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This paper summarizes work performed under a collaborative research effort between the National Aeronautics and Space Administration (NASA) and the German Aerospace Research Establishment (DLR, Deutsche Forschungsanstalt für Luft- und Raumfahrt). The objective is to develop and demonstrate advanced technology for system identification of future large space structures. Recent experiences using the Eigensystem Realization Algorithm (ERA) for modal identification of Mini-Mast are reported. Mini-Mast is a 20-m-long deployable space truss used for structural dynamics and active-vibration-control research at the Langley Research Center. A comprehensive analysis of 306 frequency response functions (3 excitation forces and 102 displacement responses) was performed. Emphasis is placed on two topics of current research: (1) gaining an improved understanding of ERA performance characteristics in theory versus practice and (2) developing reliable techniques to improve identification results for complex experimental data. Because of nonlinearities and numerous local modes, modal identification of Mini-Mast proved to be surprisingly difficult. Methods available with ERA for obtaining detailed, high-confidence results are illustrated.

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Abstract

This paper summarizes work performed under a collaborative research effort between the National Aeronautics and Space Administration (NASA) and the German Aerospace Research Establishment (DLR, Deutsche Forschungsanstalt für Luft- und Raumfahrt). The objective is to develop and demonstrate system identification technology for future large space structures. Recent experiences using the Eigensystem Realization Algorithm (ERA) for modal identification of Mini-Mast are reported. Mini-Mast is a 20-m-long deployable space truss used for structural dynamics and active-vibration-control research at the Langley Research Center. A comprehensive analysis of 306 frequency response functions (3 excitation forces and 102 displacement responses) was performed. Emphasis is placed on two topics of current research: (1) gaining an improved understanding of ERA performance characteristics in theory versus practice and (2) developing reliable techniques to improve identification results for complex experimental data. Because of nonlinearities and numerous local modes, modal identification of Mini-Mast proved to be surprisingly difficult. Methods available with ERA for obtaining detailed, high-confidence results are illustrated.

Introduction

Successful design and qualification of current spacecraft structures rely heavily on ground test programs and resulting test-verified analytical models. Throughout the development process, modal tests are performed on individual structural components as well as on fully assembled spacecraft to provide the required information (ref. 1).

The difficulty of performing these modal-identification tests depends significantly on the dynamic complexity of the structure. Identification of small, individual components is often simple and straightforward. A variety of different techniques will provide comparable results. And the accuracy of identified mode shapes is readily substantiated by analysis, intuition, or both. However, modal identification of large, assembled structures—comprised of dozens of individual components—can be much more difficult. Hundreds of sensors and weeks of testing may be required, mode shapes are often highly coupled and nonintuitive, analytical predictions may be significantly inaccurate, and different identification techniques will generate different results. As an example, a recent state-of-the-art modal test of the Upper Atmosphere Research Satellite (UARS) used 240 accelerometers to measure important degrees of freedom, required 3 weeks for data acquisition, and generated 197 different mode estimates (not all unique) using several different excitation and identification techniques (ref. 2). A comparison of experimental and pretest analytical mode shapes showed significant differences based on cross-orthogonality calculations (ref. 3).

Future large space structures, such as Space Station *Freedom,* will be even more difficult than UARS to characterize experimentally (ref. 4). Overall size and the number of individual components will increase, clusters of modes with low frequencies will occur due to numerous flexible appendages (ref. 5), and ground tests will be affected to a greater degree by gravity and test-article suspension forces (ref. 6). Verification of analytical predictions will require increased testing of large components, subassemblies, and scale models (ref. 7). Some form of on-orbit identification is also likely to be used (ref. 8). These factors represent significant departures from existing qualification practices. Recognizing the importance and difficulty of these new challenges, considerable research has been underway within NASA and DLR in the areas of improved ground test methods and system identification techniques for these future structures (refs. 9–12). The work reported here represents one phase of this activity.

The Eigensystem Realization Algorithm (ERA) is a modern modal-identification technique capable of providing accurate results with complex structures, even in the presence of high modal density and structural nonlinearities (refs. 13 and 14). However, a considerable amount of interpretation and comparison with other analyses is typically required with complex structures to fully understand the results and accept them with confidence. This interpretation phase distinguishes structural modal identification from "black-box modeling," which may be adequate for control design (ref. 15). Understanding of structural characteristics (in terms of modal parameters) is required for dynamic qualification of future large space structures and is the objective of the work discussed in this paper.

The organization of the paper is as follows. A brief overview of Mini-Mast is provided in the next section, followed by a summary of data acquisition procedures and finite-element analytical predictions. The body of the paper focuses on two topics of current research: (1) comparison of ERA performance characteristics in theory versus practice and (2) improvement of results for complex experimental data. Preparation for on-orbit identification experiments is based on the analysis of simulated data. The first research topic examines the suitability of using only simple, linear analytical data for such studies. As an initial step, identification results obtained using noise-free analytical data are compared against corresponding experimental results. The second research topic is self-explanatory. Examples are given of methods available with ERA to develop high confidence in the identification results. These methods are typically applied in sequence, with initial findings providing information to guide subsequent analyses. The paper closes with a summary of best identification results obtained for the primary modes of Mini-Mast.

Acronyms

CMI	consistent-mode indicator
CSI	controls-structure interaction
EMAC	extended modal amplitude coherence
ERA	Eigensystem Realization Algorithm
FFT	fast Fourier transform
FRF	frequency response function
IRF	impulse response function
MAC	modal assurance criterion
MIMO	multiple input, multiple output
MPC	modal phase collinearity
MSR	modal strength ratio

Mini-Mast

Mini-Mast is a 20-m-long, deployable/retractable truss located in the Spacecraft Dynamics Laboratory at NASA Langley Research Center. It is used as a ground test article for research in the areas of structural analysis, system identification, and control of large space structures. The name "Mini-Mast" is derived from the name "MAST" given to a longer, 60-m version of a similar design once considered for a Space Shuttle-attached flight experiment. Constructed with graphite-epoxy tubes, titanium joints, and precision fabrication techniques, Mini-Mast was designed and built to the high standards typical of spaceflight hardware (ref. 16).

Figure 1 shows an artist's sketch of Mini-Mast superimposed on a photograph of the laboratory. The truss is deployed vertically inside a high-bay tower, cantilevered from its base on a rigid foundation. The total height is 20.16 m, containing 18 bays in a single-laced pattern with every other bay repeating. The design uses a triangular cross section with vertices located on a circle of diameter 1.4 m.

The deployment of Mini-Mast is shown in figure 2. The structure unfolds from a stowed configuration less than 1 m in height. An overhead crane was used to extend the truss inside the tower. During deployment both the longerons and the diagonal members rotate about hinges located at their ends. Near the end of rotation of the diagonals, center-span hinges latch to provide structural stability. This design—using middiagonal hinges—permits high packaging efficiency by allowing the diagonal members to fold into the center of the stack during storage. However, from a structural dynamics point of view, these massive middiagonal hinges introduce many additional low-frequency modes. A total of 108 additional modes appear in the frequency range from approximately 15 to 20 Hz due to the x and y first-bending modes of each of the 54 diagonal members of the truss.

For controls-structures interaction (CSI) experiments (ref. 17), two large instrumentation platforms and a tip cable have been added to Mini-Mast. Neither the platforms nor the cable are visible in figures 1 and 2. These additional components have a significant effect on the structural dynamic characteristics. The platforms are located at bays 10 and 18 (the tip) and hold feedback sensors and actuators for active-vibration-control experiments. Three large torque-wheel actuators, each weighing approximately 80 lb (compared with a total truss weight of 210 lb), are attached to the tip platform. To off-load this additional weight, a 5-m-long tensioned steel cable was added. The cable extends upward from the center of the tip platform and is tensioned to approximately the weight of the platform plus equipment. Use of the tip cable increases the fatigue life of the truss members.

Data Acquisition

Figure 3 provides a summary of the data acquisition process. With practical engineering structures, the performance of modal identification techniques can be affected significantly by the method of excitation used and by the type of data functions selected for analysis. The decision was made in this project to use multiple-input random excitation and frequency response functions (FRF's)—common selections in laboratory modal tests (ref. 18). Generation of FRF's using multiple-input random excitation minimizes the influence of nonlinearities compared with other excitation methods (ref. 19). Uncorrelated random signals were applied for a period of 15 min *simultaneously* to each of three shakers located at bay 9 of Mini-Mast. Displacement responses were recorded along the full length of the truss, together with the applied excitation forces. These time histories were processed into FRF's with 2560 spectral lines from 0 to 80 Hz by using the commercially available Test Data Analysis Software (TDAS, ref. 20). Fifty ensemble averages were made, applying standard Hanning window and overlap processing techniques. Inverse fast Fourier transformation (FFT^{-1}) was used to obtain impulse response functions (IRF's) for ERA. Digital filtering was applied in some cases to concentrate the ERA analysis in a selected frequency band of interest.

