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Temperature Control in Continuous Furnace by Structural Diagram Method

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Abstract

In this paper, the fundamentals of the structural diagram method [1] for Distributed Parameter Systems (DPS) are presented and reviewed. An example is given to illustrate the application of this method for control design.

Acknowledgement

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I. Introduction

For lumped systems, practical control methods are developed based either on the transfer function or the state space representation of the system [3]. Although there are many inherent connections between these two methods, there exists many differences between them in terms of mathematical analysis tools, design methods, and engineering interpretation, etc. For DPS, however, control methods are predominantly based on the state space representation of the system [2]. It seems that this approach parallels that developed for lumped systems, but it is difficult to understand for practicing engineers, and to implement a physical controller without an intensive functional analysis background [2]. The structural diagram method for the DPS (the counterpart of transfer function method for lumped systems), on the other hand, should be easier for practicing control engineers due to its simplicity, convenience, and clarity of representation of the systems. Also, the popularity of this method should be enhanced by the fact that control engineers are already familiar with complex variable methods from their experience in dealing with lumped system.

In this paper, the fundamental concepts of the structural theories are reviewed in an attempt to explore the methodological value of a generalization of the structural theory of DPS for practical control design.

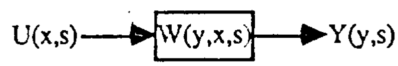
II. Structural Diagram Method

The structural diagram method developed in [1] is briefly reviewed here.

1. Distributed signal

In DPS, a distributed signal depends not only on time but also on certain spatial variables. It then has the form $w(x,t)$, $x \in D$, where D is a spatial domain. The Laplace transform of this signal with respect to time is $w(x,s)$.

2. Distributed block



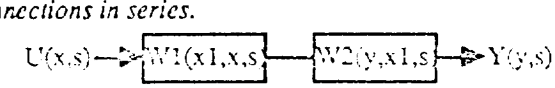
In order to describe the input/output relationship of a DPS, a distributed block, an analog to the transfer function of a lumped system, has the form $W(y,x,s)$, $x \in D_1$, $y \in D_2$ for a stationary process (the coefficients in a PDE do not change with time), where D_1 and D_2 are the domain for the distributed input and output signal respectively. Therefore, the relationship between any two places in the space is reflected by inserting in their position in $W(y,x,s)$. This is then the Laplace transfer of the Green's function of the given DPS. The relationship of the input and output is not simply a product of the input signal and transfer function as in the lumped system, but instead, a *composition* of the two, which is defined by:

$$Y(y,s) = W(y,x,s) \otimes U(x,s) = \int_D W(y,x,s)U(x,s)dx$$

2. Connecting blocks

A complicated DPS may involve more than one block, The operations among them then need to be clarified. The following three kinds of operations often arise in control design.

Connections in series.

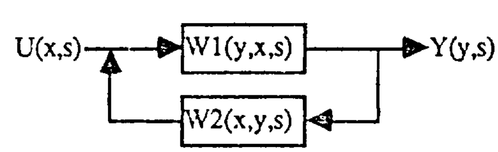


If two blocks are connected in series as shown above, the overall block is equivalent to:

$$W(y,x,s) = W1(x_1,x,s) \otimes W2(x_1,x,s) = \int_D W1(x_1,x,s)W2(y,x_1,s)dx_1$$

This is again, different from the lumped system where the overall transfer function for two systems connected in series is simply the product of two individual transfer functions. In addition, the operation of composition is non-commutative in general. If both block functions are known, the overall block is obtained after conducting the conventional integral operation.

Closed loop system.



Based on the block operation introduced before, it is easy to show that the closed loop transfer function W(y,x,s) has the following general form:

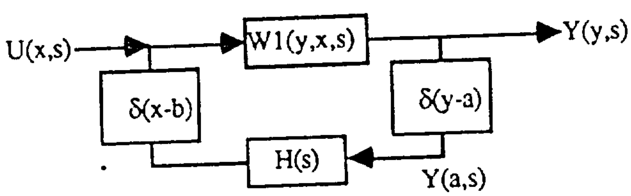
$$W(y,x,s) = W_{21}(y,x,s) \otimes W(y,x,s) + W_1(y,x,s)$$
$$W_{21}(y,x,s) = W_1(y,x,s) \otimes W_2(y,x,s)$$

For special types of $W1(y,x,s)$ and $W2(x,y,s)$, $W(y,x,s)$ may be simplified greatly.

