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RIGID LEVITATION, FLUX PINNING, THERMAL DEPINNING AND FLUCTUATION IN HIGH-T_C SUPERCONDUCTORS

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Abstract: The motionless and very stable levitation of high-T_c superconductors above a not rotationally symmetric magnet, and of a magnet above a flat superconductor, demonstrates that pinning and forced depinning of the magnetic flux lines (the Abrikosov vortices) in these type-II superconductors plays a decisive rôle here. Flux pinning causes a *hysteresis* of the magnetization curves, and thus hysteretic force–displacement curves when the superconductor is moved in an inhomogeneous magnetic field. This hysteresis stabilizes the levitation of type-II superconductors and strongly damps its vibration, rotation, and orbiting in fields of low symmetry. In perfectly rotationally symmetric magnetic fields, however, any superconductor can rotate without such friction; this allows the construction of superconducting bearings.

At temperatures sufficiently close to the transition temperature T_c, *flux creep* caused by thermally activated depinning may *decrease* the levitation force. A sharp transition to an effectively pin-free reversible state is observed above a (frequency and geometry dependent) *depinning line* in the induction–temperature plane. This onset of *thermally assisted flux flow* (TAFF), with the flux-line lattice obeying a linear diffusion equation, has been erroneously interpreted as "melting" of the flux-line lattice. It is not clear at present whether an ideally pin-free and strongly fluctuating vortex lattice would melt, what this melting really means (flux lines can cut and reconnect), and how it could be detected.

1. SUPERCONDUCTOR LEVITATING ABOVE A MAGNET

Since the discovery of high-T_c superconductors which stay superconducting above the temperature of liquid nitrogene [1], the free levitation of a superconducting disk above a permanent magnet is one of the most impressive demonstration experiments [2-15]. A ceramic disk of $YBa_2Cu_3O_7$ may be cooled in liquid nitrogene and put above an appropriately shaped magnet where it floats motionless (or rotates, orbits, or vibrates) for several seconds until it warms up to its transition temperature $T_c \approx 92K$ and then drops on the magnet (Figure 1). This experiment demonstrates superconductivity directly on a table, not behind the windows of a helium cryostat.

By permanently cooling the superconductor in a stream of evaporating nitrogene gas, the levitation continues as long as desired. Alternatively, a magnet at room temperature may be levitated above a large flat superconductor which may sit in a closed vessel with liquid nitrogene [2, 15]. Apart from cooling, superconducting levitation requires no energy input. Applications of superconducting levitation for frictionless bearings or contact-free positioning of samples will be dealt with in other contributions to this conference. Levitation of tiny superconducting crystallites may be used to separate these or to measure their magnetization [14].

Perhaps the most fascinating feature of levitating high-T_c superconductors is that, above magnets with strong transverse field gradients and without rotational symmetry, the floatation is *completely motionless.* One may push the levitated disk to a wide range of positions and orientations where it will continue to levitate rigidly [3]. This levitation is thus very stable. A superconductor levitating above a low-symmetry magnet [4], and also a magnet levitating above a ceramic superconductor [2], feels an invisible friction as if it were embedded in sand. The levitating body "digs its own potential well" wherever it sits (self trapping). The friction is so strong that the force may become *attractive.* A superconductor may thus be suspended *below* a magnet [6–10] or a magnet below a superconductor [12–13]. Various types of levitation and suspension with superconductors are visualized in Figure 2.

In general, a levitating superconductor will not rotate frictionless even if it is perfectly round [9–10]. On the other hand, a levitating round magnet will rotate frictionless above a flat or arbitrarily shaped superconductor. Rotation of a levitated superconductor (round or of any shape) is frictionless only when it levitates in a position where the magnetic field exhibits rotational symmetry. A residual, velocity-independent friction is then caused only by deviations from perfect rotational symmetry of the magnetic field. A further weak friction proportional to the velocity, in principle may originate from eddy currents which are induced in the surrounding conductors by the magnetic field of a not perfectly symmetric rotating superconductor or magnet.

In this paper I will show that the strong velocity-independent frictional force on a levitating superconductor, and on any type-II superconductor moving in an inhomogeneous magnetic field, is caused by pinning and depinning of the magnetic flux lines in its interior. Levitation may thus be used to investigate the pinning properties of a superconductor, and friction in a superconducting bearing may be minimized by choosing appropriate materials and geometries.

