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EQUILIBRIUM TEMPERATURE OF LASER COOLED ATOMS IN SQUEEZED VACUUM

Y. Shevy
Applied Physics, California Institute of Technology, 128-95
Pasadena CA-91125

ABSTRACT:

It is shown that by squeezing the vacuum fluctuations of the electromagnetic field the quantum fluctuations of the optical forces exerted on laser cooled two-level atoms, can be dramatically modified. Under certain conditions, this modification in concert with the enhanced average forces can lead to equilibrium temperatures below those attained under normal vacuum fluctuations.

INTRODUCTION:

Laser cooling of atoms in a quasi-resonant standing laser wave has been attracting considerable attention during the past few years[1]. Another exciting subject has been the modification of the statistical properties of the vacuum fluctuations of the electromagnetic field. Reduction of these fluctuations in one quadrature phase of the field by almost an order of magnitude has been already realized in the laboratory. It is well accepted that the minimum equilibrium temperature of laser cooled two level atoms is determined by the interaction with the vacuum fluctuations of the electromagnetic field.This raises the question whether the equilibrium temperature of two level atoms in squeezed vacuum can be lowered below the normal vacuum level and in particular below the so called "Doppler limit" of $K_bT= \hbar\Gamma/2$ for two level atoms.

In the following, the physical origin of the optical forces in a standing laser wave is described and an intuitive model of the effects in a squeezed vacuum is offered, the modified force in squeezed vacuum is presented and compared to the force in a normal vacuum. In order to find the equilibrium temperature the modification of the fluctuations of these forces in squeezed vacuum is calculated. This calculation show, under certain conditions, a dramatic modification of these fluctuations relative to the normal vacuum state. it is found that, *in an intense standing wave*, the reduced fluctuations in concert with the enhanced average cooling force can lead to equilibrium temperatures below those obtained under normal vacuum fluctuations. Moreover, under certain ideal conditions even sub-Doppler temperatures may be reached. In the *running wave* case, however, the temperature can not be lowered below the normal vacuum level. In addition to being of potential use for laser cooling, these results offer an interesting glimpse into the quantum nature of the momentum exchanges between the atoms and the field.

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A slowly moving atom ($kv<\Gamma$) in a low intensity standing laser light wave experiences a velocity dependent force. This "radiation pressure" force is well understood in terms of absorption and spontaneous emission. As first envisioned by Hänsch and Schawlow[2], the atom experiences an increased absorption of photons from the laser beam which is shifted closer to resonance due to the Doppler effect. This velocity dependent differential absorption can provide a cooling force for laser detuning to the red side of the atomic transition or a heating force for blue detuning. At high intensity, however, stimulated emission can change the sign of the force to a heating force at red detuning and to a cooling force at the blue side of resonance[3-4].

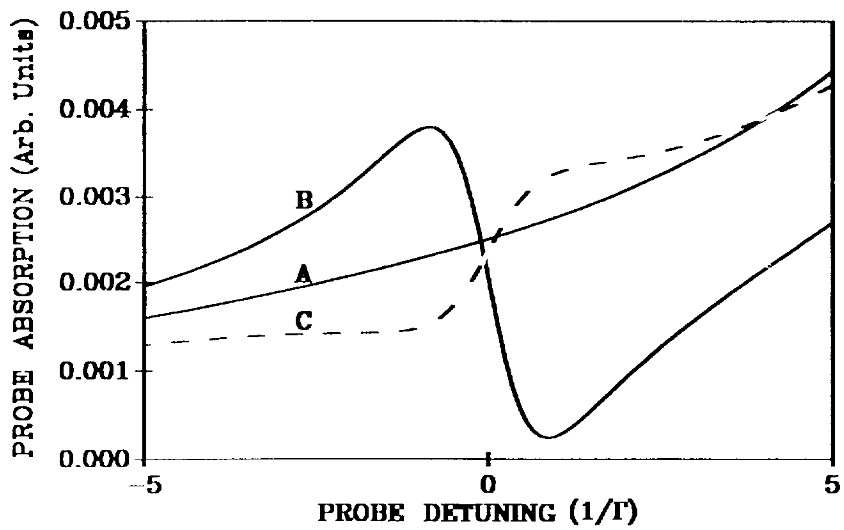

Figure 1: Probe absorption as a function of its detuning from a pump tuned 20Γ to the red of an atomic transition.A)at low pump intensity the probe sees higher absorption at positive detuning closer to the atomic transition B)In normal vacuum at high pump intensity the TWM process is induced leading to less absorption for frequency shifts closer to the atomic transition . C) At the same high intensity as in trace B but in squeezed vacuum the TWM process can change its lineshape leading to an additional cooling force.(Ref. 8 eq. 12 with $N=0.1$, $M=0.33$, $\phi=\pi$ and $\Omega=8\Gamma$).

