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**Structural Deterministic
Safety Factors Selection
Criteria and Verification**

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TECHNICAL PAPER

STRUCTURAL DETERMINISTIC SAFETY FACTORS SELECTION CRITERIA AND VERIFICATION

I. INTRODUCTION

As emphasis in the aerospace industry extends from optimum performance to high reliability and low-cost life cycle, technologies for reducing structural failures are being assessed and new ones proposed. Basic to these are the conventional deterministic safety factor and the emerging probabilistic safety index. Though probabilistic methods promise to provide more reliable structures at reduced weight, they are not expected to dominate general safety practices in their present forms. In the interim, the conservative and arbitrary selection of the conventional deterministic safety factor might be alleviated by rethinking its concept and capitalizing on its inherent probabilistic properties.

Probabilistic safety methods are highly regimented, rooted in progressive statistical techniques, and demanding of definitive data format and high fidelity engineering models. In their evolving state, they have the potential for rendering unique and optimum predictions, but these same demands do not make them particularly compatible with general designing-room dynamics. However, recent approaches are proving to be superior in cumulative damage failure modes. It is foreseeable that when methods and data banks become more adaptable to common design processes, their potential for consistently reliable predictions will reduce verification test requirements which will compensate for the computational intensive techniques.

Current application of the deterministic method is loosely structured and is predicated on a virtually zero failure rate. Safety factor selections are arbitrary and subjective, based on related corporate experiences and the designer's personal judgment. Nevertheless, it basks in decades of success, and its simplicity has made it adaptable to all structural designs and all levels of designer competence. However, this simplicity may be its own weakness and ultimate fall. Because factors may be arbitrarily and unaccountably specified, inconsistencies of quality, completeness of analyses, and unnecessary imbalances of safety measures slip into supposedly high performance structures.

The purpose of this document is to present a more coherent guiding philosophy in designing safe aerostructures. Leading any safety analysis discussion is the identification of common sources and causes of failure, followed by an appreciation for statistical techniques and data analysis supporting safety methods. A fundamental probabilistic method is presented as a basis for understanding the failure concept. The deterministic method is shown to conform to a probabilistic concept consisting of three safety factors involving materials, loads, and stress. These safety factors are combined into an index to support trades among the safety factors and to compare safety of structural regions. Bases for formulating safety criteria are proposed, and safety verification is discussed. Cumulative damage and instability phenomena operate on different material properties, and though safety concepts are similar, they are not treated in this document.

II. SOURCES OF STRUCTURAL FAILURE

Failure occurs when the applied stress on a structure exceeds the resistive stress of the structural material. In this very simple concept rests the problem of defining the material resistive properties from measured data and of predicting applied stresses using measured and assumed data. The uncertain nature of data is best characterized as a probabilistic density distribution. Where the resistive and applied stress distributions intercept, failure occurs. This failure concept is illustrated by figure 1.

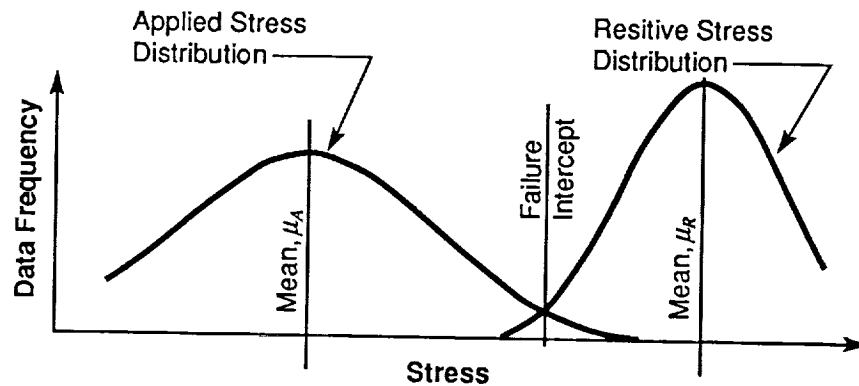


Figure 1. Failure concept.

Failure of any kind is costly, especially when the structure survives development tests and then fails during the operational phase. In the extreme, failures have paralyzed payload traffic, rendered patched and inefficient hardware, placarded operations, tarnished reputations, and generally burdened analysts with paper controls of dubious deterrence. Fortunately, failure investigations have clearly revealed that few failures are caused by ignorance and sneak phenomena; **most failures are caused by avoidable incomplete analyses and poor reasoning.** Then only after avoidable causes of failure are identified, thoroughly understood, and completely analyzed can safety factors be wisely selected and applied.

This section establishes engineering methods to support responsible judgment and promote more complete analyses for designing safer structures. It begins by defining failure, and proceeds to conceptualize sources of structural failure through all phases of design, analysis, manufacturing, verification, and operations to construct a failure source tree. Measured and assumed data from failure sources are discussed, and statistical techniques are demonstrated for evaluating data distributions and obtaining tolerance limits. Uncertainties of induced loads and stress math models are discussed. Limits of analyses are noted as unavoidable sources of failure which are compensated through design safety factors.

A. Failure Tree

At the core of static stress failures of metallic structures is the material stress-strain relationship that embodies both yield and ultimate failure modes (fig. 2). It is the easiest of properties to obtain from a simple uniaxial tension test and, from it, all other required mechanical properties may be derived.¹ The yield or ultimate stress distribution represents the resistive side of figure 1, and other material properties used to calculate operational stresses are modeled on the applied stress side.

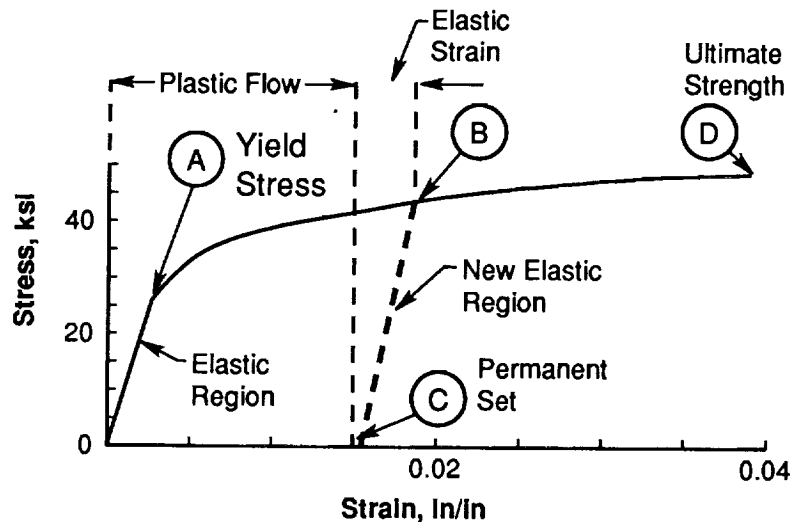


Figure 2. Material dual failure modes.

Structures are designed to operate under the worst predictable environments within the elastic region of a material, typified by the straight line 0A. If the real operating applied stress exceeds the elastic limit, point A, the material will deform inelastically to point B. In effect, the applied stress exceeds the yield failure intercept of figure 1 and operates in the yield resistive stress side. Upon relaxing the stress to zero, the structure is permanently deformed to point C, resulting in dimensional and boundary load changes. Exceeding the elastic limit may constitute a yield failure mode, if the excessive deformation produces an operational malfunction, such as leakage, interference, binding, or critical misalignment on repeated cycles.

If the applied stress continues to increase and exceeds the ultimate strength of the material, point D, the structure will fracture and fail in the second mode, the ultimate failure mode, which may compromise an operation or destroy life and equipment. The resistive stress is the uniaxial ultimate stress property. The applied stress is the multiaxial predicted stresses converted to uniaxial tension stress through the minimum distortion energy theory.²

To identify the most probable cause of a premature structural test failure, Dr. George McDonough devised a failure tree³ to screen an assortment of possible failure sources from material acceptance through design, operations, and final test. Since it was so effective in finding the cause of a genuinely experienced failure, it would seem more rewarding to apply it up front, in the design phase, to prevent failures.

Figure 3 is a modification of that tree, but each tree should be tailored to reflect the corporate experience with its own unique class of products. A tree need not be exhaustive, but must include a select list of sources to spark the inquiring process for the generic, the unusual, and the recurring. Not all possible sources are of equal fatality. Sensitivity methods are very apt for discriminating most crucial sources. Sensitivity analyses are also useful for optimizing design modifications related to the failure source, or to identify and specify operational parameter limits on submarginal structures.

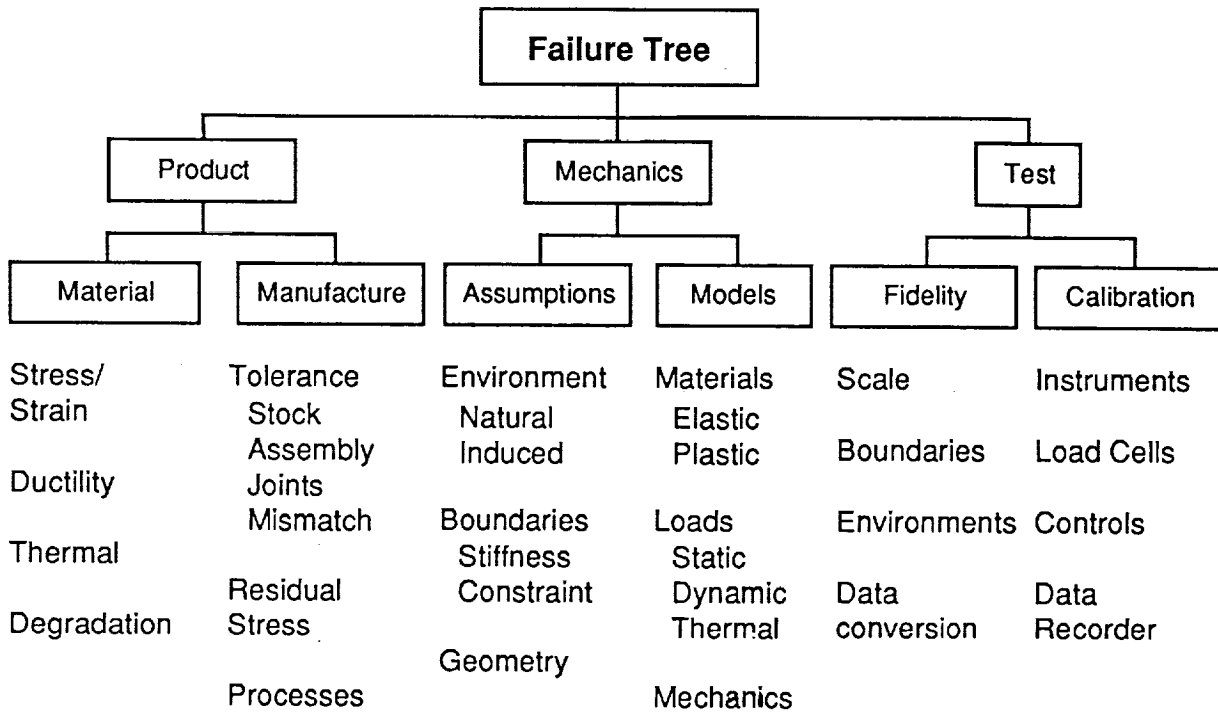


Figure 3. Structural failure source tree.

B. Data Evaluation

Failures occur at the weakest source which realizes variations in that source from article to article and from one operation to another. Distribution of these variables forms the data base used to develop structural design parameters. Failure tree sources that generate data are in materials, manufacture, environments, and test categories. Once a potential failure source is identified, test data are collected and applied to analytical techniques that support judgment as to the sufficiency of data sample size, its distribution, its expected design tolerance limit, and the bases for them.

The conventional approach to alleviate empiricism in data evaluation is through statistical methods. Statistics deals with data analysis and the application of data in decision analysis. There is an abundance of literature⁴ on the subject, and all who obtain, develop, and use data should have a good working knowledge of the subject.⁵ Consequently, only those features of statistics supporting and underscoring judgment based on complete engineering analysis techniques are elaborated.

The best way to summarize a table of raw data of any distribution is to define its centroid about which the data are scattered. This variable is the first moment of the independent variables commonly known as the sample mean, or sample average, and is defined by

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad (1)$$

where x_i is the i th specimen value, and n is the total number of specimens. The sample mean is calculated from a limited sample size and is, therefore, an estimate of the population mean. A measure of the dispersion of the data about the mean is the second moment, known as the sample variance, and it is calculated from

$$s^2 = \frac{\sum_{i=1}^n [x_i - \bar{x}]^2}{n-1} . \quad (2)$$

The square root of the sample variance of equation (2) is called the sample standard deviation " S ," which is a measure of the actual variation in a set of data.

The coefficient of variation is the relative variations, or scatter, among sets and is defined as the ratio of the standard deviation and the mean,

$$CV = \frac{S}{\bar{x}} . \quad (3)$$

The coefficient of variation is an effective technique for supporting judgment through comparison with other known events. Coefficients of variation are known to be small for biological phenomena, but are large for natural materials. Coefficients of variation are small for highly controlled man-made materials, and are larger for brittle materials. A knowledge of typical coefficients of recurring sources may provide an estimate of data distribution in preliminary design phases. That same knowledge may serve as another source for judging acceptability of data. Its application expands with experience and ingenuity.

Another technique used to evaluate raw data is the population probability density distribution. Normal distributions are most widely used because the mean of " n " independent observations is believed to approach a normal distribution as " n " approaches infinity (central limit theory). It is also a good representation of many natural physical variables or for small samples with no dominating variance. The equation of the normal probability density is

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp - \frac{1}{2} \left[\frac{x - \mu}{\sigma} \right]^2 , \quad (4)$$

where μ and σ are the population mean and standard deviation, respectively. These are the true values of a very large sample size. Normally distributed phenomena are sometimes disguised as non-normal when data samples are selected from casually broadened and unscreened sources. Most metallic mechanical properties are known to be normally distributed. Fatigue properties are not.