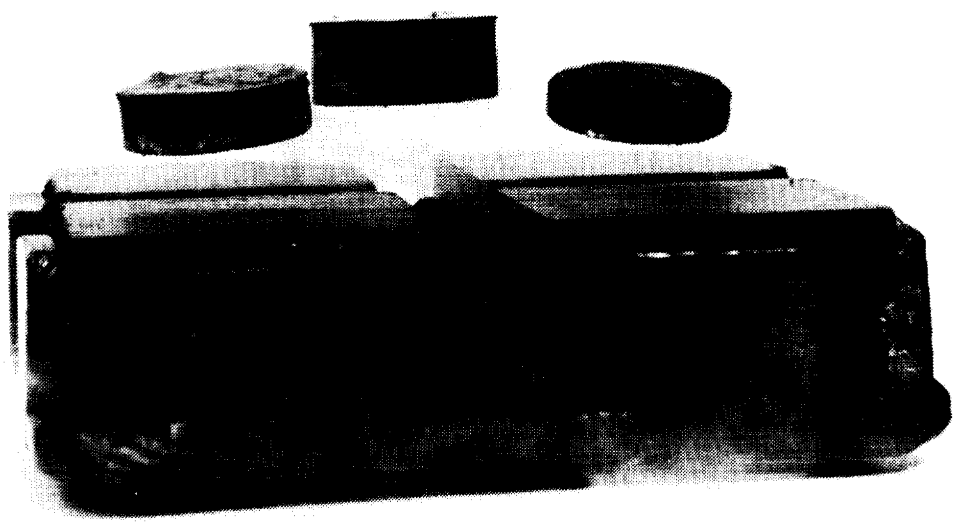

Fig.1. Disks of the high-T_C superconductor $YBa_2Cu_3O_7$ (12 mm diameter) levitating above four strong 2×2 cm magnets sitting with north pole up on an iron plate above a bowl with liquid nitrogene. A screw in the middle is the south pole. Above such a magnet with no rotational symmetry the disks can float motionless in a continuous range of positions and inclinations as if they were stuck in sand.

2. FLUX FLOW

All high-T_c superconductors, even as most superconducting alloys and superconducting compounds, are type-II superconductors. In magnetic fields exceeding the lower critical field of the material $B_{c1} \approx 0.02$T, magnetic flux penetrates a type-II superconductor in form of magnetic flux lines. These are tiny current vortices which repel each other and tend to arrange into a more or less perfect triangular lattice. Each vortex carries a quantum of magnetic flux $\Phi_0 = h/2e = 2.07 \times 10^{-15}\mathrm{Tm}^2$. The radius of the (usually strongly overlapping) flux tubes is $\lambda \approx 2 \times 10^{-7}$m (the penetration depth for weak magnetic fields) and the radius of the vortex core is $\approx \xi = (\Phi_0/2\pi B_{c2})^{1/2} \approx 1 \times 10^{-9}$m in the oxide superconductors, where both λ and ξ are highly anisotropic (by factors of ≈ 5 or more).

When the external field is increased, e.g., by pushing the superconductor closer to the magnet, more flux lines penetrate. In high-T_c superconductors, due to their very large Ginzburg-Landau parameter $\kappa = \lambda/\xi \approx 200$, the internal magnetic field B_{rev} in equilibrium practically equals the applied field B_a as soon as B_a moderately exceeds B_{c1}. Ideal (pin-free, reversible) superconductors at $B_a > 2B_{c1}$ exhibit a reversible magnetization

$$-\mu_0 M_{rev}(B_a) = (B_a - B_{rev})/(1-N) \approx 0.3\,(1 - B_a/B_{c2})\,B_{c1} \qquad (1)$$

where $B_{c2} \approx (2\kappa^2/\ln\kappa)B_{c1}$ is the upper critical field of the material and N is the demagnetization factor of the specimen ($N = 0$ for cylinders or slabs in longitudinal field, $N = 1/3$ for a sphere, $N = 1/2$ for cylinders in transversal field, and $1 - N \ll 1$ for flat disks in transversal field). $B = |\mathbf{B}(\mathbf{r})|$ is the magnetic field inside the superconductor averaged over several flux-line spacings and assumed to be spatially constant in (1).