This stimulated force(or "dipole force") has been explained within the framework of the dressed atom model [5] and equivalently as resulting from Two Wave Mixing[6] (TWM). The TWM resonance appears in pump-probe spectra as a dispersive lineshape (as a function of the probe's detuning from the pump). This feature has a width of the excited state decay rate, Γ, and shows decreased absorption at probe detuning from the pump closer to the atomic transition (see figure 1b). In this process the atom absorbs one photon from one wave and emits a photon into the counter- propagating wave, thus acquiring a momentum kick of $2\hbar k$. This process usually requires high laser intensity ; however, it has been shown to occur at lower intensity when the normal relation between the dipole decay rate Γ_2 and the excited state decay Γ ($\Gamma_2=0.5\ \Gamma$) is modified by the inclusion of phase interrupting events ($\Gamma_2>0.5\ \Gamma$). This effect is due to the appearance of a TWM term at lower order in laser intensity proportional to $[\Gamma_2/\Gamma- 0.5]P$ where P is the saturation parameter. This phenomenon is closely related to the dephasing induced extra resonances in Four Wave Mixing. These resonances , which originally have been studied by Bloembergen and co-workers, are induced whenever the normal decay rates of the the atom are modified. Their relevance to the stimulated force is discussed in more detail in reference 6.

Armed with this insight into the connection between TWM and the stimulated force, it is instructive to find the effect of squeezing on the TWM process. Gardiner[7] has shown that in general squeezing the vacuum fluctuations results in two different decay rates for the two quadratures of the atomic dipole, one of which is larger and the other smaller than the normal $\Gamma/2$ value in ordinary vacuum. Hence, after the decay of the fast decaying quadrature component of the atomic dipole one is left with the slowly decaying component which means that Γ_2 can be much smaller than $0.5\ \Gamma$. One can therefore immediately see that the "extra resonant" TWM process can also be induced in squeezed vacuum. Calculation of the lineshape[8-9] of this process shows that indeed the TWM lineshape becomes phase dependent and can even change to a "dispersive" lineshape with opposite sign (larger absorption closer to the atomic transition) as demonstrated in figure 1c . This indicates that in squeezed vacuum the stimulated force can be induced at low laser intensity . Moreover, it can change sign to provide an additional cooling force instead of heating for red laser detuning from resonance.

THE AVERAGE OPTICAL FORCES IN SQUEEZED VACUUM

The physical system under investigation is a slowly moving two level atom ($\mathbf{k}\ \mathbf{v} << \Gamma$) in either a standing or a running wave (a motionless atom is considered in the fluctuations analysis). The atom is embedded in a broad band squeezed light, so that all of the modes coupled to the atoms are squeezed. The bandwidth of the squeezing is broad enough to appear to the atom as a δ correlated squeezed white noise. The correlation functions for the multi-mode squeezed field can then be written as [7]:

$$\langle b^\dagger(t)b(t')\rangle = \Gamma N\delta(t - t') \quad , \quad \langle b(t)b^\dagger(t')\rangle = \Gamma(N+1)\delta(t - t') \qquad \text{Eq.1}$$

$$\langle b(t)b(t')\rangle = \langle b^\dagger(t)b^\dagger(t')\rangle^* = \Gamma M e^{(-2i\omega_0 t\ +\ 2i\mathbf{k}\cdot\mathbf{r})}\ \delta(t - t')$$

Where $b, b^\dagger$ are the operators defined in terms of the positive and the negative frequency parts of this field, N and M are the squeezing parameters, N is proportional to the number of photons in the squeezed vacuum, while $M \leq N(N+1)$ signifies the amount of correlation between the sidebands and the equality maximum squeezing. In the following we will choose M to be real and positive .

The Hamiltonian describing the interaction of the atom with the quantized multimode radiation field and a classical coherent field is given in the electric-dipole and rotating-wave approximation by [10]: Eq.2

$$H = \frac{1}{2}\hbar\omega_0\sigma_{22} + H_{rad} - \left(\mu E_0 e^{-i\omega t}\sigma^\dagger + \sigma\mu^* E_0^* e^{+i\omega t}\right) + \hbar\left(\sigma^\dagger b + b^\dagger\sigma\right)$$

where ω_0 is the atomic resonance frequency, σ_{22} , $\sigma=\sigma_{12}$ and $\sigma^\dagger=\sigma_{21}$ are the atomic operators , μ is the atomic dipole moment , H_0 is the free Hamiltonian of the field, and E_0 is the amplitude of the coherent field. One can then find the master equation for an atom in squeezed vacuum and derive equations of motion for the atomic operators whose expectation value is given by:

$$< \dot{\sigma}_{12} > = - \gamma < \sigma_{12} > - \Gamma M < \sigma_{12} >^* + \Omega < D > \qquad \text{eq.3}$$

$$< \dot{D} > = - \Gamma(2N+1) < D > + \Gamma - 2[\Omega^* < \sigma_{12} > + \Omega < \sigma_{12} >^*]$$

