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Least Reliable Bits Coding (LRBC) for High Data Rate Satellite Communications

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LEAST RELIABLE BITS CODING (LRBC) FOR HIGH DATA RATE SATELLITE COMMUNICATIONS

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I. ABSTRACT

An analysis and discussion of a bandwidth efficient multi-level / multi-stage [1] block coded modulation technique called Least Reliable Bits Coding (LRBC) is presented. LRBC uses simple multi-level component codes that provide increased error protection on increasingly unreliable modulated bits in order to maintain an overall high code rate that increases spectral efficiency. Further, soft-decision multi-stage decoding is used to make decisions on unprotected bits through corrections made on more protected bits.

Using analytical expressions and tight performance bounds it is shown that LRBC can achieve increased spectral efficiency and maintain equivalent or better power efficiency compared to that of Binary Phase Shift Keying (BPSK). Bit error rates (BER) vs. channel bit energy with Additive White Gaussian Noise (AWGN) are given for a set of LRB Reed-Solomon (RS) encoded 8PSK modulation formats with an ensemble rate of 8/9. All formats exhibit a spectral efficiency of 2.67 = $(\log_2 8)$*8/9 information bps/Hz. Bit by bit coded and uncoded error probabilities with soft-decision information are determined. These are traded with code rate to determine parameters that achieve good performance.

The relative simplicity of Galois field algebra vs. the Viterbi algorithm and the availability of high speed commercial VLSI for block codes indicates that LRBC using block codes is a desirable method for high data rate implementations.

II. INTRODUCTION

Future on-board processing communications satellites have been proposed using Frequency Division Multiple Access (FDMA) uplinks and Time Division Multiplexed (TDM) downlinks [2]. In some uplink cases, it will be found that accommodating a large number of FDM channels through a limited transponder or channel allocation bandwidth will require increased spectral efficiency. Bandwidth efficient modulation and coding techniques are necessary to achieve this efficiency without requiring an increase in signal power. In most downlink cases, the transmit power allocated for each high data rate TDM downlink beam will pose a more significant constraint on the system design than will the available spectrum allocation. Similar combined modulation and coding techniques optimized this time for power efficiency can satisfy this need. Further, attention must also be given to minimizing the complexity (weight, cost, and reliability) on-board the satellite and the complexity (cost) to the ground terminals. In an on-board processed FDMA/TDM system, a high rate multi-channel block decoder on the satellite would be an efficient method to protect hundreds or thousands of low to medium rate uplinks. Such a decoder would immediately follow a Multi-Channel Demultiplexer Demodulator (MCDD) that converts channelized carriers into demodulated time staggered data streams. A single, time shared encoder and modulator on board the satellite would enable power efficient TDM downlink processing in low cost ground terminals.

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III. ANALYSIS

III.1 : 8PSK Error Regions

A meaningful performance measurement of M-ary digital communications systems is the bit error probability in AWGN. The bit error probability is dependent on both the Euclidean distance properties of the signalling constellation and the Hamming distance of the bit to symbol mappings. A general equation that describes this relationship is

$$P_b = \frac{1}{M \log_2 M} \sum_{x=1}^{(\log_2 M)} \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} \left(b_{jx} \oplus b_{ix}\right) P_{|i-j|}$$

where M is the order of the M-ary scheme and x is one of the $\log_2 M$ bits being examined. The b_{jx}'s and b_{ix}'s are the logical value (1 or 0) of the x'th bit in the i'th or j'th decision region. The P term represents the probability that decision region i was received, given that decision region j was transmitted. These decision regions are illustrated in Figure 1 for 8PSK. The 8PSK decision region probabilities can be determined as sums of half-plane and quarter-plane probabilities written in terms of Q-functions. The Q-function definition and method of calculation is presented in Appendix A. A half-plane probability is given as [3]

$$h_\theta = Q\left[d_k \sin\theta\right]$$

where d_k corresponds to the signal to noise ratio and θ is an angle defining the beginning of the half-plane. In the case of 8PSK, d_k is the same for all signalling points and can be written as

$$d_k = d_{\log_2 M} = \sqrt{\frac{2\left(\log_2 M\right) E_b}{N_o}} = \sqrt{\frac{6E_b}{N_o}}$$

where E_b is the received bit energy and the zero mean white Gaussian noise has a two-sided power spectral density of $N_0/2$. A quarter-plane probability is the joint probability of two appropriate overlapping half-planes

$$q_\theta = h_\theta h_{\theta + \frac{\pi}{2}}$$

Using the above equations and the symmetry of the 8PSK constellation and the odd symmetry of the sine function, the decision region probabilities can be determined. For the case that the constellation point associated with the A_0 region was transmitted, P_0 is the probability that no symbol error occurred and is written as

$$P_0 = 1 - P_s$$

where P_s is the symbol error probability as calculated in Appendix B. To determine P_1, the first quadrant probability $P_{0,1}$ is

$$P_{0,1} = \left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right]\right)\left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{3\pi}{8}\right)\right]\right)$$

and P_1 is simply

$$P_1 = P_{0,1} - P_0$$

Similar equations are used to determine the remaining decision region probabilities and are given as follows

$$P_{1,2} = \left(Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right]\right)\left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{3\pi}{8}\right)\right]\right)$$

$$P_2 = P_{1,2} - P_1$$

$$P_{2,3} = \left(Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{3\pi}{8}\right)\right]\right)\left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right]\right)$$

