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Chapter II

A Variational Assimilation Method for Satellite and Conventional Data: MODEL II (Version 1)

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1. Introduction

The MODEL II variational data assimilation model is the second of the four variational models designed to blend diverse meteorological data into a dynamically constrained data set. MODEL II differs from the MODEL I developed during Phase I in that it includes the thermodynamic equation as the fifth dynamical constraint.

Thus MODEL II includes all five of the primitive equations that govern atmospheric flow for a dry atmosphere. The reason for delaying the introduction of the thermodynamic equation until MODEL II is as follows. Courant (1936) showed that the number of subsidiary conditions (dynamic constraints) must be at least one less than the number of adjustable dependent variables. The five primitive equations form a closed set of equations with five dependent variables. Inclusion of the same number of constraints as dependent variables overdetermines the problem and a solution is not guaranteed. Achtemeier (1975) attempted to circumvent this problem through a parameterization of the tendency terms of the velocity components and the temperature that required the exact solution of the integrated continuity equation. This method, a variational adjustment within a variational adjustment, was considered a failure after an extensive analysis (Achtemeier, 1979) found unrealistically large velocity component tendencies where actual velocity changes over a 12-hr period were small.

The approach taken in the development of MODEL I was to make possible the inclusion of the five primitive equations by increasing the number of dependent variables. We defined two new dependent variables, the developmental components of the horizontal velocity tendencies, which increased the number of dependent variables from five to seven. Though this solves the problem of the number of subsidiary conditions, the extent of internal coupling among the variables and within the equations could not be determined fully until the development and evaluation of MODEL II.

2. MODEL II: Thermodynamic Equation as a Dynamic Constraint

Upon defluxing and omitting the dissipation term of the thermodynamic equation in Anthes and Warner (1978), the thermodynamic equation as it appears as a dynamical constraint in MODEL II is,

$$\frac{\partial T}{\partial t}+m\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\dot{\sigma}\frac{\partial T}{\partial \sigma}-\frac{RT\omega}{c_p p}-\frac{Q}{c_p}=0 \tag{1}$$

The omega-term (term 4) of the thermodynamic equation can be transformed into the nonlinear sigma coordinate system through the definition,

$$\sigma=\beta(p-p^*)^3+\sigma^*\frac{p-p_u}{p^*-p_u} \tag{2}$$

where the superscript, *, and the subscript, u, identify, respectively, the variables at the reference pressure level and at the top of the model atmosphere. For more information on the nonlinear vertical coordinate system, refer to Appendix A.1. Furthermore,

$$\beta = [1-\sigma^* \frac{p_s - p_u}{p^* - p_u}]\,(p_s - p^*)^{-3} \qquad \textbf{(3)}$$

where the subscript, s, refers to quantities measured at the surface. We differentiate (2) with respect to time. If

$$\alpha = \frac{\sigma^*}{p^* - p_u} \qquad \textbf{(4)}$$

and

$$J = [3\beta\,(p-p^*)^2 + \alpha]\,(p-p^*) \qquad \textbf{(5)}$$

then we may define two coefficients such that

$$q_3 = \frac{p-p^*}{Jp} \qquad \textbf{(6)}$$

and

$$q_4 = \frac{J_s}{Jp}\left(\frac{p-p^*}{p_s-p^*}\right)^4 \qquad \textbf{(7)}$$

for $p>p^*$, and

$$q_3=\frac{1}{\alpha p}=\frac{p^*-p_u}{\sigma^* p} \tag{8}$$

and

$$q_4=0 \tag{9}$$

for $p < p^*$.