The average force can now be found by calculating, the expectation value of the atomic variables and subsequently the first order corrections due to the atomic motion [3]. This gives rise to the following expressions[11] for the expectation value of the optical forces acting on the atom in the running$\langle F\rangle$ and standing wave$\langle F^s\rangle$:

$$\langle F\rangle = \frac{\hbar k\Gamma P}{2(2N+1+P)}\left[1+\frac{2(2N+1)\Delta}{\chi(2N+1+P)}(\mathbf{k}\cdot\mathbf{v})\right] \qquad \text{Eq.4}$$

$$\langle F^s\rangle = -\frac{\alpha\hbar\Lambda P}{\Phi\_(2N+1+P)}\left[1-\frac{4\chi M\cos(2\phi)P+\Gamma_1^2\Phi\_(2N+1-P)-2\chi P^2}{\Gamma\chi\Phi\_(2N+1+P)}(\alpha\cdot\mathbf{v})\right]$$

where P is the modified saturation parameter in squeezed vacuum given by $P= 2|\Omega|^2\Phi\_/\chi$, $\Omega=e^{i\phi}\mu E/\hbar$ and the other quantities are defined by: $\Gamma_1=(2N+1)\Gamma$, $\Phi\_ = 2N+1 - 2M\cos(2\phi)$, $\chi = |\gamma|^2 - \Gamma^2M^2$, $\gamma=\Gamma_1/2 - i\Delta$, $\Lambda = \Delta + \Gamma M\sin(2\phi)$, where Δ is the laser detuning from the atomic resonance.

It is instructive to examine the new expression of the force in the standing wave by comparing it to the force in ordinary vacuum. In this limit (N =M=0) the force is reduced to the well known expression of the force (ref.3 eq.18) given by:

$$\langle\vec{F}\rangle = -\vec{\alpha}\hbar\Delta \frac{P}{1+P}\left[1 - \frac{\Gamma^2(1-P) - 2|\gamma|^2P^2}{\Gamma(1+P)2|\gamma|^2}\vec{\alpha}\cdot\vec{v}\right] \qquad \text{eq.5}$$

Note that in this limit the first term in the numerator of the velocity dependent part of eq.4 is zero while the other two terms are reduced to those of eq.5. The striking appearance of the additional term in squeezed vacuum is analogous to the result of ref.6. In this case, the introduction of classical phase noise results in the appearance of an extra term $-4|\gamma|2[\Gamma 2/\Gamma - 0.5]P$, ($\Gamma_2=\Gamma/2+\Gamma\phi$ where $\Gamma\phi$ is the rate of the phase interrupting events).This term can give the stimulated force at lower intensity when $\Gamma_2 / \Gamma>0.5$ as phase noise is added.

Notice that in the case of $\Gamma_2 / \Gamma< 0.5$, this term can also be induced but with opposite sign. This is indeed the case with quadrature squeezing, which can result in either larger or smaller phase noise than the vacuum level.This in turn introduces two different decay rates for the two quadratures of the atomic dipole.One of these, $\Gamma_{2\,x}= \Gamma(N+M+0.5)$, is larger and the other, $\Gamma_{2\,x}=\Gamma(N - M+0.5)$, is smaller than the normal $\Gamma_2=\Gamma/2$ value. Therefore, the sign of the extra term in eq.2 $4[|\gamma|2-\Gamma^2M^2]M \cos(2\phi) P$ can be controlled by the relative phase ϕ of the driving field with respect to the squeezed vacuum. Hence, the stimulated force can not only occur at lower laser intensity, it can change sign to provide an additional cooling force at red laser detuning from resonance.This modification of the force can be further correlated with the TWM lineshape which becomes strongly dependent on the laser phase ϕ and can even change sign as indicated by our intuitive analysis .

Other important modifications of the force in squeezed vacuum are described by the term, $\Delta+\Gamma M\sin 2\phi$. This term gives rise to *a force at zero detuning* as well as strong variations of the force at small detuning ($\Delta<\Gamma M\sin 2\phi$). These effects can be understood by noting that the dephasing induced lineshape of TWM at resonance is absorptive in normal vacuum, but it can be transformed to a dispersive lineshape in squeezed vacuum, giving rise to a force at resonance . In addition it has been shown that the TWM can have sub-natural linewidth at small detuning [7-10]. This indicates that one can obtain arbitrarily large cooling forces at small detuning as the number of photons in the squeezed vacuum N, and therefore the amount of squeezing , is increased. This can be understood by noting that $\Gamma_{2y}=\Gamma$ (N - M+0.5) in the limit of N>>1 becomes arbitrarily small, $\Gamma_{2y} = \Gamma / 8N$.