$$P_3 = P_{2,3} - P_2$$

$$P_{3,4} = \left(Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right]\right)\left(Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{3\pi}{8}\right)\right]\right)$$

$$P_4 = P_{3,4} - P_3$$

Alternatively, it is noticed that the probability of symbol error can also be written as the sum of two half-planes minus their overlapping area

$$P_s = Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right] + Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{\pi}{8}\right)\right] - P_4$$

and combining with the equation for Ps (from Appendix B) it is seen that

$$P_4 \approx e$$

From the symmetry of the signalling constellation the following decision region probabilities immediately follow

$$P_5 = P_3 \qquad P_6 = P_2 \qquad P_7 = P_1$$

Three symbol mappings defined in Table 1 are evaluated for their probability of bit error as a function of their decision regions. The following three expressions result

$$P_{b\,NM} = \frac{1}{24}\left(28P_1 + 24P_2 + 36P_3 + 8P_4\right)$$

$$P_{b\,GC} = \frac{1}{24}\left(16P_1 + 32P_2 + 32P_3 + 16P_4\right)$$

$$P_{b\,DP} = \frac{1}{24}\left(24P_1 + 16P_2 + 40P_3 + 16P_4\right)$$

Symbol Number	Phase Value	Natural Mapping	Gray Coding	Decision Parsing
0	0	000	000	000
1	$\pi/4$	001	001	001
2	$\pi/2$	010	011	010
3	$3\pi/4$	011	010	011
4	π	100	110	110
5	$5\pi/4$	101	111	111
6	$3\pi/2$	110	101	100
7	$7\pi/4$	111	100	101

Table 1 : 8PSK bit to symbol mappings

The BER vs. E_b/N_o curves of these 8PSK mappings are shown in Figure 2. Due to the relative minimization of the dominating decision region probability P_1, gray coding exhibits the lowest E_b/N_o to obtain a given BER. However, these equations do not indicate which bits are contributing to the weights on the decision regions. To accomplish this, P_b can be written in a matrix form that illustrates the individual bit contributions as follows

$$P_b = \frac{1}{24}\begin{bmatrix} W_{Mrb\,0} & W_{mrb\,0} & W_{lrb\,0} \\ W_{Mrb\,1} & W_{mrb\,1} & W_{lrb\,1} \\ W_{Mrb\,2} & W_{mrb\,2} & W_{lrb\,2} \\ W_{Mrb\,3} & W_{mrb\,3} & W_{lrb\,3} \\ W_{Mrb\,4} & W_{mrb\,4} & W_{lrb\,4} \end{bmatrix}\begin{bmatrix} Mrb \\ mrb \\ lrb \end{bmatrix}$$

where the W_{bd}'s are the number of ways in which a bit error b can be caused by a decision region d multiplied by the probability of being in decision region d. The lrb (least reliable bit), mrb (middle reliable bit), and Mrb (most reliable bit) represent the rightmost, middle, and leftmost bits in the bit to symbol mappings, respectively. Matrices for the three mappings studied are

$$P_{b\,NM} = \frac{1}{24}\begin{bmatrix} 0P_0 & 0P_0 & 0P_0 \\ 4P_1 & 8P_1 & 16P_1 \\ 8P_2 & 16P_2 & 0P_2 \\ 12P_3 & 8P_3 & 16P_3 \\ 8P_4 & 0P_4 & 0P_4 \end{bmatrix}\begin{bmatrix} Mrb \\ mrb \\ lrb \end{bmatrix}$$

$$P_{b\,GC} = \frac{1}{24}\begin{bmatrix} 0P_0 & 0P_0 & 0P_0 \\ 4P_1 & 4P_1 & 8P_1 \\ 8P_2 & 8P_2 & 16P_2 \\ 12P_3 & 12P_3 & 8P_3 \\ 8P_4 & 8P_4 & 0P_4 \end{bmatrix}\begin{bmatrix} Mrb \\ mrb \\ lrb \end{bmatrix}$$

$$P_{b\,DP} = \frac{1}{24}\begin{bmatrix} 0P_0 & 0P_0 & 0P_0 \\ 4P_1 & 4P_1 & 16P_1 \\ 8P_2 & 8P_2 & 0P_2 \\ 12P_3 & 12P_3 & 16P_3 \\ 8P_4 & 8P_4 & 0P_4 \end{bmatrix}\begin{bmatrix} Mrb \\ mrb \\ lrb \end{bmatrix}$$

Reading the columns, with special notice of the P_1 terms that occupy the second row verifies that some bits are more likely to be in error than others. The less reliable bits result in the bulk of the errors. For example, Figure 3 shows the individual bit error rates of gray coded 8PSK where the lrb contributes more to the overall BER than the mrb and Mrb.

III.2 : LRBC Based on Reed-Solomon Codes

The remainder of this paper focuses on coding that takes advantage of the unequal bit by bit error probabilities. For this high data rate application, Reed-Solomon (RS) block codes are studied over convolutional codes because of the commercial availability of RS codec VLSI chips at data rates in the hundreds of megabits per second. This compares to Viterbi algorithm convolutional decoders available at tens of megabits per second. Also, as individual codewords are independent, block codes are well suited to time-shared decoding and independent burst decoding .