An analytical advantage in using normal probability distributions is that many of their characteristics are well established and tabulated. The area within a specified number of standard deviations of a probability density plot represents the proportion of the data population captured. One standard deviation about the average of a normal distribution is calculated to capture 68.3 percent of the data. Two standard deviations include 95.5 percent of the data, and three standards include 99.7 percent.

The mathematical test for the normality of data distribution is rather laborious, and a quick basic program⁶ is provided in the appendix. Figure 4 is a plot of the "D" critical values for a one-sided distribution. The distribution is not normal if the program test result exceeds the "D" critical value. Most engineering data distributions are one-sided, occurring in the lower or upper sides.

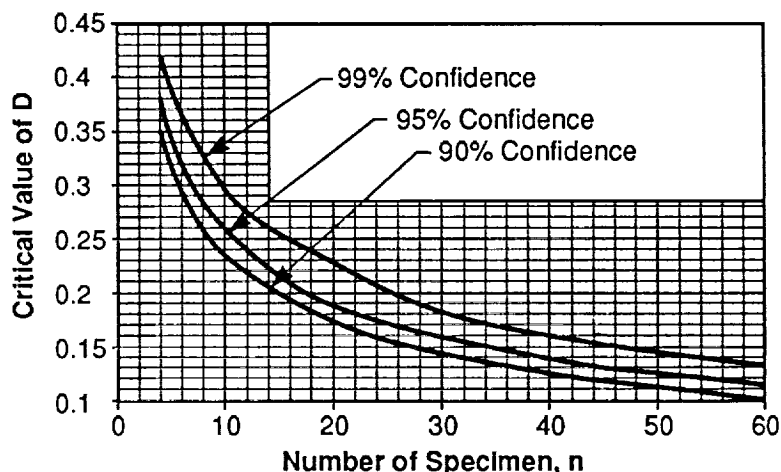


Figure 4. One-sided test critical value of *D*.

Tolerance limit is a quality control specification of a product. Statistical tolerance limits may be determined from a probability density plot for any given proportion of data. As an example, 1.96 true standard deviations are required to capture 95 percent of data from a plot of equation (4). However, true values of the mean and the standard deviation are not generally known from small sample sizes, because they may not contain a given portion of the population estimated by equations (1) and (2). In other words, the same test conducted on the same number of specimens by different experimenters will result in different means and standard deviations because of the inherent randomness in the specimens and testing. The population must contain results from all these experiments.

To insure, with a certain percentage of confidence, that the given portion is contained in the population, a *K*-factor is determined to account for the sample size and proportion. Figure 5 provides the *K*-factor for random variables with 95-percent confidence levels and three probabilities (0.90, 0.95, and 0.99) in a one-sided normal distribution. Other confidence *K*-factors may be computed from a program⁶ provided in the appendix. Through the *K*-factor, a maximum or minimum design value may be determined for a specified probability and confidence. That allowable design value is the lower or upper tolerance limit defined by

$$F_d = \bar{X} \pm K \times S. \quad (5)$$

A common usage of equation (5) is the specification of material properties. Most of NASA and Department of Defense (DOD) material properties are specified by "A" and "B" bases. The "A" basis allows that 99 percent of materials produced will exceed the specified value with 95-percent confidence. The "B" basis allows 90 percent with the same 95-percent confidence.

All of these statistical techniques are applicable in evaluating raw data and completeness of analysis which may be best understood by example. While these techniques are equally applicable in evaluating most data from sources listed in the failure tree, only the stress-strain data will be completely evaluated.

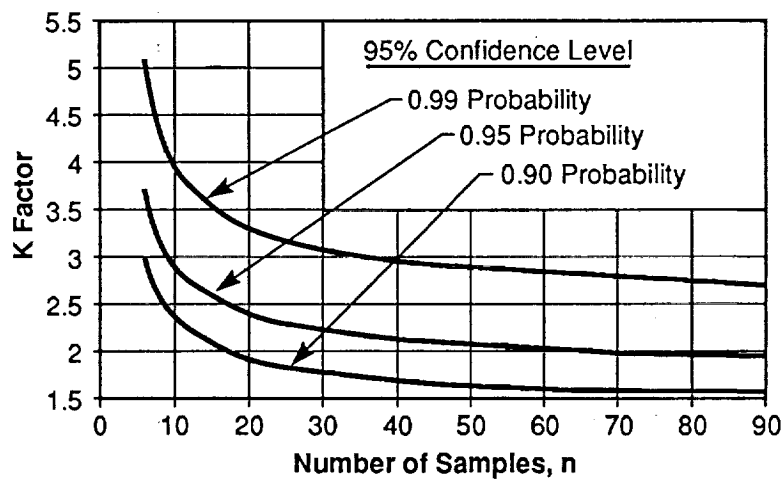


Figure 5. *K* factors for normal distribution.

1. Material Stress-Strain Data: From experience, the failure source of a butt-welded structure is the weld joint as listed in figure 3. Basic properties to be developed are the design maximum allowable yield and ultimate stresses and their respective strains shown in figure 2. Since there should be no difficulty or contention in calculating the ultimate stress from uniaxial tension test data, it is a logical place to start. It should be acknowledged that weld property development from raw data was selected because it offered the most bountiful opportunities for practicing a succession of judgments founded on the above statistical techniques.

Multipass butt-weld properties vary significantly with design geometry and manufacturing tooling which influence the weld heat intensity and distribution across the width. This uniqueness requires that weld specimens be designed and processed as much like the operational structure as practical. Usually, wide plates of a parent material are butt-welded from which a large number of specimens is cut to form a set. Material batches, dimensions, machining, tooling, weld passes, and heat treatment are expected to vary within each set and even more among different sets. Variance tolerances may be reduced and controlled through manufacturing process controls within economic limits.

Table 1 lists the ultimate stresses of 36 butt-weld specimens in 3 sets. The analysis is made more interesting because all the specimens were sliced from three independent seam welds from an existing structure, each seam joining an aluminum shell section to a thick forging.³ Specimens from each continuous seam denote a set, and all specimens are numbered in the order in which they were sliced from the set. The forging thickness and configuration mass are noted to decrease with increasing specimen number.

Table 1. Ultimate stress (ksi) data on 2219-T87 aluminum TIG weld.

Set	Spec. No.	1	2	3	4	5	6	7	8	9	10	11	12	13
1	Ult. Stress	45.5	46.6	48.5	49.7	49.6	48.9	49.5	49.4	49.4	50.0	49.8		
2	Ult. Stress	41.2	49.1	49.3	49.4	49.3	50.1	49.9	49.3	50.5	51.6	48.5	50.1	51.1
3	Ult. Stress	46.7	45.2	45.0	48.0	49.1	48.2	49.5	49.7	49.4	49.5	50.1	50.0	

The first data judgment to be made is the accuracy of the ultimate stresses obtained from specimens in table 1. Cross-sectional dimensions are measured to 0.001 in, so that cross-sectional area errors increase with decreasing area, but they are less than 0.3 percent for this specimen shape. Uniaxial testing machines are expected to produce about 0.4-percent error caused by load cell, calibration, and dial readout. The total inaccuracy of less than 1 percent is within the decimal point round off of tabulated data.

The thicker end of the welded forging engulfed the greatest weld heat, and weld heat is known to affect the weld strength. Since the weld heat sink varies with specimen number, it is necessary to separate that portion of each set which may not represent the worst-case weld-heat phenomena. The grouping of lower strength samples in figure 6 clearly shows that the first four specimens of each set are fitting candidates of high-stress, low-strength design data. The selection was based on strength property because it is the most accurately measurable variable, and, once made, the same sample specimens are used to obtain other weld properties.

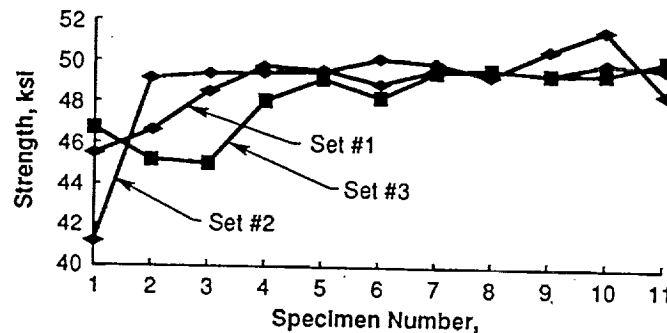


Figure 6. Specimen strength trends versus varying heat sink.

Raw test data of all properties obtained from the selected specimens were similarly processed through the above statistical techniques, and results are listed in table 2. Each condition was judged for appropriateness and completeness before establishing design values.

Table 2. Weld properties.

Line	Items	Stresses		Elongation %
		Ultimate	Yield	
1	Sample Size, n	12	12	11
2	Lowest Value	41.2	29.2	5
3	Mean	47	31.4	7.8
4	Standard Deviation, S	2.51	1.09	2.45
5	Coeff. of Variation, CV	0.053	0.034	0.32
6	Normalizing Test, D	0.143	0.095	—
7	K -Factor One-Sided	3.74	3.74	—
8	A-Basis Allowable	37.6	27.2	4

The first critical observation to be made from table 2 is the sample size on line 1, and this is a good place to begin shedding arbitrariness in data collection. The sensitivity of sample size on the critical value of " D " is inferred by the steepness of curves in figure 4. The steep slopes at

any confidence level would suggest that less than 30 specimens cannot adequately define a normal distribution without risk of over-predicting the lower limit. Moving to figure 5, the risk is reduced by increasing the K -factor. The available sample size of 12 requires a K -factor increase of over 15 percent for an A-basis weld. That small sample could squander material performance between 5 and 9 percent, which is something to think about when considering resources involved in developing higher strength welds.

Proceeding along the ultimate stress column of table 2, the next line item flags the lowest strength observed from the selected specimen. It is listed as a reminder that the calculated design allowable must not exceed it.

The estimated mean value of line 3 is a significant distribution parameter used in tolerance limit calculations. Taken with the low standard deviation of 2.5 ksi in line 4, it provides a very useful perception that the weld will fail very near the mean value. Therefore, predictions based on average property values are more applicable for tracking instrument data on test articles than design allowables. It also implies that most welded structures will have a higher probability of safety than specified by the tolerance limit of equation (5).

The low coefficient of variation, CV , of line 5 is a good index of quality control. A low coefficient of variation affirms a tight tolerance control of the whole welding process. It also implies a ductile fracture which makes it less sensitive to flaws and stress discontinuity regions.

Using figure 4, or the program in the appendix, the weld strength data passed the normalizing test, line 6, which allowed for the one-sided K -factor selection from figure 5. The A-basis design allowable was calculated from equation (5), and results are listed in lines 7 and 8. Note that the design allowable of 37.6 ksi is less than the lowest specimen value of line 2, as should be expected.

Figure 7 shows the one-sided normal distribution of the weld ultimate stress data using equation (4). Superimposed is the A-basis specification of 99-percent probability with 95-percent confidence, having small and large sample sizes.⁸ The 95-percent confidence distributions are not to scale but are intended to illustrate the reduction of allowable design strengths using figure 5 and defined by equation (5) when using smaller data sample sizes.

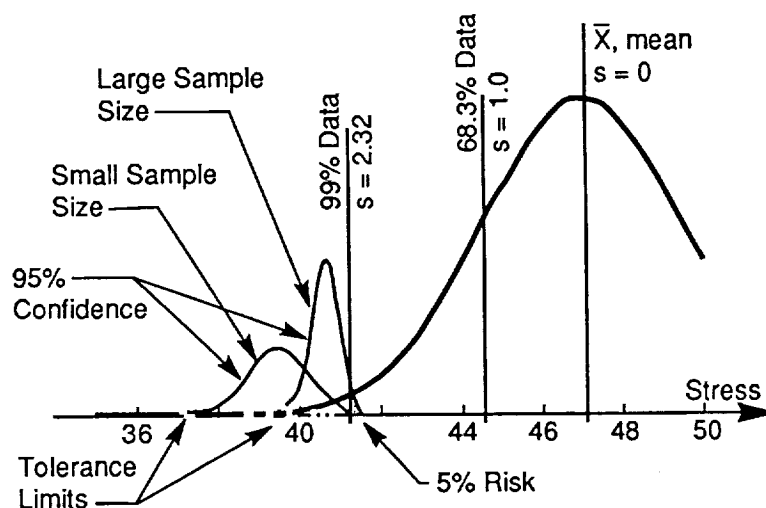


Figure 7. One-sided normal distribution with A-basis.

The preceding process generated many analytical gates from which to judge the evolution of raw data to a specified tolerance limit. This process is not all conventional, but it is suggested as a means of understanding and wringing the complete nature of the data. Completeness of analysis is a necessary condition to avoid marginal designs and potential failures, and it may be demonstrated by comparing this result with the allowable based on an incomplete analysis.

If specimens had not been separated by figure 6, and all 36 samples in table 1 were used, their distribution would have fallen 25 percent short of passing the normalizing test. But assuming further that the normalizing test had been ignored, and a K -factor of 2.98 had been selected from figure 5 for the 36 specimens, the resulting A-basis design allowable would have been 42.8 ksi which would have exceeded the lowest specimen value of 41.2 ksi. Comparing this design allowable with that of table 2, line 8, results in a nonconservative error of about 4 percent, which is one small avoidable error that deducts from the overall structural safety. Analysts with casual appreciation for statistics flirt with incomplete results.

Defining the design maximum allowable elastic limit uses the same 12 selected specimens and statistical techniques as for the maximum allowable ultimate stress just developed. It further requires the weld strain property to be recorded simultaneously with the associated stress in order to locate the inelastic initiation point A of figure 2. However, Vaughn⁷ observed through hardness tests and electrical strain gauge data measured along the width of the weld specimen that, in fact, properties varied and could be correlated with heat affected zones (HAZ), work hardening, filler interfaces, and other manufacturing related processes. Where should the limit of the elastic properties be measured?