In the analytic solution shown above the force is calculated only to first order in velocity (i.e a linear velocity dependence is assumed), this is correct only for small velocities $kv \ll \Gamma$. Numerical solution of the OBE, however, can provide the full velocity dependence of the force. This solution is shown in figure 2 for ordinary (trace A)and squeezed vacuum (trace B) . This figure demonstrates that the stimulated force which gives a heating force in normal vacuum for velocities in the order of $kv<\Gamma/2$ (as expected from the TWM lineshape, fig.1b), can be transformed to a cooling force in squeezed vacuum. The dashed lines in the figure are the results of the spatially averaged analytic solution which show good agreement with the numerical solution at small velocities.

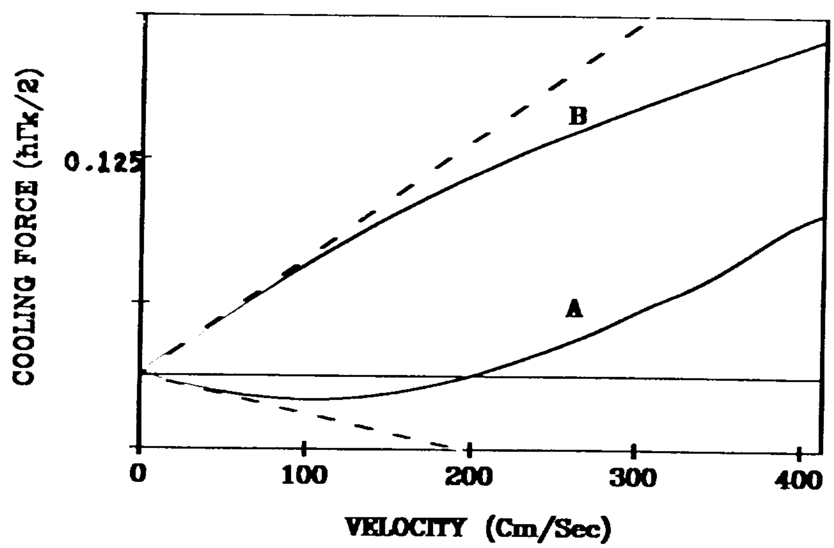

Figure 2: The velocity dependence of the spatially averaged force, in normal vacuum trace A and squeezed vacuum trace B, obtained by numerical solution of the OBE. The dashed lines are the result of the analytic solution. The parameters used for this figure are: $\Delta=-3\Gamma$, $\Omega=1.36\Gamma$, $\Gamma=10^7$ Hz and $\lambda=5890$ A for both traces and N=1., $M=\sqrt{2}$ and $\phi=0$ for trace B.

Figure 3 demonstrate the interesting dependence of the force on the driving laser phase ϕ (using the analytic solution eq.2 with $kv=\Gamma/2$).This is shown for a constant number of photons in the squeezed vacuum, N=1, but for various values of the squeezing parameter M. Trace A plots the force for thermal light M=0 (i.e no correlation between the sidebands) with no variation on the phase, as expected. Traces B-D , however, show large variations of the force for increasing degree of squeezing up to the maximum value of M, ($M^2=N[N+1]$). This dependence is due to the different amount of noise that the induced dipole sees at different quadrature phase. Figure 3 also shows that even a modest amount of squeezing induce large effects on the force.

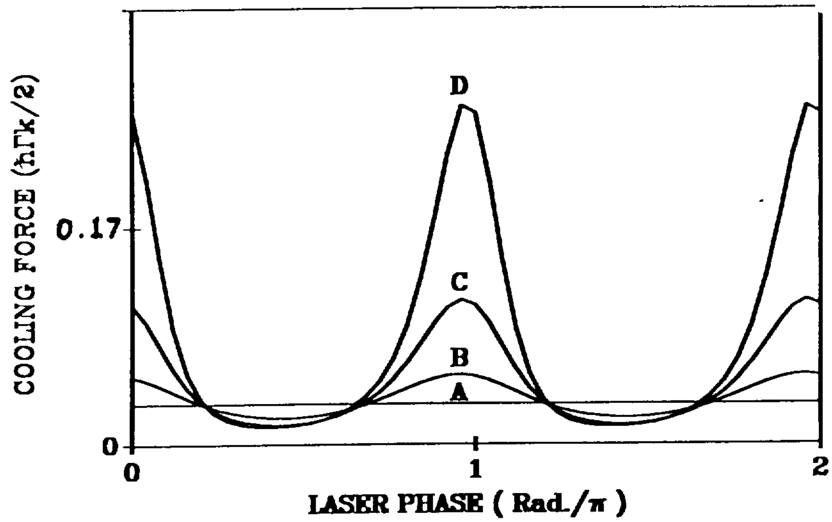

Figure 3: The spatially averaged force as a function of the laser phase ϕ for increasing amount of correlation between the sidebands A)M=0 (thermal light no correlation), B) M=0.5 , C)M=1 and D)M=$\sqrt{2}$ (maximum squeezing). Other common parameters used are: $\Delta=-3\Gamma$, $\Omega=1.5\Gamma$ and N=1.

