

## USE OF ENTANGLEMENT IN QUANTUM OPTICS\*

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**Abstract**

Several recent demonstrations of two-particle interferometry are reviewed and shown to be examples of either color entanglement or beam entanglement. A device, called a number filter, is described and shown to be of value in preparing beam entanglements. Finally we note that all three concepts (color and beam entanglement, and number filtering) may be extended to three or more particles.

**Introduction**

In recent years there has been a variety of demonstrations of two-particle interferometry. By a two-particle interferometer we mean an arrangement without polarizers whereby the coincident count rate in a pair of detectors exhibits sinusoidal oscillations (two-particle fringes) as some apparatus parameter is uniformly varied, but the singles rate in each detector is constant. Whereas the earliest demonstrations<sup>1,2</sup> of two-particle fringes employed pairs of photons from atomic cascades and employed polarizer orientation as the parameter, the new experiments employ photon pairs produced by down-conversion and usually employ mirror translation as the parameter.

Two-particle interference fringes occur only when the quantum mechanical state of the particles is entangled. By entanglement we mean that the two-particle state does not factor into a product of single particle states, but is a sum of at least two terms, each of which is a product. Note that when two-particles are so entangled, neither particle separately has a state. Because particles in an entanglement do not have states or even some properties, independently of each other, we will often refer to them not as two particles, but simply as a two-particle, i. e., a single entity.

The present paper reviews a selection of the recent demonstrations of two-particle interferometry, in order to point out the central role of entanglement. The experiments are selected so as to especially emphasize two important types of entanglement: *color entanglement* and *beam entanglement*. Although each of these types of entanglement have previously been separately discussed (but without these names) in earlier papers and conference proceedings, we thought this an appropriate place for a review. In the course of the review of existing experiments, we also describe a device, which we call a *number filter*, that may be of use in experimentally preparing entanglements in the future. Finally, we note that all three ideas discussed here (color

entanglement, beam entanglement, and number-filtering) also apply to a three-particle, a four-particle, etc.

Before starting the review, we emphasize two other aspects of our point of view, both essential to the way we use entanglement. First, we distinguish beams or paths (labelled A, B, C,...) from particles or detectors (labelled 1,2,3,...). Second, we apply elementary quantum-mechanical concepts as follows. Amplitudes and kets will be assigned to the particles and not to the beams. Total amplitude is the sum over all contributing amplitudes (i.e., the Feynman-Wheeler rule). Note then that our approach is unorthodox in that quantum optics usually employs a quantum field theory in which states (e.g., kets) are assigned not to the particles, (i.e., the photons) but to the beams (i.e., the field modes).

### A Color-Entangled Two-Photon

When a single particle decays into two, as for example in a down-conversion of an ultraviolet photon into a pair of red photons, energy conservation, together with a suitable apparatus, produce entanglement. Fig. (1) depicts an arrangement for producing entanglement in this way during down conversion.<sup>3,4</sup> Suppose, for simplicity that the incident photon is ideally monochromatic with wavenumber  $2k_0$ , so that its state is

$$\Phi(k) = \delta(k - 2k_0) \quad (1)$$

Suppose that the outgoing pair, 1 and 2, are selected as to direction by the symmetrically placed slits and as to color by filters of wavenumber width  $\sigma$ , centered at  $k_0$ , and that  $\sigma$  is narrower than any feature in the down-conversion spectrum. Then from eq. (1) and energy conservation, the state of the down-conversion photons after the filters is

$$\Phi(k_1, k_2) = \delta(k_1 + k_2 - 2k_0) e^{-(k_1 - k_0)^2 / 2\sigma^2} e^{-(k_2 - k_0)^2 / 2\sigma^2}. \quad (2)$$

Because of the  $\delta$ -function, this state cannot be factored, i. e., the two red photons are actually a *color-entangled two-photon*.

In general, a state entangled in  $k$ -space is also entangled in  $x$ -space. For example, eq. (2) in  $x$ -space, with the time dependence included, becomes

$$\Psi(x_1, x_2, t_1, t_2) = e^{ik_0(x_1 - ct_1)} e^{ik_0(x_2 - ct_2)} e^{-\frac{\sigma^2}{4} [(x_1 - ct_1) - (x_2 - ct_2)]^2}, \quad (3)$$

along the outgoing beams. Here it is the real exponential that does not factor, i. e., the photons are still entangled and with spectacular consequences. If detectors are placed in the beams equally far from the source ( $x_1 = x_2$ ), the joint probability density is, from eq. (3),

$$\Psi^* \Psi = e^{-\sigma^2 (c\tau)^2 / 2}, \quad (4)$$

where  $\tau \equiv t_2 - t_1$  is the time difference in the arrival of the photons. In short, the color entanglement implies that the distribution in time separation of the photons is dictated by the filter width.<sup>4</sup>

For even more spectacular consequences, consider the expanded arrangement first proposed by Franson<sup>5</sup> and shown in Fig. (2). Here each beam of Fig. (1) has been fed into a single particle interferometer. Kwiat, et al.,<sup>6</sup> and Ou, et al.,<sup>7</sup> have confirmed Franson's prediction that two-particle fringes can be exhibited with this arrangement. These Franson fringes follow easily from the color-entangled two-photon state (2) and (3), as has been shown in an earlier conference proceedings.<sup>8</sup> The argument is as follows. For simplicity, suppose that the path lengths are adjusted so that the two interferometers are identical, with the long path longer by  $\Delta$  than the short path. Place detectors in the corresponding output beams of the interferometer and monitor for coincidences. From Fig. (2), the state falling on these detectors is

$$\Psi(0, 0, t_1, t_2) + \Psi(\Delta, \Delta, t_1, t_2) + \Psi(\Delta, 0, t_1, t_2) + \Psi(0, \Delta, t_1, t_2), \quad (5)$$

where  $\psi$  is given by eq. (3).