The thermodynamic equation in the nonlinear sigma coordinates is, upon substitution for the omega-term,

$$\frac{\partial T_W}{\partial t}+m(u\frac{\partial T_W}{\partial x}+v\frac{\partial T_W}{\partial y}+\dot{\sigma}\frac{\partial T_W}{\partial \sigma}-\frac{RT\omega}{c_p}(q_3\dot{\sigma}+q_4\omega_s)-\frac{Q}{c_p}=0 \tag{10}$$

Here the subscript, W, refers to the whole temperature, $T_W = T_R + T$, where T_R is a reference temperature for the layer and is always in hydrostatic balance and T is the departure from the reference temperature that is subject to adjustment within the variational model. Substitution for the whole temperature yields the thermodynamic equation in the adjustable part of the temperature,

$$\frac{\partial T}{\partial t}+m(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})+\dot{\sigma}\frac{\partial T}{\partial \sigma}-\frac{R}{c_p}(T_R-T)(q_3\dot{\sigma}+q_4\omega_s)+\dot{\sigma}\frac{\partial T_R}{\partial \sigma}-\frac{Q}{c_p}=0 \tag{11}$$

Now nondimensionalize the thermodynamic equation. Letting

$$u=Uu',\ v=Uv',\ \Delta t=(L/C)\Delta t',\ \Delta x=L\Delta x',$$
$$T_R=\theta T_R'=(gH/R)T_R',\ \Delta T=(gH/R)(F/R_o)\delta T' \qquad (12)$$
$$p=Pp',\ \dot\sigma\sim(C/L)\dot\sigma',\ \omega_s\sim(PC/L)\omega_s'$$

and dividing through by $(C/L)(gH/R)(F/R_o^2)$, the nondimensionalized thermodynamic equation with primes suppressed is,

$$\begin{aligned}&R_o\left[\frac{\partial T}{\partial t}+m\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\dot\sigma\frac{\partial T}{\partial\sigma}\right]\\&-R_o\frac{R}{C_p}\left(\frac{R_o}{F}T_R'+T\right)(q_3\dot\sigma+q_4\omega_s)-\left[\frac{LRR_o^2}{CgHF}\right]\frac{Q}{C_p}=0\end{aligned} \qquad (13)$$

Dividing by the additional R_o renders (13) into the same order of magnitude as the other dynamic equations of MODEL II. In addition, it can be shown that the two terms that include T_R combine to form the static stability,

$$\sigma_\sigma=\frac{R_o^2}{F}\left[\frac{\partial T_R'}{\partial\sigma}-q_3\frac{R}{C_p}T_R'\right] \qquad (14)$$

Therefore, the thermodynamic equation reduces to

$$\begin{aligned}&R_o\left[\frac{\partial T}{\partial t}+m\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\dot\sigma\left(\frac{\partial T}{\partial\sigma}-\frac{R}{C_p}Tq_3\right)\right]\\&+\dot\sigma\sigma_\sigma-\left[\frac{LRR_o^2}{CgHF}\right]\frac{Q}{C_p}-R_oq_4\omega_s\frac{R}{C_p}\left(\frac{R_o}{F}T_R+T\right)=0\end{aligned} \qquad (15)$$

Next, the thermodynamic equation is converted to finite differences and made compatible with the Arakawa D-grid finite difference template developed for MODEL I (Achtemeier, et al., 1986). Fig. 1 shows the template with the locations of the variables that appear in the thermodynamic equation. Note that the local tendency of temperature has been defined as the dependent and adjustable variable, E_T. The finite difference version of the thermodynamic equation is,

$$
\begin{aligned}
&R_o[E_T+(\overline{m})^{xy}(\overline{u})^{x\sigma}(\overline{T_x})^{y}+(\overline{m})^{xy}(\overline{v})^{y\sigma}(\overline{T_y})^{x}\\
&+\dot{\sigma}[(\overline{T_\sigma})^{xy\sigma}-\frac{R}{c_p}(\overline{q_3})^{xy}(\overline{T})^{xy})]]+\sigma_\sigma\dot{\sigma}\\
&-R_o(\overline{q_4})^{xy}\omega_s\frac{R}{c_p}[\frac{R_o}{F}(\overline{T_R})+(\overline{T})^{xy}]-[\frac{LRR_o^2}{CgHF}]\frac{Q}{c_p}=0
\end{aligned}
\qquad (16)
$$

where the various overbar averages are defined in Achtemeier, et al., (1986).

