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HELIUM JET DISPERSION TO ATMOSPHERE

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by

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I. INTRODUCTION:

1.1 General Statement of the Problem

On the event of loss of vacuum guard of superinsulated helium dewar high rate of heat transfer into the tank occurs. Rapid boiling of liquid helium causes burst disk to rupture at four atmospheres and consequent passage of helium to atmosphere through the vent lines. Gaseous helium exiting the vent line forms vertical buoyant jet in a stagnant environment.

Characterization of the gaseous jet is achieved by detailed analysis of axial and radial dependence of the flow parameters. Unsteady flow pattern at the jet exit influences the developing profile downstream. Figure 1.1 is a schematic representation of the asymmetric jet. Three identifiable regimes of the jet are illustrated in the figure. Such trend of the radial profiles are observed in constant density jets and can also be adapted for variable density jet through some corrections in effective jet parameters. The potential core is a part of the developing shear layer where the jet is assumed to be uniform and inviscid. Velocity, temperature, density and concentration at the axis of the potential core is assumed to be equal to jet discharge values. Pressure recovery from the upstream throat pressure to atmospheric pressure occurs in this region.

The process of jet development include several complex phenomenon including turbulence, while overall character of the jet is determined by the strength of the global forces effective in the fluid motion. The fluid motion in a buoyant jet is in general governed by buoyant, viscous and inertial forces. The relative magnitude of these forces define the local character of the jet in different regions. On the other hand, the overall character is determined by the magnitude of these forces existing at the jet source and the ambient condition. Exit Reynolds number, Froude number and Grashof number are generally used to characterize the jet as explained in section 2.1.

The final regime illustrated in figure 1.1 is called the self-similarity regime. A flow field can be called self-similar when only one geometrical variable is required to characterize the nondimensionalized, time averaged behavior of velocity, temperature or concentration throughout the region. Complete self-similarity is never achieved by a developing flow. Yet, in an analogous way, local self-similarity can be defined through local geometric variables. For example, the radius of $U_{max}/2$ can play the role of local length scale used to normalize the radial distribution functions.

Entrainment of surrounding air into the jet causes broadening of the jet width and creates a mixture zone. Since vaporization of helium is completed upstream from the jet nozzle, single phase analysis is adequate to describe the dynamics of the jet. On the other hand, excessive variation of temperature induces predominant variation of gas density. Hence variable density single phase model has been considered here.

Theoretical considerations given to helium jet dispersion analysis are illustrated in Chapter II. The results and analysis obtained from the computer program developed for this project are illustrated in section 3.1. Prediction of axial and radial distribution of temperature and velocity are emphasized for illustraton. Axial velocity distributions indicate nonlinear decay profiles while axial temperature distributions indicate asymptotic increase from jet exit to surrounding temperature. Additional analysis using a second solution scheme is performed with computer program GENMIX. The results of this analysis are illustrated in section 3.2. Single phase helium jet analysis is considered with identical initial and boundary conditions used in both analysis. Jet discharge conditions are variable parameters to the programs.

An user's guide for the HEJET program is provided in appendix A. Input and output variable sequences are illustrated using corresponding file descriptions.

1.2 Previous Related Studies

Jet dispersion analysis has been studied widely under different hydrodynamic and thermodynamic circumstances. Research interest has focused on single and two phase jet characterization through effects of temperature, velocity and jet to ambient density ratios. A detailed review of the experimental data on single vertical buoyant jets are given by Chen and Rodi (1). Similarity and scaling laws are discussed by the author for jet characterization. Axial and radial distribution of mean velocity, temperature and concentration of jets are compared for a wide range experiments.

Two phase jet measurements with emphasis in spray evaporation has been reviewed by Shearer and Faeth (2). The authors have considered evaporation of a well-atomized liquid jet where homogeneous equilibrium model has been assumed and validated. Second order turbulence modeling including fluctuating density effects was employed for detailed analysis. The authors have also produced measurements of single phase variable density jets using sulfur hexafluoride gas jet. Freon-11 spray was produced by air atomizing injector in order to produce an evaporative jet.

A complete review of jet characterization measurements is given by Pitts (3). Emphasizing the effect of the ratio of jet fluid density to surrounding air density, the entire spectrum of experiments were summarized. Velocity of the similarity analysis was demonstrated for majority of the experiments. Rayleigh scattering technique was utilized for flow visualization and centerline concentration measurements. Gases of wide range of density were considered for analysis and compared to existing data.

Helium jet analysis is shown in figure 1.2. Characteristics of rate of spread of the jet has been demonstrated through the similarity parameters.

II THEORETICAL CONSIDERATIONS

2.1 Description of the model

Similarity analysis has been employed in this study as a means of solving the characteristic flow parameters of a buoyant helium jet. With the usual boundary layer approximations, the differential equations governing the mean flow quantities in a vertical buoyant jet can be written as

Continuity Equation:

$$\frac{\partial(\rho U r)}{\partial Z} + \frac{\partial(\rho V r)}{\partial r} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial(\rho U^2 r)}{\partial Z} + \frac{\partial(\rho U V r)}{\partial r} = -g(\rho-\rho_a)r - \frac{\partial}{\partial r}(r\rho\overline{uv}) \tag{2}$$

Thermal Energy Equation:

$$\frac{\partial(\rho U T r)}{\partial Z} + \frac{\partial(\rho V T r)}{\partial r} = -\frac{\partial}{\partial r}(r\rho\overline{vT'}) \tag{3}$$

Concentration:

$$\frac{\partial(\rho U C r)}{\partial Z} + \frac{\partial(\rho V C r)}{\partial r} = -\frac{\partial}{\partial r}(r\rho\overline{vc'}) \tag{4}$$

Integration of equations 1 to 4 over the jet cross-section area yields the integral form these equations:

Continuity Equation $$\frac{d}{dZ}\int_0^{r_E} \rho\, U r\, dr = E \tag{5}$$

Momentum Equation $$\frac{d}{dZ}\int_0^{r_E} \rho\, U^2 r\, dr = g\int_0^{r_E} (\rho_a - \rho) r\, dr \qquad (6)$$

Thermal Energy $$\frac{d}{dZ}\int_0^{r_E} \rho U(T-T_a) r dr = -\frac{dT_a}{dZ}\int_0^{r_E} \rho U r dr \qquad (7)$$

Concentration: $$\frac{d}{dZ}\int_0^{r_E} \rho U(C-C_a) r dr = -\frac{dC_a}{dZ}\int_0^{r_E} \rho U r dr \qquad (8)$$

Here E refers to entrained mass per unit length of the jet (divided by $2(\pi)$) and r_E is the radius of the jet from the axis of symmetry.

In order to solve the set of equations (5) (6) (7) & (8), certain non-dimensionalized local similarity parameter is specified, such that

$$\eta = r/(r½)_u \qquad (9)$$

where $(r½)_u$ = is the radius of $U_{max}/2$. Using the local similarity parameter η, it can be shown,

$$C = C_m\, f_1\, (\eta) \qquad (10)$$

and

$$U = U_m\, f_2\, (\eta) \qquad (11)$$

where C_m and U_m refer to the maximum values of mean concentration and velocity in the radial profiles. Assuming Gaussian distribution,

$$f_1\,(\eta) = e^{-\frac{.693}{R^2}\eta^2}$$

and where $R = \frac{(r½)_c}{(r½)_u}$

$$f_2\,(\eta) = e^{-.693\eta^2}$$

$(r½)_c$ = radius of $C_m/2$ = radius $T_m/2$

Implementing equations (10) and (11) into equations (5) - (8), and noting that $\frac{dC_a}{dZ} = 0$ and $\frac{dT_a}{dZ} = 0$ in non-stratified environment for gaseous jets, we have generalized jet fluid concentration

$$\boxed{\frac{C - C_a}{C_o - C_a} = \frac{T - T_a}{T_o - T_a} = \frac{\rho_o}{\rho}\ \frac{\rho - \rho_a}{\rho_o - \rho_a}} \quad (12)$$

should be

$$\frac{\rho_\infty - \bar{\rho}}{\rho_\infty - \rho_o}\ \frac{\rho_o}{\bar{\rho}} = \frac{\bar{T} - T_\infty}{\bar{T}_o - T_\infty} = \bar{c}$$

(a) Integral momentum equation

$$M_o = 2\pi\rho_a\ (r½)_u^2\ U_m^2 \int_0^\infty \frac{f^2{}_2\ \eta\ d\eta}{1 + C_m(\rho_a/\rho_o - 1)\ f_1} \quad (13)$$

$$= \pi r_o^2\ \rho_o\ \bar{U}_o^2$$

= momentum of the jet at discharge

(b) Integral mass deficit equation

$$N_o = 2\pi\ \rho_a\ (r½)_u^2\ U_m C_m \int^\infty \frac{f_1\ f_2\ \eta\ d\eta}{1 + C_m\ (\rho_a/\rho_o - 1)\ f_1} \quad (14)$$

$$= \pi r_o^2\ C_o\ \bar{U}_o$$

= Mass deficit of the jet at discharge

(c) Entrainment law

$$\dot{m}_e = k_e\ Z\ M_o^{½}\ \rho_o^{½} = 2\pi\rho_a\ (r½)_u\ U_m \int^\infty \frac{f_2\ \eta\ d\eta}{1 + C_m(\rho_a/\rho_o - 1)\ f_1} \quad (15)$$

The left hand side of equation (15) has been obtained from measurements of Reference 4, and confirmed by dimensional analysis.

Equations (13), (14), and (15) are hence to be solved simultaneously in order to obtain the unknown parameters U_m, C_m and $(r½)_u$ which define the

at each axial position

characteristics of the axial profiles. The radial distribution of the flow parameters are then achieved through equations (10), (11) and (12) respectively.

2.2 Thermodynamic Properties of Helium

Complete summary of useful thermodynamic and physical properties of helium are available in Reference 5. Considering that the gaseous jet attains atmospheric pressure immediately after the initial shock progression, the analysis has been performed at atmospheric pressure. Only the discharge temperature therefore influences the physical and thermodynamic characterics of the helium jet.

Linearity of equation of state is well documented for wide range of pressures (5).

Table 2.1 Thermodynamic and physical properties of helium

Helium Jet Discharge Property	Case 1	Case 2
Temperature	12 K	120 K
Density	4.0523 $\frac{kg}{m^3}$	0.40523 $\frac{kg}{m^3}$
Pressure	1 bar	1 bar
Viscosity	2.23x10^{-6} poise	114x10^{-6} poise
$P_{critical}$	2.3 bar	2.3 bar
$T_{critical}$	5.2 K	5.2 K
Mass flow rate	0.8 $\frac{kg}{sec}$	0.8 $\frac{kg}{sec}$

2.3 HEJET Computer Program Description

The HEJET computer program is written to solve equations (13), (14) and (15) simultaneously in order to obtain the axial distributions of centerline velocity (U_m), concentration (C_m, and hence temperature T_m and density) in addition to radius of $U_m/2$ and $T_m/2$. Subsequently, the program computes radial distributions of concentration (temperature or density) and velocity from equations (10) and (11).

The structure of the program is shown in the tree diagram of Table A.1. Function of input and output files are illustrated in Appendix A. Here, the main program and subroutine AINTG1 will be discussed.

Calculation begins with evaluation of the dimensionless numbers such as Reynolds number, Froude number and Grashof number. Implication of the magnitudes of these numbers are discussed here. The dimensionless forms are:

(a) Reynolds number $\quad R_e = \dfrac{U_o \rho_o D}{\mu_o} \quad \left[\dfrac{\text{Inertia force}}{\text{Viscous force}}\right]$

(b) Froude number $\quad F = \dfrac{U_o^2}{gD(\rho_a-\rho_o)/\rho_o} \quad \left[\dfrac{\text{Inertia force}}{\text{Buoyant force}}\right]$

(c) Grashof number $\quad G_r = \dfrac{g(\rho_a-\rho_o)D^3}{\rho_o \, \nu^2} \quad \left[\dfrac{\text{Buoyant force}}{\text{Viscous force}}\right]$

Purely turbulent jets have high Reynolds number caused by increased inertia relative to viscous effects. On the other hand, pure turbulent plumes are characterized by high Grashof number due to buoyancy effects. A turbulent buoyant jet is a combined effect of the two which is characterized by high

R_e and high G_r resulting in intermediate value of Froude number ($0 < F < \infty$). This type of jets are created by discharging fluid of density lower than the density environment.

The program then calculates the potential core which is defined as the region dominated by inertia and characterized by uniform jet velocity equal to U_o. The length of the potential cone is estimated by

$$\boxed{x_c = 2.13\ D\ (R_e)^{.097}}$$

If the integrals of equation (13), (14) and (15) are denoted by I_1, I_2 and I_3 respectively, then simultaneous solution of these equations provide

$$\frac{1}{C_m} = \frac{K_e Z}{\pi^{1/2} r_\epsilon} \frac{I_2}{I_3} \tag{16}$$

for each axial position

$$\frac{U_o}{U_m} = \frac{K_e Z}{\pi^{1/2} r_\epsilon} \frac{I_1}{I_3} \tag{17}$$

$$(r^{1/2})_u = \frac{K_e Z}{(2\pi)^{1/2}} \frac{I_1^{1/2}}{I_3} \tag{18}$$

where $r_\epsilon = r_{jet}\ (\rho_{jet}/\rho_a)^{1/2}$

Subroutine AINTG1 performs the integrals I_1, I_2, and I_3 using the supplementary subroutines QROMO, POLINT, FUNC1, FUNC2, and FUNC3. Here QROMO performs Romberg integration on an open interval where one of the limits may extend to infinity. POLINT is used to obtain polynomial of the functions FUNC1, FUNC2, and FUNC3 representing the functional forms of I_1, I_2 and I_3 respectively. The iterative method is continued until convergence of C_m is reached, such that $C_m^{i+1} - C_m^{i} \leq \epsilon\ C_m^{i}$, where ϵ is the desired convergence criteria.

III RESULTS AND ANALYSIS

Self similarity of turbulent buoyant jet has been assumed in the present analysis using HEJET computer code. The theoretical background and solution scheme has already been discussed in Chapter II. Self similarity of the profiles imply that the velocity U and the temperature T can be expressed as

$$\boxed{U = U_m \, f_1(\eta) \quad , \quad T - T_a = (T_m - T_a) \, f_2(\eta)}$$

where $\eta = r/(r½)_u$

subscript m indicates maximum radial value appearing at jet axis and subscript a corresponds to atmospheric condition.

Accordingly, it is assumed that the dimensionless form of time averaged quantities of a two-dimensional (axisymmetric) buoyant jet can be well described by a single normalized length parameter η. By incorporating the above distributions of velocity and temperature into mass, momemtum and energy balance equations (1,2 and 3 respectively), simultaneous solution of axial distribution of U_m, T_m and $(r½)_u$ are performed. The asymptotic value of jet temperature approaches the ambient temperature, which is assumed to be stagnant. Radial distributions of velocity and temperature are thereby calculated using the distribution functions f_1, and f_2 with independent parameter η.

Computer program HEJET is used to perform the analysis described above. The results obtained from HEJET are provided in section 3.1.

Considering the fact that similarity criteria is not satisfied under certain flow conditions, additional analysis is done with turbulent boundary layer code GENMIX (6). This program solves parabolic differential equations evolving from mass fraction, momentum and energy balance equations for single or multi-component system. Prandlt mixing length model is used for turbulence modeling with single equation. Analysis using GENMIX has also been presented in section 3.2 and compared to HEJET for the particular cases analyzed here.

3.1 Analysis using HEJET program

Two sample cases have been chosen for calculation with HEJET program. The upstream conditions corresponding to these two cases are shown in Table 3.1. Mass flow rate of 0.8 kg/sec at a pressure of 1 bar is the common criteria for the two cases where the jet exit temperature is allowed to be 12 K and 120 K, respectively.

Table 3.1 Helium Jet Properties used by HEJET Program

	Jet Velocity (m/sec)	Jet Pressure (bar)	Jet Temperature (k)
Case 1	10.82	1	12
Case 2	108.24	1	120

Pcor
1.38 m
0.94 m

Axial temperature profile for the two cases are illustrated in figure 3.1. The asymptotic value of temperature approaches the surrounding air temperature of 299k. While the temperature for case 2 reaches the asymptotic value before 9 meters, corresponding temperature of case 1 is lower at the same axial distance.

The potential core is characterized by the axial distance where jet velocity is equal to discharge velocity and temperature is equal to jet temperature. The potential core calculated for cases 1 and 2 are 1.38 and 0.94 meters, respectively. Radial temperature distribution for cases 1 and 2 are shown in figures 3.2a and 3.2b respectively. Both cases show sharp gradient in temperature profiles which equilibrate to surrounding temperature of 299k at a normalized radial distance of 1. Radial profiles of temperature at several axial locations are shown in these figures.

