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STATISTICAL PROPERTIES OF SQUEEZED BEAMS OF LIGHT GENERATED IN PARAMETRIC INTERACTIONS*

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ABSTRACT

Fluctuation properties of squeezed photon beams generated in three wave mixing processes such as second harmonic generation, degenerate and nondegenerate parametric oscillations, and homodyne detection are studied in terms of photon sequences recorded by a photodetector.

Photon number fluctuations and photon number correlations are fundamental properties of a light beam. These properties are different for different light sources and can be used to characterize photon beams. In this short communication we discuss our work on statistical properties of squeezed photon beams generated in three wave interaction processes in terms of counting and waiting time distributions. We summarize some of the interesting results obtained for these systems. Processes that we consider here include second harmonic generation, and degenerate and nondegenerate parametric down conversion (DPO and NDPO). Squeezed state of light have been realized in these systems experimentally (Ref.1). Homodyne statistics when squeezed light produced by the DPO is mixed with coherent light from a local oscillator are also discussed. A dynamical model for these beams is used and photon sequences recorded by a photodetector are calculated.

We use positive-P representation (Ref.2) to map quantum mechanical equations of motion for the annihilation and creation operators onto a set of C-number stochastic equations for the complex field amplitudes. Using simple transformation of field variables it can be shown that the field produced in these processes can be described in terms of independent real Gaussian stochastic processes (Ref.3-4).

We use a generating function technique to obtain the statistics of the photons emitted by these light sources. The generating function $G(s,t,T)$ for the photon statistics measured by a detector with a parameter s is given by (Ref.5)

$$G(s,t,T) = \langle e^{-s\eta \int_t^{t+T} I(t')dt'} \rangle . \qquad (1)$$

Here η is detector efficiency and $I(t)$ is photon flux emitted by the source. Generating function $G(1,t,T)$ is simply the probability of detecting no photon in the time interval t to $t+T$. In order to obtain generating function we express $I(t)$ in terms of the c-number field variables. Statistical averaging is performed by making Karhunen Loéve expansion of the field variables in terms of a set of orthogonal functions. Following the method developed in our earlier investigations (Ref.3) we derive an analytical expression for the generating function $G(s,t,T)$ for the photon statistics. From this generating function various statistical quantities such as factorial moments, photon counting and waiting time distributions can be obtained.

The photon counting distribution $p(m,t,T)$ is the probability of counting m photons in the time interval t to $t+T$. It can be obtained from the generating function by using the relationship

$$p(m,t,T) = \frac{(-1)^m}{m!}\left[\frac{d^m}{ds^m}G(s,t,T)\right]_{s=1} . \qquad (2)$$

The waiting time distribution $w(t,T)$ is the probability density for two successive photoelectrons to be separated by the time interval T and it is given by

$$w(t,T=t'-t) = -(\eta I(t))^{-1}\frac{d^2}{dtdt'}G(1,t,t'-t) . \qquad (3)$$

In the stationary regime these quantities are independent of initial time t. Here we only summarize photon statistics only in the steady state regime. The field from the DPO can be expressed in terms of two independent Gaussian random variables with mean zero and different variances

$$< u_i(t)u_j(t+T) >= \frac{1}{4}\frac{|\kappa\epsilon|}{\lambda_i}\delta_{ij}\, e^{-\lambda_i T} . \qquad (4)$$

Here κ is the mode coupling constant, and ϵ is the dimensionless amplitude of the pump beam incident on the cavity. The decay constants λ_1 and λ_2 are given by

$$\lambda_1 = (\gamma - |\kappa\epsilon|), \qquad \lambda_2 = (\gamma + |\kappa\epsilon|) . \qquad (5)$$