Since 33 of the 36 specimens failed at the interfaces, the interface is the weakest region and should be the design characteristic source of the weld. However, the interface consists of parent and weld filler of the same base material with different stress-strain responses beyond the elastic limit. Figure 8 illustrates the bifurcation of strains experienced by the tandem materials when loaded beyond the weld elastic limit by a common uniaxial tension stress. Reconciling mismatching strains at the interfaces causes local distortions and discontinuity stresses¹ which are as much as 10 percent higher than the externally applied uniaxial stress.

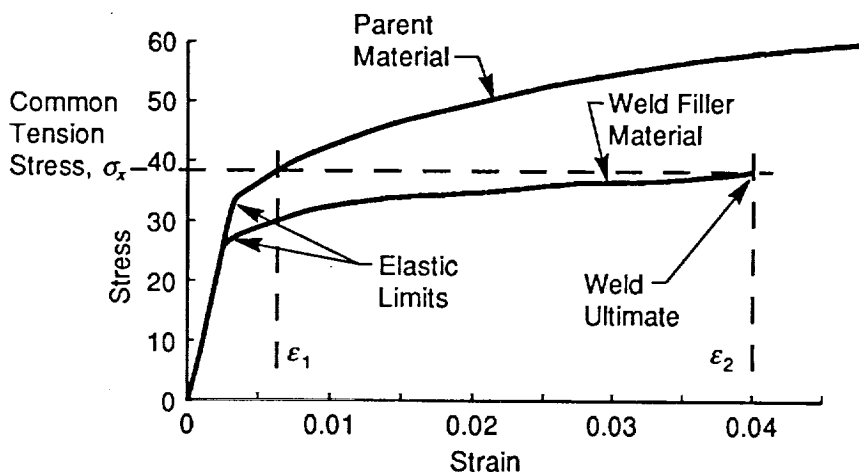


Figure 8. Weld and parent properties under common stress.

This discontinuity stress implies that the weld yields first at the interface with a local stress greater than that measured by the testing machine, while the filler stress at midwidth is as measured by the testing machine through the elastic range and beyond. Applying a gauge at the interface would read mixed response of parent and filler materials. However, using the slightly lower stress experienced at the filler midwidth may be compensated by the slightly higher stress related to the standard 0.2-percent yield strain offset. Therefore, applying an electrical strain gauge on the filler midwidth having a gauge length less than the filler width appears to be the most appropriate method for obtaining the weld stress-strain data of the weakest weld region.

Weld yield stresses for the 12 specimens were developed in that manner from 1/8-in gauges centered on a 3/16-in weld width and using a 0.2-percent offset. Having established the appropriate test data, the design allowable yield stress was developed following a similar process as for the ultimate stress. Results are listed in table 2. Young's modulus was obtained from the 0.2-percent offset slope data which averaged at 11,400 ksi. Dispersions about the average stress-strain related slopes were negligible. The average and allowable yield strains are calculated from the Young's modulus and related stress.

The final property required to characterize the weld stress-strain relationship is the elongation. Unfortunately, weld elongations exceeded electrical strain gauges' capability, and gauges failed before the weld fractured. The elongation data in table 2 were constructed by mating the fractured surfaces of the specimen together and mechanically measuring the ultimate growth over a prescribed gauge length. However, an unreasonably large coefficient of variation of over 30 percent was obtained, which makes the test method suspicious. It may be reasonable to assume that the fractured surfaces cannot mesh tightly because microscopic separations at the interface propagated at different rates, according to the nonuniform discontinuity stress intensities. The only recourse left was to estimate the elongation by assuming the strain to be less than the lowest value obtained from the sample size and not to exceed the published parent material value.¹¹ The consequence of this approximation should be acceptable because elongation is a necessary but not a sensitive property for mechanics modeling.

Though the above example demonstrated the unique resourcefulness and judgment required of analysts in contriving an uncharted approach, some rather general observations may be summarized to achieve more complete analyses versus the liability of incomplete analyses. (1) Data are only as accurate as are measuring instruments and calibration of measuring instruments. (2) Normally distributed phenomena may sometimes be missed when using too broad a data source (fig. 6). (3) Sample size of less than 30 specimens does not necessarily define a statistical distribution. (4) Tolerance limits must include the worst-case raw data variable.

Coefficients of variation for common structural materials are listed in table 3, which may be useful in preliminary structural designs. Tabulated coefficients reflect ductility and quality control. The data are approximate and should be replaced when the coefficients are developed for the specific material and design conditions. A *K*-factor of 3 is suggested to be used with table 3 for an *A*-basis property, and a factor of 1.8 is suggested for a *B*-basis. Others may be interpolated from figure 5.

Table 3. Coefficients of variation of structural metals.

Material	Coefficient of Variation		Material	Coefficient of Variation	
	Yield	Ultimate		Yield	Ultimate
Aluminum			Stainless		
plt, sheet, bar	0.03	0.03	310 Rm	0.09	0.04
sand casting	0.08	0.06	-400 °F	0.02	0.05
Magnesium	0.05	0.06	forging	0.06	0.02
Titanium			347 Rm	0.06	
sheet, bar	0.05	0.06	-300 °F	0.05	
forging -400 °F	0.02	0.02	800	0.07	
Steel			2,000	0.08	
comm.	0.09	0.04	430	0.06	0.03
Cr-Mo-V	0.02	0.02	17-7 pH	0.07	0.06
Ni-Cr-Mo	0.03	0.03	Super Alloy		
4340 Rm.	0.03	0.03	A-286 bar	0.09	0.06
900 °F	0.04	0.03	forging	0.07	0.06

Given yield and ultimate mean stresses, lower tolerance limits " F_a " may be estimated. Given an A-basis, or a B-basis property F_a and equation (5), the mean may be estimated from

$$\bar{x} = \frac{F_a}{1 - K \times CV}, \quad (6)$$

and the standard deviation is estimated from equations (3) and (6),

$$s = \bar{x} \times CV. \quad (7)$$

Application of other data generated from the failure tree source to statistical techniques will not be repeated, but variations and their cause must always be understood to support judgment. Requirements for data accuracy and completeness must be weighed against other parameter sensitivities and combined effects on stress.

2. Thermal Properties: Thermal material properties are included on the resistive side of failure and may be developed similarly to the weld stress-strain properties. The accuracy of the specimen temperature, its exposure, strain rate, creep, and other effects on applied stress must be thoroughly examined. Thermal coefficient of expansion is an example of a material property to be statistically determined and is used in combinations with geometric constraints and material parameters to predict the applied thermal stress side of failure. Unconstrained thermal strains do not cause failure.

3. Scaling Properties: Scaling is always present from different causes and is always significant when applied to properties on the resistive side of failure. Increasing weld thickness increases the weld heat sink and decreases the strength. Castings and forgings may demonstrate the same characteristics and sensitivities as welds. Milled thin sheets are stronger than thick sheets and plates of the same material and process because of the depth of work hardening

and capacity for heat treatment. The strength may vary more than 5 percent. Filament wound case stiffness and strength decrease with increasing size,¹⁰ depending on process control of compaction and of epoxy premature curing during winding operations.

4. Degradation: Fatigue, fracture growth, aging, erosion, and corrosion are all sources of failure, requiring time-dependent test data to be generated and evaluated and the mechanics to be defined. These are cumulative damage failure modes which operate on different material characteristics than static stress and are not covered by this study.

5. Manufacturing Tolerances: Manufacturing incurs a boundless list of failure sources of which dimensional tolerance is common to most parts and assemblies. Actual dimensions within a specified tolerance have a statistical distribution which may or may not need to be evaluated completely. The maximum guaranteed tolerances of milled sheet thicknesses range from 10 percent for thin sheets to less than 5 percent for thicker plates. One-third of the specified tolerance may be assumed as one standard deviation. The mean and assumed standard deviation may be used to approximate the tolerance distribution. Sometimes the minimum guaranteed thickness may be conservatively used as design allowable over small acreage to compensate for minor blemishes incurred during manufacturing and handling.

Tolerances between rivets and holes are generally not critical since rivets are impacted tightly into the hole, which helps to load them more uniformly under applied external loads. Aerospace industries have compiled extensive design allowable tables based on statistical test data for a variety of rivet and sheet sizes and for hole patterns. Rivet strength distributions and design allowables derived from statistically treated data may be substituted into the resistive side of the failure concept. Efficiency of bolts in shear is very sensitive to tolerance buildup from bolt-to-hole diameters, through-hole alignments, and in-line hole tolerance. Butt-weld mismatches vary along the weld seam and are very critical to pressure vessels.

6. Manufacturing Residual Stresses: Residual stresses produced in manufacturing cannot be quantified but may be minimized through tooling and process controls. Dimensional buildup and final assembly force-fits may produce preloads in operationally critically stressed regions. Excessively impacted rivets impose residual stresses that may add to basic rivet hole concentrated stresses. Weld heat distortions on long continuous seams may produce residual stresses that exceed test specimen data. Typical residual stresses should be duplicated on structural test articles for evaluation.

7. Processing: Manufacturing processes consist of altering a structural property through a simple heat treatment, a machining operation, drawing, or a complex filament winding process. Processing effects may promote many of the failure sources already cited, or may be unique to a particular product. Inplane stiffness of filament wound pressure vessels depends on the acceptance control of the filament strength and stiffness, the tow tension, the helical angle, and the total winding time before curing. Significant effects on processing are identified through shop observations and analyses.

8. Environments Data: Natural and induced environments produce loads that are dominant sources of applied stress. A complete environmental data analysis would include the identification of all conceivable natural and induced sources. Only that data judged to be load-sensitive should be statistically developed. Natural environments include temperature, density, winds, and gravity. Aerodynamics, thermal, propulsion, acoustics, and vehicle control are induced environments.

Some natural environment distributions over the calendar year may not be normal, but the worst design month may exhibit a normal distribution. Wind speed, shear, frequencies, and gusts bear dispersions with time and altitude. Thrust and thrust misalignment exhibit a dispersion from one unit to another at common altitudes. Propellant loading and residuals vary from flight to flight. Similar dispersion cases may be made for all natural and induced environmental parameters. Most environments are defined by their mean and tolerances which may be used to approximate a distribution as described in manufacturing tolerance.

9. Test Data: Inaccurate verification test data may contribute to structural failure during the operational phase. Common causes are when measured applied loads on the test article are higher than actually applied or when measured strain responses are less than actual. Most of these error sources are calibration types. Of particular interest are the calibration accuracies of electronic displacement indicators, load cells, and pressures associated with active and reactive load lines. The accuracy of systems used to calibrate them must also be accounted. Automatic load control and data acquisition systems measurement accuracies must be checked and cross-checked by different methods. Strain gauge tolerances are normally less than 5 percent but must be checked for temperature compensation error. Strain data conversion to stress is another source of error, especially beyond the yield point. Spurious data may be resolved with tenable math model predictions.

C. Prediction Uncertainties

All aerostructures and components are designed for ground through flight environments. By expressing the structural mass, stiffness, damping, and forces in terms of normal modal properties, the static and dynamic loadings and induced stresses may be determined. The ultimate sources of failure emanate from uncertainties accumulated in the loads and the structural response prediction models which make up the applied stress side of figure 1.

1. Loads Modeling: Single valued static loads are seldom of single parameter source because of inherent variations in duplicating structural articles and predicting day-to-day operations. Even the design load of a simple pressure vessel must include pressure relief tolerances and variations in environments and surges. Gravity-induced loads may be deterministic for one article and one application but have a statistical distribution over paths from one copy and operation to another.

Aerostructural load sources of uncertainties occur in environments, aeroelastic models, controls, and operational agenda. Environmental sources vary with flight regimes and have the most potential for disparity between predictions and flight verifications. Consider the load variations along the vehicle throughout the flight time because of atmospheric changes with altitude and thrust parameter changes with atmosphere. Mass changes with time because of propellant consumption and mixture ratio dispersion. Design wind shear, gust, speed, and direction data are evaluated for the worst month, but launch month data are used for operations determination. Aerodynamic load distributions primarily induce variations in center of pressure, forces, and moments, while mass distribution defines variations in the center of gravity and moment of inertia. The vehicle is controlled through a host of devices with innate variations which ultimately induce variations in bending moments and shears.

The classical approach¹² for determining structural system dynamic loads for each flight regime is to couple dynamics models of each structural element and substructure to produce a global dynamics model containing many associated modes and frequencies. Integrating the model

provides discrete load responses at different structural regions for each different combination and variation of structural and environmental parameters.

A complete dynamics analysis¹³ should include enough load response cases to define loads distribution at all critical structural regions within a specified probability, as well as time consistent sets of balanced loads for the total structure. Reference 14 is a standard guide to determining internal structural loadings induced by transient disturbances. Reference 15 uses payload models in modal or matrix form to calculate carrier-payload coupled accelerations and forces to derive payload design loads and stresses.

Regardless of the approach used, machine time increases with increases in number of finite elements used in the finite element method (FEM) models, load inducing parameters, and their variations. There lies the challenge, to reduce computational time while probing for worst design cases. Reducing response cases is risky without an analytical basis. One approach is reducing the number of parameters through a preliminary stress sensitivity analysis and statistically developing those significant parameters for a finally applied stress analysis. Pareto's distribution points out that a majority of failures is caused by a minority of reasons.

2. Stress Modeling: Analytical elastic methods are known to model a select few classical structural elements exactly. Analytical techniques for predicting practical structural systems behavior of combined elements are only as accurate as their modeled boundaries. However, systematically modeling constraint, sketching load paths, and free body diagrams is not only necessary to the process, but provides the designer with clear knowledge of its rudimentary behavior.