The character error correcting capability of the RS(n,k) in the absence error location information is

$$t = (n - k)/ 2$$

where n is the total number of characters per codeword and k is the number of information characters per codeword. The average channel character error rate P_c , is determined from the channel bit rate by

$$P_c = 1 - (1 - P_b)^m$$

where P_b is the channel bit rate and m= $\log_2(n+1)$ is the number of bits in an RS character. An approximate expression for the decoded character error probability of a t error correcting RS code is [4]

$$P_{cd} \approx \sum_{i=t+1}^{n} \frac{i}{n}(P_c)^i (1 - P_c)^{n-i}\binom{n}{i}$$

and the resulting average decoded bit error probability is

$$P_{bd} = \frac{2^{(m-1)}P_{cd}}{(2^m - 1)}$$

These expressions are used to determine the performance of the three 8PSK modulation schemes that are RS coded in a bit by bit fashion. In each case the ensemble code rate is 8/9, n=255 and k is shown in Table 2.

Bit	Case 1	Case 2	Case 3	Case 4	Case 5
lrb	171	199	207	213	227
mrb	NONE	241	237	233	227
Mrb	NONE	241	237	233	227

Table 2 : k Values for RS(n,k)

Figures 4, 5, 6 show the performance of the various coding schemes for each bit mapping. Figure 7 shows the best scheme from each mapping. Again, because of the minimization of the dominate P_1 term, the best mapping uses gray coding.

The use of soft-decision information [5] can be used to further improve the performance of LRBC. When a coded bit correction is made, the received decision region is necessarilly in error. The likely symbol transmitted is the nearest one (in terms of Euclidean distance) with the corrected coded bit(s). Knowing this, the uncoded bits can be modified as necessary, resulting in a reduced probability of bit error.

To quantify the performance of LRBC with soft-decision information, further parsing of the decision regions is needed. This structure of sixteen decision regions is shown in Figure 8. To find the probabilities associated with each minor decision region, denoted Pm, half and quarter-plane probabilities are again used. The following equations result

$$Pm_0 = P_0 /2$$

$$Pm_1 = \frac{1}{2}\left\{\left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{2\pi}{8}\right)\right]\right)^2 - P_0\right\}$$

$$Pm_2 = P_1 - Pm_1$$

$$Pm_3 = \frac{1}{2}\left\{\left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{4\pi}{8}\right)\right]\right)^2 - P_0 - 2P_1\right\}$$

$$Pm_4 = P_2 - Pm_3$$

$$Pm_5 = \left(1 - Q\left[\sqrt{\frac{6E_b}{N_o}}\sin\left(\frac{2\pi}{8}\right)\right]\right)^2 - P_0 - P_1 - P_2 - P_1$$

$$Pm_6 = P_3 - Pm_5$$

$$Pm_7 = P_4/2$$

$Pm_8 = Pm_7$ $\quad Pm_9 = Pm_6$ $\quad Pm_{10} = Pm_5$
$Pm_{11} = Pm_4$ $\quad Pm_{12} = Pm_3$ $\quad Pm_{13} = Pm_2$
$Pm_{14} = Pm_1$ $\quad Pm_{15} = Pm_0$

Three types of error events are considered to determine the effect of soft-decision information when only the lrb is coded. First, the probability that an uncoded mrb or Mrb is in error given a correction has been made on the coded lrb, called $P_{b1\ mrb\ ci}$ and $P_{b1\ Mrb\ ci}$ is calculated as

$$P_{bx} = \frac{1}{M}\sum_{j=0}^{M-1}\left[\sum_{\substack{i=0\\ i\,even}}^{2M-1}\left(b_{jx} \oplus b_{\frac{i-2}{2}x}\right)\left(b_{j\,lrb} \oplus b_{\frac{i}{2}lrb}\right)Pm_n + \sum_{\substack{i=0\\ i\,odd}}^{2M-1}\left(b_{jx} \oplus b_{\frac{i+1}{2}x}\right)\left(b_{j\,lrb} \oplus b_{\frac{i}{2}lrb}\right)Pm_n\right]$$

where i-2j cannot equal 0 or 1 (not an error event) and the minor decision region index n is

$$n = \begin{cases} i-2j, & i - 2j \geq 0 \\ 2j-i-1, & i - 2j < 0 \end{cases}$$

and thus

$$P_{b1\ mrb\ ci} = P_{b\ mrb}\left(1 - P_{db\ lrb}\right)$$
$$P_{b1\ Mrb\ ci} = P_{b\ Mrb}\left(1 - P_{db\ lrb}\right)$$

where $P_{db\ lrb}$ is the decoded bit error probability of the lrb. The second type of error event is the probability of bit error when no lrb correction or error has ocurred. This is written as

$$P_{bx} = \frac{1}{M}\sum_{j=0}^{M-1}\sum_{i=0}^{M-1}\left(b_{jx} \oplus b_{ix}\right)\left(\overline{b_{j\ lrb} \oplus b_{i\ lrb}}\right)P_{|i-j|}$$

$$P_{b2\ mrb\ ci} = P_{b\ mrb}$$
$$P_{b2\ Mrb\ ci} = P_{b\ Mrb}$$

The third type of error event is the probability of bit error given a lrb decoding errror or erasure has occurred. This is evaluated as

$$P_{bx} = \frac{1}{M}\sum_{j=0}^{M-1}\left[\sum_{\substack{i=0 \\ i\ even}}^{2M-1}\left(b_{jx} \oplus b_{\frac{i-2}{2}x}\right)\left(\overline{b_{j\ lrb} \oplus b_{\frac{i}{2}\ lrb}}\right)Pm_n + \sum_{\substack{i=0 \\ i\ odd}}^{2M-1}\left(b_{jx} \oplus b_{\frac{i+1}{2}x}\right)\left(\overline{b_{j\ lrb} \oplus b_{\frac{i}{2}\ lrb}}\right)Pm_n\right]$$