It follows from the state (5) that the coincident count rate is

$$A(1 + \cos 2k_0\Delta) + B\cos k_0\Delta + C, \quad (6)$$

where  $A$ ,  $B$ , and  $C$  are elementary (error) functions of  $\sigma$ ,  $\Delta$ , and the coincidence "window"  $T$ . In the ideal limit that

$$\sigma\Delta \gg 1 \quad (7a)$$

and

$$cT \ll \Delta, \quad (7b)$$

the third and fourth terms of the state (5) do not contribute and as a consequence,  $B=C=0$  in expression (6). That is, (6) reveals that under ideal conditions two-particle fringes of visibility unity can be exhibited by varying  $\Delta$  or  $k_0$ . Although experiments cannot achieve such ideal contrast, several groups at the present conference report continuing investigation of fringe contrast in Franson interferometry. We propose that some of the relationships in (6) be compared with experimental data.

### A Beam-entangled Two-photon

The collapse of the state (5) to just two terms, when the conditions of the inequalities (7) are satisfied, has already produced an example of *beam entanglement*. That is, the beams taken by the detected pair of photons were either both short or both long, the other two cases being impossible because of the choices of filter width  $\sigma$ , path difference  $\Delta$ , and coincidence window  $T$ . Beam entanglement may also be produced by directly exploiting momentum conservation<sup>9</sup> during the decay process, instead of the energy conservation that was built into the color entanglement of eq. (2). In general, this approach requires that the two beams of Fig. (1) be brought together. Fringes were first produced in this way by Alley and Shih,<sup>10,11</sup> but since their arrangement involved polarization manipulation, we will review instead the simpler arrangement of Ghosh and Mandel,<sup>12</sup> shown in Fig. (3).

As indicated in that figure, Ghosh and Mandel uncovered some of the fun of two-particle quantum mechanics in the small region of beam overlap. Consider, as shown in the Fig. (3) insert, two small detectors placed in this region. When coincident counts occur in these detectors, the count in detector 1 could have been caused by a photon 1 that took route A, in which case, by momentum conservation at the source, photon 2 took route B. Equally likely, the routes taken could have been reversed. Consequently, the state falling on the detectors is the beam-entangled two-photon state

$$\frac{1}{\sqrt{2}} \left[ |A\rangle_1 |B\rangle_2 + |B\rangle_1 |A\rangle_2 \right] \quad (8)$$

where  $\text{ket } |A\rangle_1$  denotes particle 1 in beam A, etc.

Assume, for simplicity, that each beam is monochromatic and monodirectional. Then, from state (8), the stationary two-particle amplitude at the detectors is proportional to

$$e^{i\mathbf{k}_A \cdot \mathbf{r}_1} e^{i\mathbf{k}_B \cdot \mathbf{r}_2} + e^{i\mathbf{k}_B \cdot \mathbf{r}_1} e^{i\mathbf{k}_A \cdot \mathbf{r}_2}, \quad (9)$$

where  $\mathbf{k}_A$  and  $\mathbf{k}_B$  are the wave-vectors of the beams and  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the positions of the detectors. It follows from the amplitude (9) that the probability of joint detection is proportional to

$$1 + \cos[(\mathbf{k}_A - \mathbf{k}_B) \cdot (\mathbf{r}_2 - \mathbf{r}_1)], \quad (10)$$

as confirmed by Ghosh and Mandel, who varied the separation of the detectors in Fig. (3).

Note that manipulation of the phases of beams A and B, either by changing their geometrical lengths with mirror motion or their optical lengths with phase plates, has no effect on the fringes (10). This is because each term of the entangled state (8) has both an A-beam and a B-beam factor, and hence the phase manipulations introduce only an unobservable overall phase in the amplitude (9).