On the other hand, computational methods can solve many practical problems approximately and are the preferred methods, especially for global structures and three-dimensional substructures. FEM can and have been a source of failure. One source of FEM inaccuracies is the lack of stress convergences caused by insufficient degrees of freedom. This is a programmer fault in not checking for convergence and resolving it. A more serious fault is an inaccurate code. One commercial code under-predicted the elastic stress by 33 percent because the plate element used³ had limited shear capability. Another FEM code¹ under-predicted strains because the plastic brick element was too stiff. In both cases, the failure might have been avoided, if beam and plate elements had been modeled in pure tension, bending, and combinations with the FEM and then compared with an analytical method. New FEM commercial codes, as well as any new analytical applications, must be tested for subtle limitations. Furthermore, it is generally recommended that all FEM critical structural predictions be backed up with classical analytical methods.

Though a common elastic structural model is used to develop global loads and stresses, they might be organized and performed as separate disciplines. In such cases, detailed loads and accelerations are provided to stress analysts, who develop load paths and independent detailed loads at critical stress zones from select loads data. Structural thicknesses, sizes, and design are modified, which changes the model mass and stiffness. Loads and stress analyses are reiterated with each modified thickness to converge on the allowed stress. An opportunity for a complete analysis could be lost if the stress analysts' finally derived detailed loads were not correlated with the load analysts' final elastic global load set.

3. Materials Modeling: Another common prediction error is to extend the elastic models to structures loaded beyond the elastic limit. It may work on single tension elements, but it has

no meaning on redundant load path structures. Redundant load intensities continue to vary along the same paths but at a decreasing rate as plastic deformation rates increase until one plastic path fractures. Then the surviving paths are abruptly loaded and further intensified to a secondary fracture. The weakest path and load intensities to failure may be determined only through an inelastic analysis.

An elastic model with multiple superimposed loads has even less meaning beyond the elastic limit when recognizing that superposition is not congruent with nonlinear material properties. Strains increase nonlinearly with stresses beyond the elastic limit. It has been shown that combined bending and normal loading will produce dominate bending strains with dominant normal stresses. Multiaxial loading produces similar surprises, which cannot be interpreted from test strain data without an inelastic analysis. The requirement for inelastic structural analysis becomes more compelling when considering that ultimate safety and reliability are based on stress analyses conducted over the entire nonlinear region of the material stress-strain relationship.

Briefly, a completely modeled structure should include a verified FEM elastic global model, an inelastic substructural model of critical regions, and a classical backup analysis. FEM models must be checked for convergence of stress. Insufficient degrees of freedom result in stiff models which render optimistic predictions. It must also be cautioned that the substructural model must be sufficiently large to ensure that the elastic boundaries defined by the global model will remain elastic as the inelastic region is loaded to fracture. Analyses should be checked by an independent party for assumptions.

D. Discontinuities

The most common regions of local fracture and its propagation are at stress irregularities and concentrations induced by abrupt changes in geometry, loads, thermal strains, and metallurgy. All four of these sources are readily identifiable and accommodated in structural design. Most are currently amenable to comprehensive elastic analyses but not to ultimate stress analysis.

Discontinuity stresses have been traditionally reconciled through concentration factors derived experimentally or from classical mechanics, and all have been based on linear behavior. Elastic concentration factors may be reasonably applied to very brittle materials through fracture, but preferred aerostructural materials are ductile. Boundary element methods (BEM) which calculate discontinuity stresses, including three-dimensional structures, are commercially available. This program, BEASY, will even calculate stresses as the geometry progressively varies with increasing stress, but it will not allow progressive changes in material properties. One option is to use the classical solution of the elastic stress concentration and piece-wise change the inelastic material property.

Benefits of high performance materials are often compromised at discontinuity end connections. Weld fillets not only were noted to be weaker than the parent material, but the abrupt metallurgical differences constitute discontinuity stresses. Riveted and bolted joints in shear are common examples of combined geometric and load discontinuities. A common bolt is the most abruptly sculptured and difficult connector to analyze completely. It would be unnecessary to do so. In the first place, only aircraft type bolts¹¹ should be used on aerostructures; and secondly, bolts should not be the weakest link unless specifically intended to control failure paths. Increasing the size of bolts to one size more than calculated may compensate for many design

and installation uncertainties with minor weight penalties. Weld inclusions and porosities are examples of manufacturing geometric discontinuities. Their shapes and orientation to the dominant stress field influence intensity of stress concentrations. Sharp surface scratches and their orientations are further examples of geometric stress discontinuities that must be accounted. Pressure vessels are pervaded with types and quantities of stress concentrations.

E. Limits of Analysis

Incomplete analyses and poor reasoning were noted to be the major causes of failure that could be avoided. Incomplete statistical analysis of data was shown to produce submarginal design allowables. Not verifying FEM codes and ignoring convergence checks are more examples of incomplete analyses. Extending elastic methods through inelastic fracture is typical of poor reasoning and is also avoidable through proper assumptions and analysis.

However, there is another potential source of failure which is common to many of the above discussed causes and is not avoidable by analysis. That potential source stems from the uncertainty of estimates. Limited measured data provides only an estimate of the mean and standard deviation. Unmeasured wind effects of loads on specific structural configuration and load effects on applied stress models are unavoidable estimates during design phases. There are limits to the physics that a math model can replicate, and there are limits to the details that a math model can incorporate. There are also limits to the fidelity of a development test. These limits are in a constant state of technological improvement but can never be avoided.

Measured and unmeasured uncertainties have been historically recognized and resolved by making the structure stronger and safer than analyses indicate. Two approaches for making an aerostructure safer are the conventional deterministic safety factor and the (still evolving) superior probabilistic safety index approach. In focusing on either method, it must be clearly understood that safety indexes, or factors, are the icing that is applied only after all data, models, and distribution parameters have been completely developed.

III. PROBABILISTIC SAFETY INDEX

Some form of the probabilistic safety method is incorporated in all failure concepts. In its purest form, it is based on detailed statistical data and format and provides a meaningful assessment of a structure in terms of reliability. Because there are many similarities between the deterministic and probabilistic methods, a basic probabilistic concept¹⁶ and application follow with the expectation that a better understanding of the deterministic concept may be obtained, and the best of the two methods may produce a versatile and unarbitrary deterministic safety approach.

A. Basic Probabilistic Concept

The concept of failure was introduced by figure 1. When the applied stress significantly exceeds the resistive stress, their distribution tails overlap, which suggests the probability that a weak resistive material will encounter an excessive applied stress to cause failure. This is to say that the probability of success is reliability and that the reliability is less than 100 percent. Therefore, it is necessary to understand the reliability of this interference of the applied and resistive distribution tails.

Figure 9 assumes a system of loads, geometry, and material parameters statistically combined which, for simplicity of presentation, happens to define the normally distributed applied stress, f_A . The distribution is synthesized by a normal probability density function defined by equation (4). The material resistive stress, f_R (yield or ultimate), is also assumed to be normally distributed. The probability of interference is the probability of failure and is governed by the difference of their means, $\mu_R - \mu_A$. Increasing the difference of the means decreases the tail interference area.

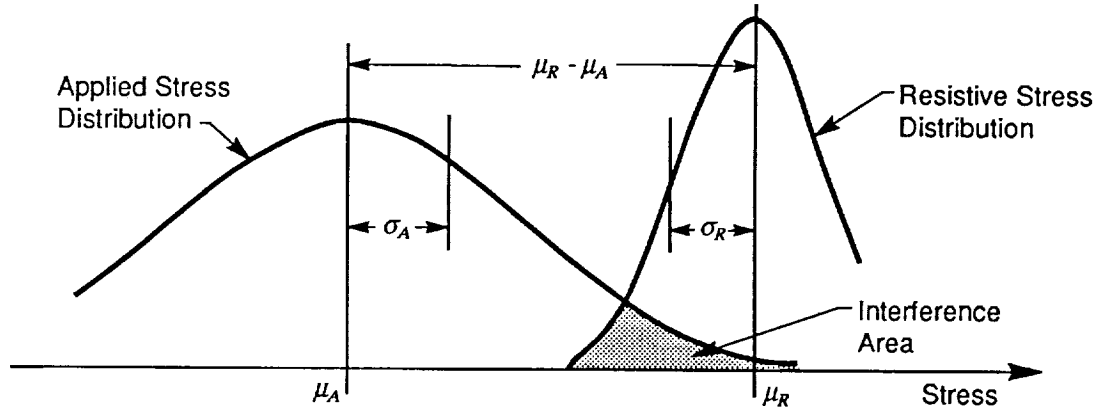


Figure 9. Applied and resistive stress interference.

Given that both probability density functions are independent, they may be combined to form a third random variable density function in $y = f_R - f_A$. If f_R and f_A are normally distributed random variables then $y = f_R - f_A$ is also normally distributed and

$$P_y = \frac{1}{\sigma_y \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left[\frac{y - \mu_y}{\sigma_y} \right]^2 \right], \quad (8)$$

where

$$\mu_y = \mu_R - \mu_A \quad \text{and} \quad \sigma_y = \sqrt{\sigma_R^2 + \sigma_A^2}. \quad (9)$$

The y -variable distribution is plotted in figure 10.

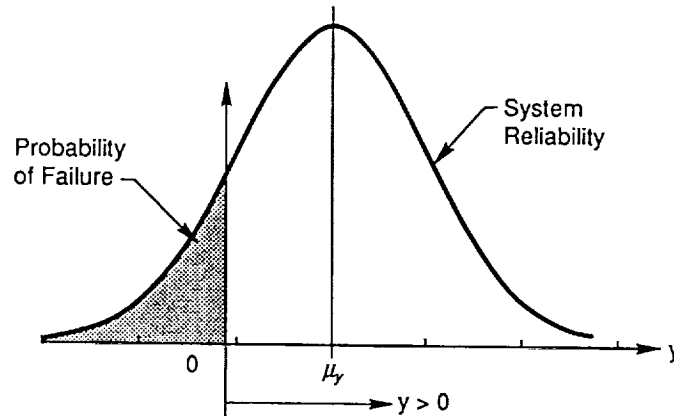


Figure 10. Density function of random variable y .

The reliability of the third density function expressed in terms of y is

$$R = P(f_R > f_A) = P(y > 0) = \int_0^{\infty} P_y dy , \quad (10)$$

where P_y is the y -density function of equation (8), and letting

$$z = \frac{y - \mu_y}{\sigma_y} , \text{ then } \sigma_y dz = dy .$$

The lower limit of z is

$$z_L = \frac{0 - \mu_y}{\sigma_y} = - \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} .$$

As y approaches infinity, z approaches infinity, and the reliability of equation (10) is reduced to

$$R = \frac{1}{\sqrt{2\pi}} \int_{z_L}^{\infty} \left[\exp\left(-\frac{z^2}{2}\right) \right] dz . \quad (11)$$

The integration of equation (11) is programmed in the appendix. Given the reliability R , the safety index " z " value is printed which may then be translated into statistical design parameters through the safety index expression,

$$z = -z_L = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} . \quad (12)$$

Equation (12) formulates the probability concept. The safety index is nothing less than a common multiplier of the resistive and applied stress standard deviations. Increasing the safety index and the standard deviations increases the means difference, which decreases tail interference area and the probability of failure. The reliability relationship with the safety index is plotted in figure 11.

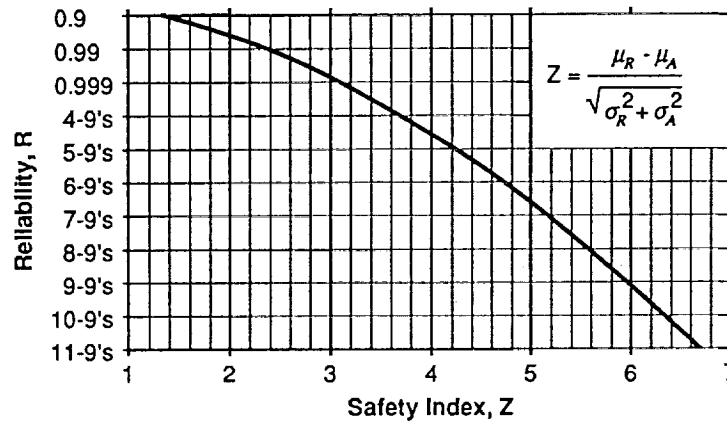


Figure 11. Reliability versus safety index.

B. Designing With Reliability

The designing process for reliability must consider the yield and ultimate failure modes and the dependence of one reliability on the other through their unique material properties. Briefly, structural reliability, configuration, size, and interfaces are specified through a systems requirements analysis. Loads are modeled from natural and induced environments, and significant drivers at critical stress regions may be identified through rough-cut deterministic stress analyses. Drivers and associated uncertainties are then developed into probabilistic distributions and applied to probabilistic failure models. A sensitivity analysis follows to further reduce trivial parameters, to increase understanding, and to select strategic drivers for improving definition. Indispensable to this process are the variety and depth of statistical methods, which is not necessarily a calling for statisticians in the design room, but for experts in mechanics first with an intrinsic attention to probability and statistical methods.

Designing for reliability is virtually developing design variables to satisfy equations (11) and (12). Once the guaranteed reliability of the structure is specified, the safety index "z" is calculated and fixed. The safety index of equation (12) must be satisfied by the four distribution variables. Note that the means and standard deviations in equation (12) refer to a population size data base. Measured and assumed data available during most of the design phase are often estimates based on small sample sizes. Since the safety index equation offers no opportunity or statistical technique to compensate for this data deficiency, resulting reliability predictions are expected to be overly optimistic. Predictions become useless when sample sizes are small. Of course, there is no lack of empirical techniques to resolve this natural shortcoming; nevertheless, a generally agreed-to correction standard is wanting.

