a}]-\frac{\gamma}{6}\lambda(\overline{\theta}-\theta_\beta\beta_{pc})-\frac{\gamma}{8}\frac{C_{d_0}}{a}
\end{Bmatrix}
$$

The constraint equations for trim condition are

$$
\frac{1}{2\pi}\int_0^{2\pi}\beta\,dt=\beta_0 \tag{3}
$$

$$\frac{1}{\pi}\int_0^{2\pi} \beta \cos t \, dt = 0 \tag{4}$$

$$\frac{1}{\pi}\int_0^{2\pi} \beta \sin t \, dt = 0 \tag{5}$$

When thrust is specified, equation (3) can be replaced by

$$\frac{1}{2\pi}\int_0^{2\pi} C_T(t)\, dt = C_{T_0} \tag{6}$$

Where

$$C_T(t) = \int_0^1 \frac{abc}{2\pi R}\left[(x+\mu \sin t)^2\theta - (x+\mu \sin t)(\lambda + x\dot{\beta}) + \mu\beta \cos t\right] dx$$

For hovering, $\mu=0$

$$C_T(t) = \int_0^1 \frac{abc}{2\pi R}\left[x^2(\theta - \dot{\beta}) - x\lambda\right] dx$$

DISPLACEMENT METHOD [1]

For displacement method, we begin with the Hamilton's law of varying action

$$\int_{t_i}^{t_f} (\delta L + \delta q^T Q_{nc})\, dt - \sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} \delta q_i \Big|_{t_i}^{t_f} = 0 \tag{7}$$

With a weak constraint of momentum, the hamilton's weak principle becomes

$$\delta \int_{t_i}^{t_f} L\, dt + \int_{t_i}^{t_f} \delta W_{nc} dt - \sum_{i=1}^{n} \delta q_i \cdot P_i \Big|_{t_i}^{t_f} = 0 \tag{8}$$

Where the trailing term

$$\sum_{i=1}^{n} \delta q_i \cdot P_i \Big|_{t_i}^{t_f}$$

can be thought of as the virtual action δA entering (or leaving) the system at time t_i and t_f

For a typical spring-damp-mass oscillator with unit mass

$$L = T - V = \frac{1}{2}\dot{x}^2 - \frac{1}{2}k\,x^2$$

$$\delta W_{nc} = (F - C\,\dot{x})\,\delta x$$

Then equation (8) becomes

$$\int_{t_i}^{t_f} (\ddot{x} + c\,\dot{x} + k\,x - F)\,\delta x\,d\,t - \left[(P_f - \dot{x}(t_f))\delta x - (P_i - \dot{x}(t_i))\delta x\right] = 0 \qquad (9)$$

For this equation to be valid for any δx, we must have

$$\ddot{x} + c\,\dot{x} + k\,x = F$$

$$P_f = \dot{x}(t_f)$$

$$P_i = \dot{x}(t_i)$$

for periodic problem, t_i=0, t_f=T, $x(0)$=$x(T)$ is a strong condition and $\dot{x}(0)$=$\dot{x}(T)$ is a weak condition enforced through P_0=P_T. By introducing Lagrange multiplier, λ , equation (8) becomes

$$\int_0^T (\dot{x}\delta\dot{x} - C\dot{x}\,\delta x - kx\,\delta x + F\,\delta x)\,d\,t - \lambda_P[\,\delta x(T) - \delta x(0)\,] = 0 \qquad (10)$$

where λ_P=P_0=P_T

For the test and trial functions, there are certain geometric boundary condition which must be satisfied[1].

STRATEGY

To solve flap-lag problem by displacement method, we use Galerkin's scheme. first, we choose the integrals of legendre polynomials $P_j(x)$ as shape functions for β, ζ, $\delta\beta$ and $\delta\zeta$ [4]

$$\beta = \sum_{j=1}^{n} \Phi_j(\eta)\, q_j \qquad \zeta = \sum_{j=1}^{n} \Phi_j(\eta)\, p_j$$
$$\delta\beta = \sum_{j=1}^{n} \Phi_j(\eta)\, \delta q_j \qquad \delta\zeta = \sum_{j=1}^{n} \Phi_j(\eta)\, \delta p_j \qquad (11)$$

$$\phi_1 = \frac{1-\eta}{2} \qquad \phi_2 = \frac{1+\eta}{2} \qquad (-1 \le \eta \le 1)$$

$$\phi_{j+1} = \sqrt{\frac{2j-1}{2}} \int_{-1}^{\eta} P_{j-1}(x)\, dx \qquad j = 2,3,4,\cdots,n-1$$

