

**SQUEEZED STATES,
TIME-ENERGY UNCERTAINTY RELATION, AND
FEYNMAN'S REST OF THE UNIVERSE**

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Abstract

Two illustrative examples are given for Feynman's rest of the universe. The first example is the two-mode squeezed state of light where no measurement is taken for one of the modes. The second example is the relativistic quark model where no measurement is possible for the time-like separation of quarks confined in a hadron. It is possible to illustrate these examples using the covariant oscillator formalism. It is shown that the lack of symmetry between the position-momentum and time-energy uncertainty relations leads to an increase in entropy when the system is measured in different Lorentz frames.

1. Introduction

In his book on statistical mechanics [1], Feynman makes the following statement on the density matrix. *When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts - the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.*

The purpose of this paper is to discuss two physical examples of Feynman's rest of the universe. We shall consider first the case of the two-mode squeezed state. In 1987, Yurke and Potasek observed that the failure to make measurements on one of the two modes will lead to non-coherent excitation of the first mode, as in the case of Einstein's calculation of specific heat in the harmonic oscillator model [2]. They observed further that this excitation is just like the thermal excitation of the ground-state harmonic oscillator. From the measurement theoretic point of view, this non-coherent excitation corresponds to an increase in entropy [3].

Let us next consider the quark model in which two quarks are bound together inside a hadron [4]. This system has a time-like separation between quarks as well as a spatial separation between them[5]. While there is no place for the time-separation variable in nonrelativistic quantum

mechanics, it plays an essential role when observations are made in different Lorentz frames. For this time-like separation, there is a time-energy uncertainty relation. It is of interest to see how this uncertainty relation is combined with the position-momentum to an observer in a different Lorentz frame.

We show in this paper that the longitudinal and time-like excitations in the relativistic quark model are exactly like two photon modes in a two-mode squeezed state [6]. We shall study how the non-measurement of the time-separation variable affects measurements along other coordinates.

In Sec. 2, we study the statistical effect on measurement and density matrices. In Sec. 3, we derive the result of Sec. 3 using the shadow coordinate system commonly used in thermo-field-dynamics [7]. In Sec. 4, the concept of entropy is introduced as a measure of our ignorance [3] [8]. In Sec. 5, the formalism of Sec. 4 is applied to the two-mode squeezed state of light.

The rest of this paper consists of the application of the concept of entropy to the relativistic quantum system in which the time-energy uncertainty relation is coupled covariantly to the position-momentum uncertainty, using the same mathematical formalism developed in Secs. 3, 4, and 5. We start this discussion in Sec. 6 by studying the time-energy uncertainty relation applicable to the time separation variable in the relativistic quark model. In this connection, the covariant harmonic oscillator formalism is presented. In Sec. 7, Lorentz-squeezed hadrons are discussed in terms of the covariant oscillator formalism. Finally, in Sec. 8, we note that the present form of quantum measurement theory does not measure the time separation variable. This incompleteness in measurement leads to an increase in entropy.

2. Statistical Decoherence

In measuring physical quantities, the accuracy of the measuring device is very important. Often, we have to face the situation where the measurement is taken on many different objects. For instance, in the case of the one-dimensional harmonic oscillator, the most general form of normalized solution is

$$\psi(x, t) = e^{-i\omega t/2} \sum_n C_n e^{-in\omega t} \psi_n(x), \quad (2.1)$$

where $\psi_n(x)$ is the solution of the time-independent oscillator equation with the energy level $\omega(n + 1/2)$. The wave function $\psi(x, t)$ is normalized:

$$(\psi(x, t), \psi(x, t)) = \sum_n |C_n|^2 = 1. \quad (2.2)$$

The expectation value $\langle A \rangle = (\psi(x, t), A\psi(x, t))$ of an operator $A(x)$ can be written as

$$\langle A \rangle = \sum_n |C_n|^2 (\psi_n(x), A(x)\psi_n(x)) + \sum_{n \neq m} C_m^* C_n e^{i\omega(m-n)t} (\psi_m(x), A(x)\psi_n(x)). \quad (2.3)$$

If we take the ensemble average for many oscillators prepared independently with different initial times, the net effect is same as that of taking the time average, and the second term in the above expression vanishes. As a consequence, the ensemble average is

$$\langle \bar{A} \rangle = \sum_n |C_n|^2 (\psi_n(x), A(x)\psi_n(x)). \quad (2.4)$$

We use the word “mixed” or “non-pure” in order to describe this ensemble average.