3. MODEL II. Variational Equations

The variational analysis melds data from various measurement systems at the second stage of a two-stage objective analysis. All data are gridded independently in the first stage and are combined in the second stage. The gridded observations to be modified are meshed with the dynamic constraints through Sasaki's (1970) variational formulation which requires the minimization of the integrand of an adjustment functional. Now it is not necessary to reproduce the full derivation of MODEL I plus the thermodynamic equation in order to get MODEL II. Each term of the equations is a linear combination with the other terms. Therefore, all that is required is to perform the variational operations upon the thermodynamic equation and add the resulting terms to the appropriate adjustment equations of MODEL I. Let,

$$I=2\lambda_5 m_5+\pi_8\left(E_T-E_T^{o}\right)^2 \tag{17}$$

where π_8 is the precision modulus weight for the temperature tendency and m_5 is equation (16). Performing the variations upon each of the dependent variables that appear in the thermodynamic equation yields the following terms to be added to the respective variational equations.

$$\delta E_T=\pi_8\left(E_T-E_T^{o}\right)+R_o\lambda_5=0 \tag{18}$$

$$\delta u=R_o(\overline{m})^{y}\left(\overline{\lambda_5(\overline{T_x})^{y}}\right)^{x\sigma} \tag{19}$$

$$\delta v=R_o(\overline{m})^{x}\left(\overline{\lambda_5(\overline{T_y})^{x}}\right)^{y\sigma} \tag{20}$$

$$\delta\dot{\sigma}=R_o\lambda_5\left[(\overline{T_\sigma})^{xy\sigma}-\frac{R}{C_p}(\overline{q_3})^{xy}(\overline{T})^{xy}\right]+\lambda_5\sigma_\sigma \tag{21}$$

$$\begin{aligned}\delta\overline{T}=&-R_o\left[(\overline{m})^{x}(\overline{u})^{x}(\overline{\lambda_5})^{y\sigma}\right]_x-R_o\left[(\overline{m})^{y}(\overline{v})^{y}(\overline{\lambda_5})^{x\sigma}\right]_y\\&-R_o\left[(\overline{\dot{\sigma}\lambda_5})^{xy\sigma}\right]_\sigma-R_o\frac{R}{C_p}\left[(\overline{\dot{\sigma}\lambda_5})^{xy}q_3+(\overline{\omega_s\lambda_5})^{xy}q_4\right]\end{aligned} \tag{22}$$

Table 1 summarizes the modifications of the existing MODEL I equations that are required to implement MODEL II. The first column labeled "variable referenced" locates the variable in the grid templet shown in Fig. 1 to which the new terms are referenced. For example, the new terms to be added to the existing function F_1 (first line in Table 1) are calculated for the location of u in Fig. 1. Also included are two new equations, the latter being the thermodynamic equation. This brings to 13 the number of linear and nonlinear equations to be solved.

4. MODEL II: Evaluation

The purpose of this section is to demonstrate whether MODEL II performs as predicted by theory. In our evaluation of the variational assimilation models, we have used three criteria which have found use in the verification of diagnostic analyses (Krishnamurti, 1968; Achtemeier, 1975; Otto-Bliesner, *et al.*, 1977). These criteria are measures of, first, the extent to which the assimilated fields satisfy the dynamical constraints, second, the extent to which the assimilated fields depart from the observations, and third, the extent to which the assimilated fields are realistic as determined by pattern recognition. The last criterion requires that the signs, magnitudes, and patterns of the hypersensitive vertical velocity and local tendencies of the horizontal velocity components be physically consistent with respect to the larger scale weather systems.

The strong constraint formalism requires that the dynamical constraints; the nonlinear horizontal momentum equations, the hydrostatic equation, an integrated form of the continuity equation, and the thermodynamic equation be satisfied exactly (to within truncation). Therefore, it is appropriate that the first evaluation of the variational model determine whether indeed the adjusted fields of meteorological variables are solutions of these physical equations.