Lumped Regulator

Following figure shows that the interaction between a DPS and a lumped parameter regulator is represented by two δ functions. The closed loop transfer function from U(x,s) to Y(a,s) is simplified to:

$$W'(a,x,s) = \frac{W1(a,x,s)}{1-H(s)\int_D W1(a,x,s)dx}$$

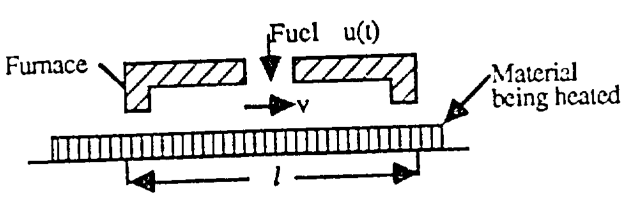


4. Transfer function of DPS

The transfer function of DPS is closely related to the attached boundary condition, as well as given PDE. This is what differentiates a DPS and lumped system. The transfer functions of many PDEs with popular boundary conditions are given in [1]. For a general PDE with arbitrary boundary conditions, one needs to find the new transfer function by solving the given PDE with its boundary conditions.

III. An Example

The following figure shows a furnace of length $l=1$ which heats a continuous strip of material passing through the furnace with speed $v=1$.



Let the temperature field of the furnace be uniformly distributed in space. Suppose the temperature distribution in the heated strip is $Q(x,t)$. Then for a normalized system, $u(t)$ and $Q(x,t)$ are connected by a first order linear partial differential equation

$$\frac{\partial Q(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} + Q(x,t) = u(t), \quad 0 \le x \le 1, \ t \ge 0$$

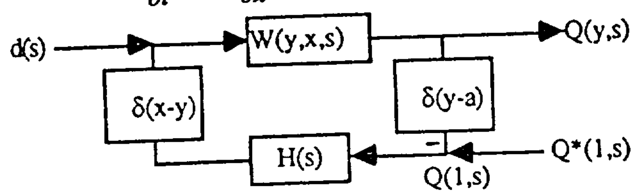
with boundary condition $Q(0,t)=d(t)$.

Two specific control tasks can now be considered. One is to regulate the temperature at the output of the furnace. This is considered first below. The other task is to heat the material to a specific spatial temperature distribution.

For the first control task, the control equation has the form $U(s)=W(s)[Q(1,t)-Q^*(1,t)]$. Then the control diagram is:

The system written in standard form is:

$$\frac{\partial Q(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} + Q(x,t) = u(t) + \delta(x)d(t)$$



The open loop transfer function $W(y,x,s)$ for the given plant and boundary conditions is easily derived or found by looking in the table in [1] to be

$$W(y,x,s) = e^{-(s+1)(y-x)}$$

The closed loop transfer function from $d(t)$ to $Q(1,t)$ is

$$W_1(s) = \frac{W(1,0,s)}{1+H(s)\int_0^1 W(1,x,s)dx}$$
$$= \frac{(s+1)e^{-(s+1)}}{(s+1)+H(s)(1-e^{-(s+1)})}$$

The closed loop transfer function from $Q^*(t)$ to $Q(1,t)$ is

$$W_2(s) = \frac{H(s)\int_0^1 W(1,x,s)dx}{1+H(s)\int_0^1 W(1,x,s)dx}$$
$$= \frac{H(s)(1-e^{-(s+1)})}{(s+1)+H(s)(1-e^{-(s+1)})}$$

The response of $Q(1,t)$ is then

$$Q(1,s) = W_1(s) \otimes d(s) + W_2(s) \otimes Q^*(1,s)$$

The characteristic equation of the system is

$$\Delta(s) = (s+1)+H(s)(1-e^{-(s+1)})$$

based on which, we can carry on control analysis and design. All this can be done using conventional control techniques, such as root locus, Bode plot, etc [3].

Consider a proportional controller where $H(s)=k$. The poles of the closed loop system are shown in Fig. 1. Notice that there is an infinite number of open loop zeros and one open loop pole at $s=-1$ that is cancelled by the zero at the same place. It is seen that for all positive k, the closed loop system is stable. The closed-loop poles for a general PID controller are shown in Fig.2 where

$$H(s) = K_p(1+\frac{1}{T_i s}+T_d s).$$

Although it is more tedious to implement control analysis for DPS using conventional techniques, DPS obey the same rules that a lumped system does. Those rules include:

(i) The damping ratio and stability decrease with the increase of K_p, system has a faster response;

(ii) Integral control reduces or eliminates steady-state errors at the expense of reduced stability;

(iii) Derivative control improves system stability.

It should be pointed out that while all of these control techniques are familiar to most of control engineers, it is the structural diagram method that makes it possible to apply this body of knowledge to DPS.

The step responses for proportional control where *(1)* $k=.5$ and two cases of PID control where *(2)* $K_p=.01, T_i=1/6, T_d=2$, *(3)* $K_p=.05, T_i=1/10, T_d=1$ are shown in Fig. 3. The simulations were performed on the transfer functions using Euler's method to replace s $(s=(z-1)/T)$ and $e^{-s}=z^{-n}$, where $n=1/T$. The time step used is *.001 sec.*

The other classical approaches, such as lead and lag compensators add little significant information.