THE QUANTUM FLUCTUATIONS

It was recognized by Einstein [12] as early as 1917 that the fluctuations of the optical force, originating from both spontaneous and induced absorption and emission processes, are important in determining the Maxwellian distribution of the atomic velocity in thermal equilibrium. A simple momentum diffusion process which comes readily to mind is due to the random direction of the spontaneous emission recoils . In addition to this geometrical source of fluctuations one should also consider the fluctuations in the number of photons emitted in a unit time . However, this process can have sub Poisson statistics, as shown by Mandel [13] in resonance fluorescence, and give rise to an anomalous contribution [3,14] which can decrease the momentum spread, as discussed by Cook [14]. An additional momentum diffusion mechanism becomes dominant at high intensities in a standing wave due to fluctuations in the stimulated emission process between the counter-propagating waves [3,14,5]. This photon exchange between the waves, results in a random transfer of $2\hbar k$ units of momentum to the atom. Finally as shown by Gordon and Ashkin [3] an atom even in its ground state can have random recoils due to the zero point vacuum fluctuations. In the following it will be shown that the dynamics in squeezed vacuum modifies the fluctuations of all of these processes.

We are now interested in finding the force fluctuations on a *stationary* atom which are given by the diffusion constant , 2Dp :

$$2D_p = 2\,\mathrm{Re}\int_0^{\infty} dt\,[<\vec{F}(0)\,\vec{F}(t)> - <\vec{F}>^2\,] \qquad \text{Eq.6}$$

Insertion of $\vec{F}(t) = -\iota\sigma\vec{\nabla}G$ + H.c. for the force (where G is the freely propagating field), using the correlation functions for squeezed vacuum,$<G>=\Omega$ (since$<b>=0$) and the commutation relation $\sigma_{ij}\,\sigma_{kl}=\sigma_{il}\,\delta(j,k)$ for the atomic operators gives :

$$\langle\vec{F}(0)\vec{F}(t)\rangle = \hbar^2\Big[\langle\sigma^{\dagger}(0)\sigma(t) + \sigma(0)\sigma^{\dagger}(t)\rangle\,|(\vec{\nabla}\Omega)^2| - \langle\sigma^{\dagger}(0)\sigma^{\dagger}(t)\rangle(\vec{\nabla}\Omega)^2 - \langle\sigma(0)\sigma(t)\rangle(\vec{\nabla}\Omega^*)^2\Big] \qquad \text{Eq.7}$$

$$+(\hbar k)^2\Big[\,\Gamma N\,\langle\sigma_{11} + \sigma_{22}\rangle + \Gamma\langle\sigma_{22}\rangle\Big]\,\delta(t)$$

Consider first the last terms which depend directly on the quantum fluctuations of the field. These terms in the limit of normal vacuum(N=0) can be transparently modeled [3] as the random instantaneous emissions of momenta $\hbar k$ at an average rate of $\Gamma<\sigma_{22}>$. In the squeezed vacuum case, this effect is enhanced, by the absorption of squeezed photons $\Gamma N<\sigma_{11}>$ and spontaneous emission $\Gamma N<\sigma_{22}>$ due to the larger number of photons in the squeezed vacuum .

In order to evaluate the remaining terms , which describe the effects of the interaction of the coherent field gradient with the atomic dipole fluctuations,we need to find the integral of the atomic dipole autocorrelation functions which after some algebra leads to the following expression for the diffusion in squeezed vacuum:

$$2D_p = D_0 + \hbar^2\beta^2\frac{\Gamma_1}{2}\,\frac{P}{(2N+1+P)}\big[1 + D_1\big] + \hbar^2\alpha^2\Gamma_1\frac{\Phi_+}{2\Phi_-}\,\frac{P}{(2N+1+P)}\big[1 + D_1^s + D_2^s + D_3^s\big] \qquad \text{Eq.8}$$

Where $\alpha=0$, $\beta=\mathbf{k}$ in a running wave; and $\alpha=-\mathbf{k}\tan(\mathbf{k}\ \mathbf{x})$, $\beta=0$ in a standing wave while the other terms are defined by : $\Phi_+ = 2N+1 + 2M\cos(2\phi)$,

$$D_0=\hbar^2k^2\frac{\Gamma}{2}\left[\frac{2N+P}{(2N+1+P)} + 2N\right] , \qquad D_1=\frac{P}{(2N+1+P)^2}\left[\frac{\Phi_+}{(2N+1)\Phi_-} - \frac{\Gamma_1\Gamma}{\chi}\right]$$

and the standing wave terms;