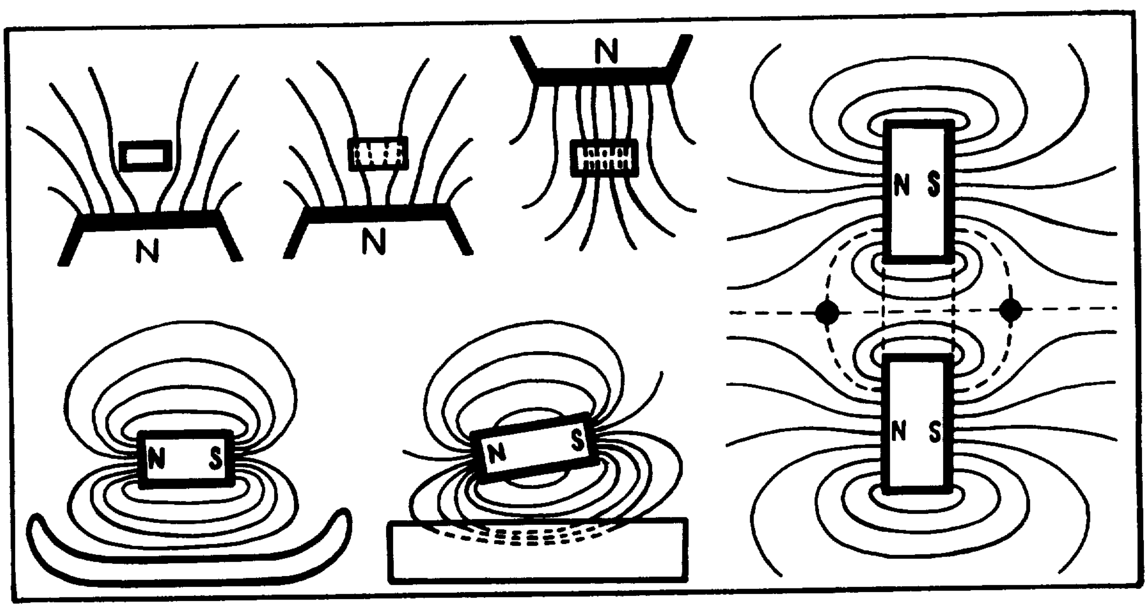

Fig.2. **Top left:** A type-I superconductor and a type-II superconductor levitating above a magnet; a type-II superconductor with positive magnetization (trapped flux) suspended below a magnet. The solid lines indicate magnetic field lines and the dashed lines flux lines. **Bottom left:** Magnets levitating above a bowl of a type-I superconductor and above a flat type-II superconductor. **Right:** The field of an axially magnetized ring magnet has two isolated zeros. In these field minima superconductors in the Meissner state (black dots) can levitate freely even below the ring. This type of suspension thus does not demonstrate attractive forces on a superconductor.

If $\mathbf{B}(\mathbf{r})$ is not constant (due to flux pinning, Section 3), or if an electric current with local density $\mathbf{J}(\mathbf{r}) \approx \mu_0^{-1}\nabla \times \mathbf{B}(\mathbf{r})$ is applied, a Lorentz force $\mathbf{P} = \mathbf{J} \times \mathbf{B}$ acts on the flux lines and causes them to drift with velocity $\mathbf{v} = \mathbf{P}/\eta_{FF}$. The viscosity η_{FF} is related to the flux-flow resistivity $\rho_{FF} \approx \rho_n B/B_{c2}$ by $\eta = B^2/\rho_{FF} \approx B\,B_{c2}/\rho_n$ where ρ_n is the normal conductivity at the same temperature T [16]. This flux drift generates an electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$. Thus,

$$\mathbf{E} = \mathbf{B} \times \mathbf{v} = \mathbf{B} \times \mathbf{P}/\eta_{FF} = \mathbf{B} \times (\mathbf{J} \times \mathbf{B})/\eta_{FF} = \rho_{FF}\mathbf{J}_\perp \tag{2}$$

is proportional to, and directed along, the current-density component $\mathbf{J}_\perp$ perpendicular to the flux lines. This means, *the presence of mobile flux lines destroys the ideal conductivity.* The resulting flux-flow resistivity is *anisotropic*: only currents perpendicular to the applied field cause a voltage drop and a dissipation JE, but currents parallel to the flux lines are loss free.

A more careful analysis reveals that, in ideal superconductors, a longitudinal current density $\mathbf{J}_\parallel = \hat{\mathbf{B}}(\mathbf{J}\hat{\mathbf{B}})$ ($\hat{\mathbf{B}} = \mathbf{B}/B$) may cause the flux lines to distort spontaneously into helices which blow up until they cut each other or the surface of the specimen [17–19]. This *helical instability* induces flux-line motion and thus dissipation. However, even as in the usual flux flow, because of flux pinning this dissipation occurs only when the current density exceeds a (longitudinal) critical current density. For a recent contribution and a compilation of references on longitudinal currents in superconductors see [20].