A. One Element Quasi-Trim Case

For this case, the collective and cyclic pitch angles are given, $\theta = \theta_0 + \theta_s \sin t + \theta_c \cos t$, so that $\beta_c , \beta_s \approx 0$ to minimize rotor hub moment. Multiplying equation (2) by ϕ_i $(i=1,2,\cdots,n)$ on both sides and integrating over $[0,2\pi]$

$$\int_0^{2\pi} [\ddot{\beta}\, \phi_i + C_{11}\dot{\beta}\, \phi_i + C_{12}\dot{\zeta}\, \phi_i + K_{11}\beta\, \phi_i + K_{12}\zeta\, \phi_i]\, dt = \int_0^{2\pi} F_1 \phi_i\, dt$$

$$\int_0^{2\pi} [\cos^2\beta\ddot{\zeta}\,\phi_i + C_{21}\dot{\beta}\,\phi_i + C_{22}\dot{\zeta}\,\phi_i + K_{21}\beta\,\phi_i + K_{22}\zeta\,\phi_i]\,dt = \int_0^{2\pi} F_2\phi_i\,dt$$

Set

$$\dot{\beta}(2\pi) = \dot{\beta}(0) = P_\beta = \lambda_\beta$$

$$\dot{\zeta}(2\pi)\cos^2\beta(2\pi) = \dot{\zeta}(0)\cos^2\beta(0) = P_\zeta = \lambda_\zeta$$

change variables

$$t = \pi(\tau + 1) \qquad -1 \le \tau \le 1$$

$$\dot{(\,)} = \frac{d}{dt} = \frac{1}{\pi}\frac{d}{d\tau} = \frac{1}{\pi}(\,)'$$

After integrating by parts, we get 2n nonlinear equations of q_i, p_i, λ_β and λ_ζ $(i=1,\ 2,\cdots,n)$. The other two equations which are required to close the system come from the periodic constraint

$$\beta(0) = \beta(T) \qquad \zeta(0) = \zeta(T)$$

$$\Rightarrow \sum_{i=1}^{n} [\phi_i(-1) - \phi_i(1)]q_i = 0 \qquad \sum_{i=1}^{n} [\phi_i(-1) - \phi_i(1)]q_i = 0$$

The final system of nonlinear equations is

$$\left[\,A(\beta,\zeta,\beta',\zeta')\,\right]_{2n+2,\,2n+2}\begin{Bmatrix} q_1 \\ \vdots \\ q_n \\ p_1 \\ \vdots \\ p_n \\ \lambda_\beta \\ \lambda_\zeta \end{Bmatrix}_{2n+2,\,1} = \{B\}_{2n+2,\,1}$$

The matrix [A] includes β, ζ, β' and ζ', which are functions of unknown variables q_i and p_i. Therefore a iteration process is required. The first guess for q_i and p_i is not difficult. In fact, the stability of this system of nonlinear equations is tremendous. Even if the initial values of q_i and p_i. are set to be equal to 100 (although there is no physical meaning), they will converge to the correct result within a few steps.

B. One Element Trim case

In this case θ_0, θ_c and θ_s are unknowns to be found such that $\beta_c = \beta_s = 0$, So we have $2n+5$ unknowns. Notice that in equation (2), the θ_0, θ_c and θ_s are included in {F}. We need to extract them from {F} and move them to the left hand side. The equations then become

$$[\,M(\beta,\theta)\,]\begin{Bmatrix}\ddot{\beta}\\ \ddot{\zeta}\end{Bmatrix}+\left[\,C(\dot{\beta},\dot{\zeta},\beta,\zeta,\theta)\,\right]\begin{Bmatrix}\dot{\beta}\\ \dot{\zeta}\end{Bmatrix}+\left[\,K(\beta,\zeta,\theta)\,\right]\begin{Bmatrix}\beta\\ \zeta\end{Bmatrix}$$

$$+\left\{T(\bar{\theta})\right\}\theta_0+\sin t\left\{T(\bar{\theta})\right\}\theta_s+\cos t\left\{T(\bar{\theta})\right\}\theta_c=\{F_T\}$$