C. Safety Index Sensitivities

Reliability predictions were noted to be solely dependent on the four distribution parameters in equation (12) whose true values are not obtainable. To assess the effects that inaccurate variables may have on reliability predictions, a sensitivity expression for each distribution parameter was obtained by differentiating the safety index with respect to the parameter of interest and dividing by the index. The decimal fraction change in safety index per change in the resistive and applied stress means and standard deviations are

$$\frac{\partial z}{z} = \left[\frac{\mu_R}{\mu_R - \mu_A} \right] \frac{\partial \mu_R}{\mu_R}, \quad \frac{\partial z}{z} = - \left[\frac{\mu_A}{\mu_R - \mu_A} \right] \frac{\partial \mu_A}{\mu_A}, \quad (13)$$

$$\frac{\partial z}{z} = - \left[\frac{\sigma_R^2}{\sigma_R^2 + \sigma_A^2} \right] \frac{\partial \sigma_R}{\sigma_R}, \quad \frac{\partial z}{z} = - \left[\frac{\sigma_A^2}{\sigma_R^2 + \sigma_A^2} \right] \frac{\partial \sigma_A}{\sigma_A}, \quad (14)$$

Relative sensitivities of parameters may be determined by substituting a common change into equations (13) and (14). Assuming a commonly quoted ultimate reliability of 0.9999, a corresponding safety index $z = 3.72$ was obtained from figure 11 (or appendix program). A 10-percent change was applied across all distribution parameters, and the resulting relative sensitivities are listed in table 4. An increased failure rate of two orders of magnitude is noted for mean parameters. Reliability is rather insensitive to material and applied stress standard deviations.

Table 4. Example of reliability sensitivities.

Assumed Value	$\mu_R = 47$	$\mu_A = 30.7$	$\sigma_R = 2.5$	$\sigma_A = 3.6$
10% change in variables will change: safety index, z reliability, R failure rate	2.64 0.996 1 in 250	3.0 0.998 1 in 500	3.6 0.9998 1 in 5 k	3.47 0.9997 1 in 5 k
Expected error Design reliability, R (to guarantee 9999)	5% 0.99999	7% 0.99999	7% 0.9999	25% 0.999999

A more relevant judgment may be drawn by changing each parameter by some estimated raw error. Material parameters are weld properties obtained from table 2. Material mean and standard deviation are calculated estimates from a small sample size, and judging from the tolerance limit in figure 7, the population true values could range between 5 and 10 percent, respectively. The standard deviation may vary up to 8 percent. An applied stress standard deviation of 25 percent was arbitrarily assumed, which includes loads and manufacturing variances. That figure could be lower in nonflight environments. The applied stress mean was calculated from equation (15) to satisfy the safety index required for a 4-9's reliability. Assuming errors in one variable at a time, the results in table 4 would suggest that a structure should be designed between 5 and 6-9's reliability to guarantee 4-9's. Combining errors or using a different set of parameters will result in different design reliability, but the point is made that a design reliability must be much higher than that specified in order to offset limited data.

The above example was based on ultimate reliability. The yield reliability analysis would substitute only the yield properties into the resistive stress parameters, which would essentially reduce the mean difference in equation (12). Designing to a 4-9's ultimate reliability, the resulting yield reliability is 0.56, which is to say that the yield point will be exceeded in half the operational use.

If the structural material is a ceramic class having no distinct yield point, then designing to an ultimate reliability is appropriate. High strength metallic structures are most likely operationally critical, in which the yield reliability should be satisfied first, and the ultimate reliability should be allowed to seek its natural bounds. For instance, if the applied stress is expected not to exceed the yield point more than once in a hundred uses, it can be shown that the ultimate reliability would stretch to 7-9's. A meaningful argument may be made that if the operational (yield) reliability is reasonably satisfied, a metallic structure will never experience fracture, because the dependent ultimate reliability will be overly compensated, as demonstrated, and the ultimate reliability specification becomes superfluous.

The probabilistic concept examples were based on normally distributed stresses because of its simplistic application and clarity of presentation. The stress-strain statistical properties are most often normally distributed. Combined loads distribution is the sum of all the independent distributions which produce normally distributed applied stresses, even when component loads are not normally distributed. Nevertheless, many similar probabilistic methods are available for non-normal distributions.

Some rather comprehensive and promising programs are evolving with stochastic methods that are applicable to cumulative damage problems. Two current probabilistic programs being funded by NASA are the Jet Propulsion Laboratory (JPL) Probabilistic Failure Modeling and the Probabilistic Structural Analysis Method (PSAM). Both are particularly applicable for determining reliability of fatigue life, flaw propagation, and wear on propulsion system components. The former is a JPL project, and the latter is a team effort managed by Southwest Research Institute.

IV. DETERMINISTIC SAFETY FACTOR

Aircraft of the early 30's were designed to a 6-g load factor which was known to include a safety factor. The 17ST aluminum alloy commonly used had a 1.5 ratio of ultimate to yield stresses. Since these aircraft performed satisfactorily without permanent deformation, the 1.5 stress ratio was arbitrarily adopted¹⁷ as the universal safety factor by civil and military communities. Commercial aircraft later imposed an additional multiplying factor of 1.15 on critical joints and 1.33 on pressurized cabins to increase fatigue life.

Though this universally accepted safety factor has since been refined by aerospace industries to incorporate statistically derived parameters, specified numerical safety factors are often based on limited criteria and virtually no philosophically supporting analyses or considerations of progress made in the aerostructural enterprise. This lack of an intellectual basis for the safety factor is reflected in its unaccountable selections and legalistic compliance. Present practices are not in stride with the progress made and the immense resources spent on developing thousands of degrees-of-freedom models to be used with safety factors, nor complex structural tests crafted to verify them. Perhaps by revisiting the safety factor concept and by understanding all its variables and limitations, a more systematic and coherent deterministic approach may be formulated.

A. Safety Factor Concept

Through improvements in modeling of materials and mechanics phenomena, the subjective element of ignorance has been largely eliminated, but the variance of phenomena may be actively reduced, though never eliminated. Therefore, the safety factor is a method to compensate for objective variances. The conventional ultimate safety factor is a numerical value by which the product of the safety factor, SF , and the applied stress, F_A , induced by the maximum expected load does not exceed the minimum ultimate strength, F_R , of the structural material,

$$SF \times F_A = F_R \quad (15)$$

It is called deterministic because each parameter is a singly determined value. The maximum and minimum limits specified imply a statistical range of variations about their most probable value² which are commonly expressed in a statistical format,

$$F_A = \mu_A + n_A \sigma_A \quad (16)$$

and

$$\begin{aligned} F_R &= \mu_R - n_R \sigma_R \\ &= SF \times (\mu_A + n_A \sigma_A) \quad (17) \end{aligned}$$

where μ and σ are the mean and standard deviations referring to the applied and resistive stresses by subscripts A and R , respectively. The number of standard deviations is noted by multipliers n_A and n_R . Equations (16) and (17) represent the probable lower and upper tolerance limits defined by equation (5) for the applied and resistive stress distributions, respectively. Consequently, statistical data analyses and distribution variables rigorously developed for application to the safety index of equation (12) are equally applicable to the safety factor of equations (16) and (17). Furthermore, progress made in data development and probabilistic safety methods should be diligently explored for application to the deterministic method.

What makes the deterministic method preferred in the drawing room is its versatile application. Though resistive and applied stresses are statistically determined, only single valued results need to be substituted into equation (15). Explicit values of distribution parameters stipulated by the safety index are not necessarily required by equation (15). This convenience allows the resistive stress to be represented by published¹¹ single values for an A - or B -base material.

Allowing the applied stress to be expressed as a single value is not only a convenience but a frequent necessity during the design phase. If the statistical distribution is not available or proves to be relatively insignificant, worst-on-worst applied stress cases are usually combined and substituted into equation (15) as a single value. At the same time, it is recognized that this versatility of accepting single value stresses may be a convenience for early design estimates, but this may be abused by allowing incompletely developed data to creep permanently into final safety analyses.

The safety factor concept and properties incorporated into equations (15) through (17) are illustrated in figure 12. The applied and resistive stress distributions are defined by probability density functions. As in the probabilistic method, their overlapping tails suggest the probability of failure. Since increasing the difference of the two means decreases their tail interference, $\mu_R - \mu_A$ expresses a measure of safety and becomes the focus of the structural deterministic safety concept.

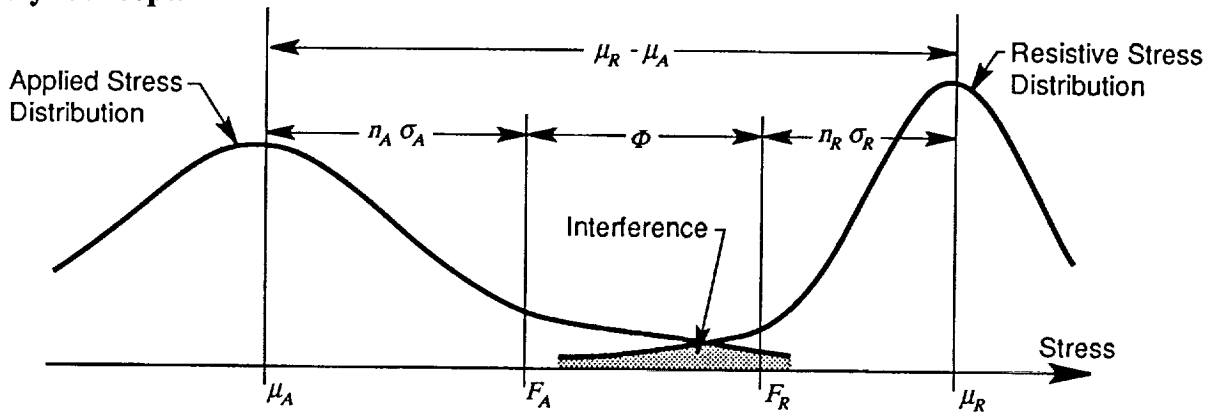


Figure 12. Deterministic concept features.

The means difference is composed of three safety zones: the two tolerance limits defined by equation (16) and the first of equations (17), and the mid-zone Φ . The contents of this mid zone are fixed by the difference of equation (16) and the second of equations (17),

$$\begin{aligned}\Phi &= SF \times (\mu_A + n_A \sigma_A) - \mu_A - n_A \sigma_A \\ &= (SF - 1) \times (\mu_A + n_A \sigma_A) ,\end{aligned}\tag{18}$$

and are noted to be dominated by the conventional safety factor. Also note that by letting the safety factor equal unity, the mid zone is eliminated, making $F_R = F_A$. But equating the maximum expected applied stress with the minimum allowed ultimate resistive stress would admit the applied stress to operate in the inelastic region of a polycrystalline material. To avoid facing a permanently deformed structure, a minimum safety factor must be specified in this zone. Using the yield failure as the upper limit of the limit design stress, $F_A = F_{ty}$, and recognizing that the maximum allowable stress is the ultimate stress, $F_R = F_{tu}$, the design lower limit of the conventional safety factor is established through equation (15) as

$$SF_{LL} = \frac{F_{tu}}{F_{ty}} . \quad (19)$$

Combining equations (15), (16), and (17) defines the difference of the two distribution means, which is the deterministic total measure of safety,

$$\mu_R - \mu_A = \mu_A (SF - 1) + SF (n_A \sigma_A) + n_R \sigma_R . \quad (20)$$

It turns out that each zone between the two stress distribution means contains a safety factor to be independently specified by loads, materials, manufacturing, and stress disciplines. These safety factors are as follows: n_A is the standard deviation multiplier of the applied stress which is specified for the desired probability that $F_A < F_{ty}$; n_R is the standard deviation multiplier, or K -factor, of the resistive stress which is specified for the desired probability that $F_R = < F_{tu}$; SF is the conventional safety factor whose minimum is specified by equation (19). None of these factors are arbitrary, and any one excessively specified may be shared with either of the other two factors.

Accordingly, the deterministic safety method is comprised of three distinct and interchangeable safety factors which may be jointly considered in formulating total safety selection criteria.

The interaction of these safety factors is best assessed through two numerical examples listed in table 5. Example No. 1 combined the universal safety factor of 1.5 with a B -basis material and a two standard deviation applied stress. The net contribution of the conventional safety factor to the difference of the means was 74 percent. Example No. 2 reduced the conventional safety factor to the equation (19) lower limit of 1.38 and increased the standard deviations of the applied stress to three with an A -basis material. The conventional safety factor contribution decreased to 42 percent with only a 6-percent change in the difference of the mean stresses. The coefficients of variations are abbreviated by the symbol " C " with subscripts referring to respective distributions. These two examples clearly demonstrate the joint effects of the three safety factors on total safety and their interchangeability.

Table 5. Safety factors interchangeability.

Example	Assumed Variables							Results	
	SF	μ_R	μ_A	C_R	C_A	n_R	n_A	$\mu_R - \mu_A$	% SF
No. 1	1.5	47	21.6	0.053	0.12	2.74	2	25.4	74
No. 2	1.38	47	20.0	0.053	0.12	3.74	3	27.0	42

Results do not suggest which of the two combinations of safety factors is preferred, but judging from the percent change of safety factors, there is indeed a preference. It would seem that the combination allowing the largest limit load stress is the more economic user of the elastic material. In comparing their limit load stresses derived from equations (15) and (17),

$$F_A = \frac{\mu_R}{SF} (1 - n_R C_R) , \quad (21)$$

example No. 2 has a slight edge, but results were too close to be conclusive. Equation (20) is the deterministic total safety equation which, unlike the probabilistic method, satisfies the yield and ultimate stress requirements simultaneously.

B. Correlation With Probabilistic Safety

Probabilistic and deterministic failure concepts are both governed by the amount of interference from their resistive and applied stress distributions, which are related to the difference of their stress distribution means. Consider their approaches to defining their mean difference. The probabilistic method uses a specified yield or ultimate reliability value derived from a systems analysis, from which a safety index is determined and the difference between the applied and resistive stress means is calculated. The deterministic method determines three safety factors independently based on their individual probability requirements and material properties. The mean difference is then the outcome of the combined three safety factors.