The dissipation caused by flux motion in type-II superconductors in general is due to eddy currents, which form a dipolar pattern around isolated vortices when $\kappa \gg 1$. Part of the dissipation originates since the induced current flows also through the normal conducting core [21]. A further contribution, approximately of the same size, originates from the relaxation of the superconducting order parameter Δ: When the flux-line core passes a fixed position in the material Δ goes to zero for a short time $\approx \xi/v$ [22]. Both dissipation effects are incorporated in the solution of the time-dependent Ginzburg-Landau theory for a moving vortex [23] or vortex lattice [24]. For a review on moving vortices see [25].

There has been some discussion recently whether dissipation in the highly anisotropic high-T_c superconductors is always caused by flux flow. Several experiments (e.g. [26]) seemed to indicate that the flux-flow concept cannot explain the measured resistivity, which was independent of the angle between $\mathbf{B}$ and $\mathbf{J}$, both chosen in the a-b plane of the crystal. However, as shown by Kes et al. [27], a small perpendicular (to the a-b plane) field component, which should always be present in imperfect crystals, can explain these observations by the usual flux-flow dissipation (Section 8).

3. FLUX PINNING

In all *real* superconductors the flux lines cannot move completely freely because they interact with material inhomogeneities, e.g., precipitates, interstitials, vacancies, dislocations, grain boundaries, and in $YBa_2Cu_3O_7$ with twin boundaries, oxygen vacancies, and, due to the extremely small vortex core, even with the atomic lattice cell and the CuO_2 planes. This means, their energy depends on the position of the vortex core. Therefore, when the driving force density P is less than a critical value BJ_c, or the current density less than a critical value J_c, the vortices will be pinned and do not move [28–30]. There is thus no voltage drop and no dissipation for $J < J_c$ (for simplicity I shall now write J for $J_\perp$). This result will be modified at finite temperatures in high-T_c superconductors, where thermally activated depinning occurs (Section 8).

The summation of elementary pinning forces (which often are estimated from the Ginzburg-Landau theory) is a complicated statistical problem [28–30]. A soft flux-line lattice is more effectively pinned since the flux lines can adjust better to the pins. J_c thus depends crucially on the elastic properties of the vortex lattice [28–31].

When J exceeds J_c the flux lines are depinned and move as discussed above, but with a nonlinear resistivity, $E \approx \rho_{FF}(J^2 - J_c^2)^{1/2}$. The dissipation EJ is still caused by the viscous motion of the vortices, but now the vortex velocity is not constant. Near some pins, the flux lines are "plucked", i.e., during depinning they jump with a much higher than the average velocity $\bar{v} = E/B$. In particular, immediately above J_c, E and thus $\bar{v}$ are small, whereas the jumping velocity v_{max} is of the order of $\rho_{FF} J_c/B$. v_{max} depends on the elastic restoring force acting during the jump but (almost) not on J or E.

4. EFFECTS OF FLUX PINNING ON THE LEVITATION

The depinning jumps forced by sufficiently large flux gradients or currents inside superconductors cause the observed frictional damping of levitating superconductors or magnets. As a consequence, the levitation is *stable in a finite range of heights, lateral positions, and orientations* of the levitating body. This behavior in principle can be calculated from the hysteretic magnetization curve $M(B_a)$ of the superconductor. The general expression for the force $\mathbf{F}$ exerted by a magnetic field $\mathbf{B_a}$ on the current density $\mathbf{J}(\mathbf{r})$ inside the superconductor (surface shielding currents and vortex currents) is

$$\mathbf{F} = \int_V \mathbf{J}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \, d^3 r. \tag{3}$$

When $B_a(\mathbf{r}) = |\mathbf{B_a}(\mathbf{r})|$ has nearly constant gradient over the volume V of the superconductor one may approximate (32) by

$$\mathbf{F} = (\mathbf{m}\nabla)\,\mathbf{B_a}(\mathbf{r}) = \nabla[\mathbf{m}\,\mathbf{B_a}(\mathbf{r})] \approx MV\,\nabla B_a(\mathbf{r}) \tag{4}$$

where $\mathbf{m} \approx \mathbf{M}V$ is the magnetic moment of the superconductor with magnetization $\mathbf{M} = -(\mathbf{B_a} - \mathbf{B})/(1-N)\mu_0$, and $M = |\mathbf{M}|$. The second identity in (3) follows from $\nabla \mathbf{B_a} = 0$, and the third one holds when $\mathbf{M}$ is nearly parallel to $\mathbf{B_a}$.