Where

$$\left\{T(\bar{\theta})\right\}=\begin{Bmatrix}\frac{\gamma}{6}\lambda\,(\theta_\beta\,\beta_{pc}-\frac{1}{2}\bar{\theta})-\frac{\gamma}{8}\\ \frac{\gamma}{6}\lambda-\frac{\gamma}{4}\lambda^2(\theta_\beta\beta_{pc}-\frac{1}{2}\bar{\theta})\end{Bmatrix}$$

$$\{F_T\}=\begin{Bmatrix}\beta_{pc}(\,P-1\,)-\frac{\gamma}{8}\theta_\beta\,\beta_{pc}-\frac{\gamma}{6}\lambda\,(\,1-\frac{1}{2}\,\theta_\beta^2\,\beta_{pc}^2+\frac{C_{d_0}}{a}\,)\\ \beta_{pc}Z+\frac{\gamma}{4}\lambda^2(1-\frac{1}{2}\theta_\beta^2\,\beta_{pc}^2-\frac{1}{2}\frac{C_{d_0}}{a})+\frac{\gamma}{6}\lambda\theta_\beta\,\beta_{pc}-\frac{\gamma}{8}\frac{C_{d_0}}{a}\end{Bmatrix}$$

Following the same scheme, we will obtain $2n+2$ equations. The trim condition. provides the extra 3 equations to close the system.

C. Multi Element Trim Case

We divide $[0,2\pi]$ into M elements, each has a length of $2\pi/M$, Within the mth element let

$$\beta^{(m)} = \sum_{j=1}^{n} \Phi_j(\eta)\, q_j^{(m)} \qquad \zeta^{(m)} = \sum_{j=1}^{n} \Phi_j(\eta)\, p_j^{(m)} \qquad m=1,2,\cdots,M$$

By continuity condition

$$\beta^{(m)}(t_{m+1}) = \beta^{(m+1)}(t_{m+1}) \qquad \zeta^{(m)}(t_{m+1}) = \zeta^{(m+1)}(t_{m+1}) \qquad m=1,2,\cdots,M-1$$

By periodic condition

$$\beta^{(1)}(t_1) = \beta^{(M)}(t_M) \qquad \zeta^{(1)}(t_1) = \zeta^{(M)}(t_M)$$

Also we set the nodal momenta to be

$$\dot\beta^{(m)}(t_m) = \dot\beta^{(m-1)}(t_m) = \lambda_\beta^{(m)} \qquad m = 2,3,\cdots, M$$

$$\dot\zeta^{(m)}(t_m)cos^2\beta^{(m)}(t_m) = \dot\zeta^{(m-1)}(t_m)cos^2\beta^{(m-1)}(t_m) = \lambda_\zeta^{(m)}$$

$$\dot\beta^{(1)}(t_1) = \dot\beta^{(M)}(t_{M+1}) = \lambda_\beta^{(1)}$$

$$\dot\zeta^{(1)}(t_1)cos^2\beta^{(1)}(t_1) = \dot\zeta^{(M)}(t_{M+1})cos^2\beta^{(M)}(t_{M+1}) = \lambda_\zeta^{(1)}$$

Then the trailing terms in equation (12) become

$$\sum_{m=1}^{M-1} [\lambda_\beta^{(m)} \phi_i(-1) - \lambda_\beta^{(m+1)} \phi_i(1)] + \lambda_\beta^{(M)} \phi_i(-1) - \lambda_\beta^{(1)} \phi_i(1)$$

$$\sum_{m=1}^{M-1} [\lambda_\zeta^{(m)} \phi_i(-1) - \lambda_\zeta^{(m+1)} \phi_i(1)] + \lambda_\zeta^{(M)} \phi_i(-1) - \lambda_\zeta^{(1)} \phi_i(1)$$