The jet centerline temperature and velocity profiles contribute the maximum radial magnitude. ? The radial distribution profiles begin at a normalized distance of 0.1. The normalization of radial distance has been achieved from the relationship.

arbitrary

$$\boxed{r_n = \frac{r}{3(r½)_u} \qquad 0 \le r_n \le 1}$$

$r_n = 1, \quad r = 3(r\frac{1}{2})u$

$r_n = 0, \quad r = 0$

Since $(r½)_u$ represents the radius of $U_{max}/2$, which is linear with z, the profiles are regular. The initial assumption of Gaussian distribution of temperature and velocity reappear through the radial profiles.

Axial velocity profile for cases 1 and 2 are shown in Figure 3.3. Very sharp decay of the velocity profile is evident from this figure for case 2. The asymptotic value of the velocity reaches the surrounding air velocity (which is stagnant in the present case). Radial velocity distribution is illustrated in figures 3.4a and 3.4b.

Radius of $T_{max}/2$ signifies the spreading rate of the expanding jet temperature profile. Figure 3.5 shows the radial $T_{max}/2$ width of the jet in terms of axial distance. The linearity in the profile is the criteria for which self similarity of the jet is assumed. Approximate correlation between the radial distance $(r½)_c$ and axial position Z is such that $(r½)_c = .105\ Z$. Figure 3.6 illustrates radial density distribution function of case 1. The asymptotic value reached by the distributions is the surrounding air density at 299k. The actual value of helium gas density at jet discharge is greater than the surrounding air density ($\rho_{jet}/\rho_{air} = 3.34$) in case 1. Hence for the mass flow rate of 0.8 kg/sec the jet velocity is low, which is equal to 10.824 m/sec. On the other hand, for case 2, ρ_{jet}/ρ_{air} = 0.334 and velocity at discharge equals to 108.24 m/sec. The asymptotic behavior of density for the two cases are therefore different as observed in figures 3.6a and 3.6b. However, radial concentration distributions are asymptotically decreasing towards 0 at the jet periphery. Radial concentration distribution for cases 1 & 2 are illustrated in figures 3.7a and 3.7b respectively.

Numerical values of the radial profiles are given in Table A.1 as a function of axial distance and normalized radial distance. The normalization in any particular axial location is based on $3*(r½)_u$, such that appropriate emphasis is given to quantities at any Z location.

3.2 Analysis using GENMIX program

Gaseous helium jet dispersion to atmosphere has also been analyzed using the boundary layer code called GENMIX(6). Heated turbulent jet with combustion is typically analyzed using this program. As an effort to estimate the predictability of HEJET program; an analysis of cryogenic helium jet is presented here. Axial temperature and velocity calculations from the two programs are then compared.

Input to GENMIX program was adjusted such that,

XLAST = 9.5 meters; XOUT = 0.0; XEND = 0.0

$R_b = 0.0$; $R_c = 0.0$; $R_d = 0.0762$ (jet radius)

Calculation has been performed for helium gas using the following constants (5).

$$\bar{R} = 8314 \text{ J/kmol.k}$$

$$R_{He} = \bar{R}/4.003 \quad ; \quad \bar{C}_p = \frac{5}{2}\bar{R} \quad ; \quad C_{p_{He}} = 5196.25 \frac{\text{J}}{\text{Kg-K}}$$

The same two reference cases as shown in section 3.1 for HEJET analysis is performed with GENMIX program, as shown in Table 3.2.

Table 3.2 Helium jet conditions used by GENMIX program

	U_{jet}(m/s)	T_{jet}(k)	Pressure (bar)
Case 1	10.82	12	1
Case 2	108.24	120	1

Comparison of axial mean temperature profiles obtained for GENMIX and HEJET programs for Cases 1 and 2 are shown in figure 3.8. Axial temperature profiles for case 1 are not in close agreement between the two analyses. Maximum deviation between the two calculations is approximately 40 K. GENMIX computer program is oriented towards parabolic solution which are dependent upon the initial conditions. The usual application of this code has been made to high temperature gaseous jets (6), where asymptotically decreasing temperature profiles are traditionally analyzed. The case considered here has not been analyzed jet by GENMIX for validation against experiments. Hence, accuracy of either program in cryogenic temperature predictions remain to be performed. As observed from figure 3.8, the asymptotic temperature calculated by GENMIX at 10 meters appear to 80 K below the surrounding temperature. The possibility of underprediction by GENMIX can only be confirmed by comparison with appropriate data. Radial temperature distribution from GENMIX analysis obtained for cases 1 and 2 are depicted in figures 3.9 and 3.10 respectively. Good agreement in axial velocity profiles of cases 1 and 2 are observed from figure 3.11. Corresponding radial velocity profiles are illustrated in figures 3.12a and 3.12b.

The above analysis using two different computer programs provide a strong validity to calculation procedures. Prediction of helium jet dispersion to atmosphere can hence be continued using unsteady upstream conditions as input to HEJET following the experimental conditions. Direct comparison of predictions by HEJET to actual available measurements is being proposed here in order to improve the working models for higher accuracy.

LIST OF REFERENCES

1. C.J. Chen and W. Rodi, "Vertical Turbulent Buoyant Jets", HMT series. Pergamon Press (1980)

2. A.J. Shearer and G.M. Faeth, "Evaluation of a Locally Homogeneous Model of Spray Evaporation" NASA report 3198 (1979).

3. William M. Pitts, "Effects of Global Density and Reynolds Number Variations on Mixing in Turbulent Axisymmetric Jets". NBSIR 86-3340. (1986)

4. F.P. Ricou and D.B. Spalding, "Measurement of Entrainment by Axisymmetrical turbulent jets". J. of Fluid Mechanics 11, 21-32 (1961).

5. Victor J. Johnson, "A Compendium of the Properties of Materials at Low Temperature", National Bureau of Standards (1960).

6. D.B. Spalding, "GENMIX - A General Computer Program for Two-Dimensional Parabolic Phenomenon" Pergamon Press (1977).

APPENDIX A

User's guide for jet analysis program HEJET.

This program calculates the centerline profiles and radial distribution profiles of

1. temperature (or density, concentration)
2. velocity

of axisymmetric jets released to atmosphere. The solution scheme is based upon integral method and iterative approach as illustrated by References 1 & 3.

a) File management

The main program will open three files for data input and output. These files are

DATA.IN	→	Input data file
JET.OUT	→	Axial profiles of temperature, velocity, concentration
RADIAL.OUT	→	Radial profiles of temperature, velocity, concentration and density

Features of the above files are given below in detail:

DATA.IN (Input data file)

(A) ICASE: 0 = Indicates steady state calculation

1 = Unsteady state calculation

Default ICASE = 0, which implies steady state upstream condition is fixed by one set of input in card group (E).

(B) DIA; ZTOT; TTOT; Diameter; total downstream distance; total time

Diameter corresponds to pipe diameter at jet release (meters). Total downstream distance ZTOT is the distance through which calculation should proceed (m). Total time of unsteady upstream condition is to be specified by TTOT. Default is TTOT = 0, which indicates steady state calculation.

(C) Calculational Constants

rr , P_i , K_e	:	Ratio of $\frac{(r½)c}{(r½)u}$, π , entrainment constant
Δz , Δt , Δr	:	Axial increment, temporal increment, radial increment
ϵ_c , KOUNT	:	convergence criteria, limit of iteration
RMID, CMAX	:	initial values of (r½)u and C_m.

(D) Surrounding air data

RATM, TATM : Air density, Temperature.

(E) Jet properties (upstream condition)

This input should follow the sequence of ICASE, such that for i = 1, ITMAX the following data are given where ITMAX = ICASE + 1 = TTOT/DT

FLOW (i), P(i), T(i): Mass flow rate (kg/sec), Pressure (bar), Temperature K

End of file DATA.IN

JET.OUT (Output of axial profiles of temperature, velocity, concentration and density)

(A) Characteristic numbers

Reynolds number

Froude number

Grashof number

Pcore : Potential core length (m)

(B) Axial output: Parameters as a function of axial distance at each time step [subscript max corresponds to centerline values].

OUTPUT Z : $R_{½}$, $Rc_{½}$, C_{max}, Rho_{max}, T_{max}, U_{max}

Z	:	axial position
$R_{½}$	:	radius of $U_{max}/2$
$Rc_{½}$	:	radius of $T_{max}/2$
C_{max}	:	Centerline concentration profile
Rho_{max}	:	Centerline density profile
T_{max}	:	Centerline Temperature profile
U_{max}	:	Centerline Velocity profile

End of file JET.OUT

RADIAL.OUT → Output of radial profiles of temperature, velocity and concentration at each time step.

For $0 \le t \le TTOT$

$0 \le z \le ZTOT$

$0 \le r_n \le 1$

$T\ (Z,\ r_n)$ Temperature

$U\ (Z,\ r_n)$ Velocity

$C\ (Z,\ r_n)$ Concentration

$R\ (Z,\ r_n)$ Density

here Z = axial distance

r_n = normalized radial distance

= $r\ /\ (3 * (R½))$

End of file RADIAL.OUT

Complete listing computer program HEJET is given in Appendix B. Output of sample cases I (discussed in Chapter 3) has also been provided in Appendix B.

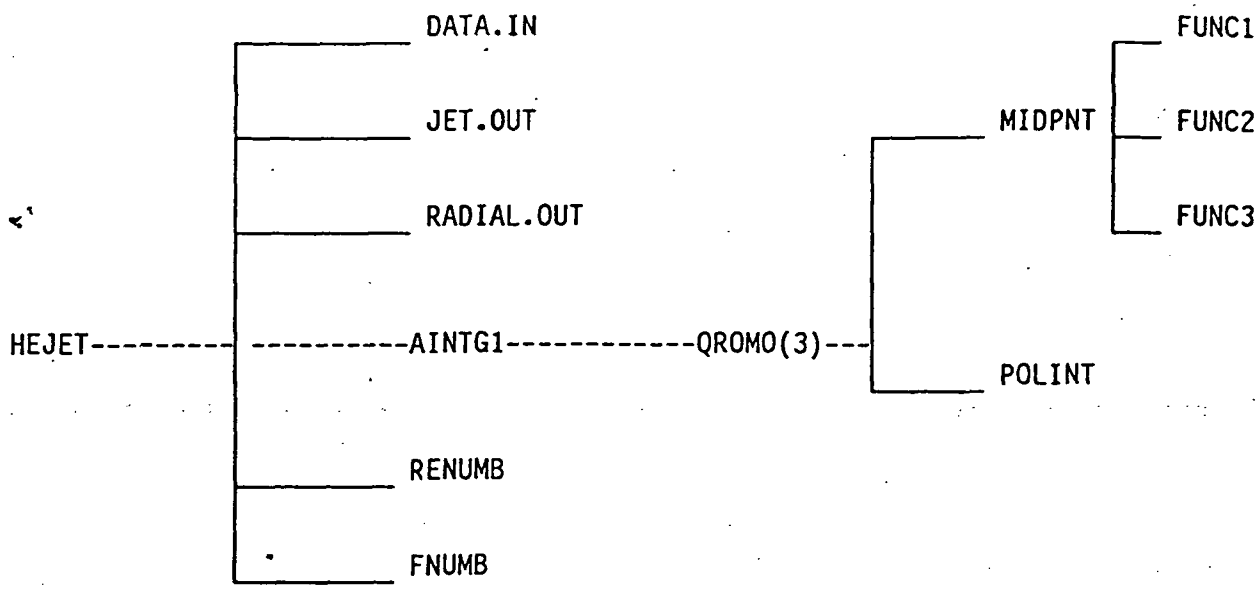

Table A.1 Structure of subroutines and I/O files of Helium jet dispersion code HEJET

Output of HEJET program used to calculate the axial and radial distribution of temperature and velocity of a gaseous Helium jet dispersed into atmosphere.

CASE 1

OUTPUT FILE : JET.OUT

```
************* JET   INPUT  CONDITIONS *************
Time step   Flow rate(kg/sec)    Pressure(bar)
Temperature   Viscosity
          1         0.80000001         1.00000000       12.00000000
2.23000006E-06
****************  OUTPUT OF HEJET *****************
****************** Dimensionless Groups*********
renolds number=   2.99772775E+06   froud numb=       112.32895700
grashof number=   3.74713672E-05   pcore =         1.37915623
 Jet Density   Jet Velocity   jet Temperature
      4.05232       10.82452      12.00000
*********** Axial distributions     **************
   z          R1/2          Rc1/2       Cmax      Rhomax        Tmax
    Umax
  1.000000   0.066777   0.078129   1.225565   4.052319  12.000000
10.824515
  2.000000   0.164225   0.192143   0.695905   2.381118  99.275238
8.539839
  3.000000   0.258746   0.302732   0.478125   1.838207 161.778152
5.911030
  4.000000   0.352793   0.412768   0.363496   1.641240 194.676529
4.508695
  5.000000   0.446674   0.522608   0.293055   1.539845 214.893234
3.641720
  6.000000   0.540477   0.632358   0.245436   1.478115 228.559860
3.053618
  7.000000   0.634236   0.742056   0.211110   1.436600 238.411407
2.628735
  8.000000   0.727972   0.851728   0.185199   1.406775 245.847824
2.307508
  9.000000   0.821691   0.961379   0.164949   1.384314 251.659683
2.056164
 10.000000   0.915398   1.071016   0.148688   1.366791 256.326538
1.854156
   Number of iterations =      40
```

OUTPUT FILE: RADIAL.OUT

Radial Distribution of Temperature

z

12.000	12.000	65.478	129.222	186.317	230.725	261.235	279.930	290.209	295.301
100.769	116.477	145.577	181.270	216.527	246.258	268.208	282.589	291.015	295.453
162.193	171.319	190.215	214.388	238.921	260.056	275.955	286.551	292.860	296.236
194.846	201.184	215.137	233.362	252.101	268.409	280.784	289.098	294.086	296.774
214.979	219.805	230.856	245.472	260.615	273.871	283.982	290.807	294.919	297.145
228.609	232.496	241.640	253.836	266.536	277.698	286.239	292.021	295.516	297.412
238.442	241.691	249.489	259.951	270.885	280.521	287.912	292.926	295.963	297.613
245.868	248.656	255.453	264.613	274.211	282.686	289.199	293.625	296.309	297.770
251.674	254.115	260.138	268.283	276.835	284.400	290.220	294.180	296.584	297.895
256.337	258.507	263.914	271.246	278.959	285.788	291.049	294.632	296.809	297.997

z

Radial Distribution of Velocity

10.172	8.438	6.179	3.994	2.279	1.148	0.510	0.200	0.069	0.021
8.453	7.549	5.952	4.142	2.545	1.380	0.661	0.279	0.104	0.034
5.887	5.356	4.301	3.049	1.908	1.054	0.514	0.221	0.084	0.028
4.499	4.128	3.344	2.391	1.509	0.841	0.414	0.180	0.069	0.023
3.637	3.354	2.730	1.962	1.244	0.697	0.344	0.150	0.058	0.020
3.051	2.822	2.305	1.662	1.058	0.594	0.295	0.129	0.050	0.017
2.627	2.436	1.994	1.441	0.919	0.517	0.257	0.113	0.044	0.015
2.306	2.142	1.756	1.271	0.812	0.458	0.228	0.100	0.039	0.013
2.055	1.912	1.569	1.137	0.728	0.411	0.205	0.090	0.035	0.012
1.854	1.726	1.418	1.029	0.659	0.372	0.186	0.082	0.032	0.011

Radial Distribution of Concentration

0.956	0.834	0.664	0.483	0.320	0.194	0.107	0.054	0.025	0.011

0.564	0.519	0.436	0.335	0.234	0.150	0.088	0.047	0.023	0.010
0.389	0.363	0.309	0.241	0.171	0.111	0.066	0.035	0.017	0.008
0.296	0.278	0.238	0.187	0.133	0.087	0.052	0.028	0.014	0.006
0.239	0.225	0.194	0.152	0.109	0.071	0.043	0.023	0.012	0.005
0.200	0.189	0.163	0.128	0.092	0.061	0.036	0.020	0.010	0.005
0.172	0.163	0.141	0.111	0.080	0.053	0.032	0.017	0.009	0.004
0.151	0.143	0.124	0.098	0.070	0.046	0.028	0.015	0.008	0.003
0.135	0.128	0.110	0.087	0.063	0.042	0.025	0.014	0.007	0.003
0.121	0.115	0.100	0.079	0.057	0.038	0.023	0.012	0.006	0.003