It is very convenient to treat this problem if we introduce the density matrix defined as [1] [6]

$$\rho(x, x') = \sum_n |C_n|^2 \psi_n(x) \psi_n^*(x'), \quad (2.5)$$

and

$$\langle \bar{A} \rangle = \int dx' \int A(x', x) \rho(x, x') dx, \quad (2.6)$$

with

$$A(x', x) = \delta(x' - x) A(x).$$

The above expression is then the trace of the matrix $A(x', x) \rho(x, x')$ often written as

$$\langle \bar{A} \rangle = \text{Tr}(\rho A). \quad (2.7)$$

If $C_n = \delta_{nm}$ for a given value of m , we say that the system is in a pure state. Otherwise, the system is in a mixed state. The information from the interference terms contained in Eq.(2.3) is lost during the process of taking the ensemble average. This information lies in Feynman’s rest of the universe.

The best-known example is the thermally excited harmonic oscillator which was used by Einstein in his calculation of the specific heat of a solid. The density matrix takes the form [1] [6]

$$\rho_T(x, x') = \sum_n (1 - e^{-\omega/kT}) e^{-n\omega/kT} \psi_n(x) \psi_n^*(x'). \quad (2.8)$$

In the zero-temperature limit, the system is purely in the ground state. As the temperature increases, $|C_n|^2$ becomes $(1 - e^{-\omega/kT}) e^{-n\omega/kT}$, but the above expression does not tell us anything about the phase of C_n . The density matrix does not give any information about the coherence of the system. In Sec. 4, we shall study how this ignorance is translated into entropy.

3. Shadow Coordinates

We discuss in this section a method of deriving the density matrix, without taking the ensemble average, by introducing an auxiliary Hilbert space consisting of $\psi_n(\tilde{x})$ and attach it to C_n [1] [7]. Let us consider the wave function of the form

$$\psi(x, \tilde{x}) = \sum_n (C_n \psi_n(\tilde{x})) \psi_n(x). \quad (3.1)$$

The auxiliary coordinate \tilde{x} is called the “shadow” coordinate in the literature [7]. It is possible to derive the result of Eq.(2.4) by treating $\psi(x, \tilde{x})$ as a pure-state wave function defined in the total Hilbert space consisting both of $\psi_n(x)$ and $\psi_n(\tilde{x})$. Because of the orthogonality relation for $\psi_n(\tilde{x})$, the expectation value of $A(x)$:

$$\langle A \rangle = \sum_{n,m} C_m^* C_n (\psi_m(\tilde{x}), \psi_n(\tilde{x})) (\psi_m(x), A(x) \psi_n(x)), \quad (3.2)$$

is the same as the ensemble average $\langle \bar{A} \rangle$ given in Eq.(2.4). It is possible to obtain the density matrix by integrating $\psi(x, \hat{x})\psi^*(x', \hat{x})$ over the \hat{x} variable:

$$\rho(x, x') = \int \psi(x, \hat{x})\psi^*(x', \hat{x})d\hat{x}. \quad (3.3)$$

The evaluation of this integral leads to the expression for $\rho(x, x')$ given in Eq.(2.5). The shadow coordinate plays the role of taking the ensemble average discussed in Sec. 2.

Let us illustrate this again using the ground-state harmonic oscillator wave function

$$\psi_0(x, \hat{x}) = \psi_0(x)\psi_0(\hat{x}) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(x^2 + \hat{x}^2)\right\}, \quad (3.4)$$

where x is measured in units of $1/\sqrt{m\omega}$, and let us now make the coordinate transformation:

$$\begin{pmatrix} x' \\ \hat{x}' \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}, \quad (3.5)$$

where

$$\cosh \eta = 1/(1 - e^{-\omega/kT})^{1/2}, \quad \sinh \eta = e^{-\omega/2kT}/(1 - e^{-\omega/kT})^{1/2}, \quad (3.6)$$

which shares the same mathematics as a Lorentz boost as we shall see in Secs. 7 and 8. Then this coordinate transformation leads to the wave function of the form

$$\psi_T(x, \hat{x}) = \left[\frac{1}{\pi}\right]^{1/2} \exp\left\{-\frac{1}{4}\left[(\tanh \frac{\omega}{4kT})(x + \hat{x})^2 + (\coth \frac{\omega}{4kT})(x - \hat{x})^2\right]\right\}. \quad (3.7)$$

