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Cosmological texture is incompatible with Planck-scale physics

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Abstract

Nambu–Goldstone modes are sensitive to the effects of physics at energies comparable to the scale of spontaneous symmetry breaking. We show that as a consequence of this the global texture proposal for structure formation requires rather severe assumptions about the nature of physics at the Planck scale.

It is an outstanding challenge in modern cosmology to explain the origin of large-scale structure. One hope is that some microphysical phenomena will give rise to macroscopic density perturbations that then grow under the influence of gravity. This is the essential idea behind both the inflation and cosmic string scenarios. Recently, Turok[1] proposed global texture[2] as a novel scenario for generating the primordial perturbations necessary for structure formation. Subsequent simulations of texture,[3] and of texture with cold dark matter[4] have yielded many promising results: with one free parameter the model can reproduce the galaxy-galaxy correlation function, lead to clustering on scales of $20h^{-1}$Mpc, and produce coherent structures and bulk motions on very large scales. A further property of texture is that they lead to a distinct signature on the microwave background,[5] which can, in principle be searched for in data now becoming available from satellite and terrestrial observations.[6] Finally, textures similar to the ones proposed in cosmological scenarios arise in condensed matter systems and can therefore be studied in the laboratory.[7]

Texture would provide a very promising alternative to conventional scenarios for generating the primordial density fluctuations if indeed they were as ubiquitous in particle physics models as claimed in earlier work.[1] In fact, texture arises in a variety of theories with nonabelian global symmetries that are spontaneously broken. However, as we shall see below, even an extremely small amount of explicit symmetry breaking will spoil the texture scenario. It is the purpose of this paper to point out this fact, and to show that having exact global symmetries involves very strong assumptions about physics at the Planck scale.

Given that so little is known about physics at extremely high energies, the only reasonable approach is to regard any particle physics model that does not include gravity as an effective field theory whose cutoff is the Planck scale M_{Pl}, and "Planck-scale" corrections should be expected. These high-mass-scale corrections in general have a very small effect upon the low-energy theory. However, Nambu–Goldstone bosons have the distinctive property that although they are massless (or very light in the case of pseudo-Nambu–Goldstone modes), they are not, properly speaking, a part of the low-energy theory because all couplings to light fields (and self couplings as well) are suppressed by a power of a large mass scale. The fact that a massless particle is part of the high-energy sector accounts for the interesting properties of Nambu–Goldstone particles, but also renders the particle susceptible to high-energy corrections in a way not true for normal particles.

This is why from the point of view of particle physics, there is little or no motivation for exact global symmetries. Chiral symmetry in QCD is only approximate; indeed this is why pions are massive. The Peccei-Quinn $U(1)$ symmetry used to solve the strong CP problem is explicitly broken by instanton effects giving mass to the axion.[8] Likewise, $B+L$ is broken by electroweak instantons, and $B-L$ may well be a gauged symmetry as in $SO(10)$ models and spontaneously broken at the GUT scale. Finally, and perhaps most importantly, there are many arguments suggesting that *all* global symmetries are violated at some level by gravity. For example, both wormholes and black holes can swallow global charge. "Virtual" black holes or wormholes,[9] which should, in principle, arise in a theory of quantum gravity, will lead to higher dimension operators which violate the global symmetry.

In this paper we explore the implications of including explicit global symmetry breaking terms for models of global texture. We find that the Nambu–Goldstone modes are very sensitive to assumptions made about the ultraviolet structure of the full theory. There are two possible assumptions one might make about the fate of global symmetries in a Universe that includes gravity. The *strong* assumption is that, despite all indications from low-energy, semi-classical gravitational physics (black holes, wormholes, etc.), it is possible to have exact global symmetries in the presence of gravity.

This is the assumption made in the standard texture scenario. The *weak* assumption is that the global symmetry is not a feature of the full theory. There are two possible realizations of the weak assumption. Either the global symmetry is approximate, in which case one must include the effects of higher-dimensional, non-renormalizable, symmetry-breaking operators, or, consistent with indications from semi-classical quantum gravity, the global symmetry is never even an approximate symmetry unless protected by gauge symmetries.

To illustrate these possibilities, consider a theory with a global $O(N)$ symmetry spontaneously broken to $O(N-1)$ by an N-vector. The theory is described by the scalar potential

$$V(\Phi) = \lambda \left(\Phi^a \Phi^a - f^2\right)^2. \tag{1}$$

Texture arises for $N = 4$, while $N = 2$ leads to global strings, and $N = 3$ to global monopoles.[10] For $N > 5$, there are no topological defects; however, even in this case gradient energy in the Nambu–Goldstone modes can lead to non-Gaussian density fluctuations, which may be interesting for structure formation.[11] Indeed, the cosmological consequences of each of these possibilities has been given serious consideration.