Radial Distribution of Density

4.052	4.052	2.834	2.086	1.687	1.469	1.349	1.285	1.252	1.236
2.364	2.202	1.954	1.716	1.532	1.405	1.324	1.276	1.249	1.236
1.835	1.776	1.665	1.542	1.435	1.353	1.298	1.263	1.244	1.233
1.640	1.607	1.539	1.458	1.383	1.323	1.282	1.255	1.240	1.232
1.539	1.517	1.468	1.408	1.351	1.305	1.271	1.250	1.237	1.231
1.478	1.461	1.423	1.376	1.330	1.292	1.264	1.246	1.235	1.230
1.436	1.423	1.393	1.353	1.315	1.283	1.259	1.243	1.234	1.229
1.407	1.396	1.370	1.337	1.304	1.276	1.255	1.241	1.233	1.229
1.384	1.375	1.353	1.324	1.295	1.270	1.252	1.240	1.232	1.228
1.367	1.359	1.339	1.314	1.288	1.266	1.249	1.238	1.232	1.228

CASE 2

I. OUTPUT FILE : JET.OUT

************* JET INPUT CONDITIONS *************

```
Time step   Flow rate(kg/sec)    Pressure(bar)
Temperature   Viscosity
          1       0.80000001       1.00000000      120.00000000
1.14000002E-04

****************  OUTPUT OF HEJET *****************

************** Dimensionless Groups*********

renolds number=   58639.76170000   froud numb=     3874.14185000
grashof number=   6.60668090E-02   pcore =        0.94162774
 Jet Density   Jet Velocity   jet Temperature
     0.40523     108.24515     120.00000
 *********** Axial distributions     **************
   z        R1/2         Rc1/2       Cmax      Rhomax       Tmax
   Umax
  0.500000   0.061999   0.072539   1.112296   0.376908 120.000000
108.245148
  1.500000   0.157886   0.184727   0.339863   0.725913 238.164536
43.170197
  2.500000   0.252165   0.295033   0.199064   0.873316 263.367493
25.150795
  3.500000   0.346118   0.404958   0.140582   0.953759 273.835815
17.716330
  4.500000   0.439950   0.514742   0.108620   1.004317 279.557037
13.667797
  5.500000   0.533726   0.624459   0.088486   1.039012 283.160919
11.123327
  6.500000   0.627466   0.734136   0.074644   1.064290 285.638672
 9.376678
  7.500000   0.721187   0.843789   0.064545   1.083523 287.446503
 8.103730
  8.500000   0.814895   0.953427   0.056851   1.098647 288.823669
 7.134884
  9.500000   0.908593   1.063054   0.050796   1.110852 289.907593
 6.372854
   Number of iterations =     28
```

OUTPUT FILE : RADIAL.OUT

Radial Distribution of Temperature

```
 120.000 135.374 169.599 205.579 237.429 261.954 278.652 288.797
294.329 297.048
 238.657 243.521 252.435 263.321 274.043 283.063 289.710 294.056
296.598 297.935
 263.481 265.871 270.791 277.073 283.440 288.920 293.039 295.782
297.414 298.286
 273.878 275.415 278.786 283.184 287.703 291.634 294.615 296.617
297.818 298.465
```

279.577	280.697	283.254	286.634	290.134	293.197	295.533	297.109	298.058	298.572
283.172	284.048	286.106	288.849	291.705	294.213	296.133	297.432	298.217	298.643
285.646	286.363	288.084	290.392	292.803	294.927	296.556	297.662	298.331	298.695
287.451	288.058	289.536	291.527	293.613	295.455	296.871	297.832	298.415	298.733
288.827	289.352	290.647	292.398	294.237	295.862	297.113	297.964	298.481	298.763
289.910	290.373	291.525	293.087	294.731	296.186	297.306	298.070	298.533	298.786

Radial Distribution of Velocity

100.714	82.747	60.012	38.419	21.711	10.831	4.769	1.854	0.636	0.193
42.693	38.053	29.940	20.794	12.749	6.899	3.296	1.390	0.517	0.170
25.041	22.764	18.267	12.939	8.091	4.466	2.176	0.936	0.355	0.119
17.675	16.212	13.127	9.382	5.919	3.296	1.620	0.703	0.269	0.091
13.648	12.583	10.240	7.356	4.665	2.611	1.290	0.563	0.217	0.074
11.113	10.279	8.393	6.050	3.849	2.162	1.072	0.469	0.181	0.062
9.370	8.688	7.110	5.137	3.276	1.844	0.916	0.402	0.156	0.053
8.099	7.522	6.167	4.463	2.851	1.608	0.800	0.352	0.136	0.047
7.132	6.633	5.445	3.946	2.524	1.425	0.710	0.313	0.121	0.042
6.371	5.931	4.874	3.536	2.264	1.280	0.639	0.281	0.109	0.038

Radial Distribution of Concentration

0.949	0.822	0.650	0.469	0.309	0.186	0.102	0.051	0.023	0.010
0.303	0.279	0.234	0.179	0.125	0.080	0.047	0.025	0.012	0.005
0.178	0.166	0.142	0.110	0.078	0.051	0.030	0.016	0.008	0.004
0.126	0.118	0.102	0.079	0.057	0.037	0.022	0.012	0.006	0.003
0.098	0.092	0.079	0.062	0.045	0.029	0.017	0.009	0.005	0.002
0.079	0.075	0.065	0.051	0.037	0.024	0.014	0.008	0.004	0.002

0.067	0.063	0.055	0.043	0.031	0.020	0.012	0.007	0.003	0.002
0.058	0.055	0.048	0.038	0.027	0.018	0.011	0.006	0.003	0.001
0.051	0.048	0.042	0.033	0.024	0.016	0.009	0.005	0.003	0.001
0.046	0.043	0.038	0.030	0.021	0.014	0.009	0.005	0.002	0.001

Radial Distribution of Density

0.391	0.430	0.497	0.596	0.722	0.863	0.996	1.098	1.164	1.199
0.728	0.753	0.803	0.873	0.956	1.038	1.109	1.160	1.193	1.210
0.874	0.891	0.929	0.982	1.042	1.100	1.148	1.182	1.203	1.215
0.954	0.967	0.997	1.039	1.086	1.131	1.167	1.193	1.209	1.218
1.005	1.015	1.040	1.075	1.113	1.150	1.179	1.199	1.212	1.219
1.039	1.048	1.069	1.099	1.132	1.162	1.187	1.204	1.214	1.220
1.064	1.072	1.090	1.116	1.145	1.171	1.192	1.207	1.216	1.221
1.084	1.090	1.107	1.130	1.155	1.178	1.196	1.209	1.217	1.221
1.099	1.105	1.119	1.140	1.162	1.183	1.199	1.211	1.218	1.222
1.111	1.116	1.130	1.148	1.169	1.187	1.202	1.212	1.219	1.222

```
      PROGRAM HEJET
c
c*************************************************************
c       this program calculates the centerline decay profiles
c       and  radial distribution profiles
c              1.   concentration  ( density , temperature)
c              2.   velocity
c       of axisymmetric jets by integral method
c       Reference :  Chen and Rodi ( Pargamom press)
c                    William Pitts ( NBS report    )
c*************************************************************

c
      parameter ( jmax = 10 ,  kmax= 10  )
c
      dimension flow( 20 ),  p(20), t(20),  uo( 20) , ro(20),
vis(20)
c
      dimension ru2(kmax,  20 ),   uc(kmax,  20  ), cc(kmax, 20)

     1     ,      tc(kmax,  20 ),   rc(kmax,  20  )

      dimension  cr(jmax, kmax, 20 ), ur(jmax, kmax, 20 )
     1     ,      tr(jmax, kmax, 20 ), dr(jmax, kmax, 20)
     2     ,     crn(jmax, kmax, 20 )
c
      data rcons , pcons , agrav /  2.077 , 1.01e2  , 9.81 /
c*************************************************************
      open (unit = 8 , file='c:\numrec\data.in',status='old')
      open (unit = 6 ,
file='c:\NUMREC\jet.out',status='unknown')
      open (unit = 7 ,
file='c:\NUMREC\radial.out',status='unknown')
c*************************************************************
c                    DATA   ENTRY
c       Specify steady state or transient clculation :
c
        read(8,*) icase
c
C A)    Read  geometry
c
        read(8,*) dia, zin,  ztot, ttot
C
c B)    Read calculation  constants
c
        read(8,*) rr,   pi,    ake
        read(8,*) dz,   dt,    delr
        read(8,*) epsc, kount
        read(8,*) rmid, cmax
c
c C)    Read atmospheric  data
c
        read(8,*) ratm, tatm
c
```

```
c D)    Read jet properties
c
         itmax = 1
c        if( icase. eq. 0) itmax = 1
         if( icase. eq. 1) itmax = int( ttot/dt )
c       if(icase.eq. 1) itmax  = 2.
c
        write(6,*)'************* JET   INPUT  CONDITIONS
*************'
        write(6,*)'Time step   Flow rate(kg/sec)    Pressure(bar)

     1   Temperature   Viscosity    '
        do  10 it = 1, itmax
        read(8,*) flow( it ), p( it ) , t ( it ), vis( it )
        write(6,*)it, flow( it) , p(it)   , t(it), vis(it)
  10    continue
        write(6,*)'****************  OUTPUT OF HEJET
*****************'
C
c      Conversion to density and velocity at the inlet
c
       area      = pi * dia**2. / 4.
       do 20  it = 1, itmax
       po        = p( it ) *  pcons
       to        = t( it )
       vsp       =  rcons * to / po
       dens      =  1./ vsp
       ro ( it ) = dens
       uzero     = flow(it)  / ( area * ro(it)  )
       uo ( it ) = uzero
 20    continue
c***************************************************************
c
c      Begin solution
c      Calculate the dimensionless groups
c
        do 300  it = 1, itmax
c
        write(6,*)'****************** Dimensionless
Groups*********'
        dens   =  ro (it)
        uzero  =  uo (it)
        visc   =  vis(it)
        reno   =  rnumb( uzero , dens, dia,  visc )
        froud  =  fnumb( uzero , dens, dia,  ratm, agrav)
        grash  =  froud / reno
        xc     =  2.13 * reno **.097
        pcore  =  dia  * xc
        write(6,*) 'renolds number= ',reno, '   froud numb=  ',
froud
        write(6,*) 'grashof number= ',grash, '   pcore =  ',pcore
c
        rjet  = ro( it )
        fac   = ro( it ) / ratm
```

```
        z     = zin
       ror    = dia/ 2.0
       reps   = ror * ( ro(it) / ratm )** 0.5
       write(6,*) ' Jet Density ',' Jet Velocity  ' ,' jet
Temperature'
       write(6,30) rjet, uo(it), t(it)
c
 30    format(3(4x,f10.5))
        kk    =  1
c      begin iterative solution  for each z location
c
       write(6,*)' *********** Axial distributions
**************'
       write(6,*)'    z          R1/2          Rc1/2      Cmax
Rhomax
     1    Tmax         Umax'
c
        do 200 k =  1, kmax
 1      cmaxo    =   cmax
c
c      calculate the integrals aint1 aint2 & aint3
       call aintg1 ( rmid, cmax, rjet, ratm,rr
     1                  ,                      aint1, aint2, aint3
)
c
c      calculate cmax
c
       cof1 = ( pi**0.5) * reps / ( ake * z )
       cmax = ( aint3 / aint2 ) * cof1
c
       delc = cmax - cmaxo
c
       if ( abs(delc).lt.epsc*cmax) then
c
c      Centerline decay profiles
         cc(k,it)     = cmax
         cof2         = ( ake * z ) / ( 2. * pi )**0.5
         ru2(k,it)    = (cof2 * aint1**0.5 ) / aint3
         rc2          = rr * ru2(k,it)
         cof3         = ( pi**0.5) * reps / ( ake*z)
         uc(k,it)     = uo(it) * cof3 * aint3 / aint1
         rc(k,it)     = ratm /( 1.+ cmax* ( 1./fac - 1.))
         tc(k,it)     = tatm + cmax*( t(it) - tatm )
c
         if( tc(k,it) .lt. t(it) ) tc(k,it) = t(it)
         if( rjet .gt. ratm. and. rc(k,it) .gt. rjet) rc(k,it) =
rjet
         if( uc(k,it) .gt. uzero ) uc(k,it) = uzero
c
        write(6,90 ) z, ru2(k,it), rc2, cc(k,it), rc(k,it),
tc(k,it)
     1        ,           uc(k,it)
  90    format ( 7(1x,f10.6) )
c
```

```
          else
       if(kk.eq.kount) go to 1000
       kk =  kk + 1
       go  to 1
          endif
c
c*****************************************************
c      calculate the radial profiles for variables
c      ur = velocity, cr= concentration
c      dr = density , tr= temperature
c ****************************************************
c
        r    =  .02
c
         delr  =  (ru2(k,it) * 3. ) / float(jmax)

        do 100   j  = 1, jmax
c
        if( k.eq.1. and.j.eq.1 ) cznorm = cc( k, it )
        ratio          =   r/ ru2(k,it)
        power          = -0.693    * ratio**2.
        ur (j,k,it)    = uc(k,it)  * exp(power)
        cr (j,k,it)    = cc(k,it)  * exp(power/rr**2.)
        crn(j,k,it)    = cr(j,k,it) / cznorm
        dr (j,k,it)    = ratm / ( 1 + cr(j,k,it)*( ratm/ro(it) -1
))
        tr (j,k,it)    = tatm +   cr(j,k,it)*( t(it) - tatm )
         r             = r    + delr
c
        if( ur(j,k,it) . gt. uzero ) ur(j,k,it) = uzero
        if( tr(j,k,it) . lt. t(it) ) tr(j,k,it) = t(it)
        if( rjet .gt. ratm. and .dr(j,k,it).gt.rjet)
dr(j,k,it)=rjet
c
  100   continue
c
c        if( k.eq. 1) z = zin
        z =  z  + dz
c
  200    continue
c
  300   continue
c
c       write the radial  profiles
c
       write(7,*)' Temperature   ',' Velocity    ','
Concentration  '
    1          , ' Density       '
c
       do 92 jj  = 1, itmax
   91  format (10(1x,f7.3))
       write(7,91)(( tr (ii,kz,jj) , ii=  1, jmax),kz= 1, kmax)
       write(7,91)(( ur (ii,kz,jj) , ii=  1, jmax),kz= 1, kmax)
       write(7,91)(( crn(ii,kz,jj) , ii=  1, jmax),kz= 1, kmax)
```

```
      write(7,91)(( dr (ii,kz,jj) , ii=  1, jmax),kz= 1, kmax)

 92   continue
c
 1000 write(6,1001)kk
 1001 format(2x,'  Number of iterations =   ',i5)
      close(8)
      close(6)
      close(7)
      end
c*******************************************************
      function rnumb(u, r, d, v )
      rnumb = u * r* d/ v
      end
c*******************************************************
      function fnumb( u, r, d, rat, g )
      anum  =  u **2.0
      deno  =  g * d * abs( rat - r ) / r
      fnumb =  anum / deno
      end
c
        SUBROUTINE MIDPNT(  A, B, ITYPE,      S, N )
c
c       This routine computes tne n'th stage of refinement of
an extended
c       midpoint rule. FUNC is input as the name of the
function
c       to be integrated between limits A and B . When called
by
c       N = 1 , the routien returns as S the crudest estimate
of
c       estimate of int f(x)dx. As  N increases the accuracy
c       increases by ( 2/3 )* 3**(N-1) additional interior
points.
c
        if ( itype.eq.1) then
C
        if (n . eq. 1 )then
           s = (b-a) * func1( 0.5* (a + b))
           it=  1
        else
           trm = it
           del = ( b-a ) / (3. * trm )
          ddel = del + del
             x = a + 0.5 * del
           sum = 0.0
        do 11 j= 1,it
           sum = sum + func1(x)
             x = x  + ddel
           sum = sum + func1(x)
             x = x  + del
  11    continue
C
           s   =  ( s+ ( b-a ) * sum/trm) / 3.
```

```
            it  =  3 * it
         endif
c
            elseif( itype. eq. 2 ) THEN
c
         if (n . eq. 1 )then
            s = (b-a) * func2( 0.5* (a + b))
            it= 1
         else
            trm = it
            del = ( b-a ) / (3. * trm )
           ddel = del + del
              x = a + 0.5 * del
            sum = 0.0
         do 22 j= 1,it
            sum = sum + func2(x)
              x = x  + ddel
            sum = sum + func2(x)
              x = x  + del
   22    continue
C
            s   =  ( s+ ( b-a ) * sum/trm) / 3.
            it  =  3 * it
         endif
C
            else
C
         if (n . eq. 1 )then
            s = (b-a) * func3( 0.5* (a + b))
            it= 1
         else
            trm = it
            del = ( b-a ) / (3. * trm )
           ddel = del + del
              x = a + 0.5 * del
            sum = 0.0
         do 33 j= 1,it
            sum = sum + func3(x)
              x = x  + ddel
            sum = sum + func3(x)
              x = x  + del
   33    continue
C
            s   =  ( s+ ( b-a ) * sum/trm) / 3.
            it  =  3 * it
         endif
C
         endif
C
         return
         end
C*****************************************************
         SUBROUTINE POLINT( XA, YA, N,X,    Y,DY)
C
```

```
C        Given arrays XA and YA , each of length N,
c        and given a value of X , this routine
c        returns a value Y and an error estimate
c        DY. If P(x) is a polynomial of degree
c        N-1 , then it returns Y= P(X)
C
           parameter (nmax = 10)
           dimension xa(n) , ya(n) , c(nmax) , d(nmax)
C
           ns   = 1
           dif  = abs( x - xa(1) )
           do 11 i = 1, n
             dift = abs( x- xa(i) )
             if ( dift. lt. dif) then
              ns   =  i
              dif  =  dift
             endif
             c(i)  =  ya(i)
             d(i)  =  ya(i)
  11       continue
           y     =  ya( ns)
           ns    =  ns - 1
C
           do 13  m= 1, n-1
C
           do 12  i  =1, n-m
                 ho  = xa(i) - x
                 hp  = xa(i+m) -x
                 w   = c( i+1) - d(i)
                 den = ho - hp
                 if( den. eq. 0.0)pause
                 den = w / den
                 d(i)= hp * den
                 c(i)= ho * den
  12      continue
          if( 2*ns. lt. n-m ) then
                dy   = c(ns + 1)
          else
                dy   = d(ns)
                ns   =  ns -1
         _endif
                Y    =   Y + DY
  13     continue
         return
         end
C************************************************
         SUBROUTINE QROMO( A, B,  ITYPE, SS,    DSS )
C
C         Romberg integration on an open interval.
c         Returns by SS the integral of the function FUNC
c         from A  to  B,  using Subroutine MIDPNT
c
          parameter (eps = 1.e-5 , jmax= 15, jmaxp=jmax + 1
    1           ,      k = 7 ,     km  = k- 1 )
```

```
c
          dimension s( jmaxp), h( jmaxp)
c
c
          h(1)    =  1.
          do  11 j =  1, jmax
            call midpnt(  a, b,itype,      sp, j )
            s(j)   = sp
            if ( j. ge. k) then
            call polint( h(j- km), s(j- km), k,0.,ss,dss)

               if (abs( dss).lt. eps* abs(ss)) return
            endif
          s(j+1) = s(j)
          h(j+1) = h(j) / 9.
 11       continue
          write(*,*) j,sp,s(j-2),s(j-1),s(j)
          pause ' too many steps'
          end
C*****************************************************
c
c
           SUBROUTINE  aintg1(rmid1,cmax1, rjet,ratm,r
     1          ,              aint1, aint2, aint3   )
c
c         This program is only to test the integration
c         routine
c
          common/com1/rmid,cmax, rj, rat,rr
          rmid = rmid1
          cmax = cmax1
          rj   = rjet
          rat  = ratm
          rr   = r
c
          a = 0.0
          b =  2.0
c
          itype = 1
           call qromo ( a,b,itype,    ss, dss )
          aint1 = ss
c
          itype  = 2
          call qromo ( a,b,itype,    ss, dss )
          aint2  = ss
c
          itype  = 3
          call qromo ( a,b,itype,    ss, dss )
          aint3  = ss
c
          return
          END
C***************************************************
          FUNCTION FUNC1(XX)
```

```
      common/com1/ rmid ,cmax, rjet, ratm, rr
c
      dcoef = cmax*(ratm / rjet -1.)
      dpow  = -.693 /rr**2.
      dpow1 = dpow * (xx / rmid)**2.
c
      deno1 =  1. + dcoef* exp( dpow1 )
c
      apow  = -.693* (xx/rmid)**2.
      anum  = exp( apow )
c
      func1 = (( anum**2. / deno1 ) / rmid**2.) * xx

c
      END
C**********************************************************
      FUNCTION FUNC2(XX)
      COMMON/COM1/RMID , CMAX, RJET, RATM, RR
c
      dcoef = cmax*(ratm / rjet -1.)
      dpow  = -.693 /rr**2.
      dpow1 = dpow * (xx / rmid)**2.
c
      deno1 =  1. + dcoef* exp( dpow1 )
c
      apow  = -.693* (xx/rmid)**2.
      apow1 = apow * ( 1. + 1./ rr**2. )
      anum  = exp( apow1 )
c
      func2  = (( anum  / deno1 ) / rmid**2.) * xx
c
      END
C**********************************************************
      FUNCTION FUNC3(XX)
      COMMON/COM1/RMID , CMAX, RJET, RATM, RR
c
      dcoef = cmax*(ratm / rjet -1.)
      dpow  = -.693 /rr**2.
      dpow1 = dpow * (xx / rmid)**2.
c
      deno1 =  1. + dcoef* exp( dpow1 )
c
      apow  = -.693* (xx/rmid)**2.
      anum  = exp( apow )
c
      func3  = (( anum  / deno1 ) / rmid**2.) * xx
c
      END
```

