WIDEBAND FINITE DIFFERENCE TIME DOMAIN IMPLEMENTATION OF SURFACE IMPEDANCE BOUNDARY CONDITIONS FOR GOOD CONDUCTORS

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SUMMARY

Surface impedance boundary conditions are employed to reduce the solution volume during the analysis of scattering from lossy dielectric objects. In a finite difference solution, they also can be utilized to avoid using small cells, made necessary by shorter wavelengths in conducting media, throughout the solution volume. This paper presents a one dimensional implementation for a surface impedance boundary condition for good conductors in the Finite Difference Time Domain (FDTD) technique.

In order to illustrate the FDTD surface impedance boundary condition, we considered a planar air-lossy dielectric interface as shown in Figure 1. The incident field has polarization TE_z, and is propagating in the +z direction. The one-dimensional FDTD grid is also shown in Figure 1. To begin our implementation for a FDTD surface impedance boundary condition, we assume that the lossy dielectric has permittivity ϵ, permeability μ, and conductivity σ; and that it is a good conductor. Thus, these constitutive parameters are real and satisfy the relation

$$\frac{\sigma}{\omega\epsilon} > 1 \qquad (1)$$

where ω is the radian frequency. We also assume that the radius of curvature is large compared to the maximum wavelength in the material and that the thickness of the material is large compared to the skin depth.

The first order (or Leontovich) impedance boundary condition in the frequency domain is [1]

$$E_x^t(\omega) = \eta_s(\omega) H_y^t(\omega) \qquad (2).$$

where $\eta_s(\omega)$ is the surface impedance of the conductor. A superscript "t" is used in equation (2) to indicate the boundary condition is applied to the total field in free space. The frequency domain surface impedance for good conductors is

$$\eta_s(\omega) = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = \sqrt{\frac{j\omega\mu}{\sigma}} \qquad (3).$$

Separating incident and scattered E_x terms in (2) yields

$$E_x^s(\omega) = \eta_s(\omega) H_y^t(\omega) - E_x^i(\omega) \qquad (4).$$

where the superscripts "s" and "i" are used to denote scattered and incident field components respectively. This equation is the required surface impedance boundary condition for scattered field components. The corresponding time domain expression involves a convolution integral and is given as

$$E_x^s(t) = \eta_s'(t) * H_y^t(t) - E_x^i(t) \qquad (5)$$

where the '*' denotes convolution and the time domain surface impedance impulse response is given by

$$\eta_s'(t) = F^{-1}\{\eta_s'(\omega)\} = F^{-1}\{\frac{\eta_s(\omega)}{j\omega}\} \qquad (6).$$

We have approximated this time domain impulse response by a series of exponentials to obtain an efficient recursive updating scheme requiring only four running sum variables similar to [2]. Figures 2 and 3 show reflection coefficient comparison versus frequency for conductivities of 10.0 S/m and 50.0 S/m. The FDTD reflection coefficients are compared against the standard analytical solution. Note that the agreement is quite good for the entire frequency band.

Overall, the surface impedance boundary condition implementation works well in eliminating the conductor volume from the solution space. This method has a distinct advantage over other possible implementations because the coefficients of the exponential approximation of the impulse response are independent of the conductivity of the scattering object and do not need to be reevaluated for different conductivities. Extensions of this surface impedance concept to two and three dimensions are currently under investigation.

REFERENCES

[1] T. B. A. Senior, "Impedance boundary conditions for imperfectly conducting surfaces," Appl. Sci. Res. B, vol. 8, pp. 418-436, 1960.