Though the probabilistic and deterministic approaches to determining the stress distribution mean differences are not alike, they both control stress distribution interference which is related to reliability. Then substituting the difference of the stress means derived in equation (20) into the mean difference represented in equations (12), a deterministic safety index is realized,

$$Z = \frac{1}{\sqrt{\sigma_R^2 + \sigma_A^2}} [\mu_A (SF - 1) + SF (n_A \sigma_A) + n_R \sigma_R] , \quad (22)$$

or

$$Z = \frac{1}{\sqrt{\left[\frac{\mu_R C_R}{\mu_A} \right]^2 + C_A^2}} \left[(SF - 1) + SF (n_A C_A) + \frac{\mu_R}{\mu_A} (n_R C_R) \right] . \quad (23)$$

Equation (23) not only defines the deterministic safety index, which establishes its structural reliability through figure 11, but it further demonstrates the probabilistic nature of the deterministic safety method.

Applying equation (23) to examples No. 1 and No. 2 in table 5, the calculated safety indexes are 6.73 and 7.40, respectively. The corresponding reliabilities are 11-9's for example No. 1 with the highest conventional safety factor and 13-9's for example No. 2 with the lower safety factor. These are ultimate reliabilities in which the included conventional safety accommodates the yield and ultimate requirements simultaneously, and partially explains the resulting high 9's. Accommodating the yield requirement in the probabilistic method was also noted to increase the ultimate reliability from 4-9's to 7-9's, but 13-9's demands examination.

Reducing example No. 2 materials from A- to B-basis and loads from 3 to 2-sigma, and substituting into equations (20) and (23) yields a deterministic safety index of 6.24 corresponding to a reliability of 10-9's. Since reducing the material and load probabilities further would be unrealistic, the deterministic high 9's cannot be charged to its conservatism. Contrarily, the low 9's associated with the basic probabilistic method should be suspected for its compatibility with static stress safety.

It should be understood that reliabilities defined by equation (11) are an approximation based on the mean and standard deviation and not on the total distribution. Reliabilities above 5 or 6-9's would seem to over extend the quality of data and the math concept. Also, a reliability of 4-9's translates into one failure in 10,000. Does specifying that reliability on a vehicle static stress imply one failure in 10,000 missions, 10,000 vehicles, 10,000 critical stress zones, or what?

Nevertheless, the safety index of equation (23) provides a technique for comparing the relative safety of one structural region to another, or a metallic structural safety with a ceramic structure. It is useful in trade studies to assess interchange of safety factors. The fact that the deterministic method produces high 9's clearly shows that the deterministic design produces structures with almost a zero failure rate. Reliability appears not to be a consideration with static loads using the deterministic method.

C. Safety Sensitivities

Safety factors selection criterion should only account for uncertainties which the designer cannot determine in a rational manner. It should include materials performance probability and confidence, anomalous operational loads, general manufacturing deviations, undeterminable residual stresses, and minor handling damages. Important to the selection of safety factors which incorporate these drivers is their effect on the total safety sensitivity. It is generally impractical for aerostructural guaranteed safety measures to compensate for errors in math and math modeling.

The deterministic difference of the stress distribution means, equation (20), is a measure of total safety which includes the conventional safety factor, SF , and the standard deviation multipliers, n . Differentiating the means difference of equation (20) and dividing by it, its sensitivities to the three safety factors are:

$$\frac{\partial (\mu_R - \mu_A)}{\mu_R - \mu_A} = \frac{SF \mu_A (1 + n_A C_A)}{\mu_R - \mu_A} \frac{\partial SF}{SF}, \quad (24)$$

$$\frac{\partial (\mu_R - \mu_A)}{\mu_R - \mu_A} = \frac{SF \mu_A n_A C_A}{\mu_R - \mu_A} \frac{\partial n_A}{n_A}, \quad (25)$$

$$\frac{\partial (\mu_R - \mu_A)}{\mu_R - \mu_A} = \frac{\mu_R n_R C_R}{\mu_R - \mu_A} \frac{\partial n_R}{n_R}. \quad (26)$$

Another safety measure discovered in section IV-B was the determinist safety index which included the same three safety factors. Differentiating the safety index of equation (23) and dividing by it for each safety factor, sensitivity expressions are:

$$\frac{\partial Z}{Z} = \frac{(1+n_A C_A)}{\sqrt{\left[\frac{\mu_R C_R}{\mu_A}\right]^2 + C_A^2}} \frac{SF}{Z} \frac{\partial SF}{SF}, \quad (27)$$

$$\frac{\partial Z}{Z} = \frac{SF \times C_A}{\sqrt{\left[\frac{\mu_R C_R}{\mu_A}\right]^2 + C_A^2}} \frac{n_A}{Z} \frac{\partial n_A}{n_A}, \quad (28)$$

$$\frac{\partial Z}{Z} = \frac{C_R}{\sqrt{\left[\frac{\mu_R C_R}{\mu_A}\right]^2 + C_A^2}} \frac{\mu_R n_R}{\mu_A Z} \frac{\partial n_R}{n_R}. \quad (29)$$

Deterministic sensitivities of equations (24) through (29) are demonstrated through the same two numerical examples used in table 5. Safety factors are changed from example No. 1 to No. 2, where the conventional safety factor is reduced to the lower limit using table 2 and equation (19). Results are listed in table 6 which show the conventional safety factor is the most sensitive factor. Also important is that the mean difference and the safety index results are identical.

Table 6. Mean difference and safety index sensitivities to safety factors.

	Mean Difference $\mu_R - \mu_A$			Safety Index Z		
	SF	n_A	μ_R	SF	n_A	μ_R
Assumed safety factors	1.5	2	2.74	1.5	2	2.74
Change in safety factors	1.38 (8.7%)	3 (50%)	3.74 (36%)	1.38 (8.7%)	3 (50%)	3.74 (36%)
Resulting change in deterministic safety	-13.7%	15.3%	10%	-13.7%	15.3%	10%

The long lived success of the conventional safety factor has less to do with its universal numerical value than with progress made in improved materials and manufacturing, in advanced loads and stress prediction techniques, in improved testing and inspection methods, and in operations management. **Any deterministic safety criterion must address experiences in all these philosophical functions plus the consequences of failure.** Though some of these functions were earlier recognized as failure and data sources, all these functions must now be assessed for risk and consequences.

1. **Consequence of failure** is the overriding consideration in any safety criterion formulation. Maximum safety would apply to loss of life, loss of vehicle from a limited fleet or heavy traffic density, loss of launch and facilities, loss of launch opportunities, and excessive insurance cost to payloads. Risk and consequence analyses apply to all critical structural regions and operational phases, such as transportation, assembly, launch, ascent, etc. Distinction between manned and unmanned rated structures should be based on consequences of a structural failure

in a crew environment. Consequences may range from crew comfort to safe return. Cost of projected payloads and launch vehicles is narrowing the consequences between manned and unmanned structures. Weight may be considered as a consequence of cost.

2. **Material** improvements have had the most significant effects on safety. Experience data based on material performance in related applications should have maximum influence on safety. New materials and new applications pose the highest risk. Development of higher specific strength materials reduces the performance sensitivity as well as the associated reduction in strength to yield ratios. Improved quality controls establish reliable mean values and reduce data scatter, which combine to improve the safety index of equation (23). Ductile materials in redundant structures sustain loads with increasing displacement better than brittle materials. Increasing sample sizes improves the mean and variance data base and increases confidence.

3. **Manufacturing** variances in machining, processing, and reproducibility may be improved to within specified economic allowance through advanced technology and quality control enforcement. It is important to know the type and control of variances allowed for the particular class of structure and the consequence of allowed variances. Also consider that tight controls promised are not always deliverable.

4. **Models** of induced flight loads are the most unpredictable of all models. Worse yet, they can be verified only through a limited sample of flight tests, which suggests their safety leans heavily on consequence considerations. Deterministic loads may require minimum safety measures; however, high energy pressure vessels must also consider serious consequences. Structural modeling inaccuracies are not considered in guaranteed safety factors if the structure is test verified. Otherwise, modeling uncertainties may be covered by safety factors, but is not accounted in the guaranteed factor. In general, prediction techniques are constantly changing and improving, and it is necessary to verify their accuracies before incorporating safety measures.

5. **Testing** is indispensable to verify guaranteed safety factors on critical structures. Improved testing techniques, instrumentation, and data recording methods have increased the thoroughness of structural verification and test data reliability. Testing diminishes uncertainties, exposes sneak phenomena, and serves as a feedback for fine tuning modeling accuracies, all of which are rationale for assessing risk.

6. **Inspection** improved methods and techniques not only promote their application to screen structures to increase reliability, but also allow significant reductions in safety measures. Inspected manufactured welds allow lower safety factors than uninspected welded articles. Scheduled inspection and maintenance reduce risk.

7. **Operations management** and mission rules have the capacity and authority to insure that the guaranteed safety is never exceeded during operations. This is true for whatever the design specified safety may be or to whatever it may degenerate. Management may choose to methodically increase the operational experience base, but never quite operates in worst-on-worst environments. A vehicle may be designed for all weather conditions, but it might be more prudent to postpone a flight that is not window critical. Measuring prelaunch environments and referring them to vehicle load indicators reduces structural risk.

Returning to the new role of the conventional safety factor, its lower limit was established, and its sole purpose in the deterministic safety index was seen to satisfy the yield and ultimate conditions simultaneously. The conventional universally adopted numerical value of 1.5

was also based on the ratio of the ultimate to yield stresses, which wisely capitalized on the "wasteland" region of stress beyond the elastic limit. Since operationally applied stresses are restricted to the linear elastic region, the excess strain energy between yield and ultimate stresses was conceded to anomalous events without penalty to material performance for the operational range.

The universal numerical factor was based on a basic material used a half-century earlier. In light of currently improved properties of common metals and development of new ones, it should be fitting to compare how well this universal factor has endured over a few select metallic materials used in contemporary aerostructures.

Figures 13 and 14 graphically compare conventional safety factor characteristics of ferrous and nonferrous materials. The first bar (darkest bar) in each group represents the lower limit safety factor defined by equation (19) and is estimated from the left scale. Properties used were published¹¹ A-basis in tension and at room temperature. A general observation about the ratios is that materials with highest heat treatment have the lowest ratio.

Dividing equation (19) by an arbitrary conventional safety factor and calculating the percent change,

$$F_{SF} = \left[\frac{F_{tu}}{SF \times F_{ty}} - 1 \right] \times 100, \quad (30)$$

introduces a significant design consideration. The second, third, and fourth bars of each material in figures 13 and 14 are based on equation (30) and represent the percent elastic stress performance denied the structure when applying arbitrary conventional safety factors of 1.5, 1.4, and 1.25, respectively. The percent loss of material performance is read from the right scale in the figures.

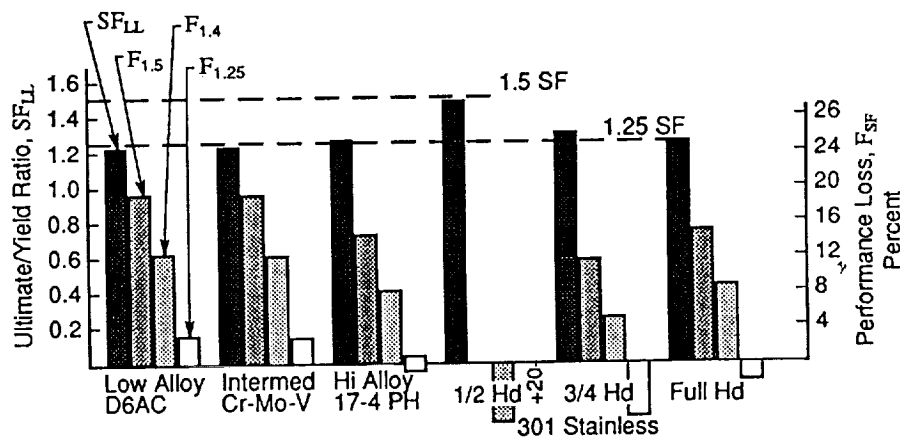


Figure 13. Safety factor characteristics of ferrous materials.

It may be noted from equation (30) and charts that selecting a safety factor in excess of the lower limit defined by equation (19) could squander any economy in structures that might have been gained from materials research and development. A safety factor of 1.5 on titanium wastes a quarter of its elastic carrying capability. On the other hand, specifying a 1.4 safety factor on one-half hard stainless steel, having a 1.5 ultimate to yield stress ratio, would allow the applied stress to operate into the inelastic region.

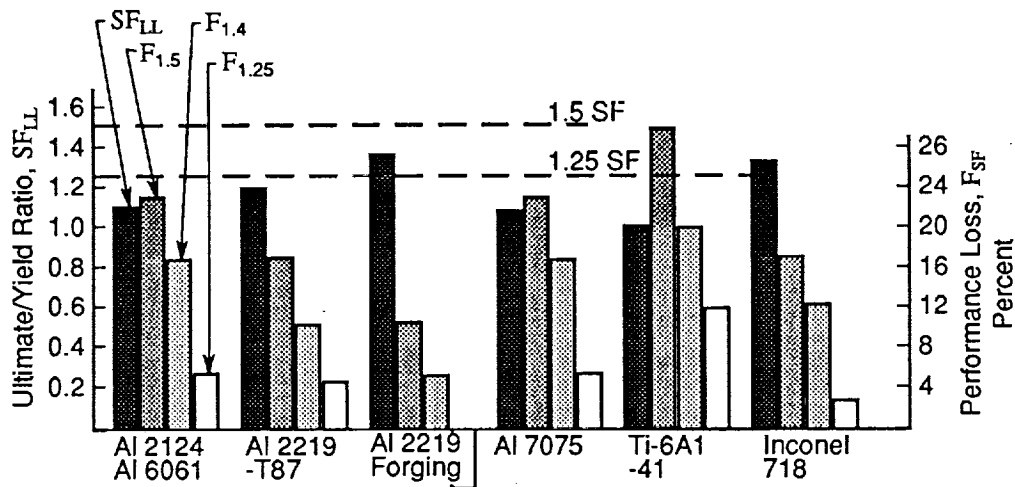


Figure 14. Safety factor characteristics of nonferrous materials.