According to Eq. (1) the magnetization M_{rev} of an ideal type-II superconductor is always *negative*. For real materials, however, $M(B_a)$ is a non-unique function due to flux pinning and may also be positive. One has $-M \geq -M_{rev}$ when B_a is increased (since flux penetration is impeded by pinning), and $-M \leq -M_{rev}$ when B_a is decreased (since the flux does not want to move out). This hysteresis of $M(B_a)$ causes a hysteresis of the force (3) as a function of $B_a(z)$ or of the height z above the magnet. There exists, therefore, a continuous range of equilibrium positions for a levitating superconductor.

In general, whenever the specimen is moved vertically or horizontally in a field gradient, flux is forced to enter or exit. This causes depinning and thus a friction which holds a levitating superconductor in place. An even stronger friction occurs when a magnet levitates above a flat superconductor. One may say that the magnetic field lines are "anchored" inside a type-II superconductor since they are materialized by pinned flux lines.

5. LEVITATION OF TYPE-I SUPERCONDUCTORS

A pinning-caused friction does not occur in type-I superconductors (e.g., lead or tin) since these do not contain flux lines. Type-I superconductors expell the magnetic flux completely when $B_a < (1-N)B_c$ where B_c is the thermodynamic critical field. In the field interval $(1-N)B_c < B_a < B_c$ type-I superconductors with demagnetization factor $N > 0$ are in the *intermediate state* [16, 30] containing superconducting lamellae or tubes (with $B = 0$) surrounded by normal conducting regions (with $B = B_c$), or normal tubes or lamellae in a superconducting matrix. These tubes, with diameters of many λ, can move and be pinned similarly as flux lines in type-II superconductors, but the pinning force is much weaker. The magnetization curves of type-I superconductors are, therefore, reversible and a levitated type-I superconductor has only one (or several discrete) positions of stable levitation but not a continuous range.

In the classical demonstration of superconducting levitation a magnet levitates over a concave bowl of lead or tin in a Helium cryostat. Levitation of a magnet over a *flat* type-I superconductor is *unstable* since the magnetic field lines are expelled from the type-I superconductor and the magnet "rolls" on its compressed field lines over the edges of the superconductor (Figure 2).

6. SUSPENSION OF A SUPERCONDUCTOR BELOW A MAGNET

When the pinning-caused magnetic hysteresis is sufficiently strong, a superconductor can be freely *suspended below* a magnet [6–7, 9–10]. When the superconductor is moved towards the pole of the magnet, more and more flux penetrates; the maximum flux density is somewhat smaller than the field at the pole. When the superconductor is removed from the pole, the internal field initially stays constant due to flux pinning; the force, proportional to $(B - B_a)\,\partial B_a/\partial z$, therefore changes from repulsive to attractive when $B - B_a$ changes sign. Even as the leviation, the suspension of a superconductor below a magnet is *vertically stable* since the attractive force *increases* when the superconductor is moved away from the pole. Horizontal stability of suspension is obvious.

In contrast, a piece of iron cannot be suspended stably below a magnet since the attractive force *decreases* when the iron is moved away from the pole. Stability is the main problem also with other types of levitation [9]: *Aerodynamic* levitation in a fluid jet; *acoustic levitation* by intensive sound waves; *radiometric levitation* by heat radiation in a low-pressure atmosphere; *optical levitation* by laser light; *electric levitation* in combined dc and ac electric fields; *magnetic levitation* of diamagnets by permanent magnets or coils; *radio-frequency levitation* of solid or liquid metals in appropriately shaped coils.

7. SUSPENSION BY A RING MAGNET

The levitation of a superconductor by an axially magnetized ring magnet is an interesting special case (Figure 2). The magnetic field of a ring has two minima (even zeros) on its axis at distances of ≈ 0.4 hole diameters from its flat surfaces [11]. Near such a minimum, the magnetic field is quadrupolar, e.g., $E = \nabla(x^2 + y^2 - 2z^2)$ in appropriate coordinates. Quite generally, electric and magnetic fields in free space cannot have isolated maxima but only minima since $\nabla^2(E^2) \geq 0$ and $\nabla^2(H^2) \geq 0$ [32].