In solving the Euler-Lagrange equations, we substituted observed or previously adjusted variables into the nonlinear terms and other terms that are products with the Rossby number or are higher order terms and treated these terms as forcing functions. This approach made the linearized equations easier to solve but several cycles with the forcing terms updated with newly adjusted variables were required for the method to converge to a solution.

The technique for determining whether the method converges to a solution is as follows. First, we note that any variable is found from the algebraic sums of all other terms of an equation. Thus the residual obtained by substituting variables back into the equation will be identically zero - the equation is satisfied exactly. This does not mean that the variational method has converged. Entirely different values for all of the variables may be found at the next cycle. Therefore, the adjusted variables are averaged over two successive cycles. Then the averaged variables are reintroduced into the dynamic constraints. Residuals are computed as remainders of algebraic sums of the terms of each constraint. The root-mean-squares (RMS) of these differences (Glahn and Lowry, 1972) vanish when variables at two successive cycles are unchanged. When this happens, the constraints are satisfied and the method has been judged to converge to a solution. A convenient measure of how rapidly the method is converging to a solution is the percent reduction of the initial unadjustment given by,

$$\Delta r(\%) = 100\left(1 - \frac{r^{o} - r^{\tau}}{r^{o}}\right) \tag{23}$$

The performance of MODEL II is assessed through the percentage reductions in the RMS differences from the initial unadjustments through the first four cycles of the solution sequence. The calculations are done for the eight adjustable levels in the model. Table 2 shows the percentages for the two nonlinear horizontal momentum equations. These results compare favorably with the MODEL I percentage residual reductions. The initial unadjustments are approximately halved at each cycle to about 90 percent after four cycles.

The percentage reductions of the initial unadjustment for the integrated continuity and hydrostatic equations are shown in Table 3. The RMS differences for the integrated continuity equation are reduced by from 96 to 99 percent at the second cycle and improve slowly to near 100 percent by the fourth cycle. These improvements are, of course, dependent upon the magnitudes of the initial unadjustment. We set the initial vertical velocity to zero. Then the initial unadjustment is equal to the divergence integrated upward. The MODEL I cyclical solution order subjects the adjusted velocity components to a second adjustment to satisfy the integrated continuity equation. In this case, the averages of the adjusted velocity components are just averages of two solutions of the integrated continuity equation. Therefore the unadjustment should approach zero by the second cycle.

The initial unadjustments for the hydrostatic equation at levels 4 through 8 are halved at each cycle and the percentage reduction increases to near 94 percent by the fourth cycle. Convergence is much slower at levels 1 and 2. There is a 65 percent reduction in the initial unadjustment at the second cycle at level 2. There is no change during the third cycle and a slight increase in the initial unadjustment is observed at cycle 4. Given that the only difference between the adjustments presented here and the adjustments presented for MODEL I is the introduction of the fifth constraint, we are led to suspect that the degradation is directly related to the thermodynamic equation.

Table 4 gives the percentage reductions of the initial unadjustment for the thermodynamic equation. Negative percentages occur where the RMS differences exceed the initial unadjustment. Table 4 shows that the initial unadjustment was reduced by nearly 90 percent by the fourth cycle at levels 2 and 9. At the remaining levels, first cycle reductions of from 48 to 63 percent were followed by increases in the RMS differences that by the fourth cycle exceeded the initial unadjustment at levels 6 and 7.

Further analysis of the behavior of the convergence of MODEL II has revealed the following:

1. The breakdown in the assimilation was almost exclusively in temperature. The initial unadjustments in the horizontal momentum equations and the continuity equation were reduced as was done with MODEL I. Only the first two levels in the hydrostatic equation showed any response to the temperature unadjustment and this was somewhat unexpected given that the most severe departures from convergence in the thermodynamic equation occurred at higher levels.

2. The patterns of winds and heights generated by MODEL II (not shown) were unchanged from the winds and heights generated by MODEL I. The pattern analysis was an additional confirmation that the breakdown in convergence in MODEL II was largely confined to the thermodynamic equation.