Fig. (4) depicts an extended arrangement of Rarity et al.<sup>13</sup> that is responsive to such phase manipulations. Here, by simply relabeling the kets in (8), the state before the first beam splitters is

$$\frac{1}{\sqrt{2}} \left[ |C\rangle_1 |D\rangle_2 + |D\rangle_1 |C\rangle_2 \right], \quad (11)$$

which evolves into

$$\frac{1}{\sqrt{2}} \left[ |A\rangle_1 |A\rangle_2 + e^{2i\phi} |B\rangle_1 |B\rangle_2 \right] \quad (12)$$

after the phase plate and before the final beamsplitter. That is, now both particles either take route A, or both take route B, and hence the phase  $2\phi$  enters. The derivation of (12) from (11) uses only the elementary rules for transmission and reflection at beam splitters, i. e.,

$$|C\rangle_1 \rightarrow \frac{1}{\sqrt{2}} (i|A\rangle_1 + |B\rangle_1), \quad (13)$$

etc. Applying this rule again at the last beam splitter, the state (12) evolves into four terms, but two of these describe both particles going into the same detector, and hence are of no interest in a coincidence counting experiment. From either of the other amplitudes, the coincident count rate is proportional to

$$1 + \cos 2\phi \quad (14)$$

i. e., two particle fringes appear when  $\phi$  is varied, as was indeed observed by Rarity et al.<sup>13</sup>

Fig. (5) proposes another arrangement for preparing the entangled state (12). Here a beam of wave number  $k$  impinges on a beam splitter. Outgoing beam A contains two non-linear crystals separated by a  $2k$  filter. Clearly the only way  $k$  radiation can pass this three-element device is for an incident two-photon to up-convert in the first crystal to a single  $2k$  photon which, after passing the filter, downconverts back to a two photon in the second crystal. Consequently, we call the device a *number filter*, since only a two-photon can pass. Of course, the two-photon could avoid the device entirely and take route B, which contains phase shifter  $\phi$ . Thus the state (12) is prepared and the two-particle fringes of (14) can be observed. Although this experiment has not been performed, we note the similarity of Fig. (5) to an arrangement of Wu, et al.<sup>14</sup> It seems that the only significant difference is that we assume the discrete counting of photons in coincidence, whereas they continuously monitored the current difference of two photo-diodes. Clearly there must exist interesting relationships between our point of view (i.e., entanglement of particles) and theirs (i.e., field quantization with "squeezing").

## A Two-Photon in Four-Beam Entanglement

Figs. (6) through (10) depict various performed or proposed experiments for exhibiting two-particle fringes by manipulating four beams of down-conversion radiation. Fig. (6) is a proposal of Horne, et al,<sup>15</sup> and an actual experiment of Rarity and Tapster<sup>16</sup> in which the four beams are taken directly from the down-conversion crystal. Fig. (7) is a proposal of Reid and Walls,<sup>17</sup> in which only two beams are taken directly from the crystal and each of these is split before suitable recombining. Fig. (8) is an experiment of Ou et al.<sup>18</sup>, in which the splitting is done before the down-conversion. Fig. (9) is a proposal for a four-beam experiment employing number-filtering. Fig. (10) is (the completion of) a figure in a recent proposal of Tan et al.<sup>19</sup>

The main point we wish to make here is that, from an elementary entanglement point of view, these five experiments are, essentially, identical. That is, in each case, the arrangement prepares the four-beam two-photon entangled state

$$\frac{1}{\sqrt{2}} \left[ |A\rangle_1 |B\rangle_2 + |A'\rangle_1 |B'\rangle_2 \right] \quad (15)$$

which, in an appropriate pair of the four detectors, produces fringes, as in (14), when the phases are manipulated. For figs. (6) and (7), the state (15) is an immediate consequence of momentum conservation at the decay. In Fig. (8), a single photon has its amplitude split at the beam splitter and then each of these amplitudes down-converts (in different crystals) to produce the state (15). Note that our description does not employ the "entanglement with the vacuum" description of Ou et al.<sup>18</sup> In Fig. (9), the state (15) is an immediate consequence of the number filter, and Fig. (10), if one ignores the extra detector marked 3, is identical to Fig. (9).

In Fig. (10), only the portion below and to the right of the dash-dot line appeared in the figure of Tan, et al.<sup>19</sup> The remainder of Fig. (10) is drawn from their verbal description, i.e., their text. It isn't clear from their text whether they propose to monitor for two particles or for three particles in coincidence. Consequently, we have included the (optional) detector 3 in order to discuss both cases here. If detector 3 is ignored, and coincidence counts are monitored in various pairs of detectors at stations 1 and 2, then the state is given by (15), since, just as in Fig. (9), the particles must either both come through the number filter, or both avoid it. Thus the phases  $\varphi_1$  and  $\varphi_2$  both enter in the second term of (15), and the joint probability of coincidence counts in appropriate detectors at stations 1 and 2 is proportional to

$$1 + \cos(\varphi_2 + \varphi_1). \quad (16)$$

On the other hand, if one does record a particle at 3, the other particle coming through the number filter can either be 1 or 2, but not both. Consequently, the phases  $\varphi_1$  and  $\varphi_2$  enter into different terms and in fact the state approaching the final beam splitters is now

$$e^{i\varphi_2} |A\rangle_1 |B'\rangle_2 + e^{i\varphi_1} |A'\rangle_1 |B\rangle_2, \quad (17)$$

instead of (15). Then the fringe pattern

$$1 + \cos(\varphi_2 - \varphi_1), \quad (18)$$

will occur in an appropriate pair of the detectors at 1 and 2. Since the fringes in (18) depend on the phase *difference*, as in the fringe equations exhibited by Tan, et al., it appears that they are proposing a three-particle coincidence experiment. In either case, it is clear that our elementary entanglement description is not compatible with their talk of a "single photon": their arrangement studies at least a two-photon and, if detector 3 is used, a three-photon.