Any diamagnetic material is attracted to a minimum in the magnetic field and can levitate there when the magnetic force exceeds the gravitational force. Therefore, the free levitation of a superconductor by a ring magnet is *not* indicative of attractive forces or a positive magnetization. Any superconductor in the Meissner state, even a type-I superconductor, can levitate *above, beside, or below* a strong ring magnet.

8. THERMALLY ACTIVATED DEPINNING

Due to flux pinning, a type-II superconductor generally is not in thermodynamic equilibrium with the applied field B_a. After a change of B_a, magnetic flux enters or exits the specimen such that the internal field $\mathbf{B}(\mathbf{r})$ exhibits a gradient which, like the slope of a sand pile, does not exceed a critical value; one has $|\nabla \times \mathbf{B}| \approx |\nabla|\mathbf{B}|| \leq \mu_0 J_c$ everywhere in the specimen [16, 28, 33].

At temperatures $T > 0$ the thermal motion of flux lines "shakes" the flux-line lattice such that some of the pinned flux lines may overcome the pinning potential [34]. The flux-density gradient and the current density $\mathbf{J} = \mu_0^{-1}\nabla \times \mathbf{B}$ will then decrease gradually by thermally activated depinning. This phenomenon of *flux-creep* (e.g. a slow decrease of trapped flux with a logarithmic time law [35]) is observed also in *"classical" superconductors* (e.g. Nb, Nb_3Sn, and Nb-Ti alloys), but only close to their transition temperature T_c, where the pinning potential is weak. In *high-T_c*

superconductors flux creep is observed in a much larger temperature interval below T_c because (a) T is higher and (b) the pinning energy $U \approx B_c^2 \xi^3/\mu_0$ is much smaller due to the very short coherence length $\xi \approx 10$Å (in Nb, $\xi \approx 1000$Å) [36-40]. The jumping probability $\propto \exp(-U/k_BT)$ is thus much higher.

According to Anderson's idea [34], the net jump rate ν of flux lines or flux-line bundels is the difference of the jump rates along and against the driving force density $\mathbf{J} \times \mathbf{B}$,

$$\nu \;=\; \nu_0 \exp[-(U-\delta U)/k_BT] - \nu_0 \exp[-(U+\delta U)/k_BT]. \tag{5}$$

In (5) ν_0 is an attempt frequency, which I interpret as the typical frequency of the thermal fluctuations of the ideal vortex lattice, $2\pi\nu_0 = \Gamma_1 = B^2/\mu_0\lambda'^2\eta = \rho_{FF}/\mu_0\lambda'^2 \approx \rho_n b(1-b)/\mu_0\lambda^2$ where $b = B/B_{c2}$ and $\lambda'^2 = \lambda^2/(1-b)$ [41]; U is an activation energy; $\delta U = JBVl$ is the work done by a jumping vortex bundle with volume V and jump width l. The interpretation of U, V, and l is not clear at present. Recent scaling theories [42] yield a current dependent $U(J)$ diverging as $J \to 0$ and thus predict zero resistivity at $J = 0$. These extensions of the successful *theory of collective pinning* [29] assume purely elastic deformation of the flux-line lattice and thus disregard its plastic deformation. Experiments yield *constant* resistivity over several decades of J at small J in $YBa_2Cu_3O_7$ single crystals [39]. A *spectrum* of activation energies is considered in [40].

Introducing *heuristic parameters* $J_c = U/BVl$ (critical current density) and ρ_c (resistivity at $J = J_c$) one gets for the drift velocity $v = \nu l$ and electric field $E = Bv$ from (5)

$$E(J) \;=\; 2\rho_c J_c \exp(-U/k_BT)\sinh(JU/J_c k_BT). \tag{6}$$

This result means that at $J \approx J_c$, $E(J)$ increases exponentially with J (flux creep) but for $J \ll J_c k_BT/U$, $E(J)$ is linear. This regime of *thermally asisisted flux flow* (TAFF) [38] is a new phenomenon observed in the high-T_c superconductors at low current densities, e.g., when the smeared transition $\rho(T)$ is measured with B as parameter [38–39]. From(6) and the flux flow approximation at $T = 0$ (Section 3) one gets the general picture for voltage–current curves (Figure 3):

$$E \;=\; (2J\rho_c U/k_BT)\exp(-U/k_BT) \;=\; J\rho_{TAFF} \qquad \text{for } J \ll J_c \quad \text{(TAFF)} \tag{7}$$

$$E \;=\; J\rho_c \exp[(J/J_c - 1)U/k_BT] \qquad \text{for } J \approx J_c \quad \text{(flux creep)} \tag{8}$$

$$E \;=\; \rho_{FF}(J^2 - J_c^2)^{1/2} \;\approx J\rho_{FF} \qquad \text{for } J \gg J_c \quad \text{(flux flow).} \tag{9}$$