again the minor decision region index n is

$$n = \begin{cases} i-2j, & i - 2j \geq 0 \\ 2j-i-1, & i - 2j < 0 \end{cases}$$

$$P_{b3\ mrb\ ci} = P_{b\ mrb}\ P_{db\ lrb}$$
$$P_{b3\ Mrb\ ci} = P_{b\ Mrb}\ P_{db\ lrb}$$

where x = mrb and Mrb. The probability that a mrb or Mrb are in error is then

$$P_{b\ mrb\ ci} = P_{b1\ mrb\ ci} + P_{b2\ mrb\ ci} + P_{b3\ mrb\ ci}$$
$$P_{b\ Mrb\ ci} = P_{b1\ Mrb\ ci} + P_{b2\ Mrb\ ci} + P_{b3\ Mrb\ ci}$$

and overall probability of bit error is

$$P_b = \frac{1}{3}\left(P_{b\ lrb} + P_{b\ mrb\ ci} + P_{b\ Mrb\ ci}\right)$$

This probability is plotted for the three mapping schemes using a RS(255, 171) code on the lrb and channel correction information on the mrb and Mrb in Figure 9. This maintains the ensemble code rate of 8/9. In this case gray coding is no longer the best mapping, in fact it is the worst. At higher E_b/N_o (greater than approximately 7 dB in the gray coded case and 8 dB in the other cases) the RS decoded lrb begins to exhibit very low probability of error. At this point the most likely error event when the mapping is gray coded is dependent on Pm_3. In the other mappings, a Pm_3 decision region results in an lrb correction and thus if necessary, mrb and Mrb corrections. The reason decision parsing is slightly better than a natural mapping has to do with the mapping of the mrb and Mrb. In decision parsing, the mrb and Mrb look like gray coded QPSK, so the most likely error event Pm_4 will result in an error on either the mrb or Mrb, but not both. With a natural mapping, Pm_4 is also the most likely error event, but can result in both a mrb and Mrb error.

For completeness, Figure 10 illustrates the effect of using shorter RS codes on the decision parsed mapping. Figure 11 illustrates a comparison between the best LRBC code with channel information, the best LRBC code without channel information, the best uncoded 8PSK scheme, point from a time domain simulation (with a union bound based extrapolated data point) of a rate 8/9 convolutional coding system [6], and for comparison, uncoded BPSK scheme. Depending on the bit error rate a service class requires would dictate the code to be chosen.

The other issue is the hardware and computational complexity. The LRBC scheme with multi-stage decoding needs one RS codec operating at 1/3 of the information data rate. Multi-stage decoding requires a buffer to wait for codeword decisions and a lookup table to make the corrections. Soft-decision quantization error is a factor, the number of quantization bits (and thus the size of the lookup table) is being studied. The LRBC without multi-stage decoding requires either one time-shared RS codec operating at the full data rate, or three parallel RS decoders each operating at 1/3 the data rate. The rate difference in the codes will also require a degree of buffering. An LRBC scheme coding only the lrb and not using side information would require only one decoder at 1/3 the data rate and some buffering. If the amount of coding gain was acceptable for the application, it would be the simplest to implement. The performance of a convolutional coding scheme plotted in Figure 11 is an existing implementation at 225 Mbps by COMSAT Laboratories [6]. It requires a set of 8 semi-custom ECL gate arrays to perform the add-compare-select function of the 16 state Viterbi decoder, a board of state memory, and hard decision logic. It is expected that the LRBC implementaion using commercially available VLSI technology will exhibit about a factor of 100 reduction in combined mass and power consumption.

IV. CONCLUSION AND FUTURE WORK

An analysis of the performance of LRBC has been presented. It is shown that a rate 8/9, spectrally efficient 2 information bps/Hz LRBC can improve power efficiency over BPSK. Comparisons were presented on the different approaches to LRBC in terms of performance and complexity. An existing equivalent convolutional coding scheme was also used for comparison.

In the future, other types of block codes will be compared to the performance of the RS codes. The use of concatenated coding schemes will also be investigated. Optimization for power efficient applications such as high rate return links from the space exploration initiative program will be addressed.

Presently, a LRBC hardware prototype is being developed at NASA Lewis Research Center. It will be integrated into a programmable modem developed by COMSAT Laboratories for NASA Lewis [7]. The LRBC hardware will be compared to COMSAT's Coded Trellis Modulation (CTM) hardware in a laboratory environment.