$$D_1^s = \frac{4P}{(2N+1+P)^2\Phi_-\Phi_+}\left[\frac{(2N+1)\chi}{\Gamma^2} - \frac{\Lambda^2}{\Gamma\Gamma_1}(1+\frac{\Gamma_1^2}{\chi})\right] \quad , \quad D_2^s = \frac{8P^2}{(2N+1+P)^2\Phi_-\Phi_+}\left[\frac{\chi}{\Gamma^2} - \frac{\Lambda^2}{\Gamma\Gamma_1\Phi_-}\right]$$

$$D_3^s = \frac{4\chi}{\Gamma\Gamma_1\Phi_-\Phi_+}\frac{P^3}{(2N+1+P)^2} .$$

In the limit of normal vacuum (N=M=0) Eq.8 corresponds to the the results of ref.3 eq. 30

$$2D_p=\hbar^2\beta^2\Gamma\frac{P}{2(1+P)}\left[1+\frac{P}{(1+P)^2}\left(1-\frac{\Gamma^2}{|\gamma|^2}\right)\right] + \hbar^2\alpha^2\Gamma\frac{P}{2(1+P)}\left[1+\frac{P}{(1+P)^2}\left(\frac{\Gamma^2}{|\gamma|^2}-3\right)+\frac{2P^2}{(1+P)^2}+\frac{4|\gamma|^2}{\Gamma^2}\frac{P^3}{(1+P)^2}\right]$$

$$+ \hbar^2k^2\Gamma\frac{P}{2(1+P)}$$

Let us first examine the terms in the running wave case by associating them with the normal vacuum limit. As we discussed previously spontaneous emission in squeezed vacuum, represented by the diffusion term D_0, gives rise to increased fluctuations as a consequence of the increased number of photons in this state. However, as can be clearly seen, the D_1 induced absorption contribution, even in the normal vacuum limit, can reduce the spread. This term has been shown to originate from sub Poisson statistics of the emitted photons. In fact in the normal vacuum limit D_1 coincides exactly with Mandel's Q parameter [13,14] :

$$Q=\frac{\langle(\Delta n)^2\rangle - \langle n\rangle}{\langle n\rangle}$$

Q=0 indicates a variance of $<n>^{1/2}$ in the number of the emitted photons(i.e no correlation between the photons) and negative Q sub-Poisson statistics. Figure 4 plots D_1 as a function of the laser detuning. In squeezed vacuum , with an appropriate phase between the laser and the squeezed vacuum, D_1 can reach a value of -1, whereas in normal vacuum the maximum effect gives D_1= -3/4. This indicates that in squeezed vacuum the photons can be emitted in an orderly manner,thus eliminating this source of fluctuations. Unfortunately, in the traveling wave case, one can not take advantage of this phenomenon due to the increased spontaneous emission term D_0. This is demonstrated in figure 5a, which shows that the equilibrium temperature, given by $K_bT= D_p/(-\partial_v<F>)$, increases with the amount of squeezing.

We now turn our attention to the more complicated case of the standing wave, as in the running wave case, we still have enhanced fluctuations due to the larger spontaneous emission term, D_0. However, the much larger average cooling force, in conjunction with the smaller fluctuations, of the higher order terms, make it possible to reach temperatures lower than otherwise obtained in normal vacuum. Unfortunately, one can not take full advantage of both of these effects

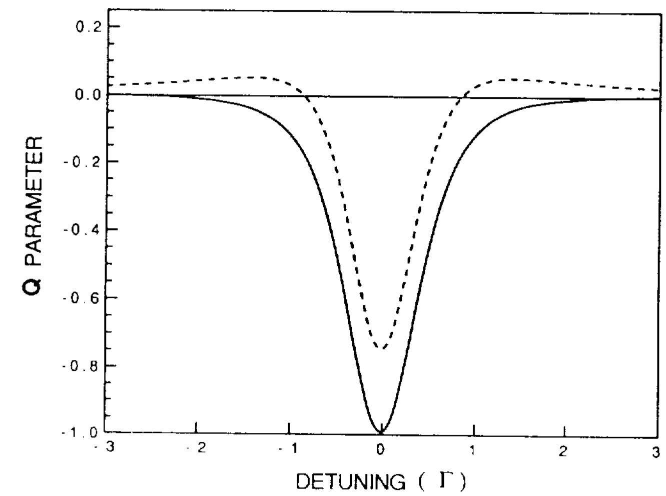

Figure 4: The deviation from Poisson statistics as a function of the laser detuning from resonance.The dashed line shows the maximum effect in normal vacuum , while the solid line indicates that almost no spread in the number of photons emitted in a unit time in squeezed vacuum can be achieved. The parameters used are Ω=0.35Γ in the normal vacuum while Ω=0.26Γ, N=2, M^2=N(N+1) and ϕ = 0.5 π in the squeezed vacuum.