In the linear regime $J \ll J_c$ (6) (even as for $J \gg J_c$) the flux density obeys a diffusion equation $\partial\mathbf{B}/\partial t = D\nabla^2\mathbf{B}$ [38] provided a term $\propto \nabla \times \mathbf{J}_{||}$ (Section 2) may be disregarded [43]. The diffusion constant is given by the TAFF resistivity, $D = \rho_{TAFF}/\mu_0$. This *flux diffusion* is discussed by Kes [38] and in a forthcoming paper [43]. As one consequence, the relaxation time $\tau_0 = L^2/\pi^2 D$ for flux-density gradients and current densities, and also the frequency $(2\pi\tau_0)^{-1}$ where maximum dissipaton occurs in ac experiments, depend on the *geometry and size (L)* of the specimen. These are thus *not* mere material parameters as is often assumed.

9. DOES THE FLUX-LINE LATTICE MELT ?

The thermal fluctuation $\langle \mathbf{u}^2 \rangle$ of the flux lines ($\mathbf{u}$ = displacement of vortex cores) is given by k_BT times the trace of the reciprocal elastic matrix $\Phi_{\alpha\beta}(\mathbf{k})$ of the vortex lattice integrated over (k_x, k_y) in the Brillouin zone and over $|k_z| < \xi^{-1}$: $\langle \mathbf{u}^2 \rangle = k_BT \int (\Phi_{xx}^{-1} + \Phi_{yy}^{-1})\, d^3k/8\pi^3$ [41, 44–46]. In high-T_c superconductors $\langle \mathbf{u}^2 \rangle$ is much larger than in classical superconductors since

(a) T can be high;

(b) the shear modulus c_{66} of the vortex lattice is small, $c_{66} \approx BB_c/4\sqrt{2}\kappa\mu_0$ ($B \ll B_{c2}$);

(c) the elasticity is highly *nonlocal*, i.e., the tilt modulus of the flux-line lattice $c_{44}(k) \approx (B^2/\mu_0)/(1+$

$k^2\lambda'^2$) depends on the wavelength $2\pi/k$ of the tilt strain, and little energy is required to tilt the vortex lattice locally [31, 45];

(d) the pronounced anisotropy of these oxides reduces $c_{44}(k)$ even more (not at $k = 0$ as argued by [44] but at $k > 1/\lambda'$ [45–46]) and thus increases $\langle \mathbf{u}^2 \rangle$;

(e) the small pinning energy cannot reduce the short-wavelength thermal fluctuations.

It has been argued [44] that the vortex lattice might *melt* when $\langle \mathbf{u}^2 \rangle^{1/2}$ reaches ≈ 0.1 times the vortex spacing $a = 1.075(\Phi_0/B)^{1/2}$. From this Lindemann criterion, and from other melting criteria (fluctuating vortex distance and shear strain) [41, 45] one may derive a "melting temperature" $T_m(B)$, which decreases when the flux density B increases. It is sometimes assumed that a "melted flux lattice" cannot be pinned since it can flow between the pins [47]. This would be true if there were much less pins than flux lines. However, in real materials and in three dimensions there are many pins per flux line, and a soft flux-line lattice is even *stronger pinned* since it can adjust better to the pins [28–30, 48]. The vibrator experiments [47], therefore, did not measure a "melting line" $T_m(B)$ but rather the usual "irreversibility line" or "depinning line" above which thermally activated depinning makes the pins ineffective (Section 8). As shown by Esquinazi [48], the data of [47] coincide with the depinning lines obtained for the same materials by measurements of resistivity, magnetization, and damping and frequency enhancement of superconducting vibrating reeds [48–49].