### A Three-Photon

Clearly the concepts of color-entanglement, beam entanglement, and number filtering may be applied to three or more particles. Fig. (11) depicts an arrangement producing a color-entangled three-photon, i.e., the generalization of Fig. (1). Here, the analogs to eqs. (1) through (3) are

$$\Phi(k) = \delta(k - 3k_0), \quad (19)$$

$$\Phi(k_1, k_2, k_3) = \delta(k_1 + k_2 + k_3 - 2k_0) e^{-(k_1 - k_0)^2 / 2\sigma^2} e^{-(k_2 - k_0)^2 / 2\sigma^2} e^{-(k_3 - k_0)^2 / 2\sigma^2}, \quad (20)$$

and

$$\Psi(x_1, x_2, x_3, t_1, t_2, t_3) = e^{ik_0(x_1 - ct_1)} e^{ik_0(x_2 - ct_2)} e^{ik_0(x_3 - ct_3)} e^{-\frac{k_0^2}{6} [(2,1) + (3,2) + (1,3)]},$$

$$\text{where } \{2,1\} \equiv \{(x_2 - ct_2) - (x_1 - ct_1)\}^2, \text{ etc.} \quad (21)$$

Note that the real exponential in this last equation implies remarkable space-time correlations among the three photons.

Fig. (12) depicts a three particle generalization of the Franson interferometer of Fig. (2). From eq. (21) and the eight-term three-photon analog of the state (5), one easily finds that the three photon equivalent of the counting rate (6), in the limit of ideal filters and detectors, is

$$1 + \cos 3k_0 \Delta. \quad (22)$$

In another paper in these proceedings<sup>20</sup>, we also consider other higher-order entanglements: the three photon generalization of the direct-beam interferometer of fig. (6) and of the number filter interferometer of Fig. (9).

Some aspects of three particle interferometry have also been explored by Choi.<sup>21</sup>

## Comments

We conclude with two comments. First, we have not attempted here a comprehensive review of all of the recent demonstrations of two-particle interferometry but have selected enough examples to exhibit the usefulness of entanglement. Consequently many beautiful experiments (and theoretical papers) have not been discussed. However, we have found that our point of view does provide simple, direct, and yet complete descriptions of all the experiments, either via color entanglement, beam entanglement, or a combination of the two. For example, one may imagine that the beams of Fig. (1) are brought together at a beam splitter, that the two filters are not inserted until downstream of the beam splitter and, moreover, that the filters are now centered on different colors,  $k_{10}$  and  $k_{20}$ . This is the arrangement of the "quantum beating" experiment of Ou and Mandel.<sup>22</sup> Our description consists of two steps. First, generalize state (3) to include two different filter colors. Second, superpose two of these generalized states to accommodate the beam entanglement aspect of the arrangement. In this way, one reproduces the key result of Ou and Mandel (their eq. (10)).

Second, note that a single quantum mechanical particle in an elementary plane-wave state has only three adjustable properties: wavenumber, propagation direction, and polarization. Consequently we claim that for two or more particles there are only three basic types of entanglement: wavenumber, propagation direction, and polarization. Clearly color entanglement is just the optical realization of wavenumber entanglement and our beam entanglement is intimately related to propagation direction entanglement. We say "intimately related to" instead of "is" because one must be on guard when idealizing a beam as monodirectional. A beam, unlike a spatially unlimited plane wave, has a finite transverse width and hence can't ever be strictly monodirectional, because of diffraction.

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- \* This work was supported in part by grants from the National Science Foundation, and (for DMG) the City University of New York CUNY-FRAP program .
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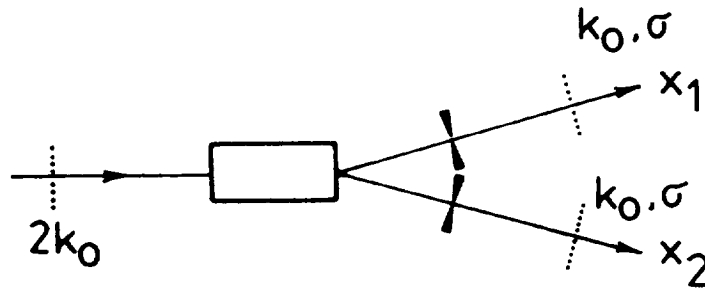


Fig. (1). An incident monochromatic UV photon of wavenumber  $2k_0$  downconverts in a crystal. Two beams of the downconversion radiation are selected by slits and by filters of center  $k_0$  and width  $\sigma$ , thereby preparing the color-entangled two-photon state of eqs. (2) and (3). Consequently, the distribution in time separation,  $\tau$ , of the two photons is given by eq. (4).