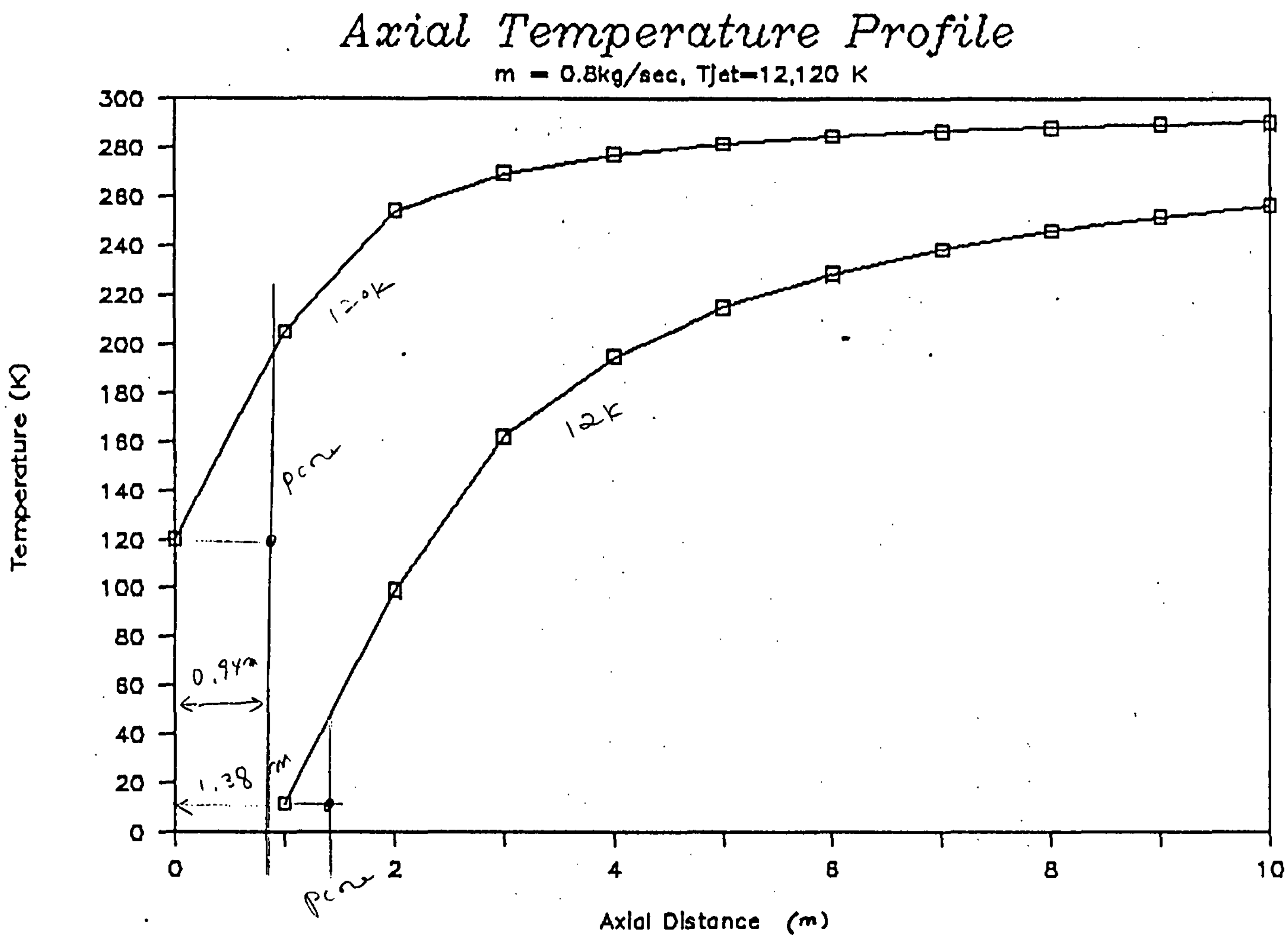

Figure 3.1 Axial temperature profile of Helium jet 12 K and 120 K

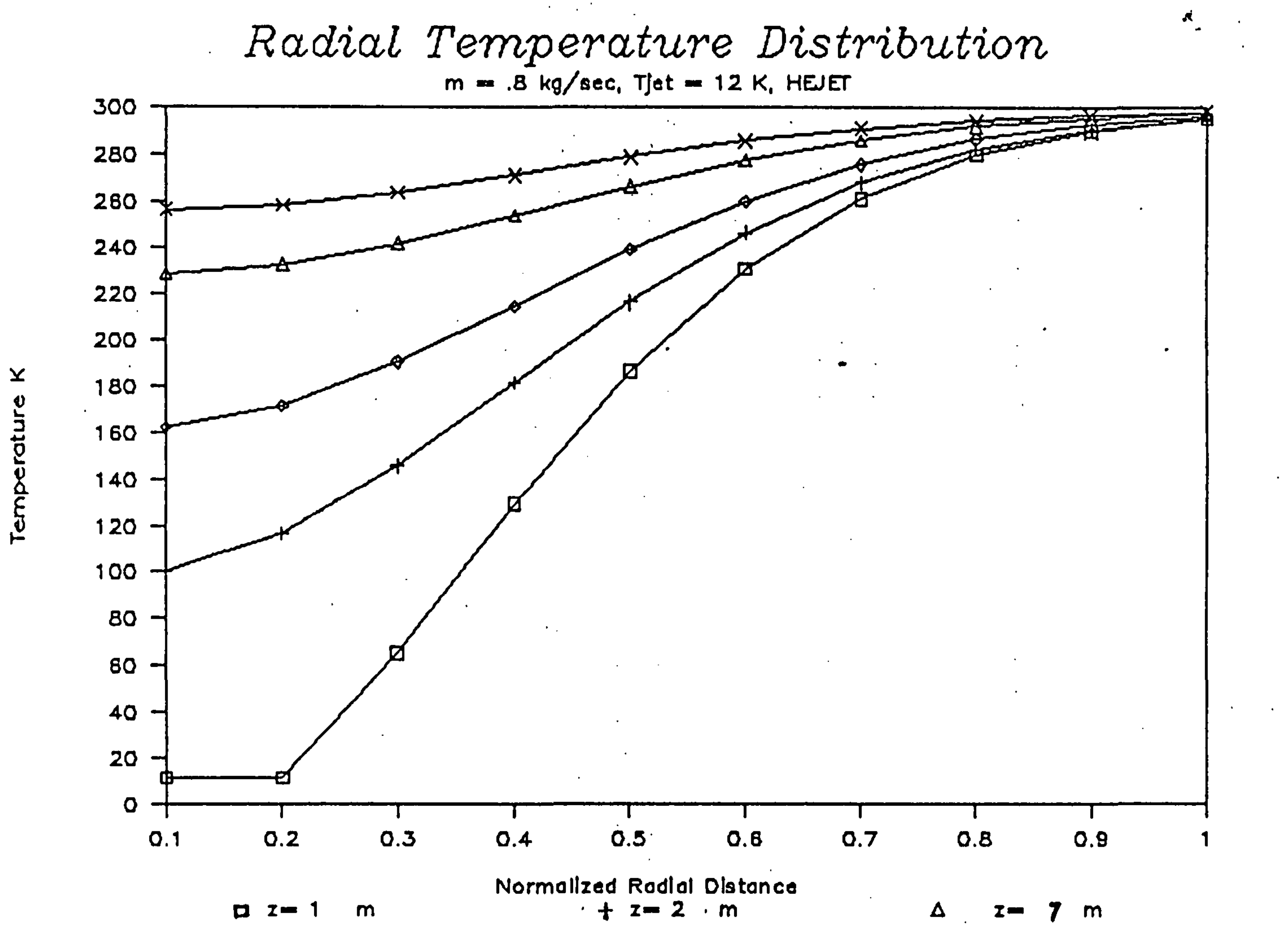

Figure 3.2a Radial temperature profile of Helium jet at 12 K

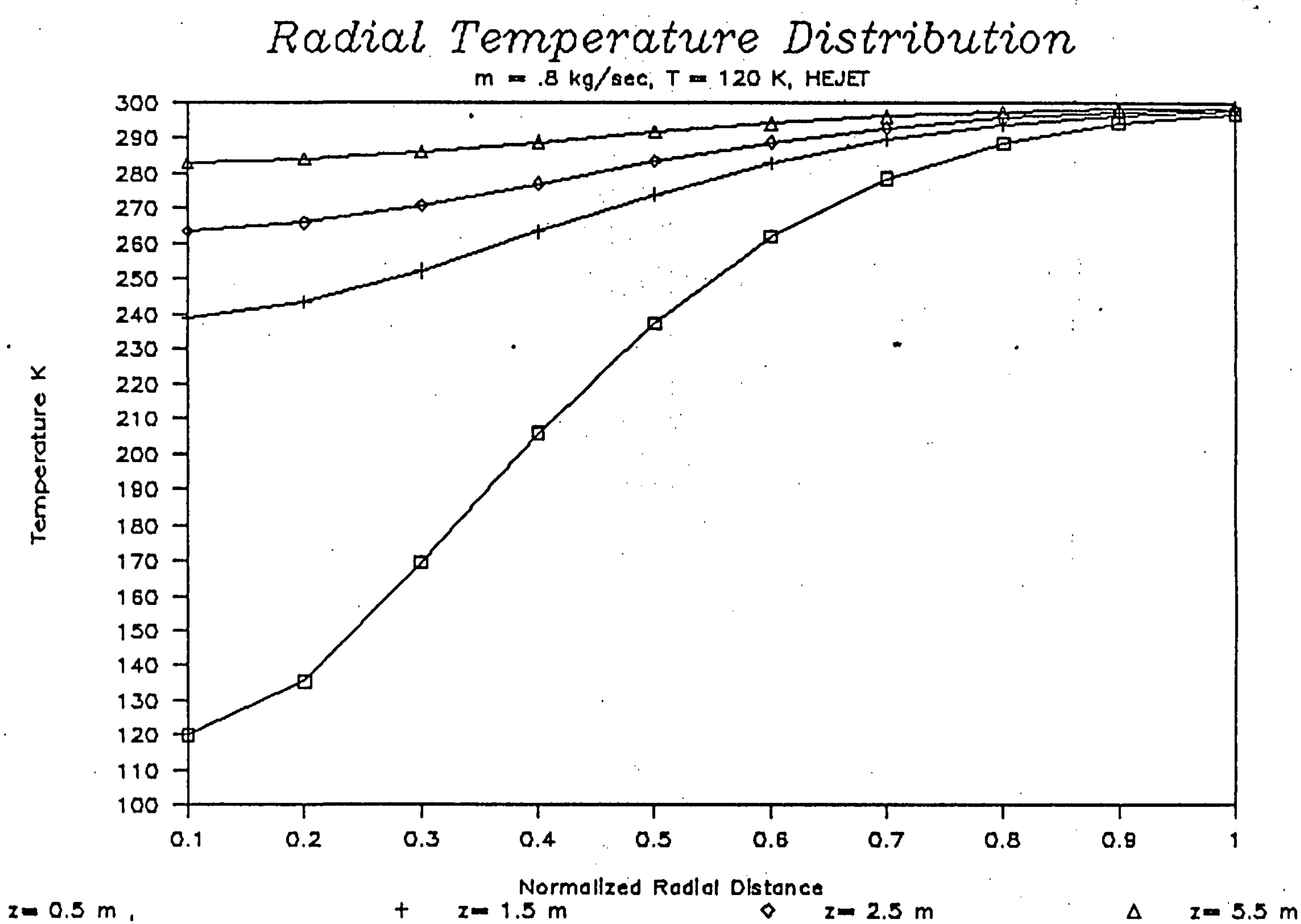

Figure 3.2b Radial temperature profile of Helium jet at 12 K and 120 K

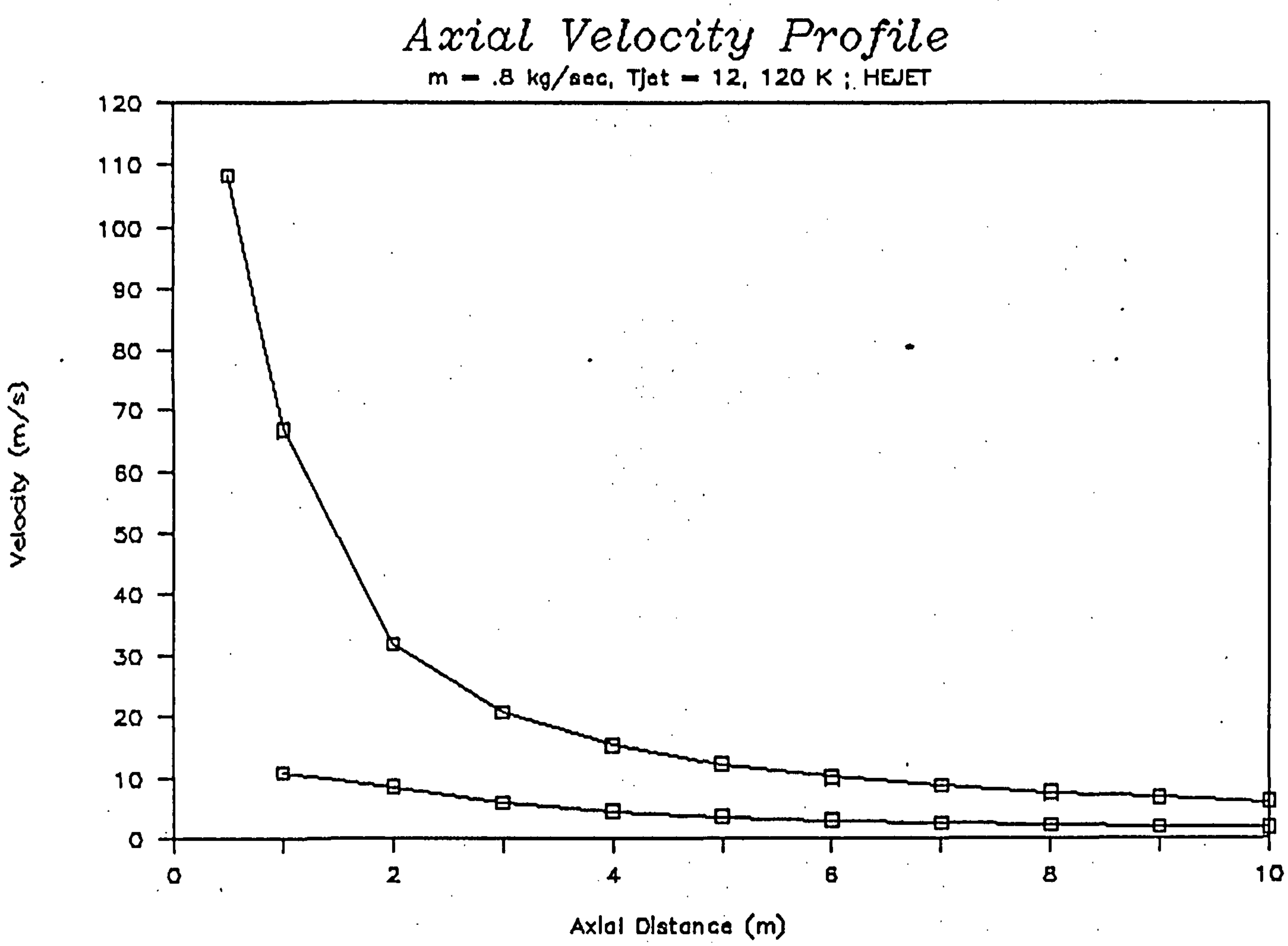