In aerospace applications, the overall objective of modal testing is usually to provide experimental results for updating finite-element models, as shown at the bottom of figure 3. Although analytical predictions are based on a mathematical representation, the predictions can provide valuable insight concerning unmeasured degrees of freedom. In this test, no sensors were available on the individual truss members, the platforms, or the tip cable. The 102 measurements made along the full length of the truss were thought to be adequate for identifying the global bending and torsional modes of the structure. Because of the highly coupled nature of many of the modes, however, this situation was found to be only marginally true.

Figure 4 shows the orientation of the shakers and 3 of the 51[1] noncontacting displacement sensors. The shakers are located circumferentially around the truss at bay 9, attached with flexible stings to the "corner body" joint at each vertex. Each excitation force is measured with a force gauge mounted to the structure. The sensors are similarly located at each vertex of the truss, from bay 2 through bay 18, with the measurement axis aligned perpendicular to the corresponding face. The sensors are eddy-current proximity devices having a resolution of approximately 0.0001 in. They were selected rather than traditional accelerometers because they are capable of measuring both static and dynamic displacements. Static deflections were measured in a series of preliminary tests.

The three shakers at bay 9 were used as disturbance sources in the CSI experiments. Their locations were selected primarily to excite the low-frequency modes below 10 Hz. Although other shaker locations could excite higher-frequency modes better, no others were used in this work. Relocation of shakers would have interfered unacceptably with ongoing CSI experiments.

[1] Figure 3 states that 102 displacement measurements were made. The 102 x, y measurements were derived using 51 skewed sensors, assuming 3 significant degrees of freedom at each bay (global x, y bending and torsion).

NASTRAN Analytical Predictions

Figure 5 shows representative modes predicted by the pretest[2] NASTRAN finite-element model. Two views of each mode are provided. The left-hand views show the full, spatially complete mode shapes. These shapes contain information at 618 grid points (3481 dynamic degrees of freedom) (ref. 21). As mentioned, it was impossible to fully measure such shapes because displacement sensors were located only at the primary joints of the truss. Based on the 51 available sensors, these predicted mode shapes would appear as shown in the right-hand views.[3] Obviously, significant differences occur in the appearance of many of the modes involving the truss members, platforms, or tip cable. Although undesirable, such ambiguities are not uncommon in modal tests of complex structures. It is often impossible, or impractical, to fully measure all structural components, such as the individual truss members of Mini-Mast.

A total of 153 modes are predicted to occur below 100 Hz. Below 80 Hz—the upper range of the experimental FRF's—the number reduces to 140. As mentioned earlier, 108 of these modes result from the bending of the 54 diagonal members of the truss. It is important to note that these "local modes" due to diagonal-member bending do *not* consist simply of only one member vibrating in each mode. In general, all 54 members participate in each mode, with complex combinations of relative motion. Many of the other modes of Mini-Mast also contain significant movement of the diagonal truss members.

The degree of similarity between mode shapes can be quantified using the modal assurance criterion (MAC, ref. 22). Based on the 102 measurement degrees of freedom, MAC values between each of the 153 NASTRAN modes and themselves were computed. The results are shown in figure 6 using a simple graphical format. Each row and column in the figure represents one NASTRAN mode shape. The value of MAC is proportional to the size of the darkened area at the intersection of the corresponding row and column. High correlation is thus expressed by large black blocks. Ideally, all the off-diagonal terms of this matrix should be small. Clearly this

[2] Posttest model adjustments have not yet been made.

[3] The mode shapes shown in the right-hand views are subsets of those shown in the left-hand views, neglecting all nodes except the 51 locations measured in the test. The displayed amplitude of motion has been normalized in both views based on the largest component among included degrees of freedom.

is not true for Mini-Mast because of the incomplete observability of modes using the 51 sensors. For example, the higher torsional modes appear as clusters of three to five modes with similar shapes, such as modes 124, 125, and 127 shown in figure 5.[4] Also, many of the local modes are similar in shape at the measurement locations. Because of this similarity of observed mode shapes, it will be difficult to uniquely associate each identified mode with a single corresponding NASTRAN mode. These difficulties will be compounded by additional identification uncertainties encountered with experimental data.

These results provide a good illustration of the difficulty of selecting a minimum number of sensors for modal identification of complex structures. The selection and placement of a minimum number of sensors are key technical challenges for future on-orbit identification experiments, such as with Space Station *Freedom* (ref. 8).

Identification Results: Theory Versus Practice

The first objective of this paper is to examine the practical performance characteristics of ERA. With linear analytical data, little effort is typically required to generate accurate, consistent results using ERA. With experimental data, however, modal identification is often not so straightforward. A primary goal of current research is to understand better the various ways in which system identification techniques behave differently in theory than in practice. Such understanding is required in order to

1. Prepare confidently for on-orbit identification experiments
2. Evaluate identification techniques using simulated data of adequate complexity
3. Improve, in a practical sense, the formulation of existing techniques

With this objective in mind, the decision was made in this project to perform each ERA analysis using two data sets. The first set is simply the collection of 306 experimental FRF's. The second set consists of corresponding analytical FRF's calculated using the NASTRAN model. The analytical FRF's were formed by a simple linear summation of all 153 modal components below 100 Hz. Each mode was assigned a damping factor of 1 percent, except modes 1, 2, and 3. Based on higher experimental estimates for these modes, a damping factor of 2 percent was assigned to modes 1 and 2 and a value of 1.5 percent to mode 3. No noise was added to the analytical data in order to generate ideal data.

Comparison of Analytical and Experimental Data

Figure 7 shows a typical comparison of analytical and experimental Mini-Mast data. FRF's are shown on the left-hand side with corresponding IRF's, computed using an inverse fast Fourier transform, shown on the right-hand side. The ERA method uses a small portion of the IRF's for modal identification. ERA is a time-domain identification technique which decomposes free-decay data into constituent modal components. ERA can also be used with other data types, including actual free-decay measurements, correlation functions, and random-decrement signatures.

Overall, the two sets of data in figure 7 look similar, particularly up to 30 Hz and up to 20 sec, respectively. The primary difference observed with the FRF's is a small amount of noise on the experimental function. Considerable averaging was used in generating the experimental data, resulting in a low noise level—estimated to be on the order of 0.1 percent. With the IRF's, the primary difference is the large "tail" occurring at the end of the experimental function. This anomaly is due to nonlinear distortion of the FRF near the frequency of the two first-bending modes (both at approximately 0.8 Hz), combined with the periodic nature of the FFT.[5]

Although these analytical and experimental data look similar, modal identification results can be affected significantly in practice by only small amounts of distortion. This is particularly true in regions of high modal density. For Mini-Mast, many of the peaks visible in the FRF's correspond to more than one mode. Modes are so closely spaced in frequency that they are not individually distinguishable in the FRF data. In theory, ERA will identify repeated eigenvalues of multiplicity m (having m independent eigenvectors) when data for at least m independent sets of inputs and outputs are included in the analysis. In practice, however, these theoretical performance characteristics are affected by data distortion.

[4] While mode 124 principally involves midplate motion and mode 125 is clearly a cable mode, only the small amount of truss torsion in these modes is detectable at the measurement degrees of freedom.

[5] The tail is *not* caused by beating of the two closely spaced modes, as might be suspected. Based on other, actual free-decay measurements, both first-bending modes have decayed to essentially zero amplitude after 25 sec. Consequently, large-amplitude beating of the two waveforms cannot occur beyond this point in time.

Selection of Analysis Parameters

Typically at the beginning of each modal survey test, information concerning the entire frequency range of interest is sought. These initial analyses are always a compromise between accuracy and speed, particularly with large data sets. With ERA, the most straightforward way of processing large data sets is to include all data simultaneously in a single analysis. The advantages of this approach are that a global, least-squares estimate is obtained using all available data and that data handling is minimized. Disadvantages include the fact that better identification results are possible for specific characteristics using alternative processing techniques (discussed later in the paper) and that computer time requirements for a single large job are usually greater than for a series of smaller jobs.