Another example of judgment based on engineering analysis is to assess the structural economy which may be equated to weight. Since conventional safety factors primarily affect structural thicknesses, the weight change with change in safety factor may be expressed as

$$\frac{\partial W}{W} \approx m \frac{\partial SF}{SF}, \quad (31)$$

where " $m = 1$ " for plates under normal loading, and " $m = 0.5$ " for plates in bending and buckling. Equation (31) should have no voting power over safety in selecting the numerical factor, but it should be a serious upper limit consideration in trade studies. Proponents to delete structural verification tests of common aerospace shell structures by increasing the lower limit safety factor from 1.4 to 2.0 must face a 42 percent weight increase. This weight penalty is conveyed through manufacturing, spares, handling, propulsion system, propellant, and payload and associated costs.

D. Safety Factors Selection Criterion

Having discovered that the deterministic safety method incorporates three safety factors, their individual purposes, risks, consequences, sensitivities, and weight penalties must be pooled into safety factors selection criteria. Though independently selected, safety factors in the three safety zones interact and combine to provide one safety index which conceptually may be developed into a criterion for each category of consequence. In the meantime, a selection criteria base follows with the prospect that each safety factor will be concisely specified by respective disciplines.

1. Resistive Stress Zone: The material ultimate stress safety factor " n_R " is identically the K -factor of figure 5 which specifies the number of standard deviations of the resistive stress distribution. It is selected on the basis of probability and confidence level required and, thus, is specified as an A or B-basis material property. The A-basis is usually specified for primary structures having high risk and severe consequences. It is specified for brittle and uncommon materials, new applications, large coefficients of variations, limited process controls and inspections. The B-basis is specified for secondary and redundant structures where performance dominates and consequences will not stall, suspend or abort missions.

There is little experience with ceramic materials as primary structures. Carbon fibers and glass plates are included among these materials. If their applications are weight and hazard critical, comprehensive test programs in related environments must be initiated using very large sample sizes, satisfying probability and confidence requirements

2. Mid Zone: The mid zone safety factor is the conventional safety factor with the lower limit defined by the ratio of ultimate to yield stresses. Its sole purpose is to ensure the applied stress does not operate in the inelastic region of the structural material. The lower limit defines the inelastic region of the material which is conceded to an extraordinary environment or missed quality control with no penalty to the operational stress region. This is the factor that specifies the upper tolerance limit of the applied stress. A factor less than the lower limit will cause the operational load to deform the structure. A factor 1 percent in excess of the lower limit will reduce structural performance an equal percent.

Ceramic materials that do not exhibit distinct inelastic regions reduce the lower limit conventional safety factor to one, and provide no reserve for extraordinary events as in metallic materials. A factor greater than one may be desirable, but should be derived only from engineering analysis including remote operational environments, manufacturing process and inspection controls, data sample size, and consequences. A more appropriate approach is to specify and develop its property to a higher than A-basis. Arbitrarily assigned safety factors do not necessarily diminish risks.

Increasing safety factors to compensate for modeling errors or deficiencies is not considered part of the guaranteed safety. If it is applied and the verification test proves it not to be necessary, the excess spills into the applied stress zone which is passed on to the operational margin.

Large safety factors specified for fittings, tubing, fasteners, etc, to survive installation, manufacturing, and handling loads do not conform to deterministic safety methods. These safety factors are uniquely specified by specialty industries or customized to user harsh ground operations.

3. Applied Stress Zone: The safety factor in the applied stress zone is the standard deviation multiplier " n_A " of the applied stress distribution. It is a composite of multipliers applied to dynamic and static load variances and to manufacturing deviations. If loads and manufacturing means and tolerances are provided, distributions may be estimated from the means and from the standard deviations assumed as one-third of the tolerances. Multipliers may be selected for each load and manufacturing deviation component based on its significance and sensitivity to combined applied stress and to its probability of occurrence, risk and consequences. Pressure load safety factors are selected for the energy content and failure consequences.

Usually, safety factor selection criteria are formulated and applied by the loads community who combine and report them as a single value 3-sigma or more limit load for an entire structure. A more useful load description would include the mean and standard deviation such that a designer may vary the number of standard deviations according to the consequence of failures at different regions. The thoroughness required to reduce the number of standard deviations depends on the performance gained.

Safety factor selection process for load, or induced stress, may sometimes involve a formidable array of considerations with risk and consequences being foremost. Selection by the

numbers is not a recommended primary approach, but having selected a set of factors, a simplistic decision analysis technique may provide a sobering backup. Using a matrix of three or more criticality columns verses rows of significant consequence sources, the column population should identify the dominant criticality. Each criticality column should be related to a specified standard deviation multiplier. Consequence source weighing factors based on relative sensitivities on the applied stress may improve safety factor selections.

E. Applied Stress Split Safety Factor

An efficient stress design may be realized by separating the applied stress safety factor into one induced by well defined loads (pressure, thrust, and inertia) and another applied to the stress induced by less certain loads (aerodynamics, dynamics, and winds). The maximum allowed applied stress is then the sum of the split induced stresses,

$$F_A = F_{A1} + F_{A2} . \quad (32)$$

The applied stress components are defined in statistical format and as rule of mixtures fractions,

$$F_{A1} = \mu_{A1}(1+n_{A1}C_{A1}) = q F_A , \quad (33)$$

$$F_{A2} = \mu_{A2}(1+n_{A2}C_{A2}) = (1-q) F_A , \quad (34)$$

where subscripts 1 and 2 refer to the well defined set and less well defined set of stresses, respectively, and "q" is the well defined mixture fraction.

Distribution parameters in equations (33) and (34) are combined into equation (32) through the basic rules of statistics for combining uncorrelated variables. The mean of the overall distribution is the algebraic sum of the means,

$$\mu_A = \sum_{i=1}^m \mu_{Ai} , \quad (35)$$

and the combined variance is the sum of the variances,

$$\sigma_A^2 = \sum_{i=1}^m \sigma_{Ai}^2 ,$$

or

$$\sigma_A^2 = \sum_{i=1}^m [\mu_{Ai} C_{Ai}]^2 . \quad (36)$$

The combined applied stress concept defined by equations (32) through (36) is illustrated in figure (15).

Solving for the means from equations (33) and (34),

$$\mu_{A1} = \frac{q F_A}{(1+n_{A1}C_{A1})} , \quad \mu_{A2} = \frac{(1-q) F_A}{(1+n_{A2}C_{A2})} ,$$

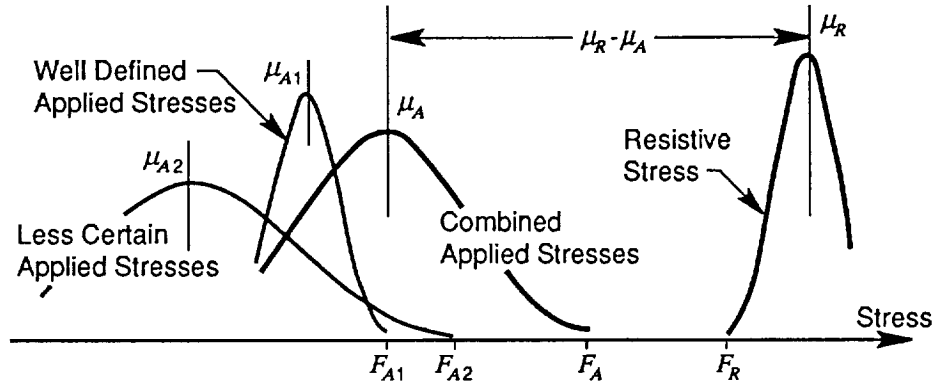


Figure 15. Dual applied stress distribution concept.

and substituting into equation (35) gives the combined mean stress

$$\mu_A = F_A \left[\frac{q}{1+n_{A1}C_{A1}} + \frac{(1-q)}{1+n_{A2}C_{A2}} \right]. \quad (37)$$

Solving for the product of the coefficient of variation and multiplier from equations (33) and (34) gives

$$n_{A1}C_{A1} = \frac{q F_A}{\mu_{A1}} - 1, \quad n_{A2}C_{A2} = \frac{(1-q) F_A}{\mu_{A2}} - 1. \quad (38)$$

Treating the product squared as a variance and combining in equation (36) gives

$$(n_A C_A)^2 = (n_{A1} C_{A1})^2 + (n_{A2} C_{A2})^2. \quad (39)$$

Substituting equations (38) into (39) yields the desired combined product of the coefficient of variation and multiplier,

$$n_A C_A = F_A \sqrt{\left[\frac{q}{\mu_{A1}} - 1 \right]^2 + \left[\frac{(1-q)}{\mu_{A2}} - 1 \right]^2}. \quad (40)$$

A similar approach may be used to combine different distributions, such as dynamic with static loads or stresses induced by loads and manufacturing variances.

V. SAFETY VERIFICATION

Requirements for structural test articles and expectations from tests are not generally understood. In light of the three safety factors discovered in the deterministic method of section IV-B, test purpose and limits need to be clarified.

After all analyses have been checked for completeness and safety criterion has been complied, there still remain four failure related prediction uncertainties that must be resolved: accuracy of structural math model, quality control on processing and manufacturing, material

properties under combined load, and induced load math models. The latter can be verified only through early operational trials which are not scoped in this document. Nevertheless, its worst case predicted load is simulated on a structural test article to verify the other three uncertainties. Conventional test procedures and generic results lead to a generalized discussion on what might be required from a structural test and how to verify deterministic safety prediction.

A. Test Article Requirements

A structural test article serves two major functions. It provides the user with an estimate of the structure's safety level and failure mode, and it offers an opportunity to verify the structural math model. A safety estimate may be achieved by simulating constraints through a test bed and by simulating predicted worst case loading through load cells. The structure is loaded to failure, and its safety is judged as to whether or not the failure loads exceeded predicted loads. It also provides visual information as to the structure failure mode: fail operational, fail safe, or catastrophic.

However, load cells alone cannot verify that the local failure was caused by the predicted combined stress. It cannot explain the cause of failure at an unanticipated region, nor can it detect a sneak phenomenon. Electrical strain gauges and stress coats provide that kind of data to be used in post test analysis with the prediction model.

Strain and displacement gauges are indispensable for verifying and fine tuning the structural math model which is later used to verify the predicted operational loads. The corrected model is particularly useful for predicting structural safety under other projected mission environments. Another purpose for a strain gauge instrumented article is the feedback learning it provides the modeler. Without that post test analysis, there is no creditable experience gained, which represents a meaningful loss.

A test plan must be preceded by a test prediction data base for tracking strain gauge data. It was indicated earlier that metallic material failures most often occur near their average values. Therefore, predictions should be extended to material average properties. It was also noted that failures occur in regions having abrupt local changes in geometry, metallurgy, and loads which pose serious limitations to obtaining direct strain data. Planning must include special strain gage applications and supporting analyses. The test structure responds to induced multiaxial combined stresses, and failure is estimated by the minimum distortion energy theory. Biaxial strain at a weld may be extrapolated by mounting gauges on adjacent parent surfaces as suggested by figure 8. Transverse shears at dome-cylinder intersections, for instance, may be analytically derived from strain gauge data obtained along the discontinuity stress wave. Combined stresses on shells induced by rivets and bolts in shear may be verified through strategic location of gauges and assisted by a detailed discontinuity analysis such as might be obtained through boundary element methods.

Ultimately, the stress analysis is the primary authority on a structure's general safety. The structural test is required only to verify prediction models in part. The best that can be expected from strain gauges is to verify one or two measured surface stress components at each critical region with model predictions. Complementing electrical strain gauges with a brittle stress coating would provide further detailed qualitative evidence on surface stress patterns for comparison with math model predicted patterns.

B. Safety Data Interpretations

The most meaningful and critical phase of the deterministic safety verification test is tracking the strain gauge data up to the structural yield point. The simulated load to the yield point includes two safety factors—the standard deviation multipliers of applied stress and the material yield stress K -factor. These are the only two that properly designed structures should be expected to encounter operationally. Yield and ultimate stresses of materials with small coefficients of variation are expected to fail near their mean values. Therefore, gauges will track linearly with the prediction analysis up to the article's yield point, which may be one or more standard deviations higher than the design allowable. If the measured yield stress " F_{AM} " on the article exceeds the design allowable yield " F_A ", the difference is the operational margin, as illustrated in figure 16.

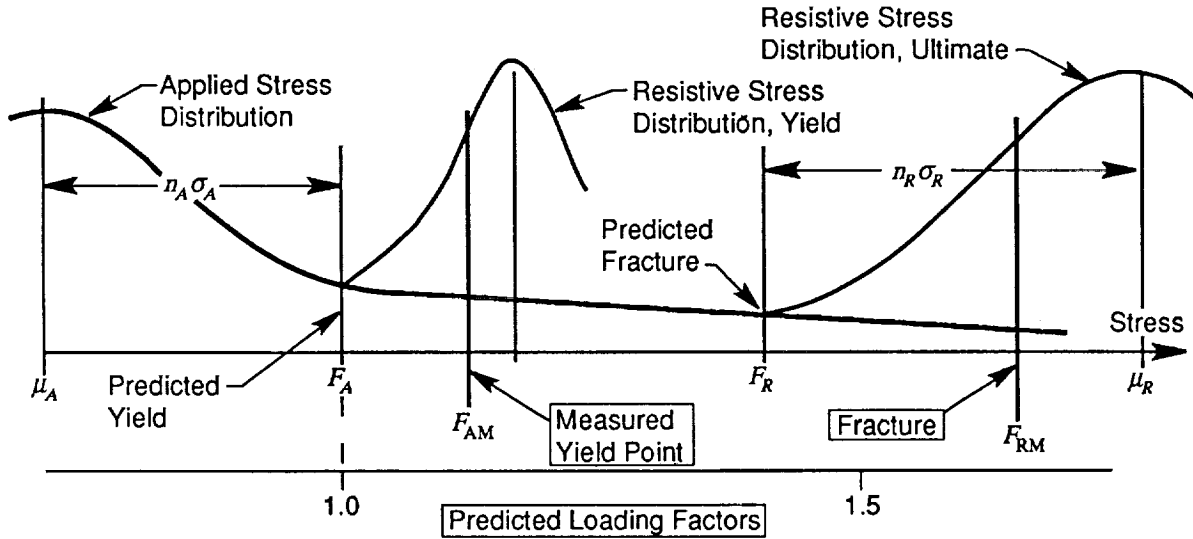


Figure 16. Structural test data interpretation.