Figure 3.3 Axial velocity profile of Helium jet at 12 K and 120 K

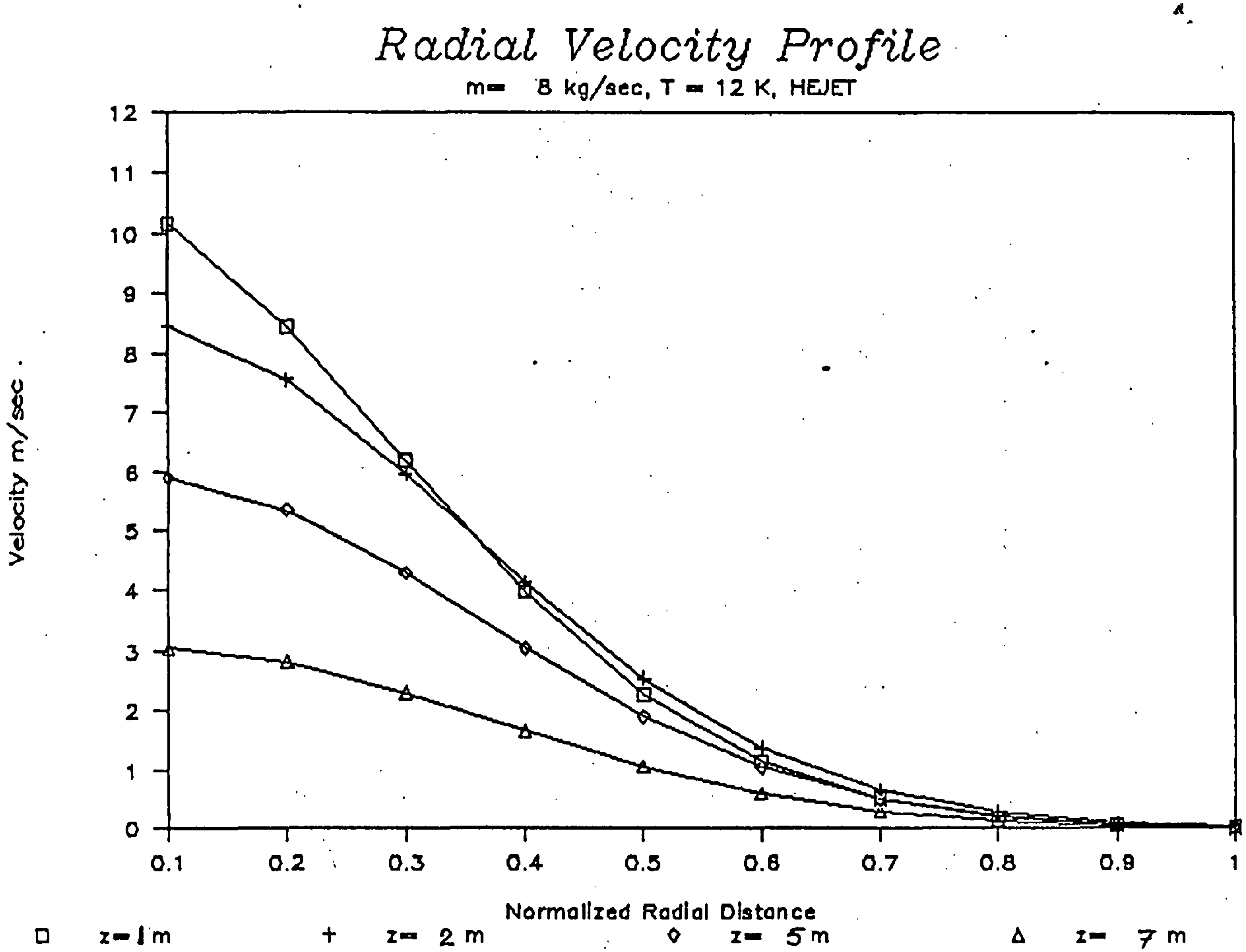

Figure 3.4a Radial velocity profile of Helium jet at 12 K

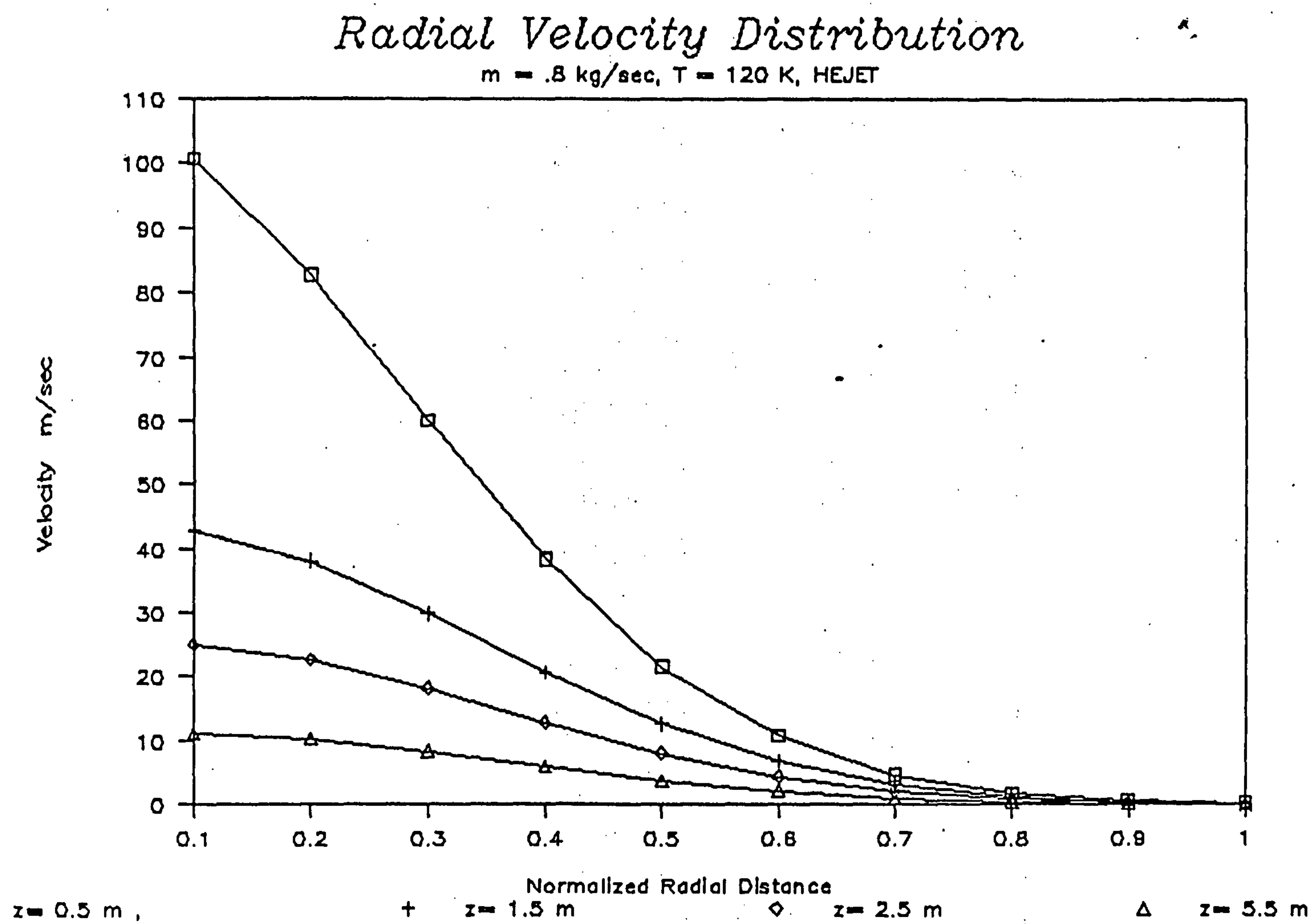

Figure 3.4b Radial velocity profile of Helium jet at 120

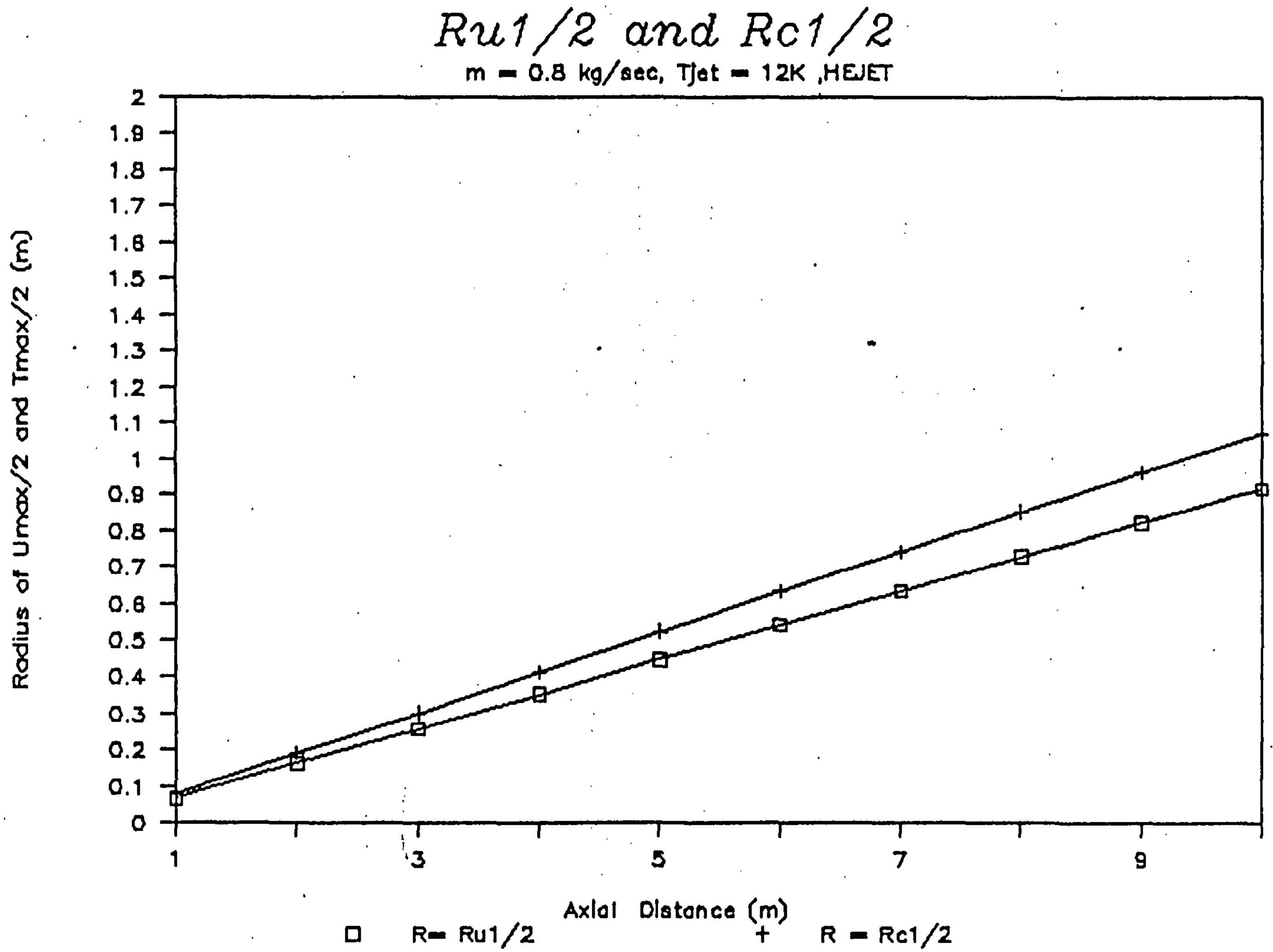

Figure 3.5 Axial profile of $T_{maximum/2}$ for Helium jet at 12 K.