3. The initial unadjustment in the thermodynamic equation was found to be approximately an order of magnitude larger than the initial unadjustments for the other dynamic constraints and was approximately two orders of magnitude larger in the stratosphere. Although this is not the cause for the breakdown in convergence, it does show that a gross imbalance existed in the initial gridded fields of meteorological variables when those variables were substituted into the thermodynamic

equation.

4. Analysis of the patterns of the residuals remaining after the fourth pass found that they were almost identical to the patterns of vertical velocity.

Our analysis of the large RMS differences in the thermodynamic equation remaining after four cycles reveals the following concerning how the initial and adjusted vertical velocity adversely impacted upon the analyses. First, the initial vertical velocity was calculated kinematically and subjected to the variational adjustment by O'Brien (1970). This method can transfer error from the lower levels into the upper levels of the troposphere and generate large and noisy vertical velocity patterns there. Furthermore, there is no consideration given for the change in static stability between the troposphere with its relatively large vertical velocities and the stratosphere with its relatively small vertical velocities. The kinematic vertical velocities were unrealistically large in the stratosphere and, when coupled with the large static stability, produced large and uncompensated terms in the thermodynamic equation. Therefore, the magnitudes of the initial unadjustments were approximately two orders of magnitude larger than were the initial unadjustments for the other dynamical constraints.

Second, further theoretical analysis has revealed that the adjustment for the divergent part of the wind is the "weak link" in this variational assimilation model. First order terms that contain the divergence adjustment cancel out in the cyclical solution formulations. The divergence adjustment must then be carried in second order terms and through other variables. Our solution for this problem has been to require the adjusted horizontal velocity components to satisfy the continuity equation constraint after each cycle, a variational model within a variational model, then allow for the second order terms and the readjusted velocity components to "nudge" the solution toward the desired dynamic balance. The result was that the RMS differences grew after the first cycle when the vertical velocity was released to converge slowly toward another equilibrium.

5. Coupling the Vertical Velocity in MODEL I.

In this section, we propose solutions for the vertical velocity related problems of very large initial unadjustments for the thermodynamic equation and the buildup of RMS differences in MODEL II.

The solution for the problem of very large initial unadjustments in the thermodynamic equation is the implementation of a blended vertical velocity algorithm such as the variational method presented by Chance (1986). This method, developed as part of this variational assimilation project but not included in the version of MODEL II evaluated as part of this study, blends the divergence of the horizontal wind with the vertical velocity calculated from the adiabatic method. The relative weighting given the horizontal and the vertical velocity is a function of the stability, relative humidity, and satellite observed cloud cover. The divergence of the horizontal wind receives the greatest weight when the conditions of low stability, near saturation, or dense cloud cover at levels with near saturation prevail. The adiabatic vertical velocity receives greatest weight at locations where stability is high. Division by large stability reduces the magnitude of the vertical velocity in the stratosphere and forces the vertical velocity to near zero at the tropopause rather than at the arbitrarily defined top of the model domain.

The formula for the modified vertical velocity is

$$\dot{\sigma}_M(k) = \frac{\dot{\sigma}_M(k-1) + D(k)\,\Delta\sigma + a\dot{\sigma}_T(k)}{1-a} \tag{24}$$

The modified vertical velocity at level k is the weighted sum of the modified vertical velocity at level k-1 plus the incremental vertical velocity obtained through the continuity equation and the vertical velocity obtained by the adiabatic method. The weight, a, carries the theoretical relative accuracies of the two methods for calculating vertical velocity as obtained through standard errors of observation for the observed variables. The weight also carries the relative importance of the vertical velocities as determined by meteorological considerations. For example, the adiabatic vertical velocities are assigned the greatest weight in the stratosphere because the adiabatic method carries information regarding static stability,

$$\sigma_o = \left(\frac{\partial}{\partial\sigma} - \frac{R}{C_p} q_3\right)\left(T + \frac{R_o}{F} T_R\right) \quad \textbf{(25)}$$

However, in the lowest layers of the analysis domain, a=0 to account for the near adiabatic conditions within the planetary boundary layer.