The parameters selected for an initial analysis of Mini-Mast data, using all 306 IRF's simultaneously, are shown in table I (job code I). The first two parameters, NCH and NRH, establish the overall size of the ERA data matrices. NCH is the total number of columns in the matrices, and NRH is the corresponding number of rows. Theoretically, both of these dimensions must be at least equal to twice the number of modes (each mode is represented by second-order dynamics). To achieve noise filtering due to singular-value truncation (ref. 23), matrix dimensions two to three times larger than the theoretical minimum are recommended with experimental data. A value of 300 for NCH was thus selected based on a preliminary estimate of 50–75 identifiable modes in the 0- to 80-Hz bandwidth. Because there are many more response measurements (102) than excitation points (3), a general guideline is to select NRH to be several times larger than NCH.[6] In this case, NRH was chosen to be three times larger than NCH, rounding up to the nearest multiple of NST (102).[7]

[6] Theoretically, these decisions concerning the size of the ERA data matrices are related to the observability and controllability indices (ref. 24). Observability and controllability are affected by the degree of linear independence of eigenvector components at the response and excitation points, and by the multiplicity and proximity of eigenvalues. In general, it is difficult to estimate these characteristics with complex experimental data sets. In most instances it is preferable to select data-matrix dimensions which are too large rather than too small.

[7] Rounding up of NCH and NRH based on the values of NIC and NST, KEYDTA, respectively, permits EMAC (extended modal amplitude coherence) to be computed for each input-output pair. EMAC is an "accuracy indicator" used to assess the consistency of the identified modal parameters. Its calculation does not affect the identification results. All values of NCH and NRH are permissible, except that EMAC may not be computed for every input-output pair.

The next two analysis parameters, NIC and NST, are simply the number of initial conditions (inputs) and the number of response stations (outputs), respectively, included in the analysis. In this case, all 306 measurements are used. The fifth parameter, NSKIP, is the number of data samples skipped at the beginning of each time history. A value of zero is normally selected for initial analyses. For linear systems, the modal frequencies, damping factors, and mode shapes are independent of the value of NSKIP. A useful method to examine the validity of this assumption is to increment NSKIP over a range of values and compare identification results. Examples of such "sliding window" analyses using Mini-Mast data are presented later in the paper.

The parameter KEYDTA can be used to select a subset of the available response measurements to include in the ERA data matrices below row number NST. In most large data sets, there is significant redundancy in the information contained in the response measurements. That is, the number of response measurements is usually several times larger than the number of generalized coordinates (modes). KEYDTA permits a subset of the NST measurements to be used in the lower portion of the data matrices. The objective is to minimize the size of the data matrices without loss of identification accuracy and/or to enhance particular modes of interest. The selection of KEYDTA affects the relative observability of modes, but does *not* reduce the number of components calculated for each mode shape (i.e., each identified mode still contains a full set of NST components). For the initial Mini-Mast analysis, all measurements were included in KEYDTA.

As mentioned earlier, digital filtering can be used to concentrate the ERA analysis in a selected frequency band of interest. For the initial analysis, the complete bandwidth of the experimental FRF's was used (F1 = 0 Hz and F2 = 80 Hz). Digital filtering will efficiently reduce the number of modes in the analysis bandwidth—reducing data-matrix size requirements—and will affect the number of cycles of data for particular modes which are included in the data analysis time window. Filtering can also be used to control the dynamic range of frequencies over which any one particular analysis is made. As a general rule, the ratio of the highest-frequency mode to the lowest-frequency mode included in a single analysis should be limited to approximately 30:1.[8]

[8] This initial analysis used a frequency ratio of approximately 100:1. Values higher than 30:1 will unfavorably affect the lower-frequency modes. With Mini-Mast, the first five modes have much greater response amplitude than higher-frequency modes, permitting larger frequency ratios to be used successfully.

The next five parameters, N1, N2, N3, N2LAST, and N3LAST, are data-shift options available to the user. N1 is the number of time samples between the two ERA data matrices. The data matrices consist of blocks of NST rows and NIC columns containing data corresponding to the same time instance. Adjacent blocks contain data at successive time instances. N2 is the number of samples between block rows in the data matrices, and N3 is the number of samples between block columns. N2LAST and N3LAST are used in the calculation of EMAC (discussed later in the "Initial Identification Results" section). They are the number of time samples by which the last block-row and last block-column, respectively, are shifted in the data matrices. The default values for all five of these parameters were used in every analysis.

The parameter IORDTU selects the order of the ERA analysis, equal to the number of retained singular values.[9] For the initial analysis, 100 singular values were retained. This selection corresponds to 50 assumed modes. The next parameter, SF, specifies the data sampling frequency in samples per second. SF equals twice the frequency bandwidth, F2 – F1.

The final parameters listed in table I, NTIM and WINDOW, are not directly selected by the user. They are determined internally by ERA based on the values entered for other parameters. NTIM is the total number of time samples used in the analysis, and WINDOW is the corresponding time interval. These values are listed to illustrate the exceptionally small amount of data typically used by ERA (in this case, 0.788 sec; compare with fig. 7). Although the use of longer time windows increases the least-squares noise reduction aspects of the analysis, *it is often possible to use too much experimental data.* Nonlinearities encountered in experimental data can cause difficulties if the data window is made too large. A general recommendation with experimental data is to use the smallest possible window capable of generating the desired results.

Singular Values

One of the most difficult aspects of system identification is the problem of determining the correct order of the system (i.e., the number of modes) based on experimental measurements. In most identification techniques, including ERA, an "appropriate" order must first be selected, after which a set of identified parameters are calculated. The identified parameters are associated with the particular order that was selected, and other choices result in other estimates of modal parameters. With complex structures, the number of modes is often ambiguous. Uncertainties arise from many sources, including high modal density, nonlinearities, local modes, weakly excited modes, and large-amplitude modes occurring outside the analysis bandwidth.

A fundamental feature of ERA is its use of singular-value decomposition (ref. 25) to calculate a minimum-order realization based on the number of observable and controllable modes. This minimum order equals the rank of the ERA data matrix (ref. 13). In theory, the desired rank is simply the number of nonzero singular values. In practice, however, numerical round-off error, measurement noise, as well as the factors mentioned above concerning complex structures all combine to introduce uncertainty into the rank-determination question. Singular-value decomposition, nonetheless, remains the best tool available to obtain quantitative information concerning the rank structure of the system. In practice, rather than simply counting the number of nonzero singular values, *the significant characteristic is the amplitude distribution of the singular values.* With the singular values arranged in order of decreasing size, the amplitude distribution provides meaningful, quantitative information concerning the number of modes and their relative strengths.

Figure 8 shows the singular values calculated for Mini-Mast in the initial ERA analysis. Results obtained using the analytical and experimental data sets are superimposed. Somewhat surprisingly, the analytical result becomes zero after the 113th singular value, although 140 modes exist in the data analysis bandwidth. (Theoretically, 280 singular values should be nonzero.) The reason is a lack of observability or controllability for many of the modes. In particular, many of the local modes are similar in shape at the measurement degrees of freedom, observed earlier by the large black regions in figure 6. Additionally, many of these modes are also closely spaced in frequency and have identical assigned values of damping (1 percent). These factors result in a significant loss of observability. Further, a loss of controllability due to the orientation of the shakers occurs for clusters of closely spaced modes with torsional characteristics, such as modes 124, 125, and 127 shown in figure 5. Because the shakers are arranged circumferentially around the truss at the same location along its length, each of these modes is excited similarly with each shaker. Loss of controllability also occurs because many of the local modes have extremely small amplitudes at the shaker degrees

[9] By default, ERA automatically chooses an analysis order based on an assumed average signal-to-noise ratio of 50 dB. The parameter IORDTU is used to select a specific analysis order.

of freedom. These modes are excited to such low amplitudes that their presence is undetectable using the single-precision format in which data are stored for ERA.

With the experimental data, all 300 singular values are nonzero and their distribution is different. Up to the 40th singular value, the analytical and experimental results agree closely, substantiating the inherent accuracy of the finite-element model. Beyond the 40th singular value, however, the experimental results diverge above the analytical results, continuing downward at a reduced slope until approximately the 150th singular value. Although the trend between the 40th and 150th singular values is nearly linear, close examination reveals small variations of slope. Beyond the 150th singular value, the slope again becomes slightly less steep and is now constant, indicative of the measurement noise floor. The rapid downturn near the 280th singular value is characteristic of reaching the numerical rank of the data matrix, governed by the product of the size of the matrix and the numerical precision.

