N92-22077

COHERENT STATES AND PARASUPERSYMMETRIC QUANTUM MECHANICS

Nathalie DEBERGH
Physique théorique et mathématique,
Institut de Physique au Sart Tilman (B.5), Université de Liège,
B-4000 LIEGE 1 (Belgique)

It is well known (Refs. 1,2) that Parafermi and Parabose statistics are natural extensions of the usual Fermi and Bose ones, enhancing trilinear (anti)commutation relations instead of bilinear ones. Due to this generalization, positive parameters appear : the so-called orders of paraquantization p (= 1,2,3,$\cdots$) and h_0 (= 1/2,1,3/2,$\cdots$), respectively, the first value leading in each case to the usual statistics. The superposition of the parabosonic and parafermionic operators gives rise to parasupermultiplets (Refs. 3-5) for which mixed trilinear relations have to be envisaged. In the particular case of quantas of the same order ($p = 2h_0$), these relations have already been studied (Ref. 6) leading to two (non equivalent) sets : the relative Parabose and the relative Parafermi ones. For the specific values $p = 1 = 2h_0$, these sets reduce to the well known supersymmetry (Refs. 7,8).

Coherent states associated with this last model have been recently put in evidence through the annihilation operator point of view (Ref. 9) and the group theoretical approach or displacement operator context (Refs. 10-12). We propose here to realize the corresponding studies within the new context $p = 2 = 2h_0$, being then directly extended to any order of paraquantization. Even if we have to take account of the two relative sets separately, the arguments are so similar in both cases that we just concentrate on the Parabose set in the following . Within the relations characterizing such a model, it is easy to prove that the operator $A = a + \frac{1}{2} a^\dagger b^{\dagger 2}$ [$a, a^\dagger$ ($b, b^\dagger$) are the usual bosonic (fermionic) annihilation and creation operators] exactly plays the role of a generalized annihilation operator i.e. satisfying the expected commutation relation with the hamiltonian and displaying the right action on the state basis of the Hilbert space (Refs. 13,14). Parasupercoherent states (Refs. 13,14) $|z>$ are then defined as eigenstates of this operator A with eigenvalues being arbitrary complex numbers. The corresponding uncertainty relation is found to be nearly 1 ($\hbar = 1$) .

The group theoretical approach asks for the consideration of a specific representation of the para-operators : the Green-Cusson Ansätze (Refs. 1,15) in which each operator is decomposed into a sum of two other ones related to the usual bosonic scaling operators and the Pauli matrices. By introducing parameters realized through two by two matrices with Grassmannian elements, we are led to the corresponding color supergroups (Ref. 16) and we are thus able to define the associated coherent states by a unitary representation of these groups. Convenient Baker-Campbell-Hausdorff relations (Ref. 12) are of particular interest in this study. Moreover the states obtained in this way are effective eigenstates of the operator A introduced before. The three usual definitions of ordinary coherent states are thus satisfied in this parasupersymmetric context.

REFERENCES

1. Green, H.S., 1953, "A Generalized Method of Field Quantization", *Phys.Rev.*, **90**, p.270.

2. Ohnuki, Y., and Kamefuchi, S., 1982, *Quantum Field Theory and Parastatistics*, University of Tokyo Press.

3. Beckers, J., and Debergh, N., 1990, "Parastatistics, Supersymmetry and Parasupercoherent States", *J.Math.Phys.*, **31**, p.1513.

4. Beckers, J., and Debergh, N., November 1990, "On Supersymmetric Harmonic Oscillators and Green-Cusson's Ansätze", Preprint Univ.Liège PTM 90/14.

5. Beckers, J., and Debergh, N., 1990, "Parastatistics and Supersymmetry in Quantum Mechanics", *Nucl.Phys.* **B340**, p.767.

6. Greenberg, O.W., and Messiah, A.M.L., 1965, "Selection Rules for Parafields and the Absence of Para Particles in Nature", *Phys.Rev* **B138**, p.1155.

7. Fayet, P., and Ferrara, S., 1985, *Supersymmetry in Physics*, North Holland, Amsterdam.

8. Witten, E., 1982, "Constraints on Supersymmetry Breaking", *Nucl.Phys.* **B202**, p.253.

9. Aragone, C., and Zypman, F., 1986, "Supercoherent States", *J.Phys. A (Math. and Gen.)* **19**, p.2267.

10. Balantekin, A.B., Schmitt, H.A., and Barret, B.R., 1988, "Coherent States for the Harmonic Oscillator Representations of the Orthosymplectic Supergroup OSP $(1/2\ N, \mathbb{R})$", *J.Math. Phys.* **29**, p.1634.

11. Fatyga, B.W., Kostelecky, V.A., Nieto, M.M., and Truax, D.R., 1991, "Supercoherent States", *Phys.Rev.* **D43**, p.1403.

12. Kostelecky, V.A., Nieto, M.M., and Truax, D.R., 1986, "Baker-Campbell-Hausdorff Relations for Supergroups", *J.Math.Phys.* **27**, p.1419.

13. Beckers, J., and Debergh, N., 1989, "On Parasupersymmetric Coherent States", *Mod.Phys. Lett.* **A4**, p.1209.

14. Beckers, J., and Debergh, N., 1989, "Coherent States in Parasupersymmetric Quantum Mechanics", *Mod.Phys.Lett* **A4**, p.2289.

15. Cusson, R.Y., 1969, "Examples of Parastatistics", *Ann.Phys.* **55**, p.22.

16 Rittenberg, V., and Wyler, D., 1978, "Generalized Superalgebras", *Nucl.Phys.***B139**, p.189.