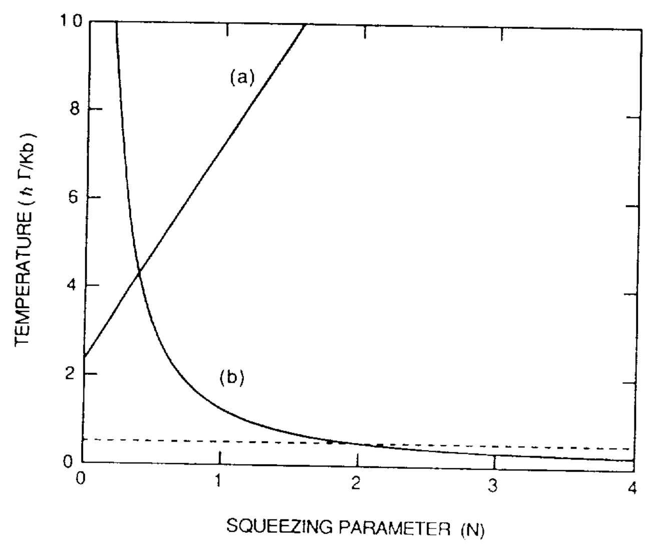

Figure 5: The equilibrium temperature as a function of the squeezing parameter N in: a) running coherent laser cooling wave with Δ=0.5Γ, Ω=Γ , ϕ = 0.25 π. b) a standing wave with Δ=0, Ω=5Γ , ϕ = 0.6 π .The dashed line is the Doppler limit temperature in normal vacuum.

at the same laser phase. Nevertheless, one can choose a particular phase which will minimize the temperature as the squeezing increases. Figure 5b demonstrates the lowering of the equilibrium temperature below the normal vacuum level (N=0) as the amount of squeezing increases.

Moreover, the temperature can be decreased to values slightly below the Doppler limit (which in normal vacuum is achieved at low intensity). Lowering the equilibrium temperature is not the only benefit of squeezing, in the above example, the average force becomes larger than the maximum value in the normal vacuum giving rise to a shorter equilibrium time. Notice, however, that high degree of squeezing is needed in order to reach sub-Doppler temperature, in addition , degradation from ideal squeezing ($M^2<N(N+1)$) results in a temperatures higher than the Doppler limit. This unfortunately makes the experimental demonstration of this effect rather difficult.

A few words are now in order to get some insight into the modification of the fluctuations in the standing wave case. We first discuss the various terms in the *normal vacuum* case. The most notable difference from the running wave is the appearance of higher order terms in P. These terms were interpreted as resulting from the fluctuations of the dipole force [3,5, 14] and become important at high laser intensity. In particular the P^3 term is the only term that does not saturate at high P. Hence, although one can use the stimulated force in normal vacuum at the blue side of resonance to give a very large average cooling force(with the advantage of very short equilibrium time) the large fluctuations make the equilibrium temperature much higher than the Doppler limit [5].

With regard to the modification of these processes in squeeze vacuum, we begin by comparing the D_1^s term to its counterpart in normal vacuum. As discussed in the running wave case, this term can be interpreted as the deviation of the fluctuations from Poisson statistics. We found that the modified D_1^s in squeezed vacuum can reach values close to -1. However, the behavior in a standing wave is quite different from the running wave, as is the case in the normal vacuum .

The next term, D_2^s , is not present in a running wave and we assume that it describes the fluctuations of the lower order stimulated force. In a previous publication [6], it was shown that while the velocity dependent stimulated force does not usually occur at low intensity, the inclusion of phase interrupting events (which increase the dipole decay rate), gives rise to a stimulated force term at lower order in laser intensity. In order to show that this identification is correct and to get some insight into this term D_2^s was calculated in normal vacuum but with increased dipole decay $\Gamma_2=\Gamma/2+\Gamma_\phi$, where Γ_ϕ is the rate of phase interrupting events, which gives:

$$D_2^s = \frac{2P'^2}{(1+P')^2}\left[\left(\frac{2\Gamma_2}{\Gamma} -1\right)\left(\frac{\Delta}{\Gamma_2}\right)^2 + 2\frac{\Gamma_2}{\Gamma}\right]$$

As can be seen an additional diffusion occurs as phase noise is added , $\Gamma_2 >\Gamma/2$. Note, that when $\Gamma_2<\Gamma/2$, this term becomes negative. Analogously, D_2^s in squeezed vacuum may be associated with the fluctuations of the extra stimulated force in squeezed vacuum (the first term in the average force in squeezed vacuum , $<F^S>$. Moreover, the fact that one of the dipole's quadrature components can decay at a rate smaller than $\Gamma/2$ suggests that D_2^s in squeezed vacuum can become negative, as indeed is the case.