The interpretation of these results is as follows. The close agreement between the analytical and experimental curves up to the 40th singular value indicates the presence of approximately 20 primary modes which are similar in both the analytical and experimental data sets. Separation of the curves between the 40th and 150th singular values suggests that the observability of the local modes is considerably different in theory than in practice. In the analytical model, all members and joints of each type are assigned identical physical properties, causing a high degree of uniformity in the predicted mode shapes. In reality, however, each component of Mini-Mast is somewhat different. This inevitable variation results in local modes which are less similar in shape than predicted. Also, the damping values for these modes are all somewhat different in practice, compared with identical values used in the analytical data. Encountering the measurement noise floor at approximately the 150th singular value corresponds to a relative amplitude decrease from the first singular value of 70 dB. This result correlates with the earlier observation of a low measurement noise level—on the order of 0.1 percent.

Initial Identification Results

For an initial analysis, singular-value truncation at 100 singular values (50 assumed modes) was selected. The objective of singular-value truncation is to retain all significant principal components (ref. 26) of the measurement signals while eliminating smaller components associated with noise. If a significant drop in amplitude occurs between any two consecutive singular values,[10] truncation should be made at this point. In data sets with a clearly defined number of modes, such drops do occur. In many applications with complex structures, however, there are no significant drops whatsoever. With Mini-Mast, there were no significant singular-value drops in most analyses.

Results from this initial ERA analysis are summarized in table II for the analytical data set and in table III for the experimental data set. Identified damped natural frequencies and damping factors, as well as several types of "accuracy indicators," are shown. Accuracy indicators are used in ERA to assess the quality of the identified parameters from a modal-identification point of view. They are derived using the identification results only (i.e., without comparison with analytical predictions) and have been developed and improved based on practical applications.

The primary accuracy indicator used with ERA is known as the consistent-mode indicator (CMI). CMI is calculated using two other parameters, the extended modal amplitude coherence (EMAC) and weighted modal phase collinearity (MPC). EMAC measures the consistency of mode-shape components identified using data from the beginning of each time history with corresponding components identified using data extended past the primary data-analysis window. A value of EMAC is computed for every input-output pair included in the analysis. As a summary of the results, an average EMAC value is then calculated for each mode. The EMAC results shown in tables II and III were obtained in this manner. Weighted modal phase collinearity, also shown in tables II and III, measures the extent of phase angle deviations from the ideal $\pm 90^\circ$ monophase behavior of classical normal modes. A value of 100 percent indicates exact monophase behavior. Although all structural modes are complex, in general, the normal-mode assumption is well approximated in many instances. With weighted MPC, the magnitude of each mode-shape component is also considered. That is, the significance of each phase result is weighted by the corresponding magnitude. This approach deemphasizes mode-shape components with small amplitudes, which typically possess a disproportionate amount of phase-angle scatter due to noise.

The usefulness of EMAC and MPC has been demonstrated in many previous applications. To provide a simpler method of highlighting those modes identified with high confidence by both EMAC and

[10] A decrease in amplitude by a factor of 20 or more is considered to be a "significant drop."

MPC, a decision was made recently to combine these two parameters into a single new parameter. The result is CMI, computed as simply the product of the average EMAC and weighted MPC values.

CMI values range from 0 to 100 percent. High values of CMI reliably indicate consistency in the identified structural modal parameters, both in terms of eigenvalues and eigenvectors. Modes identified with high confidence in the initial analysis are highlighted in tables II and III using asterisks and plus signs. An asterisk indicates CMI values in the range of 95–100 percent, a dagger in the range of 90–95 percent, and a double dagger in the range of 80–90 percent. Using these categories, a summary of the number of modes in each group is also provided. For the analytical data, 29 of the 45 identified modes have a CMI of at least 80 percent. CMI values in this range indicate that the identified eigenvalues and eigenvectors are highly consistent with the characteristics of valid structural modes. All 15 of the global bending and torsion modes—labeled in the left-hand column—had a CMI of at least 98 percent, indicating essentially perfect identification.

In contrast, the CMI values obtained using experimental data, shown in table III, are considerably lower. Only 4 of the 45 identified modes have values of at least 80 percent. Also, CMI results for the global modes are considerably lower and more widely distributed than corresponding results obtained using analytical data, ranging from a maximum of 97.4 percent for mode 1B-Y to a minimum of 0.1 percent for mode 4B-X.[11] *These large differences in CMI values between the analytical and the experimental data highlight the increased difficulty of modal identification with experimental data, compared with ideal analytical data.* With experimental data, identification results cannot be taken verbatim; the confidence associated with each result must always be considered.

In the remainder of this paper, examples are given of many methods available with ERA for improving identification results compared with these initial findings. Using additional analyses, CMI values for the 15 global modes of Mini-Mast were increased from an average of 65 percent for the initial analysis to an average of 86 percent, with 11 modes having a CMI of at least 80 percent. A summary of these improved results is provided in the final section of the paper.

The last parameter listed in tables II and III is the modal strength ratio (MSR). MSR is computed by dividing the root-mean-square (rms) amplitude of each identified mode by the total rms value of the data included in the ERA data matrices. It provides a useful indication of the relative strength of each mode. Strongly excited modes, in general, display less identification scatter than weakly excited modes.[12] MSR results in tables II and III show that the first five modes of Mini-Mast have significantly greater amplitude than most of the other modes. The analytical and experimental trends are similar.

Reconstruction

The natural frequencies, damping factors, and scaled mode shapes identified by ERA can be used to calculate structural response due to arbitrary excitation. One method for examining the accuracy of identified modal parameters is to compare calculated impulse response functions (IRF's) with corresponding experimental IRF's. This process of generating IRF's using the set of identified modal parameters is referred to as reconstruction.

Figure 9 shows typical reconstruction results obtained in the initial analysis for both the analytical and the experimental data. In each case the time history used in the ERA analysis is plotted above the corresponding reconstructed time history. Although only 0.788 sec of data were used in the analysis, a longer time interval is shown. In general, the reconstruction results will always match the original data closely over the time window analyzed. *Matching data and reconstructions over later time intervals is a much more challenging requirement.*

With the analytical data (fig. 9(a)), the reconstructed time history closely matches the original data over the entire 4-sec interval. Comparison of time histories, however, provides information concerning only the few strongest modes in the data. A more detailed comparison is obtained by examining corresponding frequency spectra, plotted on the right-hand side using a logarithmic magnitude scale. These results (obtained from the time histories using an FFT) show excellent agreement in both amplitude and phase characteristics over the entire 0- to 80-Hz analysis bandwidth.[13] With the experimental data (fig. 9(b)), the reconstructed time history is also similar to the original data over the entire 4-sec interval; however, the disagreement between data and

[11] The mode labels shown in the left-hand column of table III were derived using MAC values presented later in figure 10(b), as well as engineering judgment.

[12] With nonlinearities, identification scatter may be greater for strongly excited modes than for weakly excited ones.

[13] Small differences between the data and the reconstruction occur between 15 and 20 Hz due to negative damping estimates for modes 9, 12, 15, and 21 (table II). Negative damping values are changed to zero when reconstructed IRF's are generated.

reconstruction grows with time. Close examination of the corresponding spectra reveals larger differences than found using the analytical data set.

Disagreements between data and reconstructions occur for two reasons. First, disagreements result from inaccurate estimation of modal parameters. This reason is obvious. A more common cause, however, is the nonideal nature of experimental data. In particular, distortions due to mechanical nonlinearities and other sources cause experimental measurements to deviate from the assumed linear form of the governing differential equations. Piecewise linear analysis over different time windows can be used to assess the extent of these deviations. Examples of such "sliding window" analyses are shown later in the paper.

Comparison of Identified and NASTRAN Mode Shapes

Correlation of experimental and predicted modal characteristics requires the comparison of mode shapes. Using the modal assurance criterion (MAC), each identified mode shape from the initial ERA analysis was compared against each of the 153 NASTRAN-predicted modes below 100 Hz. Figure 10(a) shows the results obtained using analytical data, and figure 10(b) the corresponding results using experimental data. As in figure 6, the MAC value for each pair of modes is proportional to the size of the darkened area at the intersection of the corresponding row and column. The calculations are based on the 102 measurement degrees of freedom.

With the analytical data (fig. 10(a)), high MAC values are obtained for most ERA-identified modes. The lower left corner shows unique correlation of the first five identified modes with the first five NASTRAN modes. In the local-mode cluster located near the center of the plot, several ERA-identified modes are similar in shape to numerous modes of the NASTRAN model. This behavior is due to the insufficient observability of modes shown earlier in figures 5 and 6. *This ambiguous result is not caused by ERA, but by the limitations of finite amounts of instrumentation.* In the upper right corner of figure 10(a), there is again a dominant diagonal trend. However, certain ERA-identified modes, particularly higher-frequency torsional modes, correlate with several NASTRAN modes. *This behavior is again due to reduced observability at the measurement degrees of freedom.* In summary, all identified global mode shapes using the analytical data correlate well with corresponding NASTRAN predictions.