[2] R. J. Luebbers et al, "A Frequency-Dependent Finite-Difference Time-Domain Formulation for Dispersive Materials," IEEE Trans. Electromagn. Compat., vol. EMC-32, pp. 222-227, August 1990.
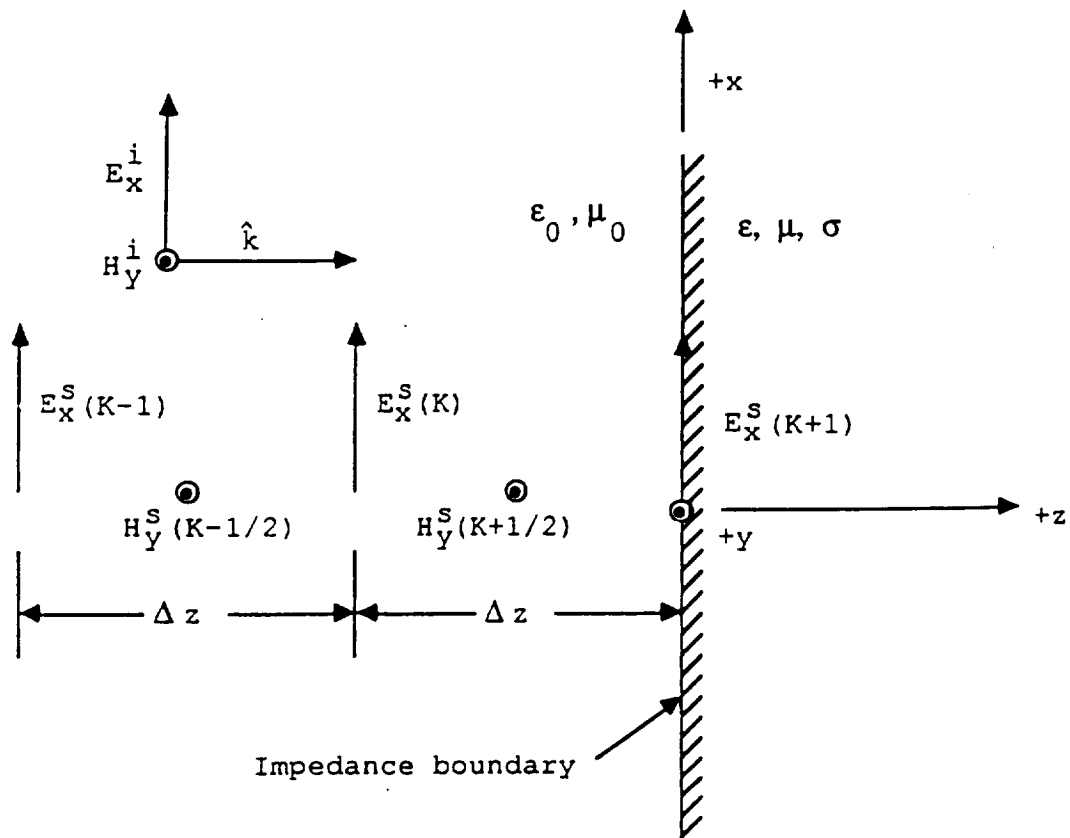
Figure 1. Problem geometry showing incident plane wave, planar air-lossy dielectric interface and one dimensional FDTD grid.

FDTD SURFACE IMPEDANCE COMPARISON
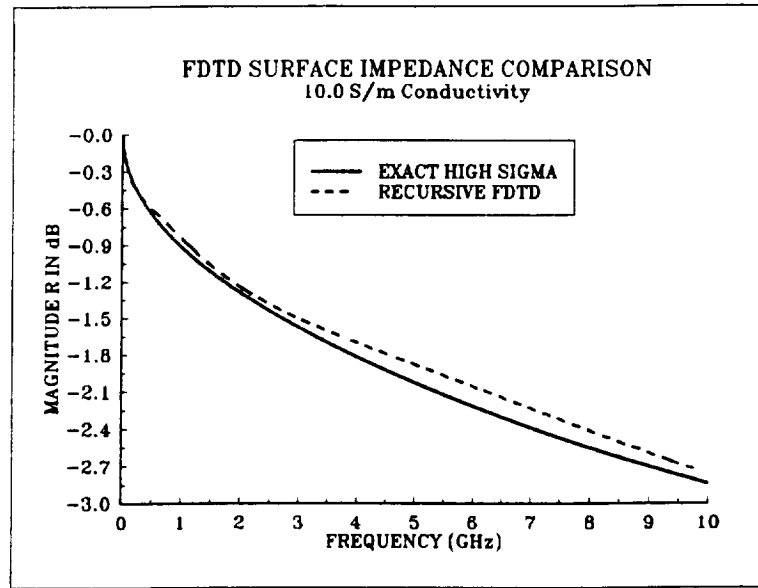10.0 S/m Conductivity

Figure 2. Reflection coefficient magnitude versus
frequency for normal incidence plane wave
calculated for 10.0 S/m conductivity using
FDTD surface impedance and analytical
evaluation of high conductivity approximation.

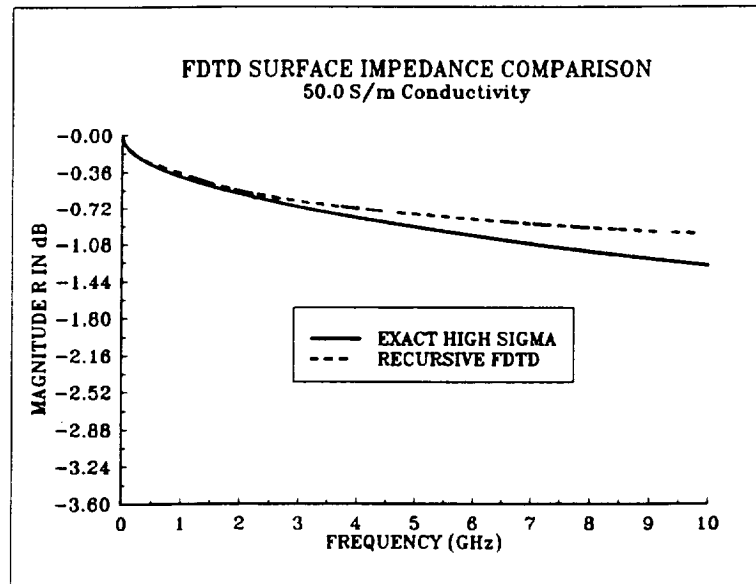**FDTD SURFACE IMPEDANCE COMPARISON**
50.0 S/m Conductivity

Figure 3. Reflection coefficient magnitude versus
frequency for normal incidence plane wave
calculated for 50.0 S/m conductivity using
FDTD surface impedance and analytical
evaluation of high conductivity approximation.