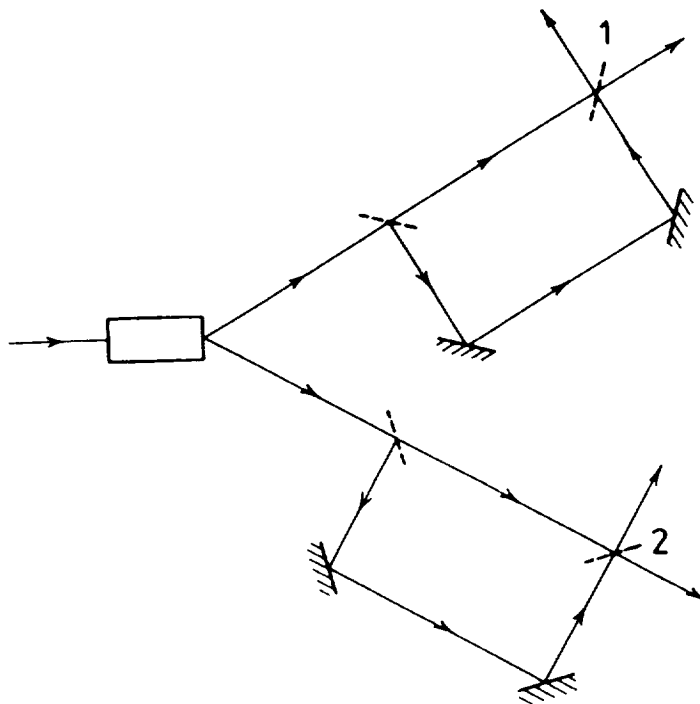


Fig. (2). Franson's two particle interferometer. The two beams of Fig. (1) are each fed into a single-particle interferometer in which one path is  $\Delta$  longer than the other and adjustable. As  $\Delta$  is varied in both interferometers, the coincident count rate in two corresponding outgoing beams (one at station 1 and one at station 2) exhibits the oscillations (two-particle fringes) given by (6).

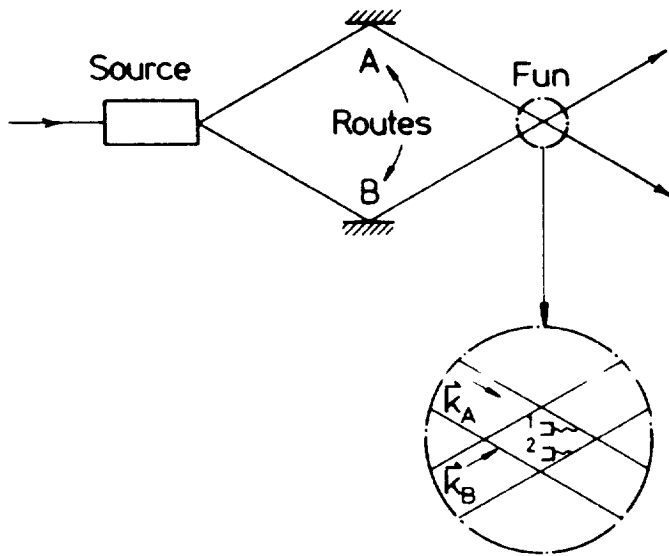


Fig. (3). The crossed-beam two-particle interferometer of Ghosh and Mandel, ref. (12). The beams of Fig. (1) here intersect so that the two-photon falling on the pair of small detectors, 1 and 2, is the beam-entangled state (8), i. e., a superposition of particle (1) in beam A and particle 2 in beam B, and vice-versa. Consequently, the coincident count rate exhibits the two-particle fringes (10);  $r_2$  and  $r_1$  are the positions of the detectors.

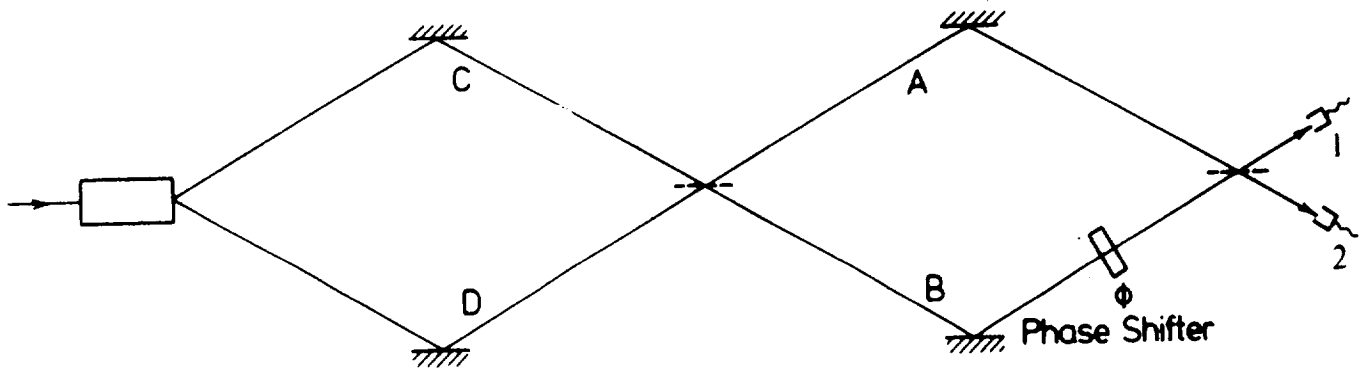


Fig. (4). A two-beam two-particle interferometer of Rarity et al., ref. (13). The beams C and D intersect on the central beam splitter and thereby prepare the beam-entangled state (12). Consequently, the coincident count rate exhibits the two-particle fringes (14), where  $\phi$  is the phase shift imparted by the glass plate in beam B.