With the experimental data (fig. 10(b)), similar trends are found. However, there are also several differences, including (1) three experimental mode shapes correlate with both first-bending modes of the NASTRAN model; (2) MAC values in the local-mode cluster are generally lower than in figure 10(a); (3) MAC values of upper-frequency bending and torsion modes are also considerably lower than in figure 10(a), particularly for Mode 5B-X; (4) one experimental mode shape at 60 Hz (attributed to electrical noise) disagrees with all NASTRAN-predicted mode shapes; and (5) some experimental modes at frequencies less than 80 Hz correlate with modes at frequencies higher than 80 Hz in the NASTRAN model.

Overview Analysis

All identification results discussed thus far were obtained in the initial ERA analysis using 50 assumed modes (100 retained singular values). Recall from figure 8 that the singular-value distribution with experimental data was essentially linear beyond the 40th singular value, providing no indication of the number of modes present.[14] This uncertainty is caused primarily by the large number of modes present, and to a lesser degree by measurement noise. For research purposes, identification results were also calculated in this project using a wide range of assumed number of modes, realized by increasing the number of retained singular values. The number of assumed modes was varied in the range from 1 to 56 modes for the analytical data and from 1 to 125 modes for the experimental data. The results from these analyses are discussed in this and the following section of the paper. They are referred to as the "Overview Analysis," differing from the initial analysis only in the parameter IORDTU (table I, job code O).

The natural frequencies identified as a function of the assumed number of modes are plotted in figures 11(a) and 11(b) for the analytical and experimental data sets, respectively. Each row in these figures corresponds to a separate ERA analysis.[15] The confidence of each result is expressed by the length of

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[14] The meaning of "the number of modes present" depends on the objectives of the analysis. For use in designing control systems, for example, it is correct to say that there are approximately 20 *significant* modes present. An adequate state-space model for control synthesis would be obtained by truncating at 40 singular values. For updating structural analyses, however, the objective of identification is to extract information for as many modes as possible from the experimental data. Modes that are "insignificant" based on the particular shaker positions used in the test may be important when disturbances occur elsewhere.

[15] Actually, the singular-value decomposition step of ERA is performed only once. Also, the lower-order **A** matrices used in the eigenanalysis are subsets of the larger **A** matrix computed for the highest order requested.

the vertical dashes. The length of each dash is drawn proportional to the corresponding CMI value for the mode, with 100 percent represented by the distance between minor tick marks on the Y-axis. High confidence is thus placed on modes appearing as solid lines in these figures, and lower confidence on modes appearing as dotted or dashed lines.

With the analytical data (fig. 11(a)), triangles have been drawn at the top of the plot to mark the known frequencies of the 140 NASTRAN modes between 0 and 80 Hz. Clearly, the ERA results align entirely with the triangles, indicating accurate frequency identification. All 15 global modes, including 2 modes each at the lines labeled 1B and 2B, are identified using approximately 25 assumed modes. No significant changes in the results for these modes are observed with increasing numbers of assumed modes. These results demonstrate the capability of ERA to identify all well-observed, linear modes over a wide range of frequency. Many of the local modes in the cluster between 15 and 20 Hz, however, are not identified. The reason is the lack of observability mentioned earlier rather than the inability of ERA to identify closely spaced but sufficiently observed modes.

In general, similar results are obtained in the corresponding identification of experimental data, shown in figure 11(b). However, a comparison with figure 11(a) discloses some differences. First, note that the results are extended up to 125 assumed modes, rather than to only 56 as with the analytical data (with the analytical data, there are only 113 nonzero singular values). At high numbers of assumed modes, some additional weak modes appear in the midfrequency range between 24 and 65 Hz. Also, a second cluster of local modes appears at frequencies between 70 and 80 Hz which is not predicted by the analysis. Mode 4B-X is identified at a much higher number of assumed modes (53) than with the analytical data (16). This difference is caused by the response of the mode being much smaller in the experimental data set than in the analytical data set. Another difference between figures 11(a) and 11(b) is that mode 5T is much closer in frequency to mode 5B-Y in the experimental results. Finally, typical of experimental data, an undamped 60-Hz mode due to electrical line sources is identified. In contrast to the analytical data, approximately 55 assumed modes are necessary to identify all 15 global modes, a 120-percent increase caused primarily by the increased difficulty of identifying mode 4B-X.

One interesting characteristic occurs in figure 11(a) for modes 2T and 4T. Notice that at low numbers of assumed modes a single frequency is identified which becomes two frequencies at higher numbers of assumed modes. In both cases, each pair of modes have shapes which are nearly identical at the measurement degrees of freedom (recall that the "true" modal parameters are known with the analytical data). Because of this similarity of shapes, ERA identifies a frequency midway between the two correct frequencies at low numbers of assumed modes. When a sufficient number of modes is allowed, ERA determines that there are, in fact, two separate modes. At this point, the lengths of the dashes also increase considerably, illustrating the capability of CMI to detect such inaccuracies.

Expanded Frequency Plots

To examine in more detail the differences between the analytical and the experimental results for the overview analysis, expanded views of figure 11 in selected frequency intervals are presented in figures 12 through 14. In addition to the identified frequencies, corresponding results for damping and weighted modal phase collinearity (MPC) are also shown. These additional results are plotted using small numerals corresponding to each mode appearing in the frequency-results figure. The damping and weighted MPC values for the lowest-frequency mode (along each row) are plotted using a "1," for the second-lowest frequency (along the same row) using a "2," and so forth.

Identification results obtained near the frequency of the two first-bending modes, 1B-X and 1B-Y, are shown in figure 12. Notice that a greatly expanded frequency scale, covering an interval only 0.15 Hz wide, is used. With the analytical data (fig. 12(a)), two different regions are clearly recognized: (1) an area in which the results for the two modes converge and (2) an area of high stability beyond approximately 45 assumed modes. Completely different behavior is seen in figure 12(b) using the experimental data. Three modes, rather than two, are consistently identified—mostly with low confidence. Also, no clear, stable identification of either frequencies or damping factors occurs, and there are numerous estimates of negative damping. Weighted MPC results scatter randomly across the entire range of 0 to 100 percent. *These significant differences between the analytical and experimental results for modes 1B-X and 1B-Y are attributed to nonlinearities. In particular, large variations of damping factor with amplitude (greater than 400 percent) occur for these modes.* These nonlinearity findings will be presented later in figure 18.

Figure 13 provides similar results for the frequency interval from 4 to 7 Hz, including modes 1T, 2B-X, and 2B-Y. Using analytical data (fig. 13(a)), the results are again well behaved. All parameters rapidly converge with high stability to their proper, known values. On the other hand, results obtained using experimental data (fig. 13(b)) show three different regions as a function of the assumed number of modes. After an initial region of convergence, an area of relative stability occurs up to approximately 75 assumed modes. This region is followed by a second area of instability, particularly in the damping results for modes 1T and 2B-X. Also, several spurious modes with low confidence are identified. These spurious modes are attributed to nonlinearities, similar to the third mode occurring in figure 12(b), but with lower intensity. Identification of the three global modes in this frequency range is optimum using a moderate singular-value truncation of approximately 60 assumed modes.

A third frequency interval, containing modes 5B-X, 5B-Y, and 5T, is presented in figure 14. With analytical data (fig. 14(a)), a fourth NASTRAN mode that is neglected in the following discussion exists near mode 5T. In general, a separation into two regions is again possible, and all results are stable and well behaved. The only unusual characteristic is a small, distinct shift in frequency for both fifth-bending modes at approximately 25 assumed modes. Notice, however, that the erroneous frequencies identified at lower numbers of assumed modes have smaller CMI values, indicated by dashed lines. The experimental results (fig. 14(b)) also display only two separate regions. At lower numbers of assumed modes, considerable identification scatter occurs, particularly in the MPC results. All results stabilize, however, at approximately 85 assumed modes. Also, rather than becoming less stable at higher values as occurred in figure 13(b), stability is maintained all the way up to 125 assumed modes. In general, the best identification results for these global modes are obtained using the full 125 assumed modes. Note that mode 5T in the experimental results is much closer in frequency to mode 5B-Y than predicted by NASTRAN.