APPENDIX A

Q(x) is defined as

$$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

For the 8PSK case x is defined as

$$x = \sqrt{\frac{3E_b}{N_o}}\,\sin\left(\frac{\pi}{8}\right)$$

The erfc(x) can be written in series notation as

$$\mathrm{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \cdot\right)$$

and thus

$$Q(x) = \frac{1}{2} - \frac{1}{\sqrt{\pi}}\left([x/\sqrt{2}] - \frac{[x/\sqrt{2}]^3}{3\cdot 1!} + \frac{[x/\sqrt{2}]^5}{5\cdot 2!} - \frac{[x/\sqrt{2}]^7}{7\cdot 3!} + \cdots\right)$$

in series notation this reduces to

$$Q(x) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\left(\frac{(-1)^k x^{(2k+1)}}{2^k(2k+1)\cdot k!}\right)$$

As k becomes large, the direct evaluation of this alternating series becomes more inaccurate. Thus, the terms of Q(x) are found by the ratio of the k and k-1 terms. This ratio is written as

$$\frac{\mathrm{term}_k}{\mathrm{term}_{k-1}} = \frac{\left(\dfrac{(-1)^k x^{2k+1}}{2^k(2k+1)\cdot k!}\right)}{\left(\dfrac{(-1)^{k-1} x^{2(k-1)+1}}{2^{k-1}(2(k-1)+1)\cdot(k-1)!}\right)} = \frac{x^2\,(2k-1)}{2\,(2k+1)\cdot k}$$

APPENDIX B

A tight bound on the probability of symbol error for 8PSK is written as [8]

$$P_s \approx 2Q\left[\sqrt{\frac{6E_b}{N_o}}\,\sin\left(\frac{\pi}{8}\right)\right] - e$$

The error term e that results in an upper bound for P_s is defined as

$$e = \frac{\tan\left(\frac{\pi}{8}\right)}{2\pi}\left[E_i(y) - \left(\frac{\frac{\exp^{-y}}{y} - E_i(y)}{2\cos^2\left(\frac{\pi}{8}\right)}\right)\right]$$

where y is

$$y = \frac{3E_b}{N_o}$$

and E_i (y) is the exponential integral defined in [9] as the series

$$E_i(y) = -\gamma - \ln(y) + \sum_{k=1}^{\infty}\frac{y^k}{k\cdot k!}$$

where γ is Euler's constant. The ratio of the k to k-1 terms is

$$\frac{\mathrm{term}_k}{\mathrm{term}_{k-1}} = \frac{\left(\dfrac{y^k}{k\cdot k!}\right)}{\left(\dfrac{y^{k-1}}{(k-1)\cdot(k-1)!}\right)} = \frac{y\ (k-1)}{k^2}$$

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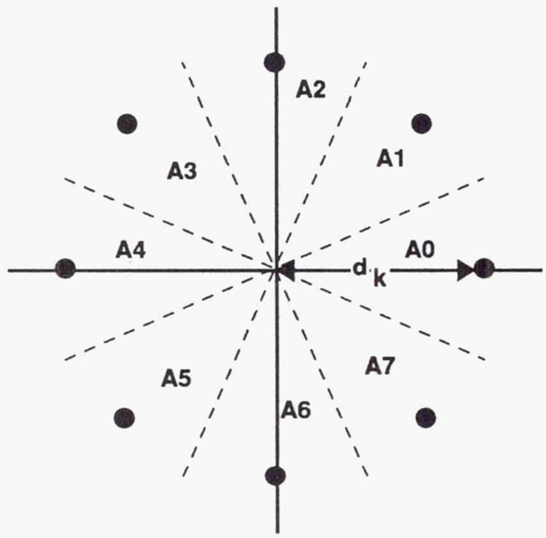