Finally we turn our attention to the highest order term in P , D_3^s, which is associated with the fluctuations of the normal stimulated force.The modification of this diffusion term is critical for achieving lower temperature at high laser intensity where the stimulated force becomes dominant. Figure 6 shows the spatially averaged total diffusion constant 2Dp, in a high intensity standing wave, as a function of the laser phase, in normal and squeezed vacuum.This comparison demonstrates the dramatically reduced fluctuations in a high intensity standing wave under squeezed vacuum conditions.

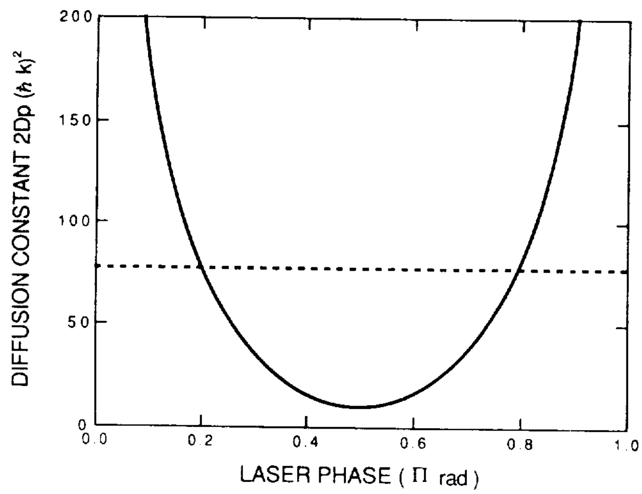

Figure 6: The diffusion constant in a high intensity standing wave as a function of the laser phase in the case of normal vacuum (dashed line) and squeezed vacuum(solid line) .The parameters used are: $\Delta=0$, $\Omega=5\Gamma$, N=5.

As to the experimental verification of these interesting phenomena. Although 90% squeezing has already been achieved in the laboratory, it is important to note, that the calculation presented here is carried out with the assumption that the atom is embedded in squeezed vacuum. In practice, the output of present sources of squeezed light (degenerate parametric oscillators) can couple only to part of the 4π steradians enveloping the atom. A possible solution to this problem has been proposed by Gardiner who suggested coupling the squeezed modes to the atom in a micro cavity. The other important assumption here is that the spectrum of the squeezing is much broader than that of the atomic transition . Theories which include finite bandwidth of squeezing[15] have shown that the essential features due to squeezing are preserved, even for a bandwidth of squeezing only a few times larger than the width of the atomic transition. It should also be noted that it will be interesting to develop the theory with bandwidth of squeezing larger than Γ but smaller than the Mollow sidebands separation . This can introduce the possibility of controlling all three of the decay rates of the atom and therefore might reduce the problem the diffusion due to of the higher rate of spontaneous emission in a broad band squeezed light [16].

In conclusion, this paper demonstrates a dramatic modification of the quantum fluctuations of the mechanical effects of light on atoms which are also embedded in squeezed vacuum. In the running wave case the temperature can not be lowered below the normal vacuum level. However, in conjunction with the modified average cooling force even sub-Doppler temperatures may be reached, under ideal conditions, for atoms cooled in a standing wave. These interesting results, in addition for being of potential use, offer some insight into the quantum statistics of the photon exchanges between the atom and the field under squeezed vacuum conditions. Further investigation of other schemes of cooling with squeezed light might also be beneficial

REFERENCES:

1) For recent advances in this field see JOSA B November (1989) .
2) T. W. Hansch and A. L. Schawlow, Opt. Commun. 13, 68 (1975).
3) J. P. Gordon and A. Ashkin , Phys. Rev. A21 ,1606 (1980).
4) A. Aspect et al, Phys. Rev. Lett., 57, 1688 (1986).
5) J. Dalibard and C. Cohen-Tannoudji, JOSA B , 2, 1707, (1985)
6) Y. Shevy, Phys. Rev. A 41, 5229, (1990).
7) C. W. Gardiner, Phys. Rev. Lett., 56, 1917, (1986).
8) H. Ritsch and P. Zoller , Opt. Commun. 64, 523 (1987).
9) S. An and M. Sargent III, Phys. Rev. A39 , 3998 (1989).
10) H. J. Carmichael, et al, Phys. Rev. Lett., 58, 2539, (1987).
11) Y. Shevy, Phys. Rev. Lett., 64, 2905 (1990).
12) A. Einstein, Phys. Z.18, 121, (1917).
13) L. Mandel, Optics Lett., 4, 205, (1979).
14) R. J. Cook, Phys. Rev. A 22,1078, (1980); Optics Commun., 35, 347, (1980), S. Stenholm, Phys. Rev. A 27, 2513, (1983).
15) A. S. Parkins and C. W. Gardiner, Phys Rev. A37 , 3867 (1988); H. Ritsche and P.Zoller Phys. Rev. Lett. 61,1098, (1988).
16)A. S. Parkins, Phys. Rev. A 42, 4352, (1990).
17) The magnitude of the force is different from Ref. 11 due to an error by a factor of 5.89