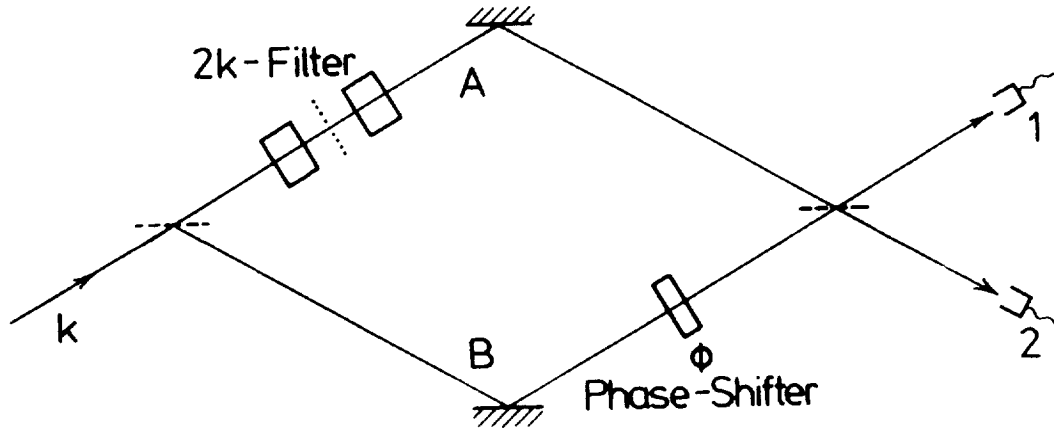


Fig. (5). A two-beam two-particle interferometer employing a number filter. Incident radiation of wavenumber  $k$  can transit beam A only if a two-photon upconverts in the first crystal, passes the filter as a  $2k$  single photon, and then downconverts in the second crystal. Alternatively, the two-photon can take route B. Consequently the state  $12$  is prepared.

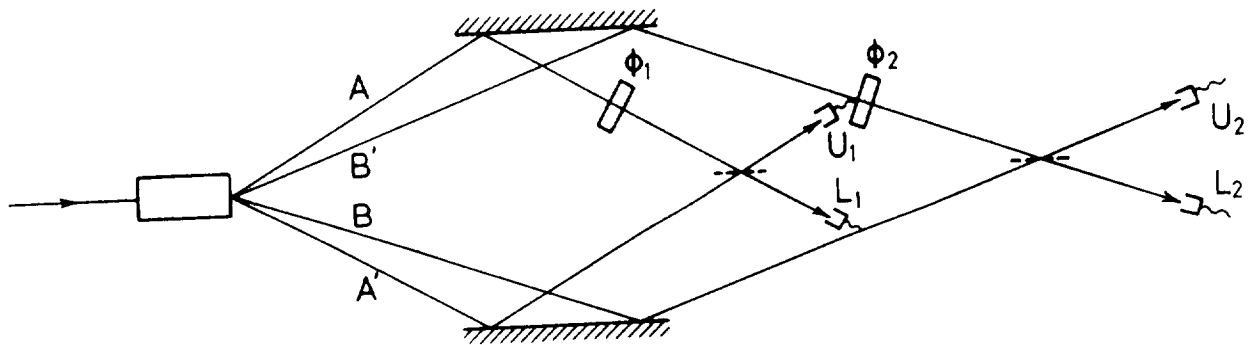


Fig. (6). Preparation of the two-particle beam-entangled state (15) by selecting four direct beams of downconversion radiation.

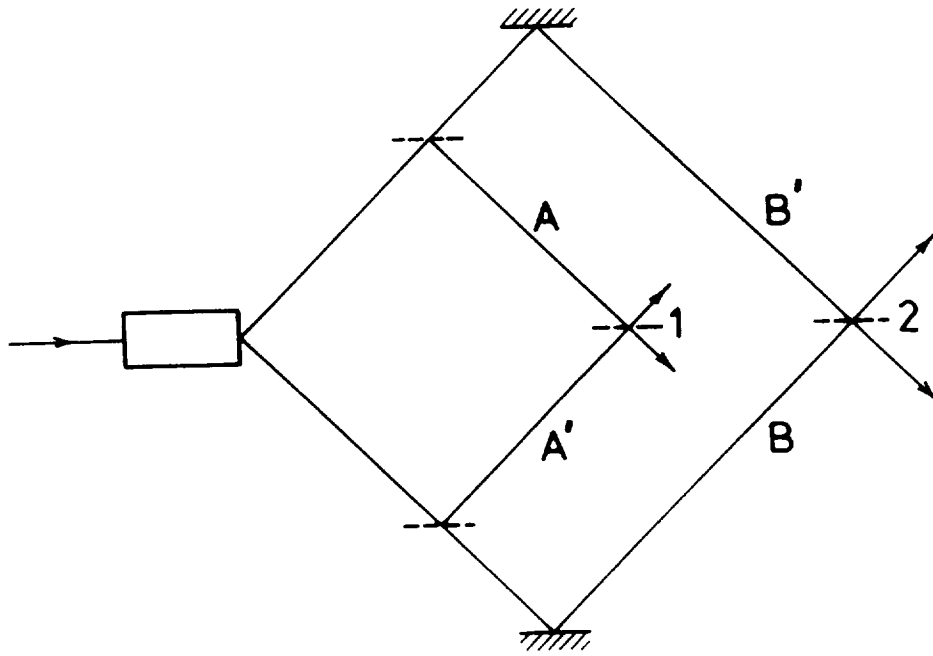


Fig. (7). Preparation of state (15) by first downconverting and then beam splitting.