Preliminary studies with the blended vertical velocity show that large magnitude centers of either sign developed by the kinematic method in the upper troposphere and lower stratosphere are reduced or eliminated. Therefore the large initial unadjustments that exist because of the use of the kinematic vertical velocities will be reduced or eliminated also.

The solution for the problem of buildup of RMS differences in MODEL II is to reformulate the MODEL I variational equations so that the solution sequence will better couple the vertical velocity with the dynamic adjustment. Achtemeier, et al. (1986) have shown that the derivations in MODEL I required to reduce the number of dependent variables and equations to a single diagnostic equation in geopotential cancel out the zero order divergence adjustment terms. The adjustment of the divergent part of the wind is therefore forced into higher-order nonlinear terms which do not sufficiently impact upon the final adjustment to bring about compatibility with the continuity equation. The continuity equation was satisfied through the second variational step which forced an adjustment of the adjusted velocity components. The problem was that the two variational steps could not be connected in a way that allowed adjustments required for satisfaction of the thermodynamic equation to feed back to the continuity equation.

This analysis of MODEL II reveals that the second variational step must be eliminated and the coupling of the vertical velocity with the remainder of the adjusted variables must be part of a single variational model. It was found that the divergent part of the wind obtained from the first step adjustment is a function of the nonlinear terms of the horizontal momentum equations. If F_5 represents the nonlinear terms of the u-component equation and F_6 represents the nonlinear terms of the v-component equation, then the horizontal momentum equations can be expressed as

$$m_1 = -v + \frac{\partial \phi}{\partial x} + F_5 = 0 \qquad \textbf{(26)}$$

$$m_2 = u + \frac{\partial \phi}{\partial y} + F_6 = 0 \qquad \textbf{(27)}$$

Forming the divergence from (23) and (24) and integrating through the depth of the analysis domain gives

$$\int (u_x + v_y)\, d\sigma = -\int (F_{6x} - F_{5y})\, d\sigma = 0 \qquad \textbf{(28)}$$

Equation (25) is an integral of the vorticity equation. The constraint upon the divergent part of the wind, and hence the vertical velocity, that must be satisfied in order for all MODEL I dynamic constraints to be satisfied is as follows. A particular solution of the vorticity equation must integrate to zero at the top of the model domain - the particular solution being that the divergent component of the same adjusted wind field must also satisfy the integrated continuity equation.

The incorporation of the integrated vorticity equation into the variational formalisms is the subject of MODEL IIB derived in Chapter III.

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FIGURE CAPTIONS

Fig. 1. The grid template for the variational assimilation model.

Table 2. Percent reduction of the initial unadjustment in the horizontal momentum equations after 4 cycles.

Cycle No.	Level 2	3	4	5	6	7	8	9
				u-component				
0	0	0	0	0	0	0	0	0
1	54	54	52	51	50	50	51	51
2	81	78	77	75	74	75	76	76
3	92	89	87	86	86	87	87	87
4	94	93	90	89	91	91	90	90
				v-component				
0	0	0	0	0	0	0	0	0
1	54	53	52	53	51	51	50	50
2	78	80	77	80	77	76	76	73
3	88	89	87	90	88	88	87	84
4	93	92	91	92	91	91	91	88

Table 3. Percent reduction of the initial unadjustment in the integrated continuity and hydrostatic equations after 4 cycles.

Cycle No.	Level 2	3	4	5	6	7	8	9
				Integrated Continuity				
0	0	0	0	0	0	0	0	0
1	-	-	-	-	-	-	-	-
2	97	98	98	99	99	99	99	99
3	96	98	98	99	99	99	99	99
4	96	98	99	99	99	99	99	99
				Hydrostatic				
0	0	0	0	0	0	0	0	0
1	51	50	50	50	50	50	50	50
2	73	65	75	75	75	75	75	75
3	83	65	88	88	88	88	88	88
4	86	62	94	94	94	94	94	94

Table 4. Percent reduction of the initial unadjustment in the thermodynamic equation after 4 cycles.