Here $(1/2\gamma)$ is the cavity lifetime. Below threshold λ_1 and λ_2 are always positive. Using the properties of field variables u_1 and u_2 we obtain the generating function for the DPO as (Ref.3)

$$G(s,T) = Q_1(s,T)Q_2(s,T) , \qquad (6)$$

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where

$$Q_i(s,t,T) = \left[\frac{e^{\lambda_i T}}{[\cosh(z_i T) + f_i(t)\sinh(z_i T)]}\right]^{\ell}, \quad (7)$$

with

$$f_i(t) = \frac{1}{2}\left(\frac{\lambda_i}{z_i} + \frac{z_i}{\lambda_i}\right), \quad (8)$$

and

$$\begin{aligned} z_1^2 &= \lambda_1^2 + 2s\eta\gamma\kappa\epsilon, \\ z_2^2 &= \lambda_2^2 - 2s\eta\gamma\kappa\epsilon. \end{aligned} \quad (9)$$

Mean photon number inside the cavity is given by

$$\bar{n} = \frac{1}{2}\left(\frac{|\kappa\epsilon|^2}{\gamma^2 - |\kappa\epsilon|^2}\right). \quad (10)$$

For the DPO ℓ is equal to half. Once the generating function is known photon counting and waiting time distributions are obtained from Eqs. (2) and (3). For small mean photon number and short counting time $p(m,T)$ decreases monotonically.

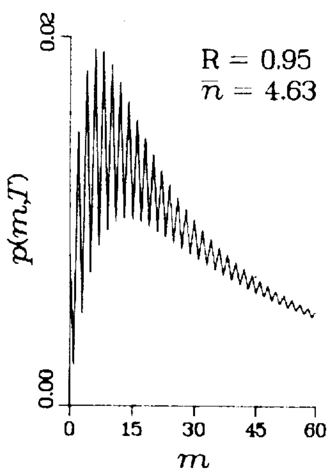

Figure 1a. Photon counting probability distribution for the DPO for $R = 0.95$, unit efficiency, and counting time interval $2\gamma T = 20$.

For long counting times $p(m,T)$ shows sharp even-odd oscillations implying that the probability of detecting odd number of photons is much smaller than the probability of detecting even number of photons. As the mean photon number $\bar{n}$ is increased these even-odd oscillations become smaller. Figure (1a) shows $p(m,T)$ for $R = \kappa\epsilon/\gamma = 0.95$ near threshold corresponding to $\bar{n} = 4.63$. These curves are meaningful only for integer values of m. We see that near threshold even-odd oscillations become less pronounced and $p(m,T)$ develops a long tail. We have also studied photon statistics for the DPO in the transient regime, that is, during its evolution from vacuum state to the steady state (Ref.4). For small transient time even-odd oscillations in photon counting distributions are even sharper than the even-odd oscillations in the stationary regime.

With the degenerate modes of a parametric oscillator, nondegenerate modes are also present. We consider the nondegenerate modes of parametric oscillator for which two nondegenerate photons have the same frequencies. These fields can be expressed in terms of four real random Gaussian variables. Here we discuss two cases, one in which amplitudes of the two nondegenerate modes are homodyned and second in which intensities of the two modes are added together. For a given pump strength the NDPO the mean photon number is much smaller than the mean photon number for the DPO. Mathematical expression for the generating function for the first case is similar to the generating function for the DPO. Thus $p(m,T)$ for the NDPO also shows even-odd oscillations. However, for the same pump strength, even-odd oscillations in the NDPO are sharper then the oscillations for the DPO. They are centered towards smaller values of m. The difference between the DPO and the NDPO lies mainly in the value of the mean photon number.

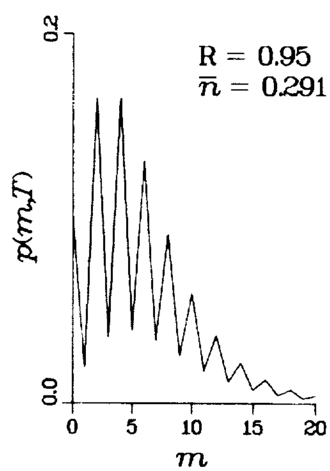

Figure 1b. Photon counting probability distribution for the NDPO for $R = 0.95$, efficiency $\eta = 1$, and counting time interval $2\gamma T = 20$.