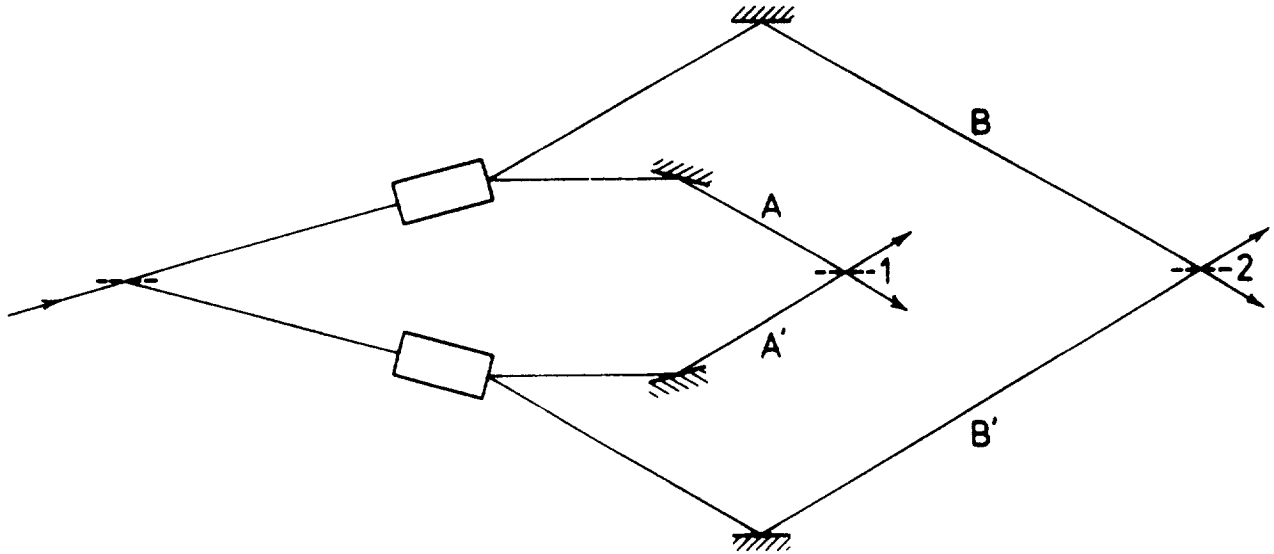


Fig. (8). Preparation of state (15) by first beam splitting and then downconverting.

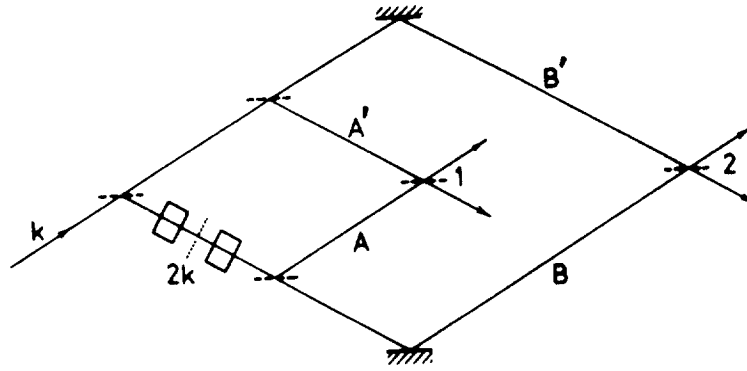


Fig. (9). Preparation of state (15) by using a number filter.