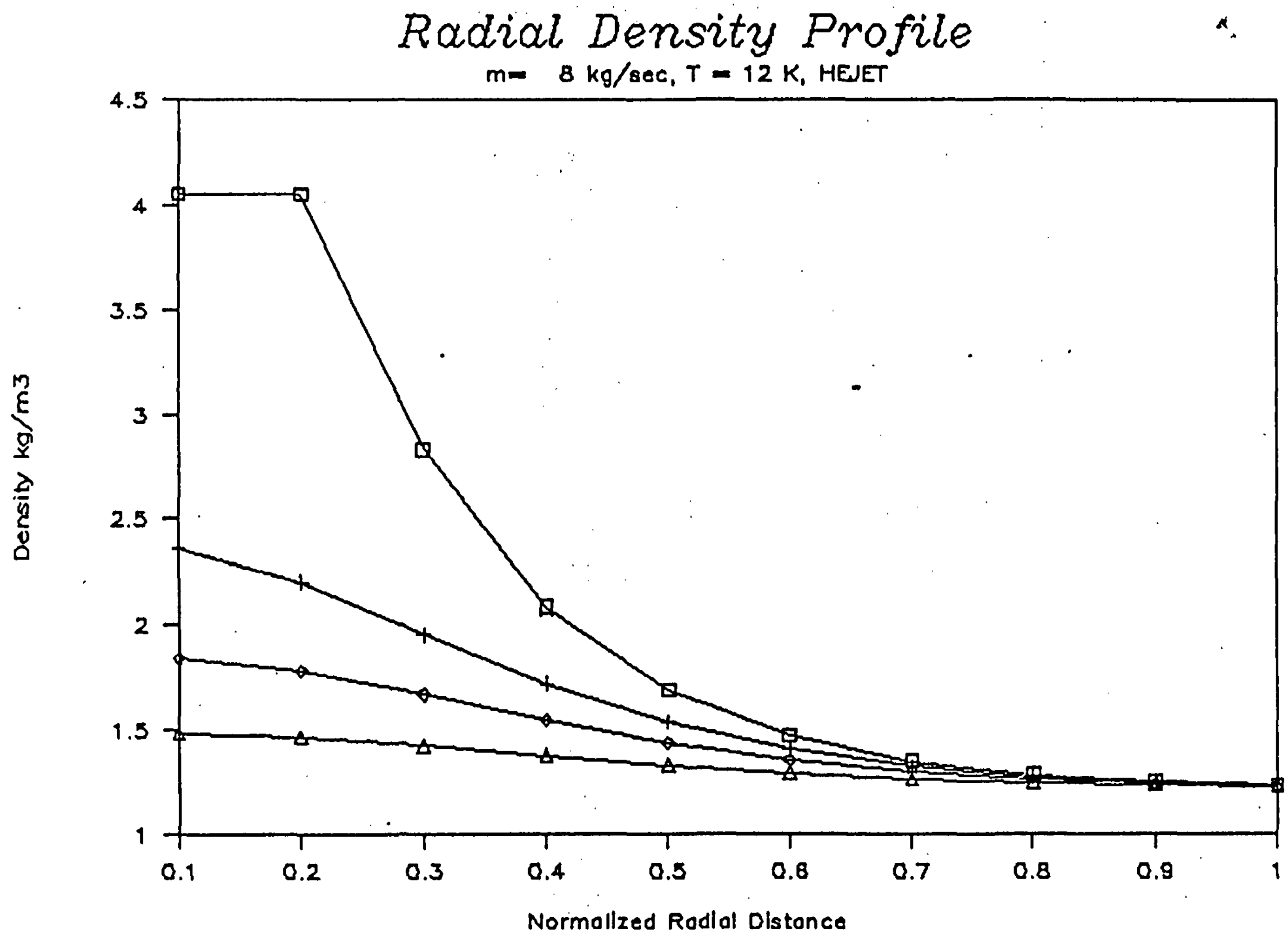

Figure 3.6 a   Radial distribution of density for Helium jet at 12 K

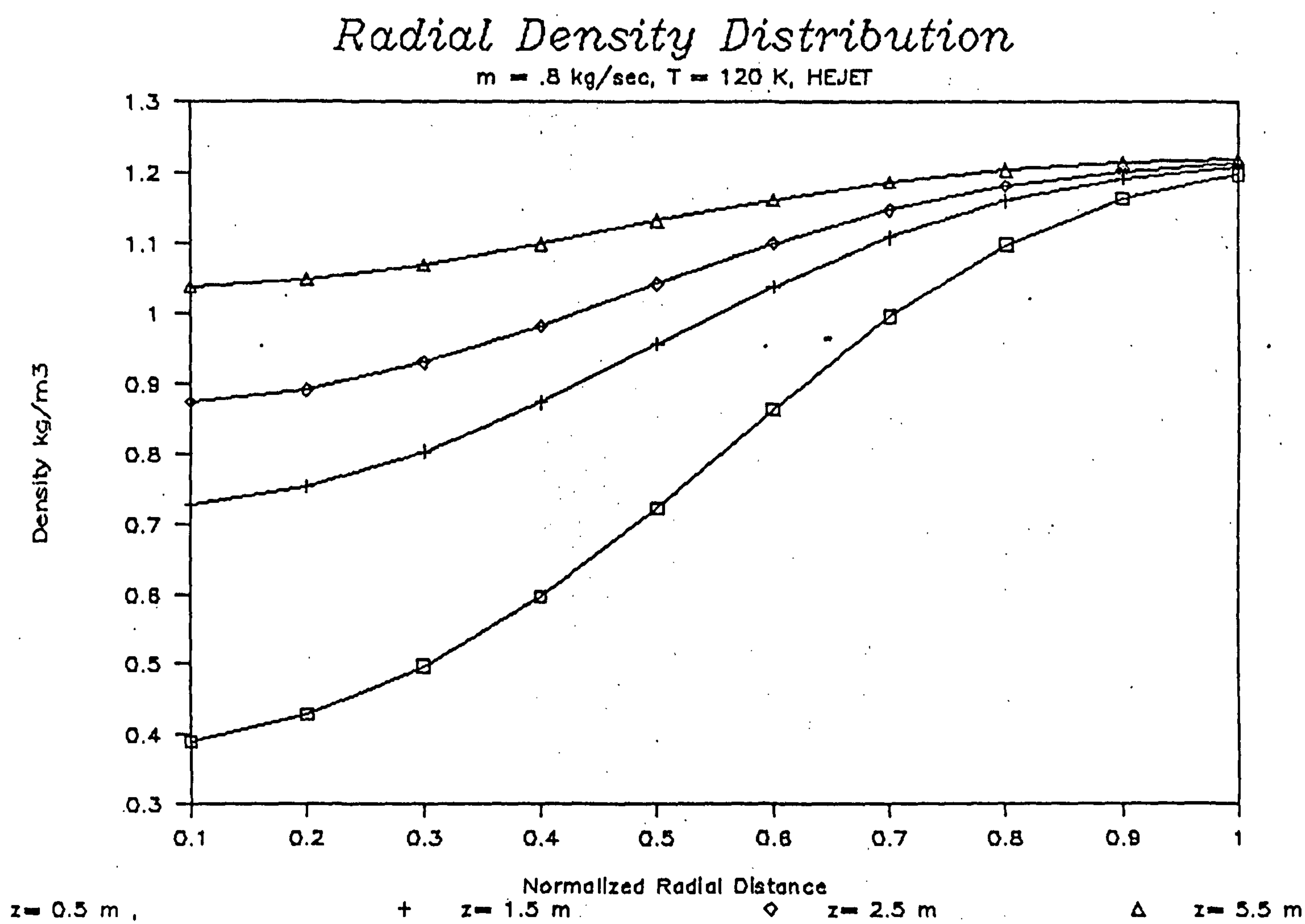

Figure 3.6b Radial distribution of density for Helium jet at 120

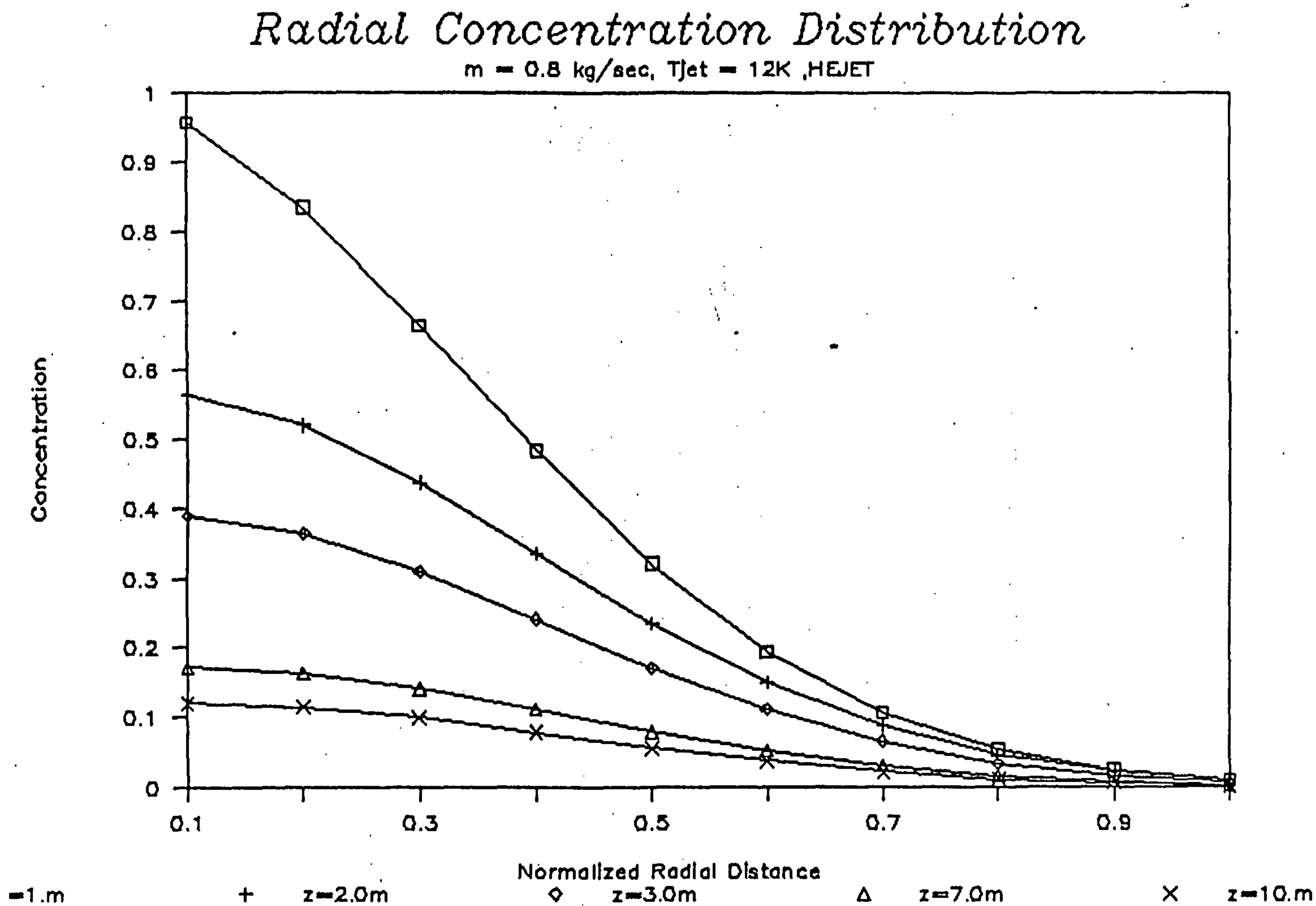

Figure 3.7a Radial concentration distribution for Helium jet at 12 K

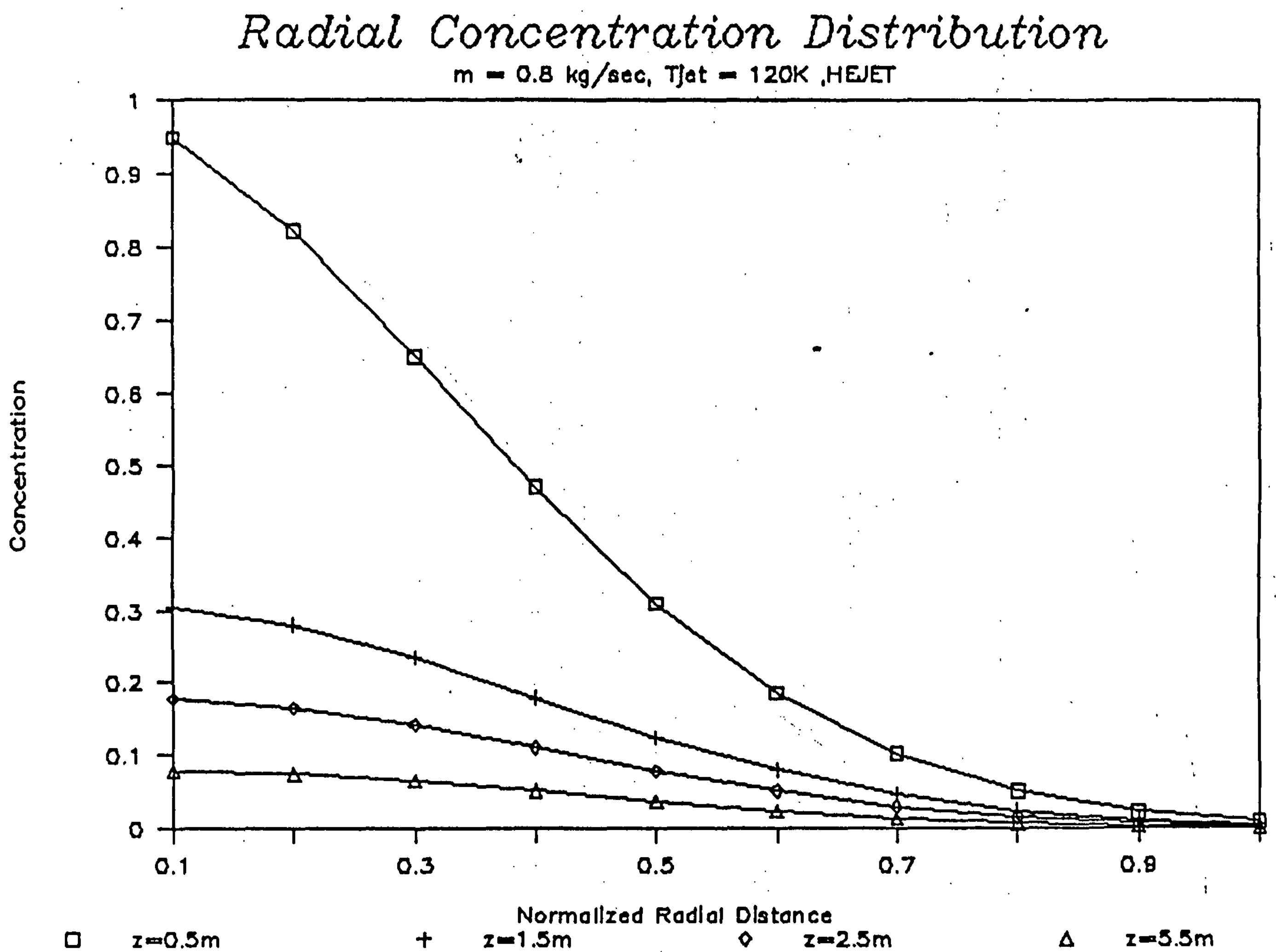

Figure 3.7b Radial concentration distribution for Helium jet at 120 K

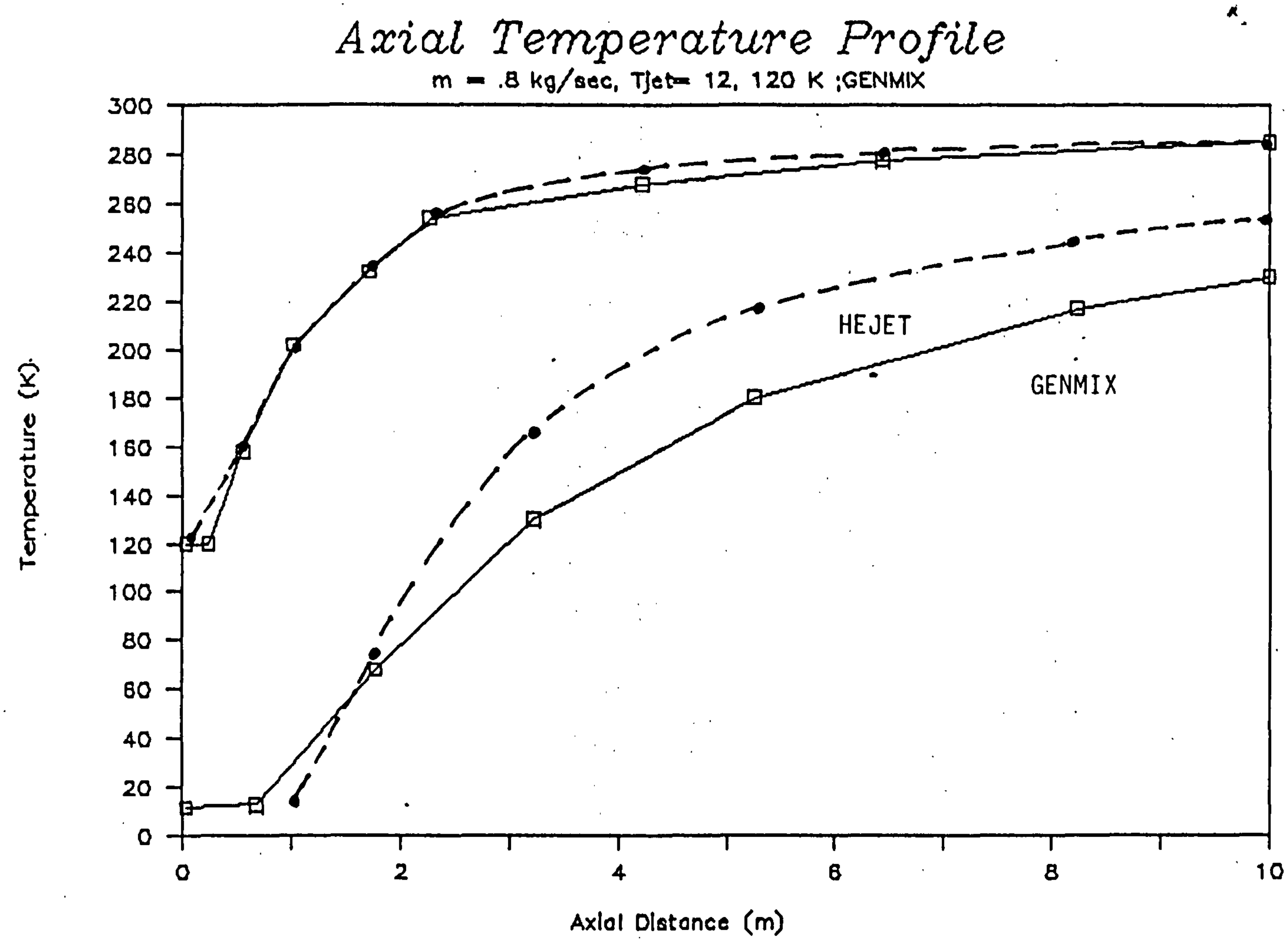