The wave function of the two variables can be expanded as [6]

$$\psi_T(x, \hat{x}) = [1 - \exp(-\omega/kT)]^{1/2} \sum_n \exp(-n\omega/2kT)\psi_n(x)\psi_n(\hat{x}). \quad (3.8)$$

The evaluation of the density matrix given in Eq.(3.3) with this form of the wave function leads to the density matrix of the form of Eq.(2.8). The same evaluation with the wave function of the form of Eq.(3.7) gives [6]

$$\rho_T(x, x') = \left[\frac{1}{\pi} \tanh \frac{\omega}{2kT}\right]^{1/2} \exp\left\{-\frac{1}{4}\left[(x + x')^2 \tanh \frac{\omega}{2kT} + (x - x')^2 \coth \frac{\omega}{2kT}\right]\right\}. \quad (3.9)$$

Then the probability distribution $\rho_T(x) = \rho_T(x, x)$ becomes

$$\rho_T(x) = \left[\frac{1}{\pi} \tanh \frac{\omega}{2kT}\right]^{1/2} \exp\left\{-\left[\tanh \frac{\omega}{2kT}\right]x^2\right\}. \quad (3.10)$$

This expression is normalized. In the $T = 0$ limit, the probability distribution becomes

$$\rho_0(x) = (1/\pi)^{1/2} \exp(-x^2). \quad (3.11)$$

The increase of temperature broadens the probability distribution. It is possible to carry out the same analysis for the momentum variable. The momentum distribution will also become widespread. The net result is the increase in uncertainty. This increase is due to our ignorance about the shadow coordinate system. Feynman's rest of the universe consists of the shadow coordinate.

4. Entropy and Ignorance

The interpretation in terms of thermal excitation was possible because the expansion coincides with the thermally excited oscillator state. There are, however, cases where the density matrix does not correspond to any state in thermal equilibrium. For instance, if we start from one of the excited harmonic oscillator states [9], the density matrix does not correspond to a thermally excited state. What then will be the variable which measures our ignorance about the second coordinate variable?

The answer to this question is the entropy defined as [8]

$$S = - \sum_n \rho_n \ln(\rho_n). \quad (4.1)$$

In general, the density matrix is Hermitian and can be diagonalized. ρ_n in the above expression is the diagonal element. If the system is in a pure state, the entropy is zero. If the system is not in a pure state, the entropy is positive. This definition of entropy does not depend on the question of whether the system is in thermal equilibrium. The definition given in Eq.(4.1) does not depend on temperature.

On the other hand, the above definition does not exclude a system in thermal equilibrium. In the case of a thermally excited harmonic oscillator, the density matrix of Eq.(2.8) is diagonal and its elements are

$$\rho_n = (1 - e^{-\omega/kT}) e^{-n\omega/kT}. \quad (4.2)$$

Thus, according to the definition given in Eq.(4.1),

$$S = \omega/kT (e^{\omega/kT} - 1) - \ln(1 - e^{-\omega/kT}). \quad (4.3)$$

This expression is the same as the one available from textbooks on statistical mechanics.

In Secs. 5 and 8, we shall study the examples which are not thermal excitations, but share the same mathematical formalism. The concept of temperature is convenient but not essential in the examples to be discussed in the following sections.

5. Entropy and Two-Mode Squeezed States of Light

As is well known, the mathematics of harmonic oscillators is the standard language for the photon-number space. The energy level in a given oscillator system corresponds to the number of photons, and the ground state corresponds to the vacuum or zero-photon state. The step-up and step-down operators in the oscillator formalism are given by

$$a^\dagger = \frac{1}{\sqrt{2}} \left(x - \frac{\partial}{\partial x} \right), \quad a = \frac{1}{\sqrt{2}} \left(x + \frac{\partial}{\partial x} \right), \quad (5.1)$$

respectively. These are now the creation and annihilation operators. Let us consider two sets of these operators: a^\dagger, a and $\tilde{a}^\dagger, \tilde{a}$ for the first and second modes of photons respectively. We are interested in the state of these photons where those created and annihilated by \tilde{a}^\dagger and \tilde{a} are not observed.