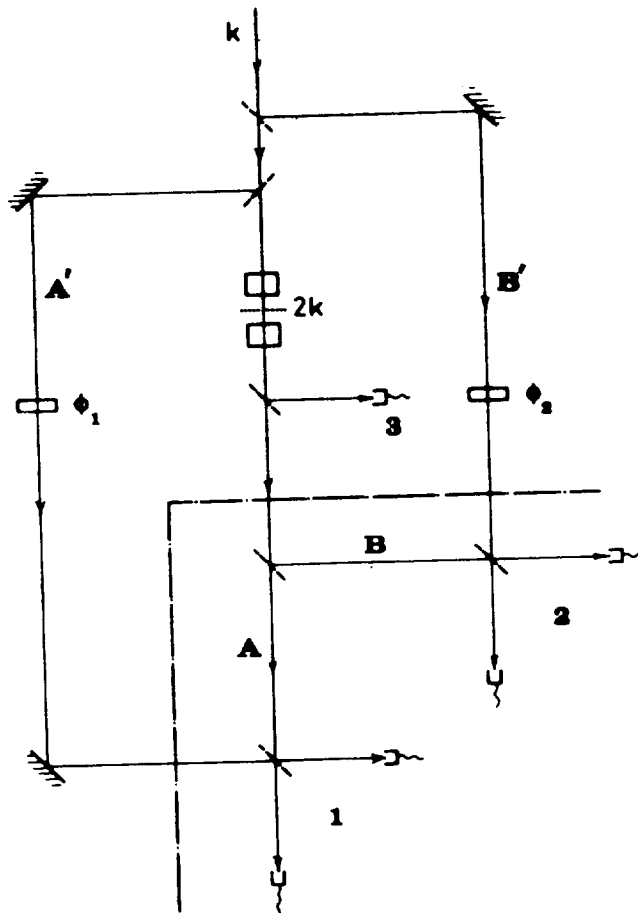


Fig. (10). An arrangement proposed by Tan, et al., ref. (19). If detector 3 is ignored, the state (15) is prepared. If detector 3 is monitored for coincidences with detectors at 1 and 2, the state (17) is prepared.

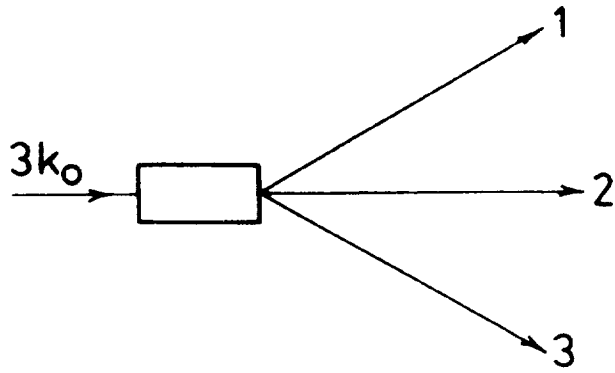


Fig. (11). Three particle downconversion to produce the color-entangled three-photon state (20) and (21). Each outgoing beam contains a filter (not shown) of width  $\sigma$  and centered at  $k_0$ .

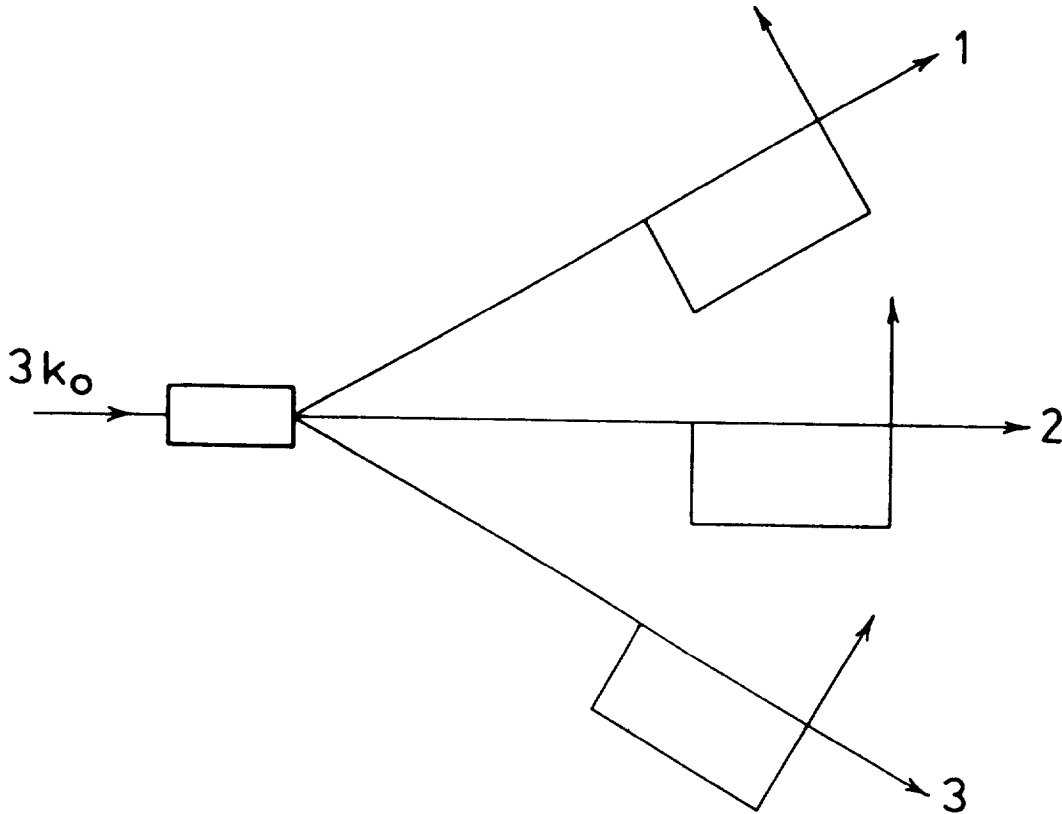


Fig. (12). A three-particle Franson type interferometer. Under ideal conditions the triple coincident count rate at corresponding outgoing beams is given by (22), where  $\Delta$  is the path difference in each of the three branches.