The theory described above should be regarded not as fundamental, but as an effective low-energy theory of the full theory including gravity. In order to arrive at Eq. (1) for the effective low-enery theory it is necessary to make the strong assumption as discussed above. If one makes this assumption, then the standard texture scenario follows.

If one makes the weak assumption, then one must include explicit symmetry breaking terms. If one assumes that gravity does not respect global symmetries at all, then renormalizable operators like

$${M_{Pl}}^2 \lambda_{ab} \Phi^a \Phi^b, \tag{2}$$

which explicitly break the global symmetry, should be included. These terms are expected, for instance, by the action of wormholes swallowing global charge. If virtual wormholes of size smaller than the Planck length are included, then we expect λ_{ab} to be of order unity. In this case it is wrong to consider an effective low-energy theory with a global symmetry.[12]

If one makes the assumption either that wormholes do not dominate the functional integral, or that the global charge is protected by gauge symmetries, then it may be possible to suppress the renormalizable operators. But even in this case higher dimension operators should be included.[13] An example would be a dimension-5 operator, which would add to $V(\Phi)$ terms like

$$\frac{\lambda_{abcde}}{M_{Pl}}\Phi^a\Phi^b\Phi^c\Phi^d\Phi^e. \tag{3}$$

Such terms explicitly break the global symmetry and lead to a mass for the pseudo-Nambu–Goldstone mode of $m^2 \propto \lambda f^3/M_{Pl}$. Of course the mass is suppressed by M_{Pl}, but we will show below that it still has a drastic effect on the texture scenario.

The implications of the strong and weak assumptions for texture are as follows: With the strong assumption, the texture scenario is unaffected. If one allows unsuppressed wormhole contributions, global symmetries (and hence texture) are a non-starter. If all effects of gravitational physics in the low-energy theory are contained in non-renormalizable terms, a more careful analysis is required. This is the possibility we explore for the rest of the paper. In this approach we are then required to include all higher dimension operators consistent with the gauge symmetries of the model and suppressed by appropriate powers of M_{Pl}.

Before turning to the implications these terms will have for the texture scenario, we review the essential features of texture. Texture occurs in any theory where the third homotopy group of the vacuum manifold π_3 is nontrivial. For texture discussed in cosmological scenarios the vacuum manifold is the three sphere. (The simplest model for texture is given by Eq. (1) with $N = 4$.) Texture corresponds to knots in the Higgs field that arise when the field winds around the three sphere. These knots are generally formed by misalignment of the field on scales larger than the horizon at the symmetry breaking phase transition because of the Kibble mechanism.[14] As the knots enter the horizon, they collapse at roughly the speed of light, giving rise to nearly spherical energy density perturbations. New knots are constantly coming into the horizon and collapsing, leading to a scale invariant spectrum of density perturbations. The magnitude of the perturbations is set by the scale of the symmetry breaking, and for scenarios of structure formation involving texture, the scale of symmetry breaking must be about 10^{16}GeV.

We now consider the effects of the higher dimension operators discussed above. These terms will break the symmetry explicitly, generating a complicated potential for the Nambu–Goldstone modes. In general, the vacuum manifold will be reduced to a point, though the potential will likely have many local minima. To see how this works, consider the theory discussed above with $N = 3$. Here, the vacuum manifold is the two sphere and the model, in two spatial dimensions, will have texture. (In three spatial dimensions, the model admits both global monopoles and texture, although the texture in this case is not spherically symmetric.[7]) We express the field as

$$\Phi = f\left(\sin\frac{\theta}{f}\,\cos\frac{\phi}{f},\ \sin\frac{\theta}{f}\,\sin\frac{\phi}{f},\ \cos\frac{\theta}{f}\right), \tag{4}$$

where θ and ϕ are the angular variables on the two-sphere which represent the Nambu–Goldstone modes of the problem.

The effect of the dimension 5 operators is to introduce 21 terms to the potential for the field which depend explicitly on θ and ϕ. (These are nothing more than the Y_{1m}, Y_{3m}, and Y_{5m} spherical harmonics.) The potential for representative choices of coefficients of the 21 terms is shown in Figs. 1 and 2. Fig. 1 shows the potential as a function of θ and ϕ, and Fig. 2 is a contour plot showing maxima (solid curves) and minima (dashed curves).[15] We see from Figs. 1 and 2 that the energy landscape is far from flat. Note that in general, the mass of the Nambu–Goldstone boson in this potential is roughly $f(f/M_{Pl})^{1/2}$.

So long as the mass of the Nambu–Goldstone mode is small compared to the Hubble parameter, the field will evolve essentially as in the original texture scenario. However, once the Compton wavelength of the Nambu–Goldstone mode enters the horizon, the field will begin to oscillate about the minimum (or rather the closest local minimum) of its potential. The field will then align itself on scales larger than the horizon and texture on all scales quickly disappear. For texture to be important for structure formation, they must persist at least until matter–radiation decoupling when $H \simeq 10^{-28}$eV.