The operational safety margin is defined by

$$MS_{OP} = \frac{F_{AM} - F_A}{F_A} \quad (41)$$

A simplistic conclusion to be drawn from one test is that this particular article has a verified operational margin defined by equation (41). A more important product of the test is the comparison of load cell and strain gauge data with the structural math model. If the linear model faithfully tracks the test data to the yield point, then the number of applied stress standard deviations may be estimated, and the operational margin is approximately determined. If the model tracked the test data in rate, but not in intensity, the model may be in error by a constant. If, on the other hand, the intensity is off in rate and intensity, the model must be examined for global or local stiffness. Local fastener stiffness is a common cause.

Beyond the yield point to failure is the inelastic stress induced by combined, nonlinear loads through successively deforming load paths. It exercises the conventional safety factor and some unknown number of standard deviation multipliers of the ultimate stress distribution. It represents the resistive stress available to a one-time anomalous loads or missed manufacturing

deviation. It is defined as the ratio of the applied test load measured at failure with the load measured at yield. In fact, it is the same method used to determine the current safety factor, but of less importance. Current structural testing techniques should not be expected to adequately verify all nonlinear predictions with all nonlinear data.

If fracture occurs in a discontinuity region, which is most likely, there are no current techniques for adequately predicting inelastic stress with material properties changing simultaneously with geometry. To estimate the number of standard deviation multipliers experienced at fracture, test data in discontinuity regions must be supported with an inelastic math model. With this ultimate stress interpretation limitation in mind, the designer is ever more compelled to insure the elastic stress verification is complete and competent, and to waive routine inelastic testing. Test to fracture criteria should consider failure source criticality and benefit of inelastic data verses cost of flight-like article. It is conceivable that structures with safety indexes exceeding a specified value may be exempted from testing beyond the yield point.

VI. HUMAN FACTOR

Though structural failures are rare compared to failures in other disciplines, there are as many structural failures today as decades ago. Recognizing progressive technological demands and corresponding advances made, the most frequent cause of failure cited in failure investigations is incomplete analysis. It is as much a human factor as it is an administrative one. There are many related lessons published by Drucker, Peters, Deming, etc. More relevant to mechanics disciplines are the lessons extensively practiced and currently published by R. Ryan.^{18,19,20}

All such discourses should include the old truism that a design is no better than the designer. Therefore, one starts with the best employees and even better supervisors—supervisors with an infectious vision and technical credentials to inspire and perpetually advance their staffs and technology. Switching to administrative roles must not be perceived as more prestigious and rewarding than improving products. World War II quartermasters wore the best boots while the infantry got trench feet. Rewarding career opportunities should be developed for specialists as specialists.²¹

All design specialists must cultivate an instinct for spotting potential hardware problems in manufacturing and operations. If hands-on experience is not an option, then on-site familiarity with the hardware process is a minimum which may require priority trades for more plant travel. There is no substitute for on-site experience. Good technologists must be nurtured. Before there is a safe design, the concept has to grow and crackle in the mind of the designer.

VII. SUMMARY

The primary purpose of this study was to research the current arbitrary application of the conventional safety factor and to approach the deterministic method from a probabilistic concept leading to a more coherent philosophy and criteria for designing safer aerostructures.

It was soon realized that focus on any safety measure must be preceded by a fundamental understanding of the source, cause, and consequences of failures and the validity and completeness of data reduction, assumptions, and math models. These were discussed, and an example

using standard statistical techniques demonstrated an incomplete analysis as a cause of avoidable failure. Likewise important, the statistical exercise was an educational lead into understanding its application to probabilistic and deterministic safety methods.

A basic probabilistic safety method was used to learn the relationship of reliability and tail interferences of the applied and the resistive stress distributions (fig. 9). It was noted that the reliability concept produced a safety index which serves as a common multiplier of the applied and the resistive stress distribution variances that defined the difference of the mean stresses of the two stress distributions. Increasing the difference of the means stresses decreased tail interferences which reduced the probability that a weak resistive material will encounter an excessive applied stress.

The current deterministic concept is defined in terms of statistical tolerance limits of applied and resistive stress distributions and a conventional safety factor. When these functions were synthesized in figure 12, comparison with the probabilistic concept was clear. A significant difference was that the probabilistic concept used a common safety factor, or safety index, for two zones, while the deterministic concept had three safety zones with as many different safety factors. They all served to specify the tail interferences.

It turned out that none of the deterministic safety factors are arbitrarily selected. They are independently specified by loads, materials, manufacturing, and stress disciplines. The applied stress safety factor is the standard deviation multiplier which specifies the desired probability that the applied stress does not exceed the material yield point. The resistive stress safety factor is the standard deviation multiplier, or *K*-factor, that specifies the *A*- or *B*-basis material property. The conventional safety factor's sole purpose is to insure that the applied stress does not operate in the inelastic region of a metallic material, and it is defined as a ratio of ultimate to yield stresses. Any safety factor excess may be shared with either of the other two factors.

A deterministic safety index was derived by equating the means difference of the deterministic and probabilistic stress distributions. Applying the deterministic safety index to a numerical example resulted in a deterministic reliability of over 10-9's. Reliabilities above 4-9's should be suspected of over extending the quality of input data and reliability model. More importantly, the very large number of 9's demonstrated conclusively that the deterministic method produces nearly zero failure rates and that the method is reliability insensitive.

Nevertheless, the deterministic safety index is still a significant contribution. It condenses all three safety factors into a single index to support trades of safety factors. The deterministic safety index may be used to calibrate or compare safety at different stressed regions or different structures. It promises to be developed into a safety and verification criterion. It also raises serious questions about the unequivocal three safety factors of the deterministic method producing many more reliability 9's than the basic probabilistic method. It questions the philosophy of specifying reliability levels to static stress problems. A 0.9999 reliability implies one failure in 10,000 what?

Rationale for formulating deterministic safety criteria is presented, and deterministic verification test requirements, expectations, and data interpretation are discussed. It would seem that a structural test article achieving the predicted yield stress plus two-thirds or more of the material yield standard deviations should be considered a successfully verified structure. Tests should not be routinely tested beyond that point, since the "unverified" resistive stress above the

yield point is a cushion to some "unpredictable" environmental or structural phenomena without penalty to the structural linear operational performance.

There were other discoveries. What was perceived as an arbitrary method was demonstrated to be totally invested in a statistical failure concept. What has been treated as an exclusive domain of the stress community is a collaboration of loads, materials, stress, and manufacturing disciplines.

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APPENDIX

Quick Basic Programs for Macintosh

```
' NORMAL DISTRIBUTION TEST [6]
' Kolmogorov-Smirnov (normality test)
' Critical values (n > 30): a=.10, d=.805
' Crit. Val: a=.05, d=.886; a=.01, d=1.031.
' Input data
```

```
CLEAR:INPUT "N=";N
DIM A(N),D(N),Z(N)
FOR I=1 TO N
INPUT A(I)
NEXT I
' sort data
K=N-1
LINE180:FOR X=1 TO K
B=A(X)
IF B<=A(X+1) GOTO line250
A(X)=A(X+1)
A(X+1)=B
Y=1
T=X-1
line250:NEXT X
IF Y=0 GOTO line300
Y=0
K=T
GOTO LINE180
line300:
PRINT "SORT DONE"
'mean and std. deviation
FOR I=1 TO N
C=C+A(I)
D=D+A(I)*A(I)
NEXT I
M=C/N
SD=((D-N*M*M)/(N-1))^.5
PRINT "MEAN=";M
PRINT "STD DEV=";SD
'standardized normal
FOR I=1 TO N
Z(I)=(A(I)-M)/SD
NEXT I
```

```
'cumulative normal
FOR I=1 TO N
X=Z(I):T=X
G=EXP(-X*X/2)/SQR(2*3.14159)
A1=.31938:A2=-.35656:A3=1.78147
A4=-1.82125:A5=1.330427
IF X<0 THEN T=-X
Y=1/(1+.2316419*T)
P=(((A5*Y+A4)*Y+A3)*Y+A2)*Y+A1)*Y
F=1-G*P
IF X<0 THEN F=1-F
DI=I/N 'empirical cumulative
D(I)=ABS (F-DI)
IF DM<D(I) THEN DM=D(I):J=I
NEXT I
'results
FOR I=1 TO N
PRINT D(I)
NEXT I
PRINT "WORST SAMPLE #";J
PRINT "ABS DIFFERENCE, D";DM
END
```

'K1-FACTOR PROGRAM [6]

MARIO:

DEFDBL A-Z

INPUT "SAMPLE SIZE=";NS

INPUT "PROPORTION";P

INPUT "CONFIDENCE=";CL

IF NS>90 THEN PRINT "SAMPLE >90"

WHILE INKEY\$="" :WEND

START=TIMER

PI=3.141592654#

'INVERSE NORMAL

Q=1-P:T=SQR(-2*LOG(Q))

A0=2.30753:A1=.27061:B1=.99229

B2=.0481

NU=A0+A1*T:DE=1+B1*T+B2*T*T

X=T-NU/DE

L0: Z=1/SQR(2*PI)*EXP(-X*X/2)

IF X>2 GOTO L3

V=25-13*X*X

FOR N=11 TO 0 STEP -1

U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V

V=U:NEXT N

F=.5-Z*X/V

W=Q-F:GOTO L2

L3:V=X+30

FOR N=29 TO 1 STEP -1

U=X+N/V

V=U:NEXT N

F=Z/V:W=Q-F:GOTO L2

L2:L=L+1

R=X:X=X-W/Z

E=ABS(R-X)

IF E>.00001 GOTO L0

'END OF INVERSE NORMAL

'CALCULATION OF FACTORIAL

N=NS:NU=N-1

MT=INT(NU/2):UT=NU-2*MT

GT=1

FOR I=1 TO MT-1+UT

KT=I

IF UT=0 GOTO L1

KT=I-.5

L1:GT=GT*KT

NEXT I

GT=GT*(1+UT*(SQR(PI)-1))

GF=GT*2^(NU/2-1)

'END OF FACTORIAL

'SECANT METHOD

KP=X:J=1:K=KP

K0=K:GOSUB INTEGRATION:SF0=SF

K=K*(1+.0001):K1=K:GOSUB INTEGRATION

SF1=SF

BEGIN:K=K1-SF1*(K1-K0)/(SF1-SF0)

IF ABS((K1-K)/K1)<.000001 GOTO RESULT

J=J+1:K0=K1:K1=K:SF0=SF1

GOSUB INTEGRATION:SF1=SF:GOTO BEGIN

RESULT:FINISH=TIMER

BEEP:BEEP

PRINT "K =";USING"###.####";K

PRINT "TIME=";FINISH-START;"SECONDS"

'END OF SECANT METHOD

WHILE MOUSE(0)<>(1):WEND

GOTO MARIO

INTEGRATION:L1=0:L2=10

IF N>40 THEN L2=20

DL=KP*SQR(N):TP=K*SQR(N)

Y=NU/2

M=2:E=0:H=(L2-L1)/2

X=L1:GOSUB FUNCTION

Y0=Y:X=L2:GOSUB FUNCTION

YN=Y:X=L1+H:GOSUB FUNCTION

U=Y:S=(Y0+YN+4*U)*H/3

START:M=2*M

D=S:H=H/2:E=E+U:U=0

FOR I=1 TO M/2

X=L1+H*(2*I-1):GOSUB FUNCTION

U=U+Y

NEXT I

S=(Y0+YN+4*U+2*E)*H/3

IF ABS((S-D)/D)>.00001# GOTO START

SF=S/GF-CL

RETURN

'END OF SIMPSON

FUNCTION: Z=TP*X/SQR(NU)-DL

T0=Z:G0=1/SQR(2*PI)*EXP(-Z*Z/2)

A1=.3193815:A2=-.3565638:A3=1.781478

A4=-1.821256:A5=1.330274

IF Z<0 THEN T0=-Z

W=1/(1+.231649*T0)

P1=(((A5*W+A4)*W+A3)*W+A2)*W+A1)*W

PH=1-G0*P1

IF Z<0 THEN PH=1-PH

Y=PH*X^(NU-1)*EXP(-X*X/2)

RETURN

'SAFETY INDEX FROM RELIABILITY

```
'NORMIN (.5,P,1)
DEFDBL A-Z
LL: INPUT "Probability=";P
PI=3.141593
PI=3.141593
Q=1-P:T=SQR(-2*LOG(Q))
A0=2.30753:A1=.27061
B1=.99229:B2=.0481
NU=A0+a1*T
DE=1+B1*T+B2*T*T
X=T-NU/DE

'CUMULATIVE NORMAL
L0: Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V
W=Q-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:W=Q-F:GOTO L2

L2:L=L+1
R=X:X=X-W/Z
E=ABS(R-X)
IF E>.001 GOTO L0
PRINT "SAFETY INDEX IS"
PRINT USING "###.####";X
GOTO L1
END
```

'RELIABILITY FROM SAFETY INDEX

```
'NORMIN (0.5,P,1)
DEFDBL A-Z
INPUT "P=";P:PI=3.141593
PI=3.141593
'Q=1-P:T=SQR(-2*LOG(Q))
'A0=2.30753:A1=.27061
'B1=.99229:B2=.0481
'NU=A0+a1*T
'DE=1+B1*T+B2*T*T
'X=T-NU/DE

'CUMULATIVE NORMAL
INPUT "X=";X
Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V:F=1-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:F=1-F
L2:
PRINT F

END
```

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