We construct the two-mode state by applying to the vacuum state the operator $\exp(-i\eta G)$, where [6] [10]

$$G = -\frac{i}{2} \left(a^\dagger \tilde{a}^\dagger - a \tilde{a} \right), \quad (5.2)$$

where the subscripts 1 and 2 are for photons of the first and second kinds respectively. The two-mode squeezed state constructed from

$$|\eta\rangle = e^{-i\eta G} |0, 0\rangle, \quad (5.3)$$

where $|n, \tilde{n}\rangle$ is the state with n photons of the first kind and \tilde{n} for the second kind. According to this definition, $|0, 0\rangle$ is the vacuum state. The power-series expansion of the exponential factor leads to

$$|\eta\rangle = (1/\cosh \eta) \sum_n (\tanh \eta)^n |n, n\rangle. \quad (5.4)$$

In order to distinguish the photons of the first and second kinds, we write the above expression as

$$|\eta\rangle = (1/\cosh \eta) \sum_{n, \tilde{n}} (\tanh \eta)^n \delta_{n, \tilde{n}} |n, \tilde{n}\rangle. \quad (5.5)$$

The mathematics which led to the above expression is exactly the same as that for the harmonic oscillator with a shadow coordinate given in Sec. 3. From the mathematical point of view, this form is the same as the expansion given in Eq.(2.8), and they become identical if we use the correspondence between T and η given in Eq.(3.6). In terms of the η parameter, an element of the diagonal density matrix is

$$\rho_n = (\tanh \eta)^{2n} / (\cosh \eta)^2, \quad (5.6)$$

which leads to the entropy:

$$S = \ln(\cosh \eta)^2 - (\sinh \eta)^2 \ln(\tanh \eta)^2. \quad (5.7)$$

This form of entropy is determined directly from the squeeze parameter η , and it is not necessary to introduce the concept of temperature. The fact is that the measurement or non-measurement of photons of one kind affects the measurement of photons of the other kind. In the present case, the non-measurement of the photon of the second kind increases the degree of ignorance of photons of the first kind, and this degree of ignorance is measured in terms of the entropy. The system of photons of the second kind is Feynman's rest of the world.

There are however special cases where the entropy can be associated with temperature. This is one of those cases. As Yurke and Potasek observed [2], it is possible to define the temperature

of this system by using the connection between the squeeze parameter and temperature. The temperature T is related to the squeeze parameter by

$$\tanh \eta = e^{-\omega/2kT}. \quad (5.8)$$

If T approaches zero, the squeeze parameter also becomes zero. As the temperature becomes very high, the squeeze parameter becomes very large.

6. Time-energy Uncertainty Relation and Relativistic Quark Model

In order to study the role of the time-energy uncertainty relation in relativistic quantum mechanics and relativistic measurement theory, we consider here a concrete physical example which gives observable effects in high-energy laboratories. Let us consider a hadron consisting of two quarks. If the space-time position of the two quarks is specified by x_a and x_b respectively, the system can be described by the variables:

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)/2\sqrt{2}. \quad (6.1)$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks. As for the four-momenta of the quarks p_a and p_b , we can combine them into the total hadronic four-momentum and the momentum-energy separation between the quarks [4]:

$$P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b), \quad (6.2)$$

where P is the hadronic four-momentum conjugate to X . The internal momentum-energy separation is conjugate to x .