Figure 3.8 Axial temperature distributon obtained from GENMIX and HEJET analysis

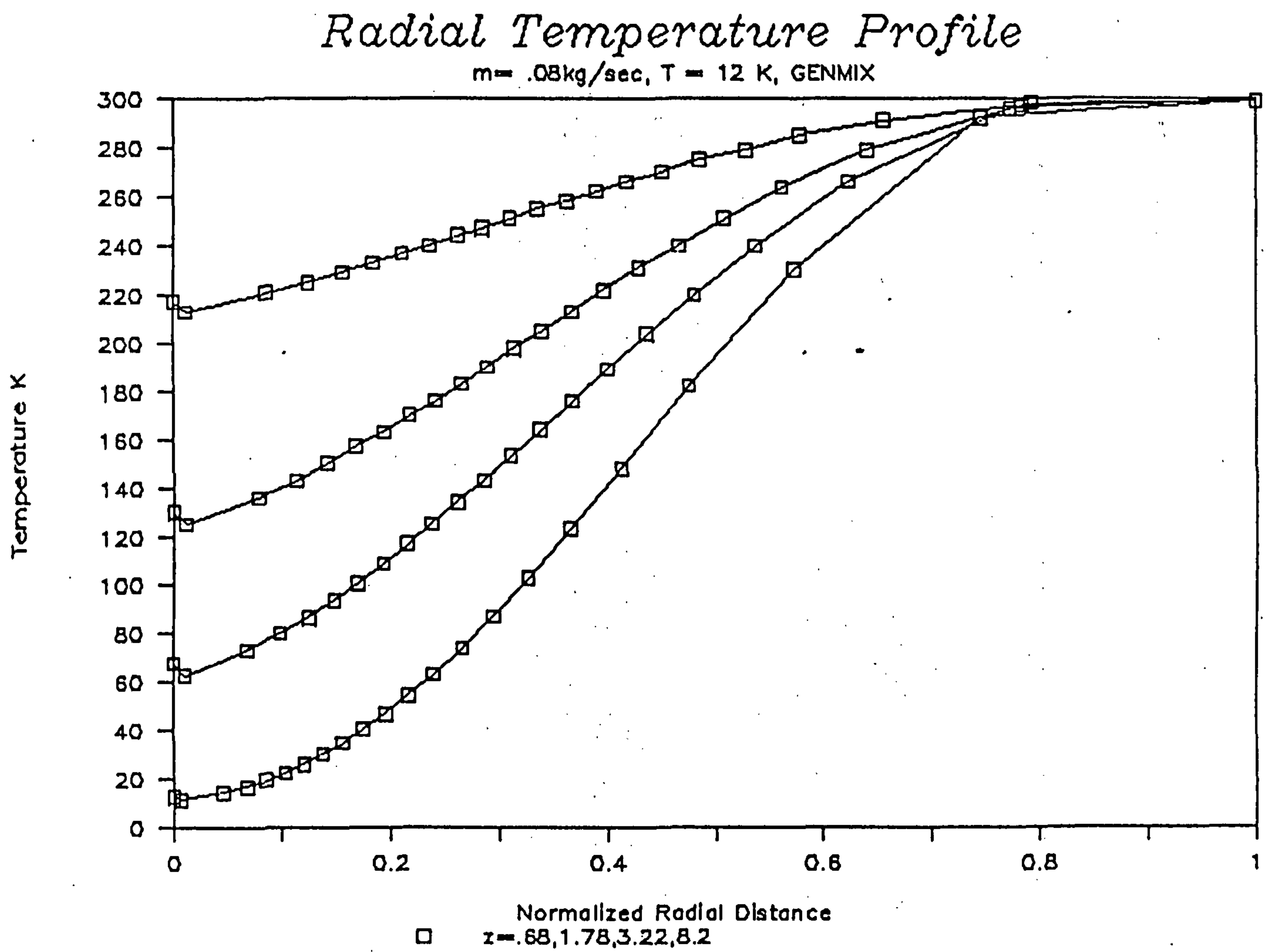

Figure 3.9 Radial temperature distribution obtained from GENMIX at 12 K

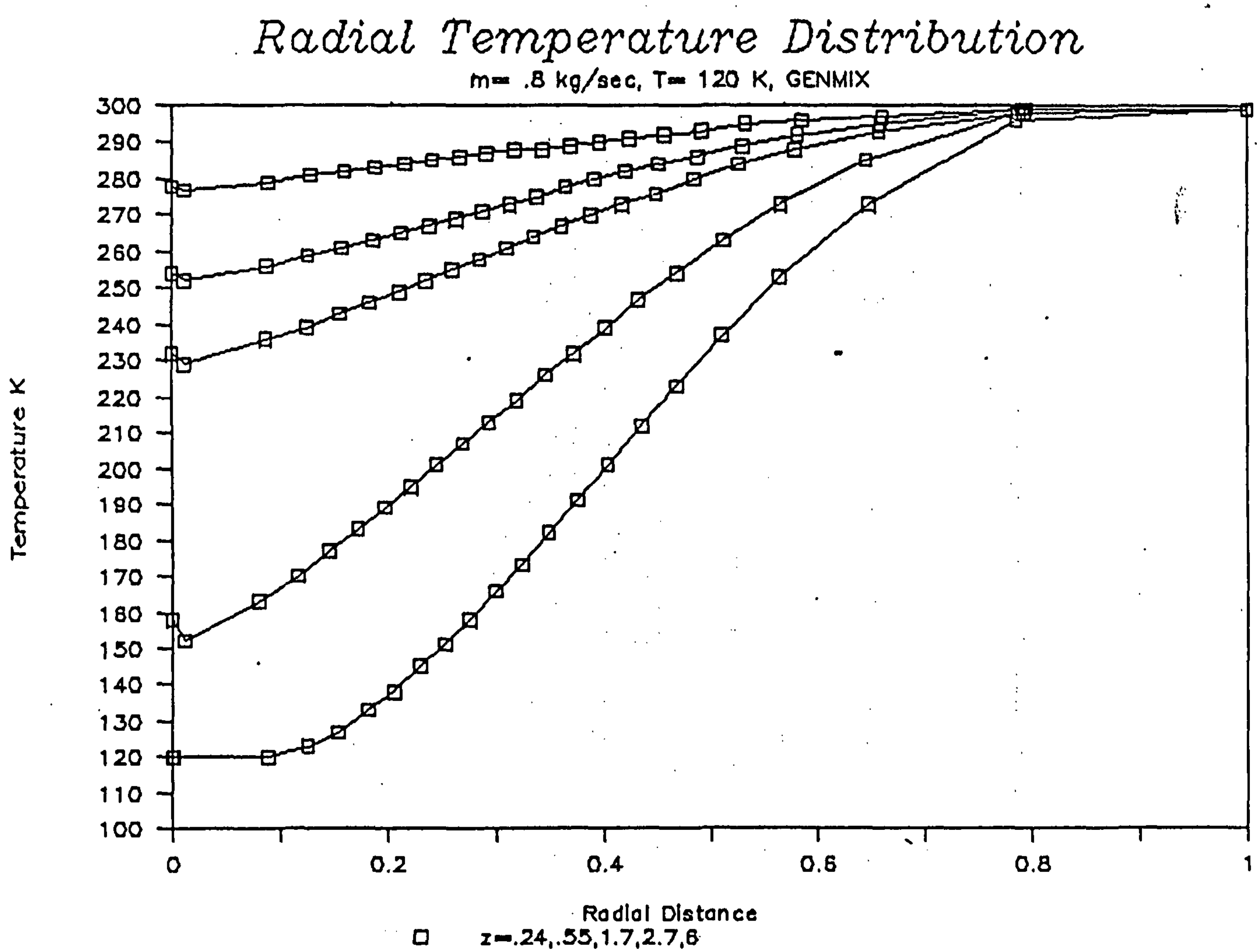

Figure 3.10 Radial temperature distribution obtained from GENMIX at 120 K

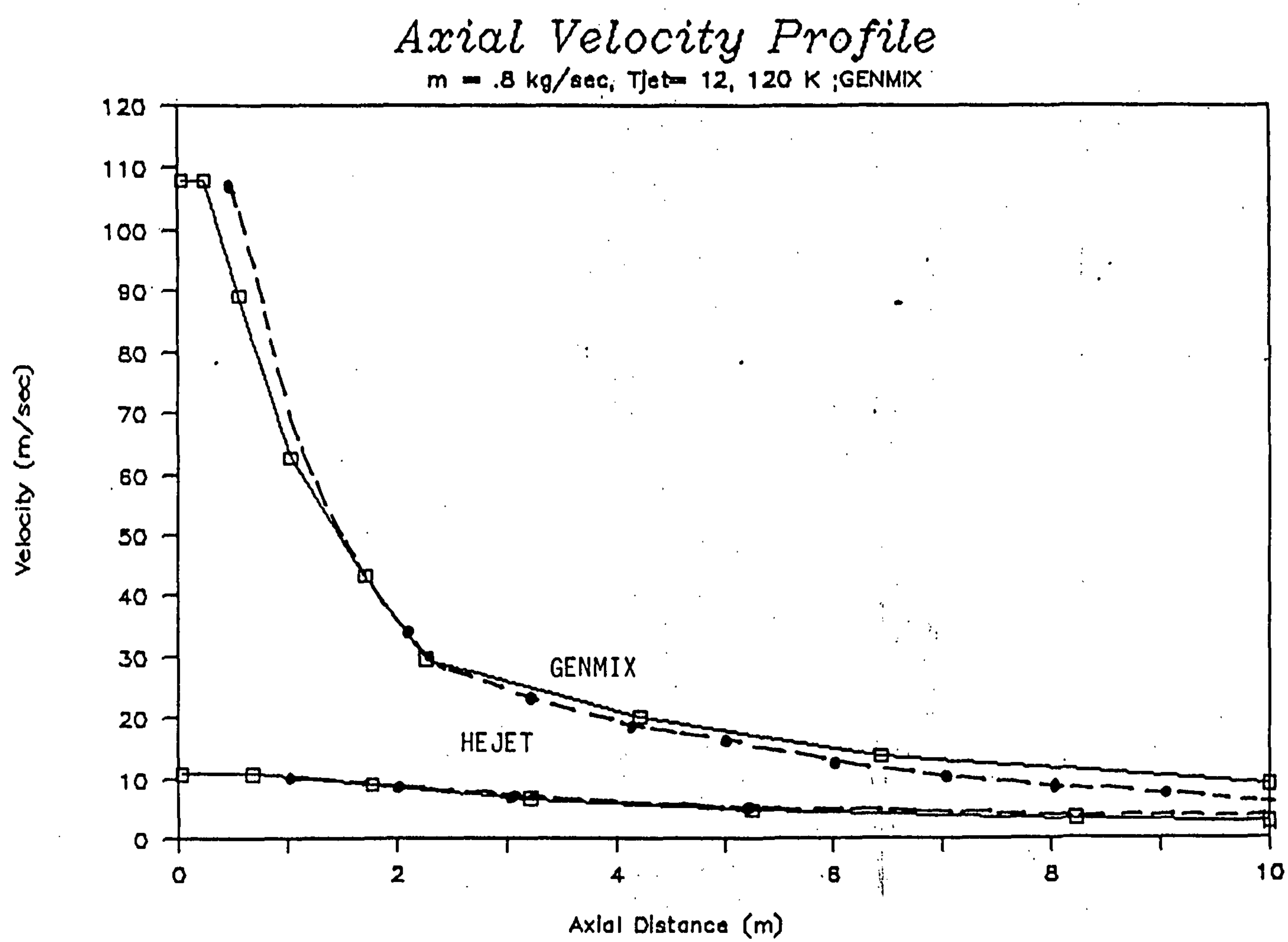

Figure 3.11 Axial velocity distribution obtained from GENMIX and HEJET analysis

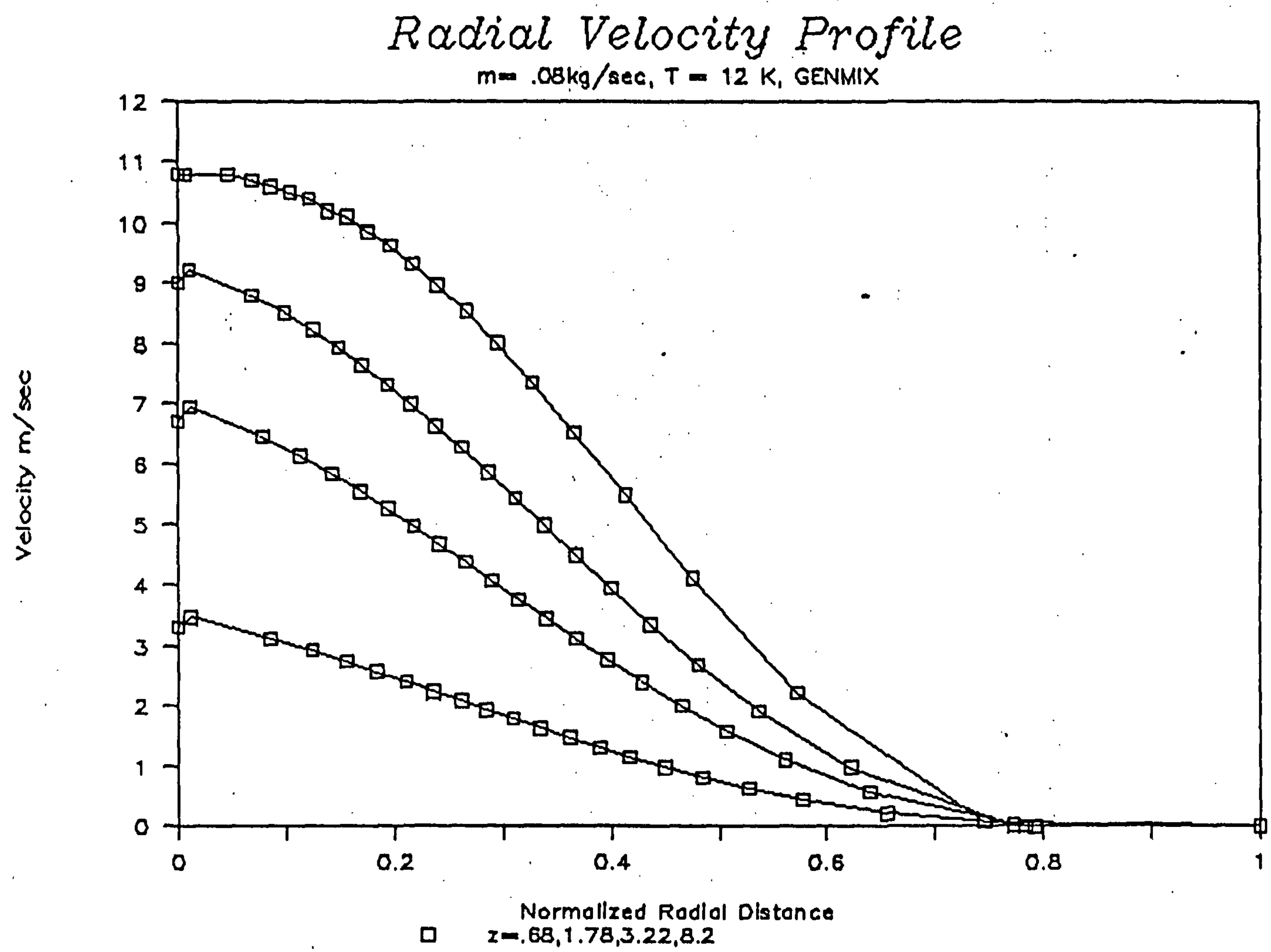