Another more complex control problem is to regulate the temperature distribution along the whole x direction. For this case, the chosen error signal has the following form:

$$e(s) = \int_0^1 (Q(y,s) - Q^*(y,s))dy$$

The control signal is then $U(s) = W(s)e(s)$, where $W(s)$ is the lumped compensator. Using the same techniques as above, the system response can be regulated to the desired specifications for a reasonably given expected signal $Q^*(y,s)$ (obviously, $Q^*(y,s)$=constant is not a reasonable choice because of the uniform distribution assumption of control signal $u(t)$). Fig.4 shows the steady states of the temperature distribution of the material being "cooled" for two cases of control design.

This system will be considered further in a later paper where robust control methods [4] and functional observers will be considered.

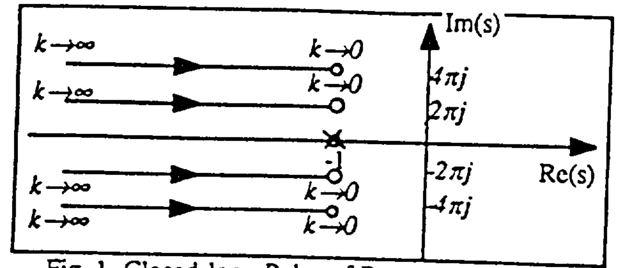


Fig. 1 Closed-loop Poles of Proportional Control

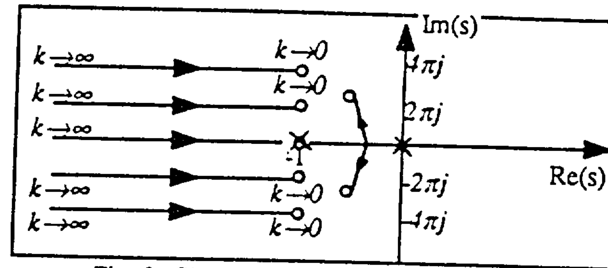


Fig. 2 Closed-loop Poles of PID Control

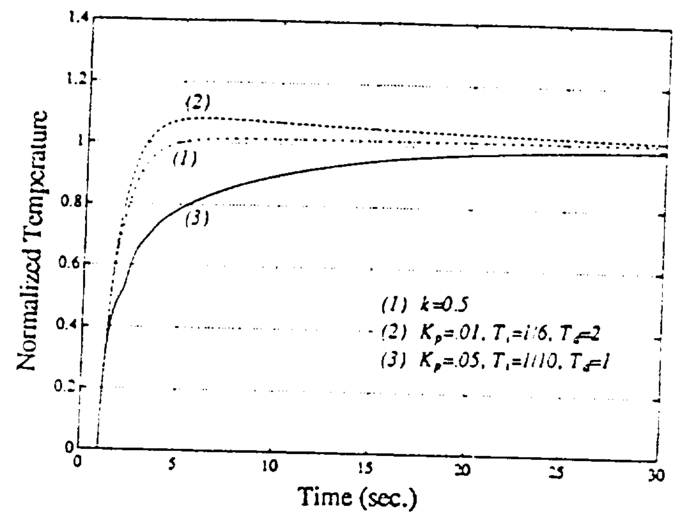


Fig. 3 Step Response of Closed-loop System

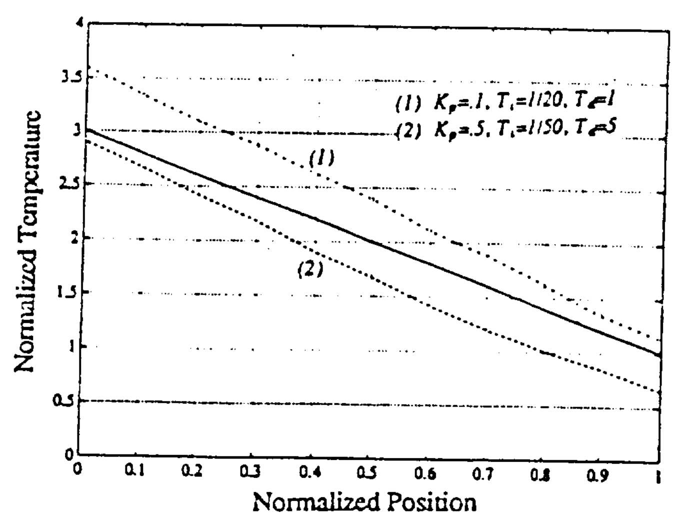


Fig. 4 Steady States of Temperature Distribution

IV. Summary

It has been shown that the structural theory for DPS has many properties similar to those of lumped systems, as well as some substantial differences. These are mostly related to having non-rational transfer functions. Unfortunately, each particular type of PDE has a corresponding non-rational function of the complex variable s (eg. $J(x,s)$, $sinh(x,s)$, etc). Thus each particular system requires unique special study to determine its closed loop poles as root locus methods will not generally be available.

V. References

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