In the convention of Feynman *et al.* [4], the internal motion of the quarks can be described by the Lorentz-invariant oscillator equation:

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x), \quad (6.3)$$

where we use the space-favored metric: $x^\mu = (x, y, z, t)$. The four-dimensional covariant oscillator wave functions are Hermite polynomials multiplied by a Gaussian factor, which dictates the space-time localization property of the wave function. The Gaussian factor takes the form

$$\exp \left\{ -\frac{1}{2}(x^2 + y^2 + z^2 + t^2) \right\}. \quad (6.4)$$

We are accustomed to the polynomial $(x^2 + y^2 + z^2 - t^2)$, but not with $+t^2$. What is the physics of the Gaussian factor of Eq.(6.4)? If the hadron is at rest, it is possible to construct three-dimensional harmonic oscillator wave functions with excited energy levels. This would be a multiplication of the appropriate Laguerre polynomial with the Gaussian factor $\exp\{-(x^2 + y^2 + z^2)/2\}$. As for the time-like separation, Eq.(6.4) contains the factor $\exp(-t^2/2)$. However, unlike the position coordinates, there is no excitation along this axis, since the time variable is a c-number

[5]. The fact that the time-energy uncertainty relation is a c-number relation is well-known and well-established. Figure 1 illustrates these features of the uncertainty relations.

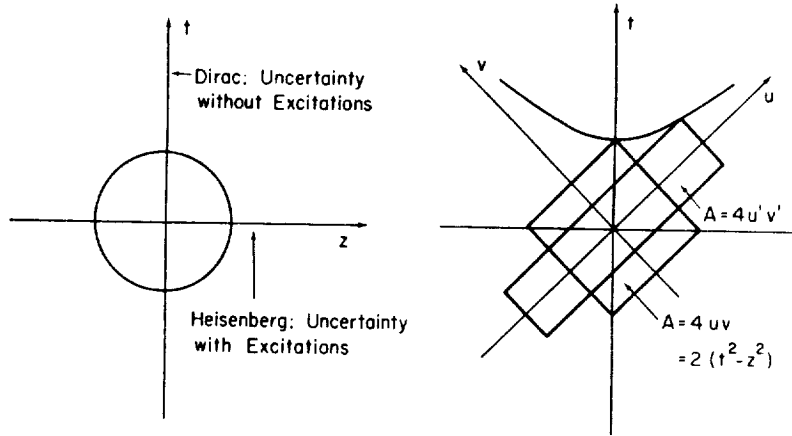


FIG. 1. Quantum mechanics and relativity. The left part of this figure illustrates that the position-momentum uncertainty relation with excitations and the time-energy uncertainty relation without excitations, as the time is a c-number variable. The right part is special relativity. In the light-cone system, it is transparent that the Lorentz boost is a squeeze transformation. One way to combine quantum mechanics with special relativity is to superimpose these two figures, as is done in Fig. 2.

Since the three-dimensional oscillator differential equation is separable in both spherical and Cartesian coordinate systems, $\psi(x, y, z)$ consists of Hermite polynomials of x, y , and z . If the Lorentz boost is made along the z direction, the x and y coordinates are not affected, and can be dropped from the wave function. The wave function of interest can be written as

$$\psi^n(z, t) = \left(\frac{1}{\pi}\right)^{1/4} \exp(-t^2/2) \psi^n(z), \quad (6.5)$$

with

$$\psi^n(z) = \left(\frac{1}{\sqrt{\pi} 2^n n!}\right)^{1/2} H_n(z) \exp(-z^2/2),$$

where $\psi^n(z)$ is for the n^{th} excited oscillator state. The full wave function $\psi^n(z, t)$ is

$$\psi_o^n(z, t) = \left(\frac{1}{\pi 2^n n!}\right)^{1/2} H_n(z) \exp\left\{-\frac{1}{2}(z^2 + t^2)\right\}. \quad (6.6)$$

The subscript o means that the wave function is for the hadron at rest. The above expression is not Lorentz-invariant, and its localization undergoes a Lorentz squeeze as the hadron moves along the z direction [5].

7. Lorentz-squeezed Oscillator Wave Functions

Let us next consider special relativity and Lorentz transformations. It is important to note that the Lorentz boost is a squeeze transformation in the zt coordinate system if the boost is made along the z axis. In order to see this point, let us use the light-cone variables, which are

defined as

$$u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}. \quad (7.1)$$

The u and v axes are perpendicular to each other. In terms of these variables, the Lorentz boost along the z direction,

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}, \quad (7.2)$$

takes the simple form

$$u' = e^\eta u, \quad v' = e^{-\eta} v. \quad (7.3)$$

This transformation is illustrated in Fig. 2. This is an area preserving transformation where one side becomes contracted while the other side is expanded in a manner that their product is constant. This is a squeeze transformation.