The contribution of a dimension $4 + d$ operator to the Nambu–Goldstone boson mass is $m \sim f(f/M_{Pl})^{d/2}$. Given that the texture scenario requires $f \sim 10^{16}$ GeV, the requirement that $m \lesssim 10^{-28}$eV implies that $d \gtrsim 35$; i.e., we must be able to suppress all operators up to dimension 40! It is rather difficult to see how this might occur; even the use of additional gauge quantum numbers could not prevent the occurrence of dimension 6 operators which break a non-Abelian symmetry (although they could protect an Abelian symmetry). We note that if we consider dimension 5 operators, then the mass becomes dynamically important immediately after the phase transition: texture therefore never exists.

In conclusion, any model which depends on the dynamics of Nambu–Goldstone modes will be extremely sensitive to physics at very high energies. Texture can by no means be considered a robust prediction of unified theories. This, in our opinion, is most discouraging for the texture scenario. On the other hand, if texture is discovered, then this will have profound implications not only for theories of structure formation, but for Planck-scale physics.

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Figure Captions:

Figure 1: The potential energy surface for the $N = 3$ model. Due to the inclusion of dimension-5 operators, the potential energy depends on θ and ϕ. The height of the potential is roughly f^5/M_{Pl}. θ ranges from 0 to πf while ϕ ranges from 0 to $2\pi f$.

Figure 2: A contour plot corresponding to the potential surface in Fig. 1. Solid lines are maxima, while dashed lines are minima.