In summary, the three examples shown in figures 12 through 14 illustrate the difficulty of selecting a single, optimum singular-value cutoff with complex experimental data, particularly with nonlinearities. Contrary to analytical data where accuracy improves uniformly with increasing numbers of assumed modes (when no noise or distortion is included), *accuracy with experimental data varies significantly from mode to mode with no single selection of singular-value cutoff being optimum for all modes.*

Improvement of Experimental Results

ERA is a tool that can be used in many different ways. With analytical or simple experimental data, good identification results are often obtained in a single analysis (provided a few basic guidelines are followed). Also, the results typically change only slightly with changes in the analysis parameters. With complex experimental data, however, significant differences can occur among different analyses. The extent of these changes depends on the specific characteristics of the data, including the degree of nonlinearity, the modal density, the level of damping, the magnitude of measurement and background noise, and the adequacy of shaker positions. The objective of modal identification is to determine the best possible estimates of structural modal parameters, recognizing that these data characteristics, and others, affect the accuracy of the results. To obtain best estimates, various analyses are normally required.

With Mini-Mast, considerable uncertainty was encountered in the initial and overview analyses with 5 of the 15 global modes, namely modes 1B-X, 1B-Y, 4B-X, 5B-X, and 5T. For modes 1B-X and 1B-Y, three modes were consistently identified rather than only two. This difficulty is attributed to nonlinearities. Based on other experimental data not shown, such as frequency responses generated using sinusoidal excitation, these fundamental bending modes are known to be appreciably nonlinear due to friction and backlash in the joints. With mode 4B-X, the identification results were weak and uncertain. The problem here is low response level attributed to a node line occurring near the shakers.[16] Mode 5B-X was identified with good confidence (CMI of 50 percent in the initial analysis); however, the MAC value between identified and NASTRAN shapes was only 26 percent—considerably lower than for the other modes. And the frequency of mode 5T is apparently identical to that of mode 5B-Y. Additional analyses are needed to substantiate this result. The remainder of the paper discusses improved identification results for these five global modes. No additional results using analytical data will be presented.

Many methods can be used with ERA to improve identification results for complex data, including

1. Digital filtering
2. Selection of emphasized data

[16] As mentioned earlier, only a single set of shaker positions was available due to ongoing CSI experiments.

3. Multiple-input versus single-input analysis
4. Sliding time-window analysis

The first method, digital filtering, is a generic capability used in conjunction with the others. It is implemented by extracting a selected frequency band of interest from the frequency response functions and then applying an inverse Fourier transform to this data only. Examples of each of the last three methods are presented individually in the following sections.

Selection of Emphasized Data

Figure 15(a) provides an expanded view of frequency, damping, and weighted MPC near mode 4B-X from the overview analysis, which used data for all three shakers and included all 102 response measurements in KEYDTA. Considerable scatter is evident in these results, particularly in the damping values. Furthermore, the minimum number of assumed modes at which each mode is identified is relatively high. This minimum number of assumed modes provides an indication of the relative strength of each mode. In this result, mode 3T is first identified at 20 assumed modes, mode 4B-X at 53 assumed modes, and mode 4B-Y at 16 assumed modes.

Improvement can be achieved by emphasizing measurements with the largest vibration amplitudes in the modes of interest. This approach provides an increased signal-to-noise ratio for the target modes, perhaps at the expense of modes not included in the target set. Results obtained by emphasizing data from sensors at bays 5, 6, 12, and 16 are shown in figure 15(b) (table I, job code E). Improvement of all three modes is clearly indicated. Initial identification of all modes occurs at smaller numbers of assumed modes (3T at 10, 4B-X at 37, and 4B-Y at 7), and CMI values, indicated by the length of the dashes, are uniformly higher. Also all damping factors are much more stable. Based on these results, an improved damping estimate for mode 4B-X of 2.0 percent was obtained.

In summary, a significant improvement can be achieved by emphasizing measurements corresponding to larger vibration amplitudes. This procedure requires estimates of the mode shapes for the modes of interest. Mode-shape estimates were obtained in this example using the initial identification results.

Multiple-Input Versus Single-Input Analysis

In theory, ERA will identify repeated eigenvalues of multiplicity m (having m independent eigenvectors) when data for at least m independent sets of inputs and outputs are included in the analysis. Also, closely spaced modes are identified better using multiple inputs and multiple outputs (MIMO). In practice, however, data inconsistencies can cause difficulties for MIMO analyses. For example, when data acquired in different tests of the same structure are analyzed simultaneously, slight changes in eigenvalues or eigenvectors between data sets can cause additional modes to be identified. Such inconsistencies are not uncommon in laboratory tests because of nonlinearities or small variations of physical properties with time.

To assess the extent of such inconsistencies with the Mini-Mast experimental data, ERA analyses were performed using various combinations of shakers.[17] Figure 16 shows typical results obtained in the frequency interval from 66 to 68 Hz. Using only a single shaker (fig. 16(a)), two modes are clearly identified. Although the frequency and damping results are stable, weighted MPC values for the higher-frequency mode (labeled 5B-Y) are only about 70 percent. Also, when MAC values are computed between the identified and the NASTRAN mode shapes (not shown), the higher-frequency mode correlates approximately 50 percent with NASTRAN mode 5B-Y *and* approximately 25 percent with NASTRAN mode 5T. The explanation for this behavior is that the identified mode labeled "5B-Y" is, in fact, a linear combination of the two actual modes. The two modes are so closely spaced in frequency that a single-input analysis is unable to separate them.

Figure 16(b) shows the improved results obtained using all three shakers simultaneously. Two modes at essentially the same frequency (within 0.001 Hz) are now identified. All results, including the weighted MPC values, stabilize at approximately 45 assumed modes. Also, MAC values computed with the NASTRAN shapes now show unique correlation. In particular, the modes labeled 5B-Y and 5T each correlate approximately 60 percent with their corresponding NASTRAN predictions. Moreover, the cross-correlation of shapes between the two pairs is now approximately zero, indicating linear independence. These identification results shown in figure 16(b) were obtained using three shakers and emphasized data (table I, job code E). Similar results

[17] Clarification of this terminology is required. All data analyzed in this project were obtained in a single test conducted using all three shakers. An "ERA analysis performed using various shakers" refers to the process of analyzing simultaneously a subset of these data corresponding to various shakers. The expressions "single shaker" and "multiple shaker" are used synonymously with "single input" and "multiple input," respectively.

were obtained in the overview analysis (fig. 14(b)) except much higher numbers of assumed modes were required.

In summary, multiple-input analysis provides a clear advantage over single-input analysis for identification of modes 5B-Y and 5T. Nonlinearities and data inconsistencies are apparently small enough for these modes that identification performance characteristics observed in practice are similar to those predicted by theory.

Sliding Time-Window Analysis

The objective of modal identification is to determine best estimates for modal parameters. Most identification techniques, including ERA, are based on the assumption of linear structural behavior. However, all mechanical structures are nonlinear to some degree. Nonlinearities can significantly affect modal identification results, particularly with closely spaced modes. Random excitation with averaging was used in the Mini-Mast tests to minimize these effects. Although this approach generates the best linear estimates of FRF's (ref. 19), residual nonlinear effects remain.

A sliding time-window analysis was performed using ERA to characterize these residual effects. The method is illustrated in figure 17 with a typical Mini-Mast IRF. Beginning at the data window labeled "1," an initial ERA analysis was performed. Then, using a time shift of 6 data samples (0.3 sec), the window was moved down the function and a second ERA analysis performed. This process was repeated 50 times for a total shift of 15 sec. With linear data, the identified modal parameters remain constant among these separate analyses. Nonlinearities or other data distortions, however, cause changes to occur. The objective is to determine the nature and size of these changes.

To concentrate the analyses on the low-frequency global modes, digital filtering was applied from 0 to 10 Hz (table I, job code W). Frequency, damping, and MPC results obtained for modes 1B-X and 1B-Y as a function of time shift are discussed in this section. Also shown are representative MAC values calculated between the identified mode shapes and each of the first five NASTRAN-predicted mode shapes.

Identification results obtained using data for all three shakers simultaneously are shown in figure 18(a). As in the overview analysis (fig. 12(b)), three modes are consistently found. Based on CMI (the height of the dashes), the confidence in these results varies randomly, and the frequencies scatter throughout the entire 0.8- to 0.9-Hz interval. The damping as well as the MPC values also show large scatter, including negative damping. A typical MAC value is plotted in the right-hand figure. NASTRAN mode 1 (1B-X) is clearly identified in this result while NASTRAN mode 2 (1B-Y) is identified twice, by experimental modes 1 and 3. Additionally, however, these MAC values vary considerably among the 50 separate analyses that were performed (not shown). At other time shifts, completely different mixtures of correlation with the two NASTRAN modes were obtained for the three identified modes. MAC results for Mode 1T and both second-bending modes (NASTRAN modes 3, 4, and 5) are high and correlate uniquely with NASTRAN predictions in all cases.