Figure 1: 8PSK Decision Regions

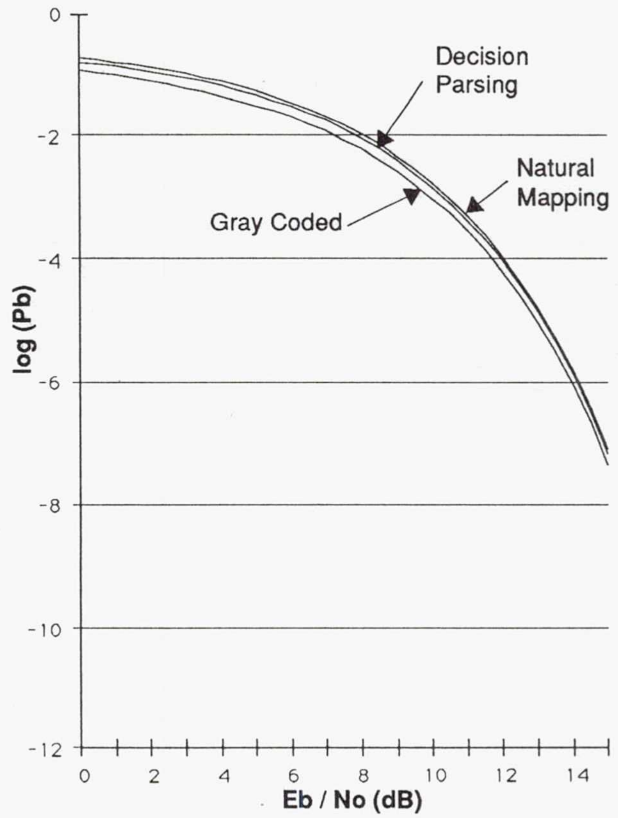

Figure 2: Uncoded 8PSK Comparison

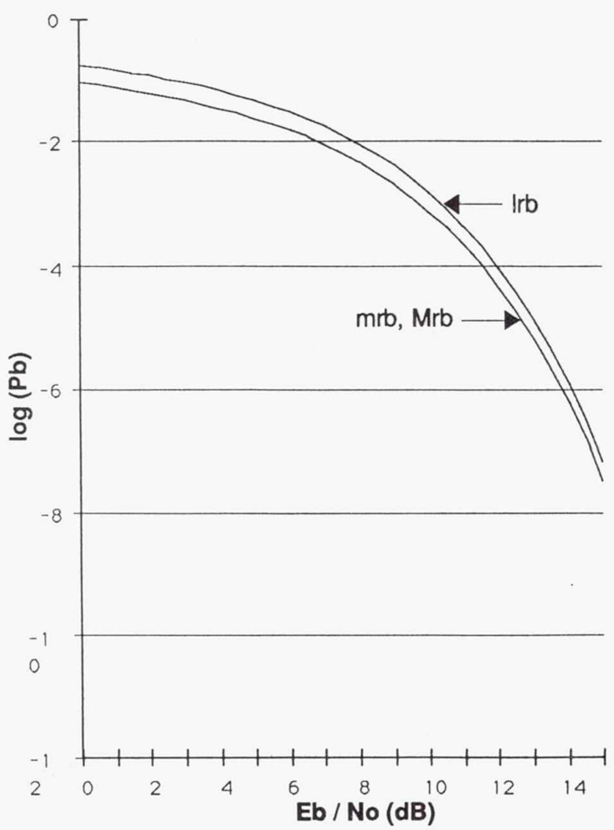

Figure 3: Gray Coding Bit by Bit Error Probabilities

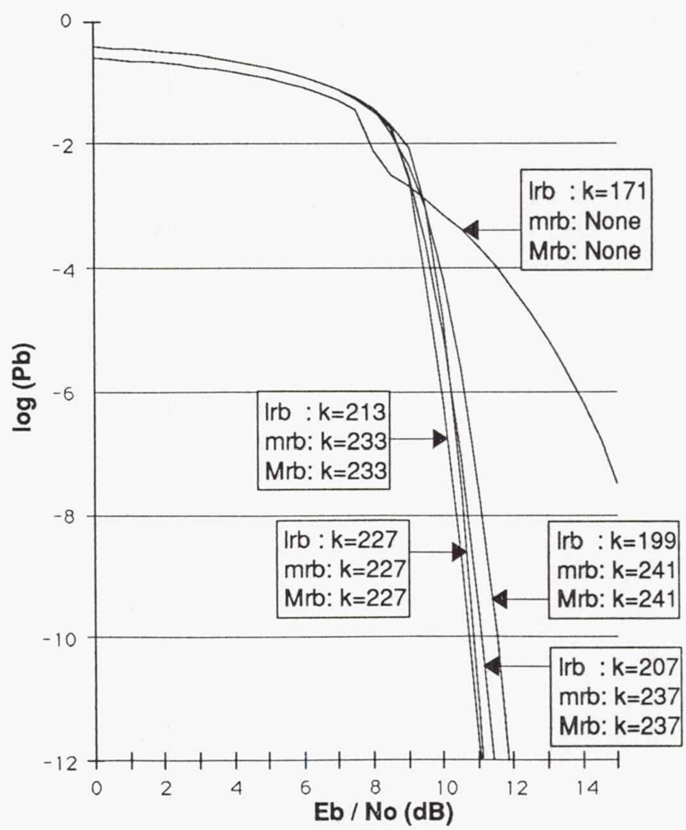

Figure 4 : LRBC with Natural Mapping

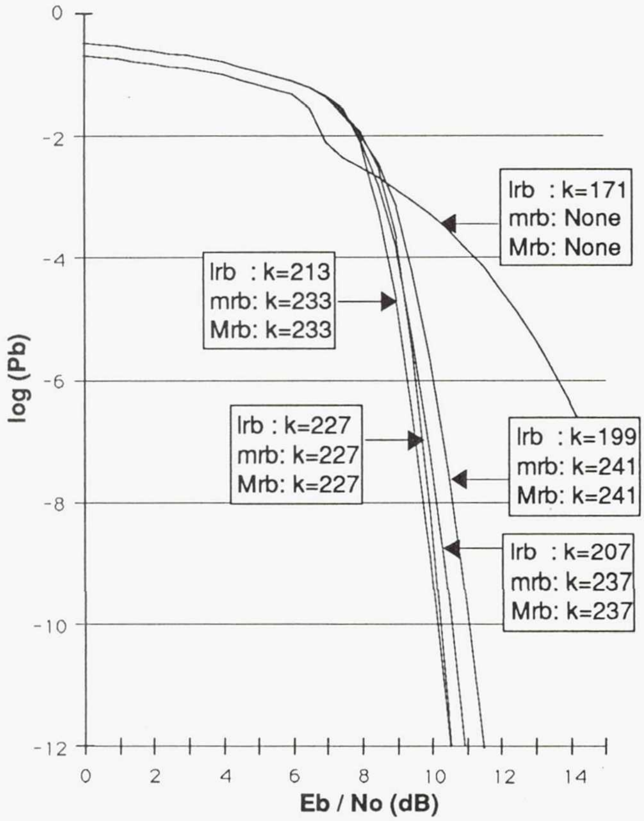

Figure 5: LRBC with Gray Coding

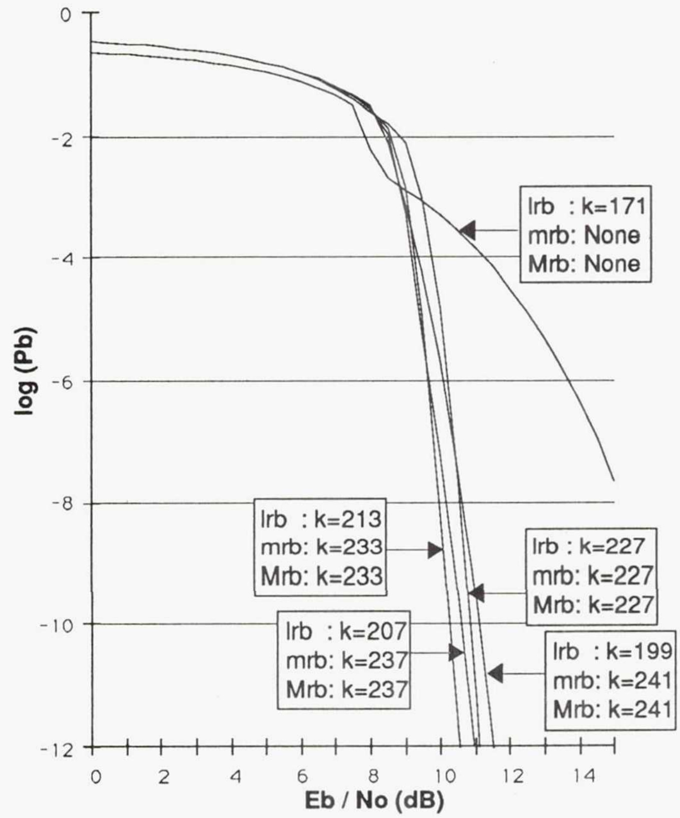

Figure 6: LRBC with Decision Parsing

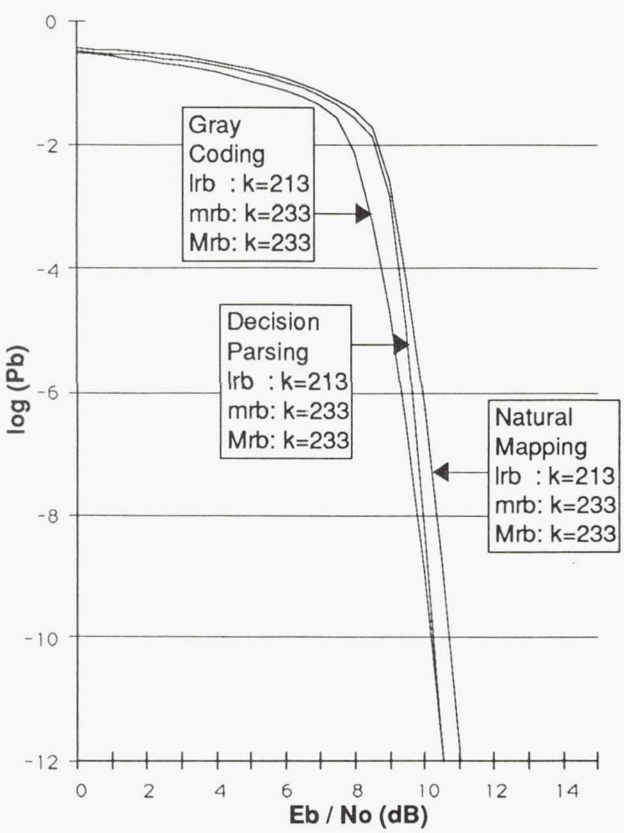

Figure 7: LRBC Comparison

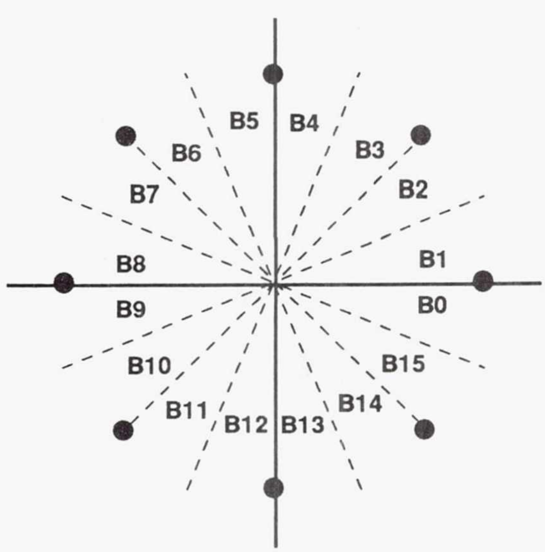

Figure 8: Minor Decision Regions

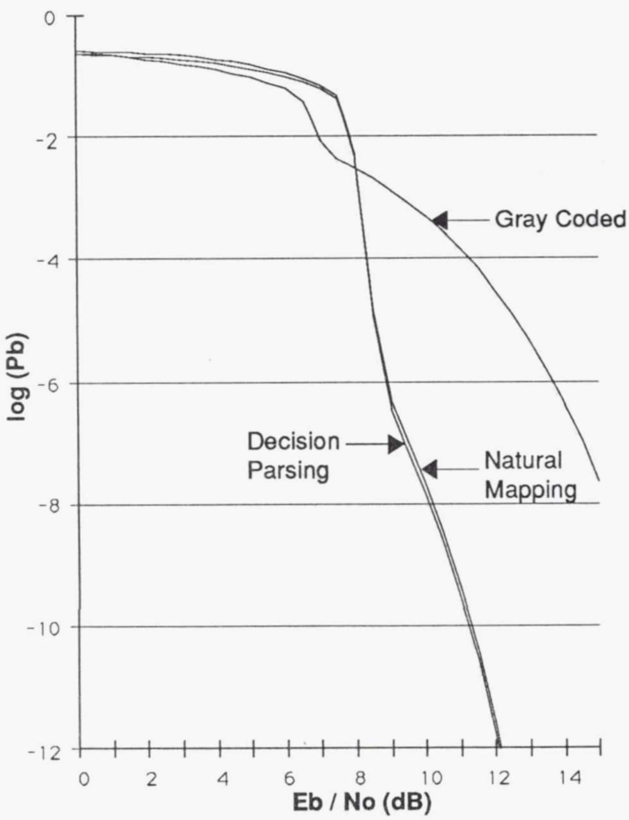

Figure 9: Multi-Stage LRBC

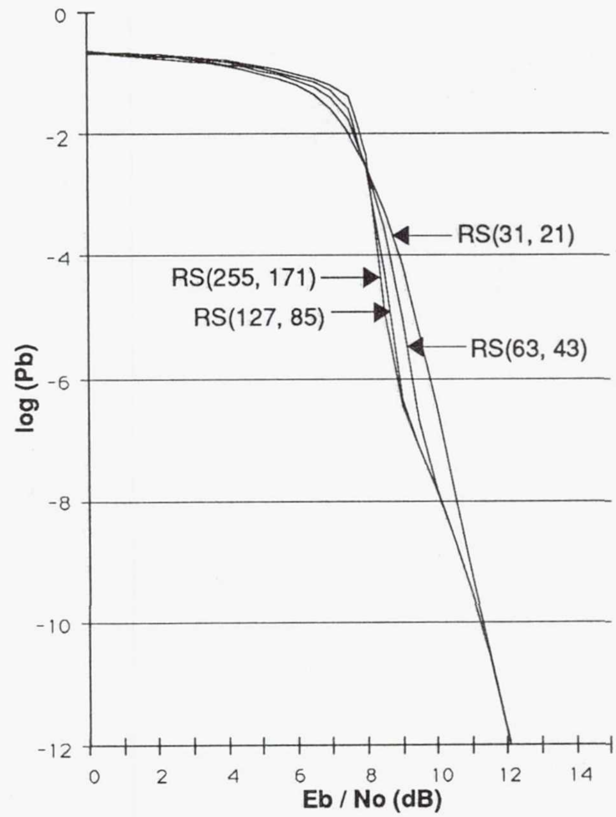

Figure 10: Multi-Stage Decision Mapped LRBC

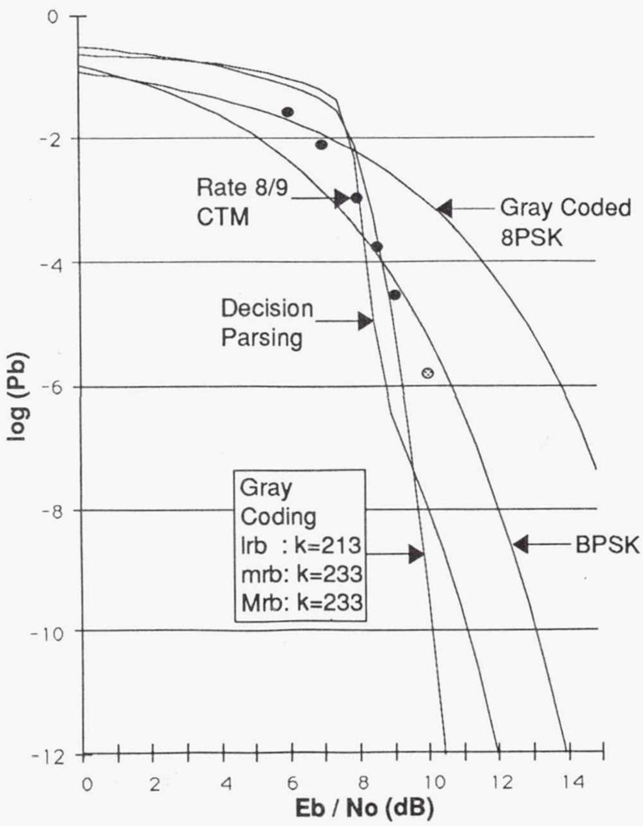

Figure 11: Overall Comparison