QUARKS \longrightarrow PARTONS

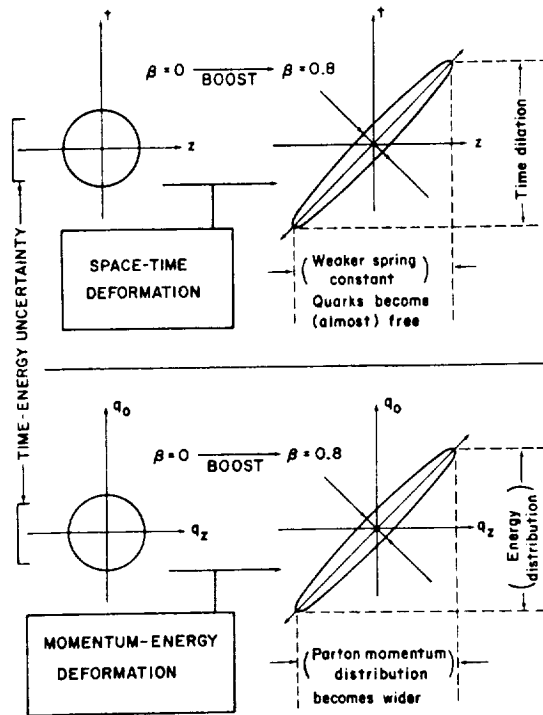


FIG. 2. Lorentz-squeezed space-time and momentum-energy wave functions. This figure is a result of combining quantum mechanics and special relativity described in Fig. 1. The physical significance is that this figure gives a unified picture of the quark model for slow hadrons and the parton model for rapid hadrons. This figure is from Refs. [5] and [6].

In the light-cone coordinate system, the oscillator wave function in the rest frame takes the form

$$\psi_o^n(z, t) = \left[\frac{1}{\pi(n!)2^n} \right]^{1/2} H_n((u+v)/\sqrt{2}) \exp \left\{ -\frac{1}{2}(u^2 + v^2) \right\}. \quad (7.4)$$

If the system is boosted, the wave function becomes

$$\psi_\eta^n(z, t) = \left[\frac{1}{\pi(n!)2^n} \right]^{1/2} H_n((e^{-\eta}u + e^\eta v)/\sqrt{2}) \exp \left\{ -\frac{1}{2}(e^{-2\eta}u^2 + e^{2\eta}v^2) \right\}. \quad (7.5)$$

This wave function can be expanded as [5]

$$\psi_\eta^n(z, t) = (1/\cosh \eta)^{n+1} \sum_k (C_{n,k})^{1/2} (\tanh \eta)^k \psi_o^{n+k}(z) \psi_o^n(t), \quad (7.6)$$

where

$$C_{n,k} = (n+k)!/n!k!.$$

Since the space-time localization property is dictated by the Gaussian factor, let us study in detail the ground state with $n = 0$. In this case, the boosted wave function is

$$\psi_\eta(z, t) = \left(\frac{1}{2\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2}(e^{-2\eta}u^2 + e^{2\eta}v^2) \right\}. \quad (7.7)$$

The quantum space-time distribution of Fig. 1 is squeezed to an ellipse described in the upper half of Fig. 2.

Let us next consider the momentum-energy wave function, which is the Fourier transform of $\psi_\eta(z, t)$:

$$\phi_\eta(q_z, q_o) = \frac{1}{\pi} \int \psi_\eta(z, t) \exp \{ i(p_z z - p_o t) \} dz dt, \quad (7.8)$$

where q_z and q_o are defined in Eq.(6.2). Since the integration measure is invariant under the boost, the evaluation of the integral is straight-forward, and the momentum-energy wave function takes the form

$$\phi_\eta(q_z, q_o) = \left(\frac{1}{\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2}(e^{2\eta}q_u^2 + e^{-2\eta}q_v^2) \right\}, \quad (7.9)$$

with

$$q_u = (q_z - q_o)/\sqrt{2}, \quad q_v = (q_z + q_o)/\sqrt{2}.$$

The Lorentz-squeeze property of the momentum-energy wave function is the same as that of the space-time wave function, as is illustrated in the lower half of Fig. 2. The significance of the Lorentz-squeeze property is that it gives observable consequences in high-energy laboratories.