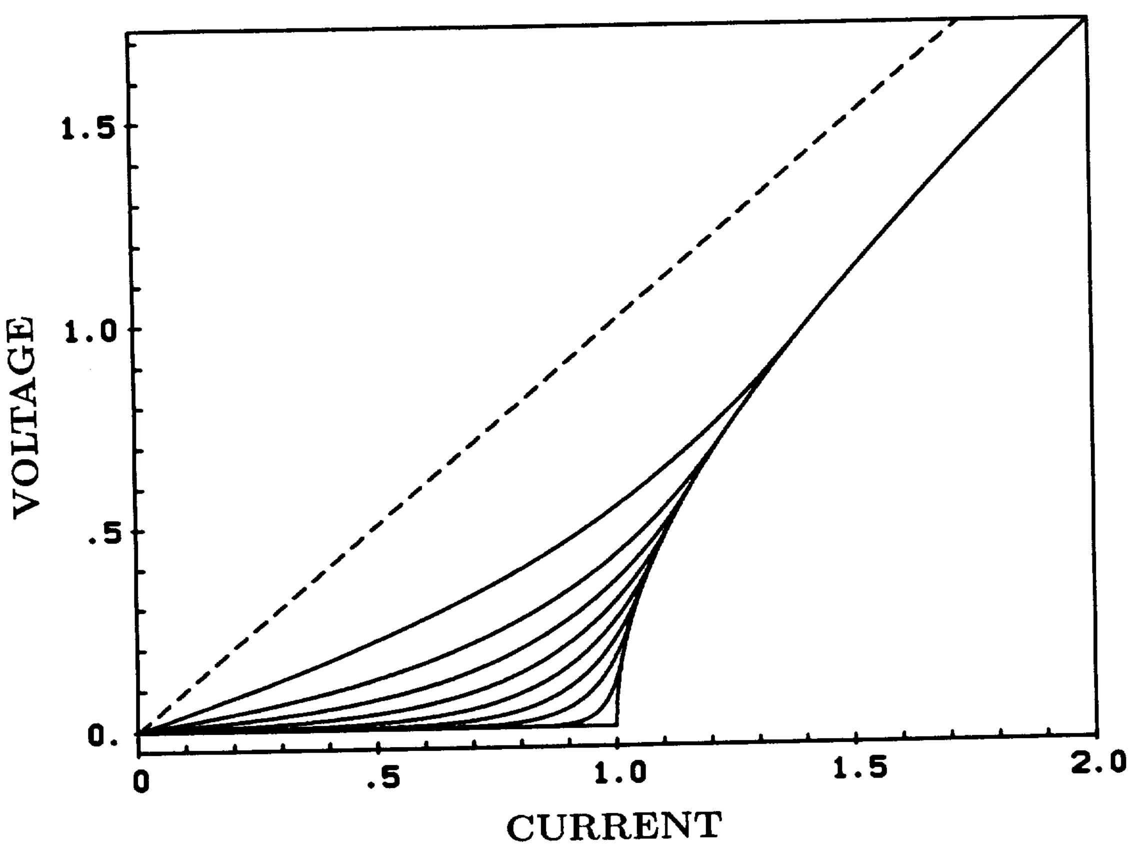

Fig.3. Voltage–versus–current curves due to flux-line drift at various temperatures T and inductions B, Eqs. (7)–(9), in reduced units $E/\rho_{FF}J_{c0}$ versus J/J_{c0} with $J_{c0} = J_c$ at $T = 0$. Larger T and B yield smoother curves. The dashed line shows $E = \rho_{FF}J$.

Due to the always present pinning-caused disorder it is not clear how one should observe "flux melting" and separate it from thermal depinning. It is even not clear what melting of a line lattice really means. In the melting theory [4] a simple stiff-vortex interaction has been assumed rather than the correct three-dimensional interaction between line elements [30–31]. If melting is defined by the vanishing of a shear modulus, one should consider that the shear stiffness of a vortex lattice containing defects depends on both *time and length scale* of the deformation since screw dislocations in the vortex lattice can move freely (they see no Peierls potential). An important difference to the atomic lattice is that some topological defects can easily vanish by cutting and reconnection of flux lines. More work has to be done in order to clarify this melting concept. Probably there is no sharp melting transition but a gradual increase of thermal disorder.

The thermally activated flux motion and the melting speculation concern monocrystals or single grains of ceramic superconductors. The influence of the granular structure [50] on the magnetic and levitation properties has not been dealt with in this contribution due to limited space, but the levitation behavior of ceramic superconductors in principle can be explained from the measurements of intergrain critical currents [51] and irreversible magnetization curves [52].

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