Figure 3.12a Radial velocity distribution obtained from GENMIX at 12 K

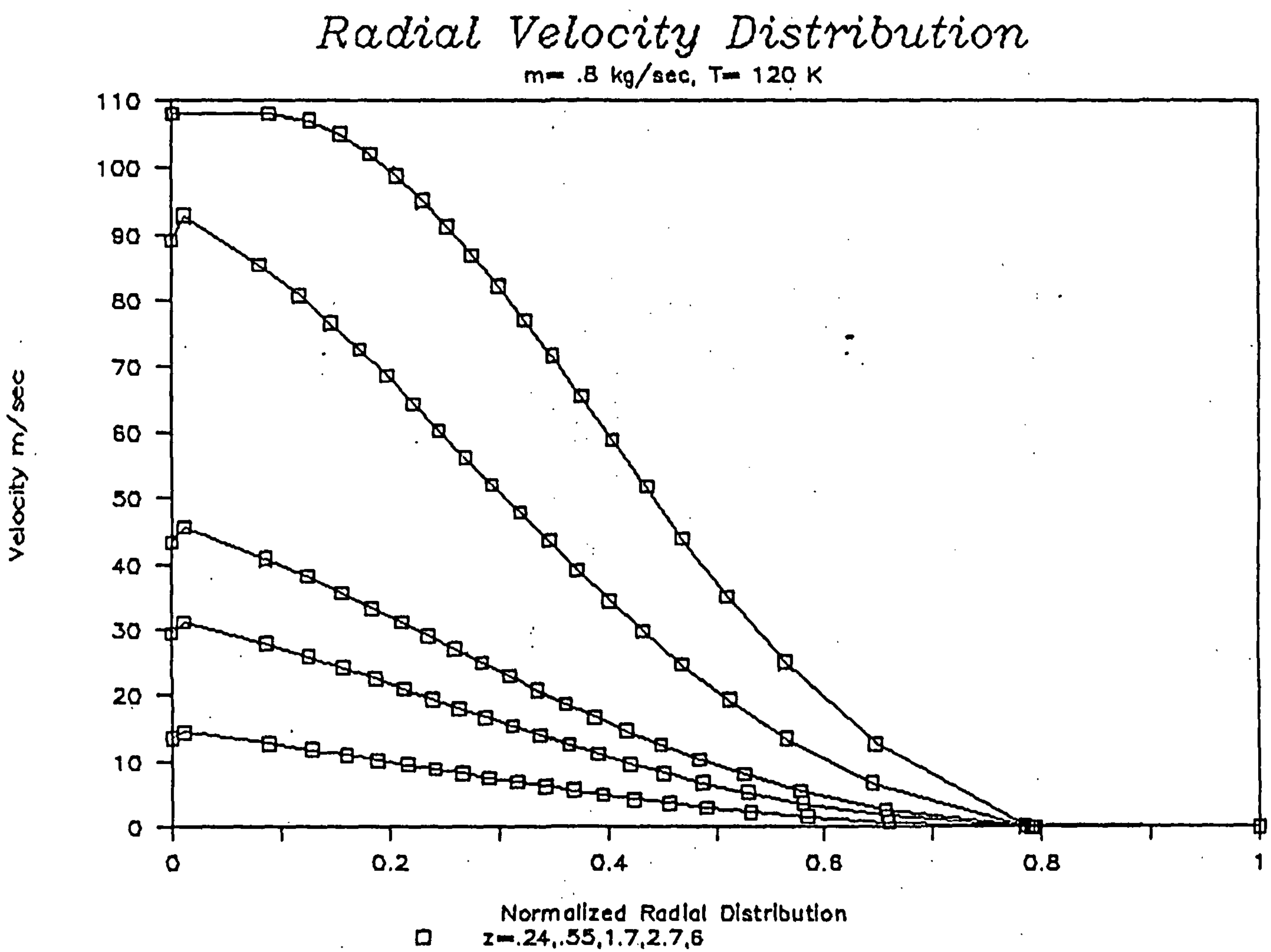

Figure 3.12b Radial velocity distribution obtained from GENMIX at 120 K

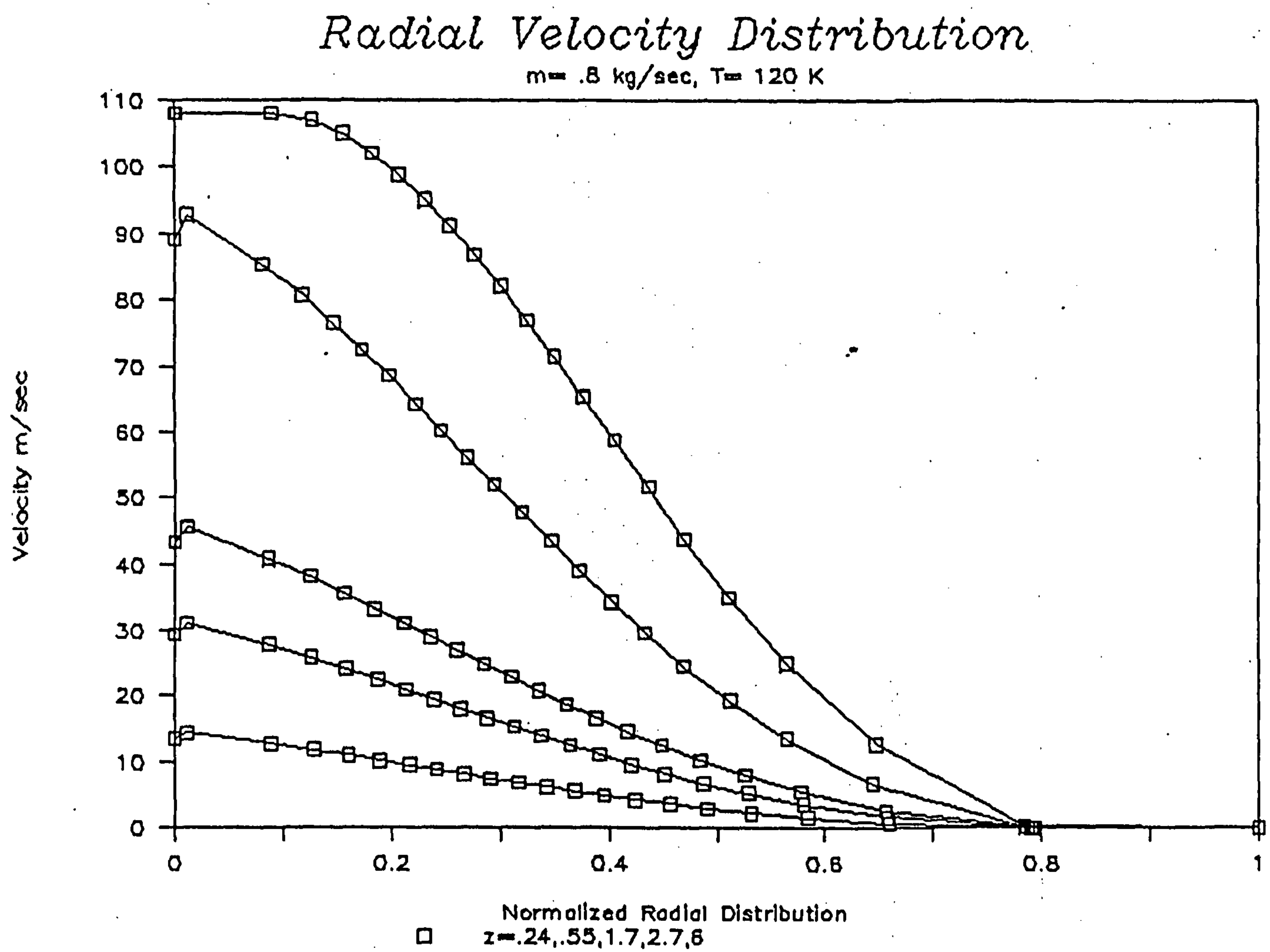

Figure 3.12b Radial velocity distribution obtained from GENMIX at 120 K

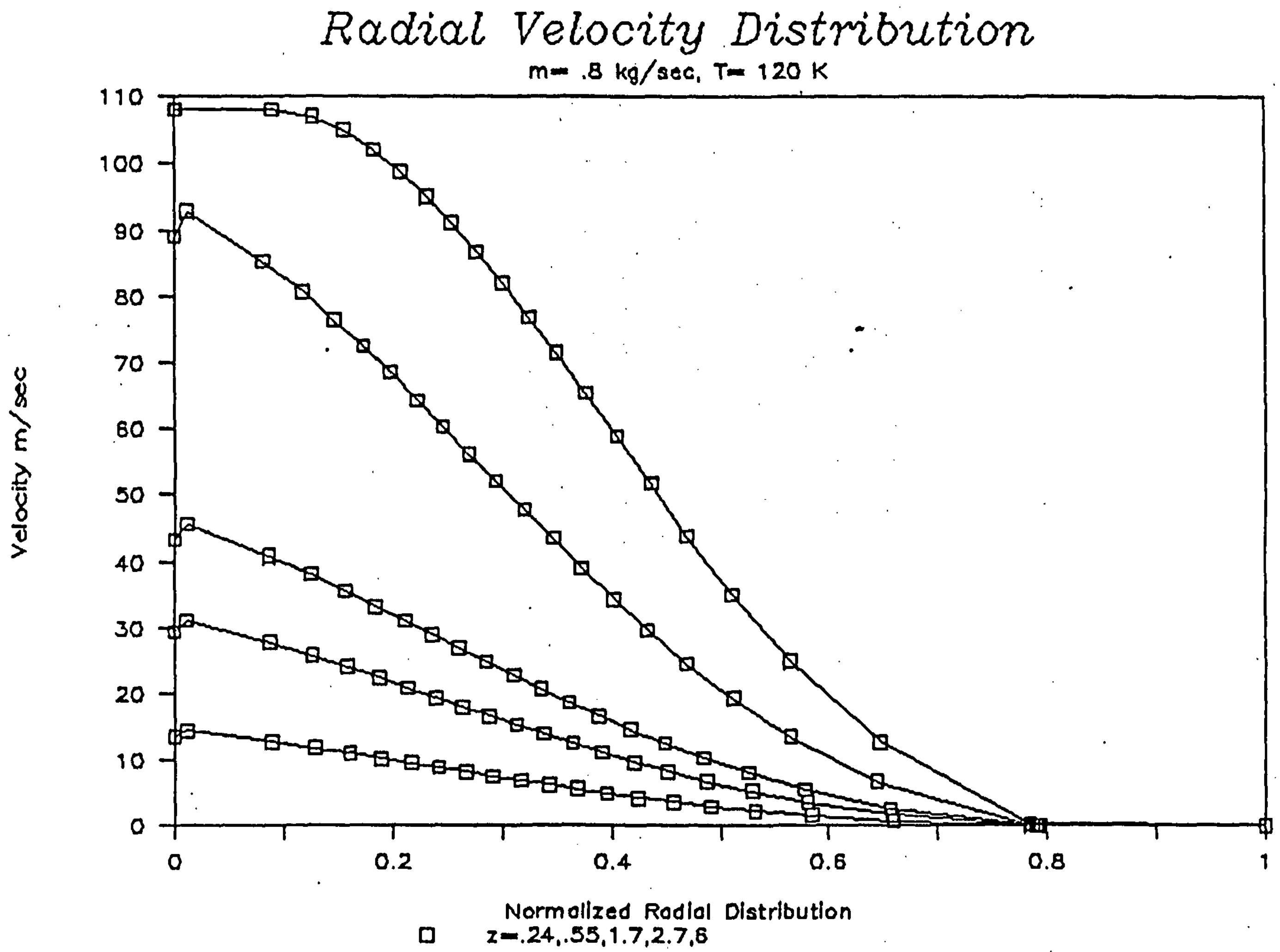

Figure 3.12b Radial velocity distribution obtained from GENMIX at 120 K

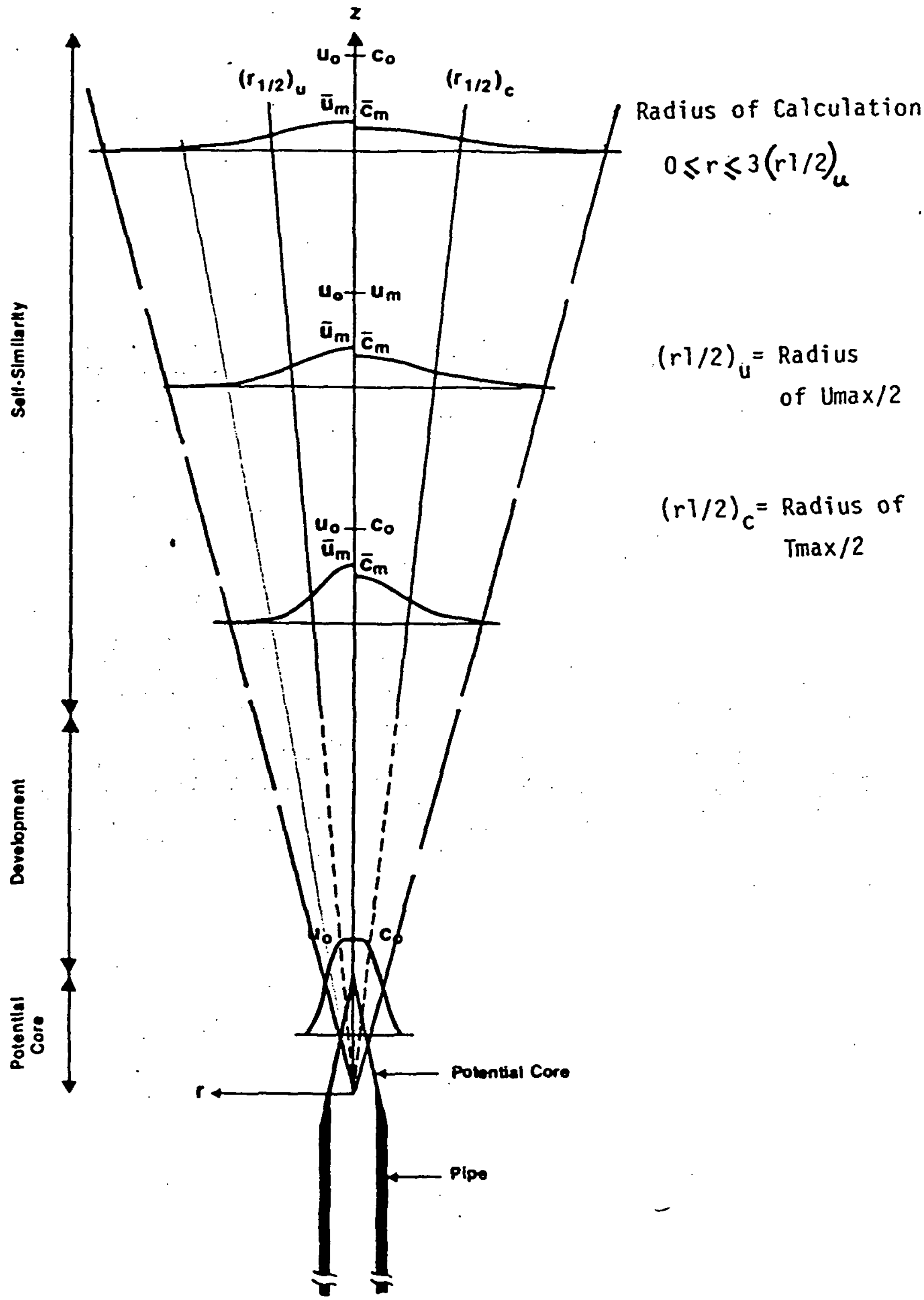

Figure 1.1 Schematic representation of an axisymmetric jet