The trim constraint for multi elements with $(N-1)^{th}$ order polynomial are [1]

$$\sum_{m=1}^{M} \int_{-1}^{1} \sum_{j=1}^{N} q_j^{(m)} \phi_j \cos\left[\frac{\pi}{M}(\tau + 2m - 1)\right] d\tau = 0$$

$$\sum_{m=1}^{M} \int_{-1}^{1} \sum_{j=1}^{N} q_j^{(m)} \phi_j \sin\left[\frac{\pi}{M}(\tau + 2m - 1)\right] d\tau = 0$$

$$\sum_{m=1}^{M} \sum_{j=2}^{N} q_j^{(m)} \int_{-1}^{1} \left\{ \left[\frac{1}{3} + \frac{\mu}{2} \sin\left(\frac{\pi}{M}(\tau + 2m - 1)\right)\right] \phi_j' + \frac{\pi}{2M} \mu^2 \sin\left[\frac{2\pi}{M}(\tau + 2m - 1)\right] \phi_j \right\} d\tau$$

$$-\frac{2}{3} \pi \theta_0 \left(1 + \frac{3}{2}\mu^2\right) - \mu \pi \theta_s = -\frac{4 \pi C_{T_0}}{a \sigma} - \pi \lambda$$

The final system of nonlinear equations includes $2M(N+1)+3$ unknowns. They are $q_i^{(m)}, p_i^{(m)}, \lambda_\beta^{(m)}, \lambda_\zeta^{(m)}, \theta_0, \theta_c$ and θ_s.

RESULT AND CONCLUSION

The results are very close to those obtained by other methods. This varifies the validity of this method. As far as the flap lag problem is concerned, there is no restriction on the choice of initial values. They always converge to the final results within a few iteration steps. Also notice that the results are almost independent of the number of elements and the order of Legendre polynomials. Therefore we can conclude that, in solving this kind of nonlinear problem, the displacement method has a very good stability. Since the flap lag problem in hovering flight is a very much simplified case compared with the flap lag problem in forward flight, additional work is required to investigate the performance of displacement method in forward flight condition. It is also a interesting topic to see how the mixed method behaves in such cases.

no.Elements/pol.order		$\beta/100$	$\zeta/1000$	θ_0	$\theta_s/1000000$	$\theta_c/1000000$
1	4	9.6825644	-6.3044904	0.2970519	1.9670248	-1.7705892
1	6	9.6825607	-6.3045006	0.2970519	1.6492519	-1.1021315
1	8	9.6825600	-6.3044997	0.2970519	1.6540506	-1.0808242
2	4	9.6825629	-6.3045016	0.2970519	2.5363567	-1.4900319
2	6	9.6825629	-6.3045030	0.2970519	1.1276192	-9.1521546
2	8	9.6825622	-6.3045025	0.2970519	9.1574357	-1.0367157
5	4	9.6825719	-6.3045076	0.2970521	1.0911533	-9.4596531
8	4	9.6825704	-6.3045053	0.2970520	-6.8381580	2.0519121
8	8	9.6825697	-6.3045057	0.2970520	-1.6251148	-3.1796359

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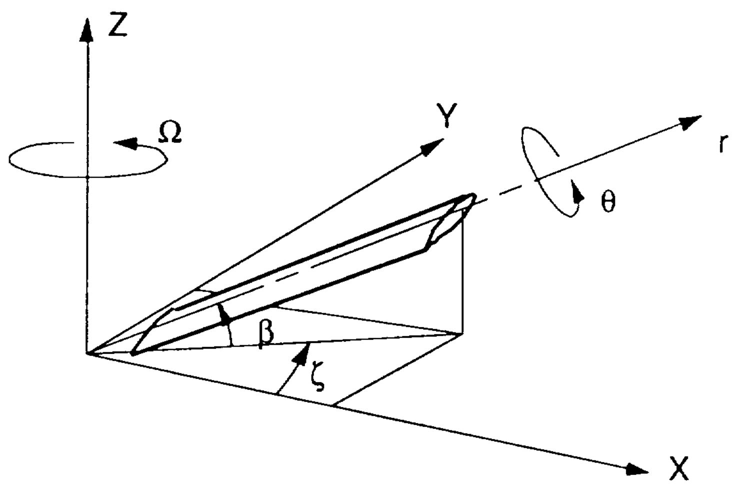

Fig. 1