Cycle No.	Level 2	3	4	5	6	7	8	9
				Thermodynamic Equation				
0	0	0	0	0	0	0	0	0
1	54	60	62	63	61	63	63	48
2	81	80	74	55	24	39	76	72
3	89	73	61	32	-12	9	62	83
4	88	65	50	14	-38	-12	49	89

Table 1. Modifications to variational equations in MODEL 1 to obtain MODEL 2.

Variable Referenced	Existing Function	New Terms to be Added
u	F_1	$R_o(\overline{m})^{y}(\overline{\lambda_5(\overline{T_y})^{y}})^{x\sigma}$
v	F_2	$R_o(\overline{m})^{x}(\overline{\lambda_5(\overline{T_y})^{x}})^{y\sigma}$
$\dot{\sigma}$	F_3	$\lambda_5\sigma_\sigma+R_o\lambda_5[(\overline{T_\sigma})^{xy\sigma}-\frac{R}{C_p}(\overline{q_3})^{xy}(\overline{T})^{xy}]$
$\overline{T}$	*Eq 34 p 39 Achtem.etal 1986*	$F_8=-\{[(\overline{m})^{x}(\overline{u})^{x}(\overline{\lambda_5})^{y\sigma}]_x+[(\overline{m})^{y}(\overline{v})^{y}(\overline{\lambda_5})^{x\sigma}]$ $+[(\overline{\dot{\sigma}\lambda_5})^{xy\sigma}]_\sigma+\frac{R}{C_p}[(\overline{\dot{\sigma}\lambda_5})^{xy}q_3+(\overline{\omega_s\lambda_5})^{xy}q_4]\}$
$\overline{T}$	*Eq 47 p 41 Achtem.etal 1986*	F_8/γ
$\dot{\sigma}$	*New Equation*	$\lambda_5=-\frac{\pi_8}{R_o}(E_T-E_T^o)$
$\dot{\sigma}$	*New Equation*	$E_T=-\{[(\overline{m})^{xy}(\overline{u})^{x\sigma}(\overline{T_x})^{y}+(\overline{m})^{xy}(\overline{v})^{y\sigma}(\overline{T_y})^{x}]$ $+\dot{\sigma}[(\overline{T_\sigma})^{xy\sigma}-\frac{R}{C_p}(\overline{q_3})^{xy}(\overline{T})^{xy}]+\frac{\sigma_\sigma}{R_o}\dot{\sigma}$ $-(\overline{q_4})^{xy}\omega_s\frac{R}{C_p}[\frac{R_o}{F}\overline{T_R}+(\overline{T})^{xy}]\}$

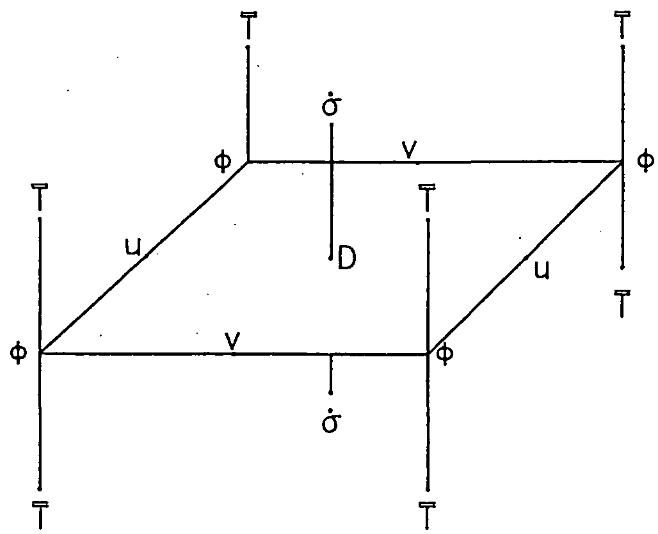

Fig. 1. The grid template for the variational assimilation model.