The second case that we consider is when intensities of the two modes are added together. In this case the power ℓ appearing in Eq. (7) is one (Ref.6). In this case the expression for $Q_1(s,T)$ is similar to the generating function for thermal light (Ref.5). The counting distribution for NDPO, however, is very different from that for the thermal light. It shows even-odd oscillations as a function of m whereas such oscillations are not seen for thermal light. Figure (1b) shows even-odd oscillations in $p(m,t)$ for $R = \kappa\epsilon/\gamma = .95$ for the second case.

Next we discuss photon statistics of the fundamental beam from an intracavity second harmonic generation (SHG). Field for this system can be expressed in terms of two real Gaussian random variables and a coherent component (Ref.7). These results are obtained by linearizing the field amplitude equations around the deterministic steady state values. The generating function for the SHG can be written as (Ref.6)

$$G(s,T) = Q_1(s,T)Q_2(s,T)e^{-f(s,T)}. \qquad (10)$$

Here

$$f(s,T) = -2s\eta\bar{n}n_0T\left[\frac{\lambda_1^2}{z_1^2}\left(1+\frac{2}{(2+\lambda_1T)}\right) - \frac{2s\eta\bar{n}v(z_1)}{z_1^2\left(1+\frac{s\eta\bar{n}T}{\lambda_1}\right)} - \frac{2}{(2+\lambda_1T)\left(1+\frac{s\eta\bar{n}T}{\lambda_1}\right)}\right], \qquad (11)$$

with

$$v(z_1) = \left[\frac{1+\left(\frac{2+\lambda_1T}{z_1T}\right)\tanh(z_1T/2)}{1+\frac{z_1}{\lambda_1}\tanh(z_1T/2)}\right]. \qquad (12)$$

Here $Q_1(s,T)$ and $Q_2(s,T)$ are given by equation (7) with $l = 0.5$ and

$$\lambda_1 = (1+3\bar{n}), \qquad \lambda_2 = (1+\bar{n}), \qquad (12)$$

For the SHG z_1 and z_2 are given by

$$\begin{aligned} z_1^2 &= \lambda_1^2 - 2s\eta\bar{n}, \\ z_2^2 &= \lambda_2^2 + 2s\eta\bar{n}. \end{aligned} \qquad (13)$$

Here $\bar{n}$ and n_0 are the average and threshold photon numbers, respectively. From this generating function various statistical quantities of interest can be calculated. Photon sequences in the SHG can be antibunched. Although the antibunching effect is very small riding on an intense coherent background it is clearly reflected in the behavior of the waiting time distribution for the SHG.

Using similar techniques we also obtain the generating function when light from the DPO is homodyned with coherent light from a local oscillator. Depending upon the relative phase between coherent light from the local oscillator and squeezed light from the DPO we can see sub-Poissonian or super-Poissonian statistics for the homodyned photon beam. Figure (2) shows waiting time distribution when the relative phase is 0^o and 90^o. We see bunched light when coherent component is added to the unsqueezed component and antibunched light when coherent component is added to the squeezed component.

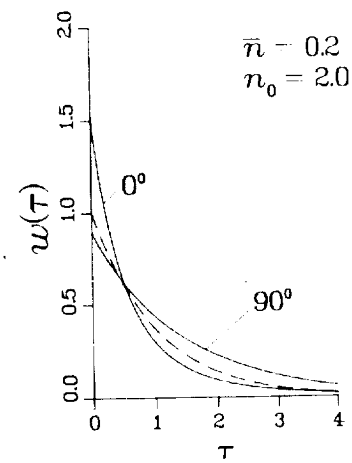

Figure 2. Waiting time distribution for DPO mean photon number $\bar{n}$ = 0.2 and local oscillator mean photon number n_o = 2. Dashed curve is for coherent light

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