Next, results obtained using all possible combinations of two shakers are shown in figures 18(b) through 18(d). In each case only two modes are identified and frequencies are relatively stable. *A strong nonlinear characteristic is also clearly observed in the damping results.* The identified damping factors increase uniformly from approximately 1 percent at zero time shift to approximately 4 percent at a time shift of 15 sec. Overall, these frequency and damping results obtained using two shakers are significantly more stable and understandable than those shown in figure 18(a) using three shakers. MPC results, however, continue to have considerable scatter. Also, MAC results show a consistent pattern of modal coupling with the first two NASTRAN modes. MAC results again vary among the 50 different analyses; however, the variation is smaller than with three shakers. In general, each of the two identified mode shapes obtained in these analyses is a linear combination of the first two NASTRAN mode shapes, with approximately 50 percent similarity to each mode. This inadequate uncoupling of identified mode shapes is attributed to the effects of strong damping nonlinearity, combined with the small separation of natural frequencies. MAC results for modes 3 through 5 are again high and correlate uniquely with corresponding NASTRAN modes.

In the final set of results (figs. 18(e) through 18(g)), data for each shaker are used individually. As with two shakers, the identified frequencies are fairly stable while damping factors again show an increasing nonlinear characteristic. Using shaker 1, the identified mode is again a coupled one. However, consistently high and unique MAC values for modes 1B-X and 1B-Y are found when shakers 2 and 3 are used alone. Also, these MAC results for shakers 2 and 3 vary only slightly among the 50 separate analyses. Decreasing, deterministic patterns are observed in the MPC results for all three cases. These trends are normal and are attributed to the decreasing signal-to-noise ratios of the modes as a function of time (i.e., the modal amplitudes decrease uniformly versus time while the noise remains constant).

Although the MAC results obtained using single shakers are significantly less coupled than using two shakers, the amount of scatter observed in the corresponding frequency and damping results is somewhat larger. In order to maintain a fixed window size among the separate cases, the parameter NCH was varied (table I). The increased scatter observed in the results with a single shaker is attributed to the reduced size of the data matrices used in these analyses. Overall, however, the identification results shown in figures 18(f) and 18(g) are considered to be the most accurate among the seven cases presented.

In summary, single-input data analyses provided improved results for the first two modes of Mini-Mast compared with multiple-input analyses. When data for all three shakers were used simultaneously, a spurious (computational) third mode was consistently identified. Using data for only two shakers generally eliminated the spurious mode, but identified mode shapes were highly coupled. The largest MAC values with NASTRAN predictions were consistently obtained using shakers 2 and 3 individually to identify modes 1B-Y and 1B-X, respectively.

Summary of Identification Results

Final identification results for all global modes of Mini-Mast below 80 Hz are listed in table IV, together with their best CMI values. For comparison with the NASTRAN model, the predicted frequencies and mode shape correlation based on MAC are also shown. Because of nonlinearities, frequency ranges for the first two modes and damping-factor ranges for the first five modes are given. Beyond 10 Hz, all modes can be assumed to be linear.

Each of the 15 global modes, except mode 4B-X, was identified with good confidence based on CMI. Corresponding MAC values are also relatively high, although a trend of decreasing correlation with increasing frequency is clearly evident. In general, natural frequencies and damping factors were all well identified, including those for modes 5B-Y and 5T, which have virtually identical frequencies. The only exception is the damping result for mode 4B-X, which has reduced confidence indicated by the low CMI value. Overall, the NASTRAN predictions agreed closely with the experimental results, the largest difference in frequency being 8.3 percent for mode 5T.

These final identification results were selected from among all analyses performed in this project. The selections correspond to the largest CMI values obtained in all analyses, unless the corresponding MAC value was unusually low.

Conclusions

The work discussed in this paper was conducted under a collaborative research agreement between the National Aeronautics and Space Administration (NASA) and the German Aerospace Research Establishment (DLR) in the area of dynamics and control of large space systems. The objective is to advance the state of the art in system identification and validation of structural analytical models. Validated analytical models of future large spacecraft are essential to assuring on-orbit performance and for designing and operating control systems.

As a result of the experiences encountered in this project, the following general conclusions are reached:

1. With complex structures, the selection and placement of a minimum number of sensors critically affects the verification of analytical models using identified modal parameters. Even with ideal identification results, correlation efforts are ambiguous without adequate sensors.

2. Differences observed in identification performance between theory and practice with Mini-Mast are attributed primarily to the large number of modes and nonlinearities rather than to measurement noise. For planning future on-orbit experiments, simulations which only use added noise may be inadequate.

3. The theoretical advantages of multiple-input data analysis with closely spaced modes are disrupted by nonlinearities or other data inconsistencies. Classical single-input analysis may offer better understanding in such situations.

4. A variety of different methods can be used to improve the accuracy of particular identified parameters, perhaps at the expense of others. The methods illustrated in this paper generated considerable improvements with Mini-Mast data; however, they require further development to become routine capabilities.

5. The consistent-mode indicator (CMI) developed in this project reliably indicates modes with classical normal-mode behavior, both in theory and in practice. Values greater than 80 percent correspond to modes identified with high confidence.

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Table I. ERA Analysis Parameters

Parameter	Job code				
	I	O	S	E	W
NCH	300	300	210	300	[a]20, 40, 60
NRH	918	918	260	912	186
NIC	3	3	1	3	[b]1, 2, 3
NST	102	102	102	102	102
NSKIP	0	0	0	0	(c)
KEYDTA	All	All	Driving points	(d)	Driving points
F1, Hz	0	0	20	20	0
F2, Hz	80	80	80	80	10
N1, N2, N3	1	1	1	1	1
N2LAST, N3LAST	10	10	10	10	10
IORDTU	100	(e)	(f)	(f)	12
SF, Hz	160	160	120	120	20
NTIM	127	127	308	164	[g]53, 48
WINDOW, sec	0.788	0.788	2.558	1.358	2.600

I Initial data analysis

O Overview data analysis

S Single-input data analysis

E Selection of emphasized data analysis

W Sliding window data analysis

[a]Single input NCH = 20; 2 inputs NCH = 40; 3 inputs NCH = 60.

[b]NIC = 1 using data for shaker 1, 2, or 3;

NIC = 2 using data for shakers 1 + 2, 1 + 3, or 2 + 3;

NIC = 3 using data for all shakers.

[c]Varied between 0 and 300 in steps of 6.

[d]Measurements used in x-direction: 10, 11, 12, 14, 15, 31, 32, 44, 45;
Measurements used in y-direction: 61, 62, 63, 64, 65, 82, 84, 94, 95.

[e]Varied between 2 and 112 singular values (SV) for analytical data and between 2 and 250 SV for experimental data.

[f]Varied between 2 and 150 SV.

[g]NTIM = 53 using 1 or 2 inputs.

NTIM = 48 using all 3 inputs.

Table II. Initial Results Using Analytical Data

[*CMI range of 95–100 percent; 20 modes.
†CMI range of 90–95 percent; 3 modes.
‡CMI range of 85–90 percent; 6 modes.
All other entries: CMI range of 0–80 percent; 16 modes.]