By now the quark model for hadrons is firmly established. The proton consists of three quarks bound together by a oscillator-like force, according to an observer in the Lorentz frame in which the hadron is at rest. On the other hand, to an observer in a moving frame, the wave function appears squeezed. If the frame moves with a speed close to that of light, the hadron appears as a collection of an infinite number of partons [5] [11]. This is called Feynman's parton picture. This phenomenon is now universally observed in high-energy laboratories, and the squeezed picture of Fig. 2 gives an explanation of Feynman's parton picture.

One of the most uncomfortable aspects of the present discussion is the time-separation variable. Without this variable, it is not possible to perform Lorentz boosts. On the other hand, there is no time-separation variable in any of the existing measurement theories of quantum mechanics. In order to reconcile this difference, we have to conclude that the time-separation exists, but is not a measurable variable. This variable is in Feynman's rest of the universe.

8. Entropy and Lorentz Transformations

Entropy is a measure of our ignorance and is computed from the density matrix, as was noted in Sec. 4. The density matrix is needed when the experimental procedure does not analyze all relevant variables to the maximum extent consistent with quantum mechanics. For the bound state of two particles, the present form of quantum mechanics does not tell us how to measure the time-separation variable, as is illustrated in Fig. 3.

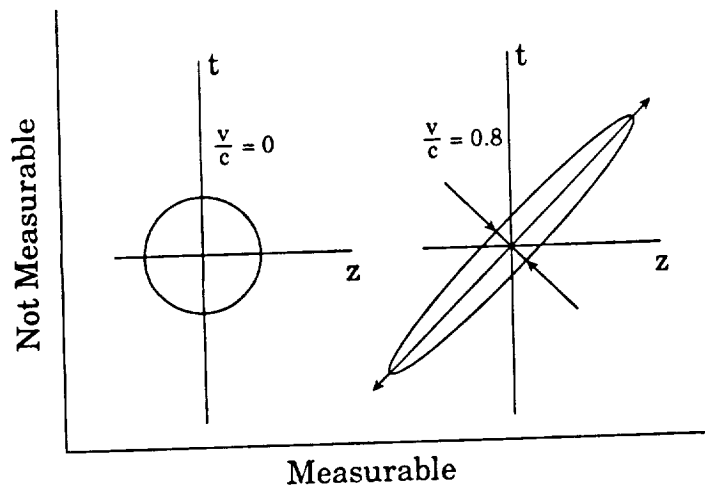


FIG. 3. Localization property in the zt plane. When the hadron is at rest, the Gaussian form is concentrated within a circular region specified by $(z+t)^2 + (z-t)^2 = 1$. As the hadron gains speed, the region becomes deformed to $e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2 = 1$. Since it is not possible to make measurements along the t direction, we have to deal with information that is less than complete. The time-separation variable lies in Feynman's rest of the universe.

If the time-separation were a measurable variable, the pure-state density matrix would be

$$\rho_{\eta}^n(z, t; z', t') = \psi_{\eta}^n(z, t)[\psi_{\eta}^n(z', t')]^*, \quad (8.1)$$

which satisfies the condition $\rho^2 = \rho$:

$$\rho_{\eta}^n(z, t; z', t') = \int \rho_{\eta}^n(z, t; z'', t'') \rho_{\eta}^n(z'', t''; z', t') dz'' dt''. \quad (8.2)$$

This pure-state density matrix is possible only if both the z and t coordinates are measurable space-time variables. On the other hand, there are at present no measurement theories which accommodate the time-separation variable t . Indeed, this time separation variable is the coordinate in the part of the universe outside the system. We do not observe the distribution outside the system. What we do then is to take the trace of the ρ matrix with respect to the t variable. The resulting density matrix is

$$\begin{aligned} \rho_{\eta}^n(z, z') &= \int \rho_{\eta}^n(z, t; z', t) dt = \int \psi_{\eta}^n(z, t)[\psi_{\eta}^n(z', t)]^* dt \\ &= (1/\cosh \eta)^{2(n+1)} \sum_k C_{n,k} (\tanh \eta)^{2k} \psi_{\circ}^{n+k}(z)[\psi_{\circ}^{n+k}(z')]^*. \end{aligned} \quad (8.3)$$