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13. ABSTRACT ***(Maximum 200 words)***

An analysis and discussion of a bandwidth efficient multi-level/multi-stage block coded modulation technique called Least Reliable Bits Coding (LRBC) is presented. LRBC uses simple multi-level component codes that provide increased error protection on increasingly unreliable modulated bits in order to maintain an overall high code rate that increases spectral efficiency. Further, soft-decision multi-stage decoding is used to make decisions on unprotected bits through corrections made on more protected bits. Using analytical expressions and tight performance bounds it is shown that LRBC can achieve increased spectral efficiency and maintain equivalent or better power efficiency compared to that of Binary Phase Shift Keying (BPSK). Bit error rates (BER) vs. channel bit energy with Additive White Gaussian Noise (AWGN) are given for a set of LRB Reed-Solomon (RS) encoded 8PSK modulation formats with an ensemble rate of 8/9. All formats exhibit a spectral efficiency of 2.67 = $(\log_2 8)*8/9$ information bps/Hz. Bit by bit coded and uncoded error probabilities with soft-decision information are determined. These are traded with code rate to determine parameters that achieve good performance. The relative simplicity of Galois field algebra vs. the Viterbi algorithm and the availability of high speed commercial VLSI for block codes indicates that LRBC using block codes is a desirable method for high data rate implementations.

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