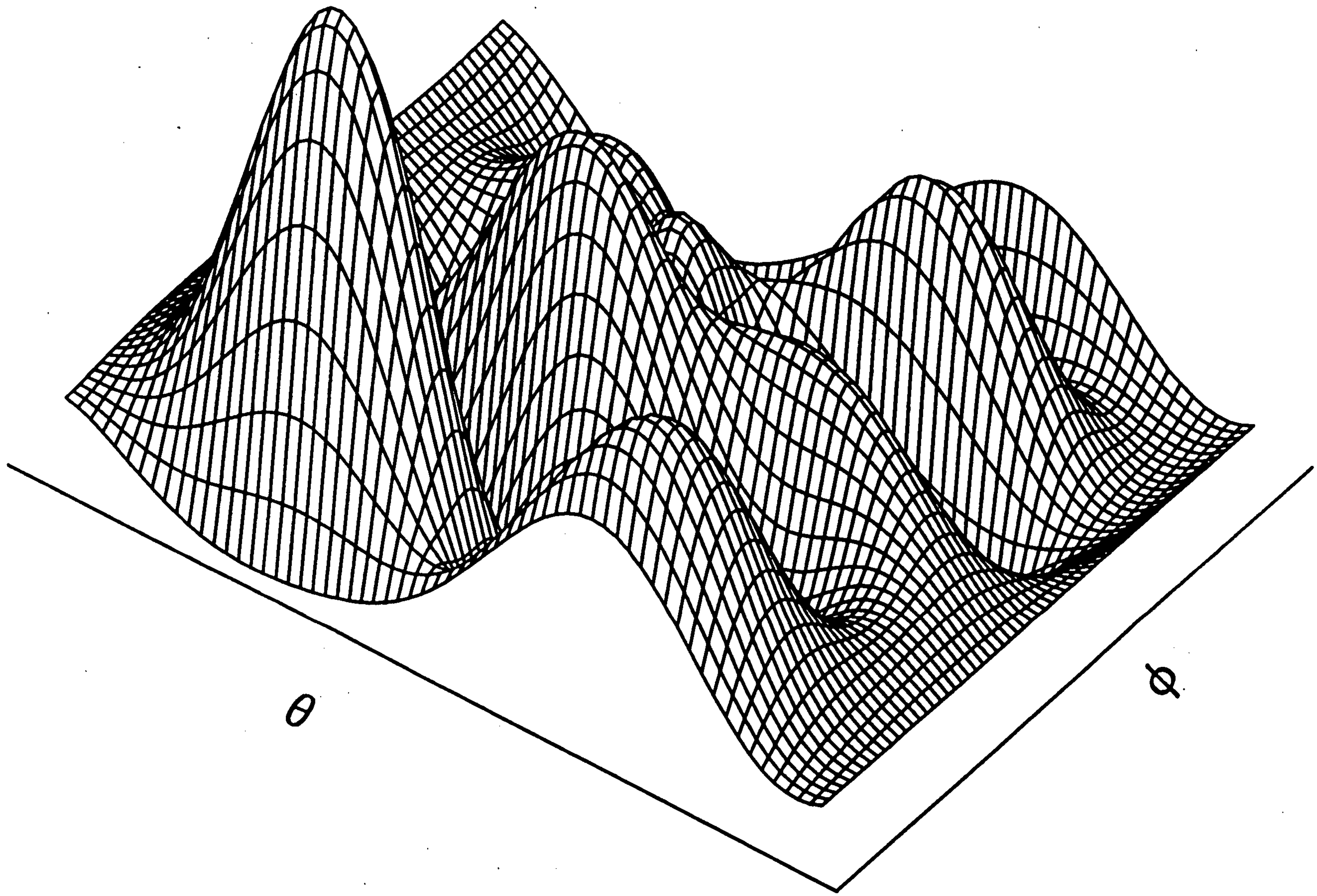

Figure 1

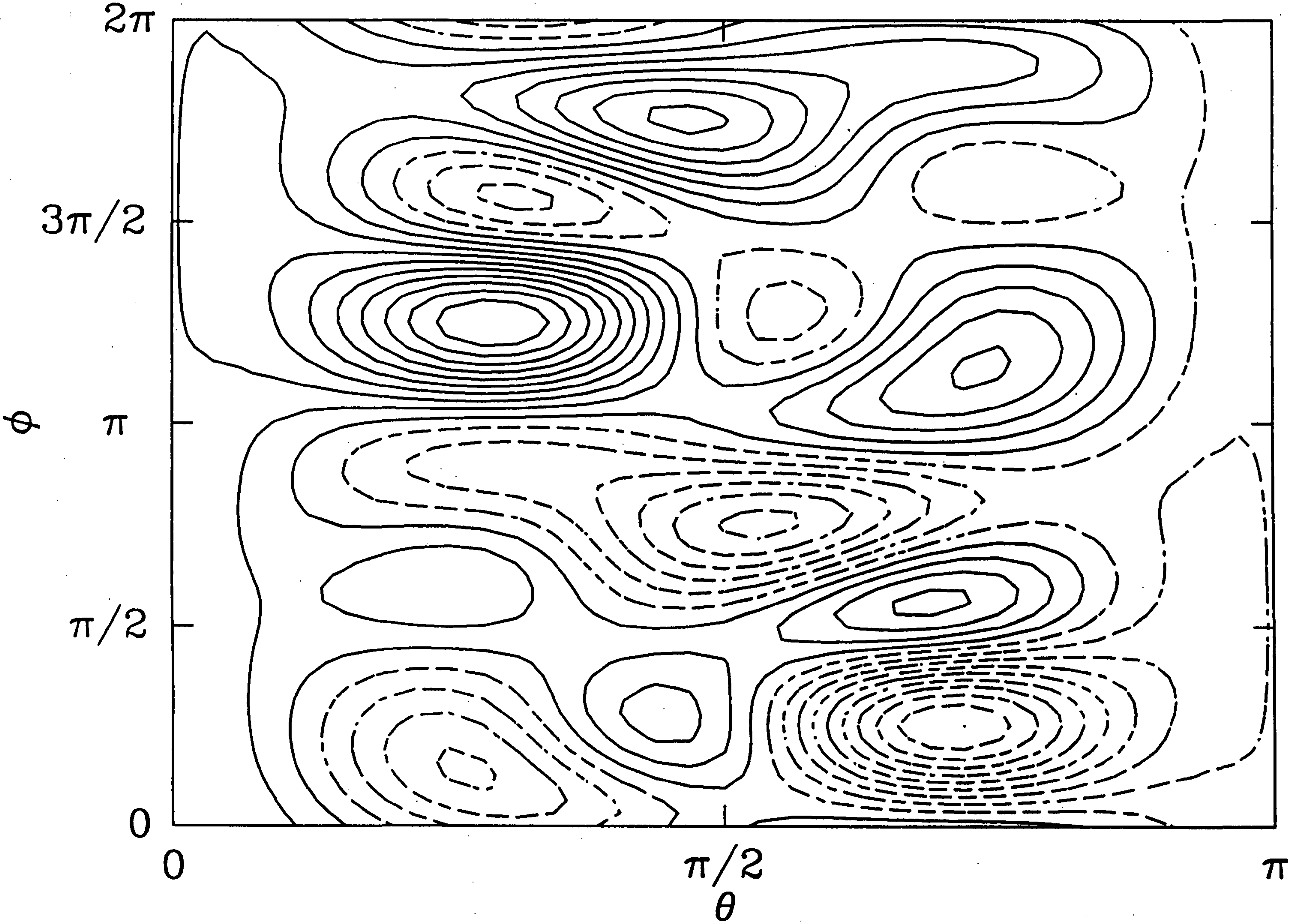

Figure 2