The trace of this density matrix is one, but the trace of ρ^2 is less than one, as

$$\begin{aligned} Tr(\rho^2) &= \int \rho_{\eta}^n(z, z') \rho_{\eta}^n(z', z) dz' dz \\ &= (1/\cosh \eta)^{4(n+1)} \sum_k (C_{n,k})^2 (\tanh \eta)^{4k}, \end{aligned} \quad (8.4)$$

which is less than one. This is due to the fact that we do not know how to deal with the time-like separation which lies in Feynman's rest of the universe. Our knowledge is less than complete.

We can now go back to Sec. 4 on entropy, and write Eq.(4.1) as

$$S = -Tr[\rho \ln(\rho)]. \quad (8.5)$$

If we pretend to know the distribution along the time-like direction and use the pure-state density matrix given in Eq.(8.1), the entropy is zero. However, if we do not know how to deal with the distribution along t , then we should use the density matrix of Eq.(8.3) to calculate the entropy, and the result is

$$\begin{aligned} S &= 2(n+1) \left\{ (\cosh \eta)^2 \ln(\cosh \eta) - (\sinh \eta)^2 \ln(\sinh \eta) \right\} \\ &\quad - (1/\cosh \eta)^{2(n+1)} \sum_k [C_{n,k} \ln(C_{n,k})] (\tanh \eta)^{2k}. \end{aligned} \quad (8.6)$$

In terms of the velocity v of the hadron, where $v/c = \tanh \eta$,

$$S = -(n+1) \left\{ \ln[1 - (v/c)^2] + (v/c)^2 \ln \frac{(v/c)^2}{[1 - (v/c)^2]} \right\}$$

$$-[1 - (v/c)^2]^{(n+1)} \sum_k [C_{n,k} \ln(C_{n,k})] (v/c)^{2k}. \quad (8.7)$$

Here again, entropy is derived as a measure of ignorance. It does not depend on the question of whether or not the system is in thermal equilibrium. The expression for S in Eq.(8.7) does not depend on temperature.

It was noted in Sec. 7 that the ground-state wave function occupies an important place in the oscillator formalism, and it will undoubtedly give a simpler and more transparent expression for the entropy. In terms of the z and t variables, the Lorentz-boosted wave function of Eq.(7.7) takes the form

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{4}[e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2]\right\}, \quad (8.8)$$

which can be expanded as

$$\psi_\eta(z, t) = (1/\cosh \eta) \sum_n (\tanh \eta)^n \psi^n(z) \psi^n(t). \quad (8.9)$$

The density matrix is

$$\rho_\eta(z, z') = \left(\frac{1}{\pi \cosh 2\eta}\right)^{1/2} \exp\left\{-\frac{1}{4}[(z+z')^2/\cosh 2\eta + (z-z')^2 \cosh 2\eta]\right\}, \quad (8.10)$$

and the entropy becomes

$$S = \ln(\cosh \eta)^2 - (\sinh \eta)^2 \ln(\tanh \eta)^2. \quad (8.11)$$

As a consequence of Eq.(8.10), the quark distribution $\rho(z, z)$ becomes

$$\rho(z) = \left(\frac{1}{\pi \cosh 2\eta}\right)^{1/2} \exp\left(\frac{-z^2}{\cosh 2\eta}\right). \quad (8.12)$$

The width of the distribution becomes $(\cosh 2\eta)^{1/2}$, and becomes wide-spread as the hadronic speed increases. Likewise, the momentum distribution becomes wide-spread [5] [11]. This simultaneous increase in the momentum and position distribution widths is called the parton phenomenon in high-energy physics [11]. The position-momentum uncertainty becomes $\cosh \eta$. This increase in uncertainty is due to our ignorance about the physical but unmeasurable time-separation variable.

For the special case of the ground state, it is possible to convert the entropy into the temperature scale, exactly as we did for the case of two-mode squeezed states in Sec. 5. The squeeze parameter η used in Sec. 5 is now the boost parameter. We can use Eq.(5.8) to establish the correspondence between the temperature and squeeze parameter [12].

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