Mode no.		Frequency, Hz	Damping factor, percent	CMI, percent	Average EMAC, percent	Weighted MPC, percent	MSR, percent
1B-X	1	*0.798	1.979	98.67	99.90	98.77	52.3
1B-Y	2	*.799	1.984	98.97	99.92	99.05	50.3
1T	3	*4.368	1.497	99.91	99.91	100.00	28.8
2B-X	4	*6.106	0.997	99.94	99.94	100.00	44.1
2B-Y	5	*6.159	.998	99.84	99.84	100.00	37.3
	6	†14.611	.895	94.54	95.94	98.54	0.8
	7	‡14.759	1.010	87.68	92.00	95.31	.4
	8	‡14.947	.959	84.57	91.08	92.85	.3
	9	15.521	–.693	1.67	78.29	2.13	.2
	10	15.716	.714	65.47	80.56	81.26	.6
	11	16.002	1.124	60.41	83.16	72.64	.8
	12	16.105	–1.143	27.27	84.03	32.45	.4
	13	16.172	1.272	77.81	88.15	88.28	1.0
	14	16.544	33.665	0	0	14.00	0
	15	16.745	–.104	52.49	87.50	59.99	.6
	16	*17.080	1.074	96.23	96.38	99.84	15.7
	17	‡17.192	.609	84.68	89.72	94.38	2.3
	18	17.727	2.499	78.16	89.86	86.98	2.8
	19	18.044	2.274	22.32	87.67	25.46	1.1
	20	18.380	.614	13.07	87.23	14.98	.4
	21	18.609	–.207	50.18	89.52	56.05	.3
	22	18.958	1.063	10.17	84.52	12.04	.1
	23	18.981	7.885	2.66	16.50	16.09	.1
	24	‡19.726	1.153	81.22	93.26	87.08	1.0
	25	‡19.746	1.041	85.79	96.60	88.81	.9
	26	†20.296	.953	92.50	92.61	99.87	1.2
2T	27	*21.569	1.001	98.75	98.75	100.00	16.3
	28	*23.472	.997	98.99	98.99	100.00	1.7
	29	*28.647	.999	98.86	98.86	100.00	.9
3B-X	30	*30.720	1.000	99.90	99.90	100.00	4.1
3B-Y	31	*32.062	1.000	99.75	99.75	100.00	3.9
	32	*37.328	1.005	95.57	95.58	100.00	.1
	33	38.313	.994	78.95	78.95	99.99	.1
3T	34	*39.010	.999	99.43	99.43	100.00	.8
4B-X	35	*42.220	1.000	99.76	99.76	100.00	1.7
4B-Y	36	*44.854	1.000	99.93	99.93	100.00	2.7
4T	37	*54.264	1.000	99.93	99.93	100.00	4.5
	38	*56.056	1.004	99.27	99.27	100.00	3.5
5B-X	39	*69.863	1.000	99.37	99.37	100.00	1.9
5B-Y	40	*70.180	.999	99.16	99.16	100.00	1.8
	41	†72.524	1.002	92.39	92.42	99.97	.1
5T	42	72.881	.999	98.90	98.90	100.00	.7
	43	‡75.194	1.001	88.43	88.44	99.99	.1
	44	78.902	20.774	0	0	83.87	.1
	45	79.985	.240	29.06	72.76	39.94	0

Table III. Initial Results Using Experimental Data

[*CMI range of 95–100 percent; 2 modes.
†CMI range of 90–95 percent; 1 mode.
‡CMI range of 85–90 percent; 1 mode.
All other entries: CMI range of 0–80 percent; 41 modes.]

Mode no.		Frequency, Hz	Damping factor, percent	CMI, percent	Average EMAC, percent	Weighted MPC, percent	MSR, percent
1B-X	1	0.827	3.880	78.10	89.08	87.67	27.9
	2	.862	−.950	3.09	97.16	3.18	80.8
1B-Y	3	*.867	1.243	97.44	97.86	99.57	73.7
	4	3.319	54.501	0	.01	32.65	.6
1T	5	*4.187	1.424	96.87	97.17	99.69	24.5
2B-X	6	†6.118	2.053	94.80	96.12	98.63	31.1
2B-Y	7	‡6.175	.993	88.18	94.45	93.36	35.0
	8	13.298	27.099	0	0	30.64	.6
	9	14.062	1.961	37.70	43.20	87.27	1.4
	10	15.325	2.157	13.86	35.73	38.79	1.5
	11	15.897	1.225	42.90	65.45	65.54	4.7
	12	16.361	1.320	26.27	65.23	40.28	4.3
	13	16.460	5.508	12.75	17.51	72.82	5.2
	14	16.682	2.413	10.00	42.04	23.78	1.0
	15	17.381	1.957	57.99	63.86	90.82	5.3
	16	18.905	18.756	0	0	46.59	.8
	17	19.607	1.793	19.92	26.04	76.50	1.5
	18	20.349	5.896	.38	.72	52.90	1.5
	19	20.636	.916	8.06	38.56	20.90	.8
	20	21.396	1.914	7.49	11.07	67.71	.8
	21	21.518	3.683	.42	17.00	2.45	.9
	22	22.372	58.914	0	0	9.82	.5
2T	23	22.891	.949	76.60	81.94	93.48	7.4
3B-X	24	31.137	1.780	60.83	68.66	88.59	3.4
3B-Y	25	32.410	1.935	64.32	76.82	83.72	3.1
	26	35.671	20.831	0	0	4.78	.4
3T	27	38.126	1.250	44.97	51.42	87.45	1.0
4B-X	28	40.172	4.872	.09	.18	49.61	.5
4B-Y	29	43.315	.701	56.57	67.72	83.54	1.7
	30	45.000	7.852	0	0	16.53	.3
	31	51.059	9.683	0	0	33.70	.5
4T	32	51.563	.705	73.77	91.52	80.60	6.1
	33	55.748	1.029	12.54	16.34	76.74	.4
	34	60.070	.102	29.34	41.06	71.46	.4
5B-X	35	66.886	.382	50.39	60.33	83.53	1.9
5T	36	67.079	.560	27.56	59.13	46.60	1.7
5B-Y	37	67.225	.393	72.91	86.87	83.94	2.5
	38	69.017	2.458	.27	.50	52.85	.9
	39	70.245	.649	3.08	52.66	5.84	.7
	40	70.792	.324	12.20	64.77	18.84	1.4
	41	71.119	1.272	1.22	7.67	15.90	.9
	42	71.233	.794	15.33	26.07	58.79	.8
	43	73.946	1.321	.69	1.75	39.70	.3
	44	76.647	2.304	.02	.19	12.55	.3
	45	79.558	.846	32.16	37.14	86.60	1.3

Table IV. Best Identification Results for Global Modes

Mode	NASTRAN	Test			MAC, percent
	Frequency, Hz	Frequency, Hz	Damping, percent	Best CMI, percent	
First X-bending	0.798	*0.856–0.870	*1.0–4.0	87.4	94.1
First Y-bending	.800	*0.862–0.868	*1.0–4.0	97.4	98.9
First torsion	4.37	4.19	*1.3–1.9	98.3	98.9
Second X-bending	6.11	6.11	*2.0–2.5	96.8	92.1
Second Y-bending	6.16	6.18	*1.1–1.4	97.1	97.1
Second torsion	21.57	22.89	0.82	92.5	92.2
Third X-bending	30.72	31.16	1.56	83.8	90.0
Third Y-bending	32.06	32.39	1.36	73.1	76.8
Third torsion	39.01	38.06	.83	79.7	85.9
Fourth X-bending	42.22	40.42	1.99	55.7	89.8
Fourth Y-bending	44.86	43.23	.43	82.2	74.4
Fourth torsion	54.27	51.55	.74	79.4	65.8
Fifth X-bending	69.87	66.92	.44	90.2	32.1
Fifth Y-bending	70.18	67.27	.33	88.0	56.4
Fifth torsion	72.87	67.27	.57	80.9	60.7

*Due to nonlinearity.

Figure 1. Artist's sketch of Mini-Mast.

Figure 2. Deployment process.

Figure 3. Data acquisition and analysis overview.

L-92-06

Figure 4. Shakers and displacement sensors at bay 9.

Figure 5. Representative NASTRAN mode shapes. Left-hand views show full mode shapes (618 nodes). Right-hand views at sensor locations only (51 nodes).

Figure 6. Self-correlation of NASTRAN mode shapes at test degrees of freedom (102 degrees of freedom).

Figure 7. Comparison of analytical and experimental data.

Figure 8. Singular values of initial analysis.

(a) Using analytical data.

(b) Using experimental data.

Figure 9. Typical reconstruction results for initial analysis.

(a) Using analytical data.

(b) Using experimental data.

Figure 10. Correlation of identified mode shapes with NASTRAN mode shapes (initial analysis).

(a) Using analytical data.

(b) Using experimental data.

Figure 11. Identified frequencies for overview analysis.

(a) Using analytical data.

(b) Using experimental data.

Figure 12. Expanded plot near mode 1B (overview analysis).

(a) Using analytical data.

(b) Using experimental data.

Figure 13. Expanded plot near modes 1T and 2B (overview analysis).

(a) Using analytical data.

(b) Using experimental data.

Figure 14. Expanded plot near modes 5B and 5T (overview analysis).

(a) Using analytical data.

(b) Using emphasized data.

Figure 15. Improvement of identification results using emphasized data.

(a) Using data for shaker 3 only.

(b) Using data for all three shakers.

Figure 16. Multiple-input versus single-input analysis.

Figure 17. Sliding window analysis.

(a) Using data for all three shakers.

(b) Using data for shakers 1 and 2.

Figure 18. Identification results using sliding window.

(c) Using data for shakers 1 and 3.

(d) Using data for shakers 2 and 3.

Figure 18. Continued.

(e) Using data for shaker 1 only.

(f) Using data for shaker 2 only.

(g) Using data for shaker 3 only.

Figure 18. Concluded.

(a) Overview analysis.

(a) Using analytical data.

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