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Annual Progress Report on the Project
"Particle Acceleration, Transport and Turbulence in Cosmic and
Heliospheric Physics"
funded under the Space Physics Theory Program
Grant number NAG-W-1573 (formerly NAG-W-2076)

The Bartol SPTP project, including PI Prof. W. Matthaeus, and a critical mass group of Co-Investigators at Bartol, has continued productive activities during its second year of its first three year grant period. Our scientfic focus is in three areas, represented in the project title: first, the physics of particle acceleration and transport, including heliospheric modulation and transport, shock acceleration and galactic propagation and reacceleration of cosmic rays; second, the development of theories of the interaction of turbulence and large scale plasma and magnetic field structures, as in winds and shocks; third, the elucidation of the nature of magnetohydrodynamic turbulence processes and the role such turbulence processes might play in heliospheric, galactic, cosmic ray physics, and other space physics applications. In this progress report we describe our long term goals, review our recent scientific progress, and finally describe recent organizational activities and personnel matters.

Research Goals

In order to orient our efforts towards we have adopted two major research goals: the development of self consistent models for the solar wind and for shock structure and particle acceleration. What we would like to achieve in a solar wind model is the inclusion of the dynamics of the large scale plasma flow and magnetic field, along with a reasonable treatment of the dynamics of small scale MHD turbulence and energetic particles. The main goals of a solar wind model would be to account for the acceleration of the wind, the radius and nature of the critical points of the flow, the radial evolution of the turbulence, and the influence of the wind and the turbulence on the energetic particles, both of solar and galactic origin, that are transported within the wind. Goals for a self-consistent shock model are quite similar. To account properly for shock structure the effects of the accelerated particles must be treated on the same footing as the shock hydrodynamics. But to treat the energetic particles in a refined manner, the effects on the particles due to the

turbulence, including pitch angle scattering and stochastic acceleration, must be included. For full self consistency, we need also to include the influences of the large scale plasma structures on transport of the turbulence and the excitation of MHD fluctuations by the energetic particles. Our recent efforts are geared towards assembly of first attempts at these models in the next one or two years. Somewhat longer terms goals include consideration of galactic transport of cosmic rays, and development of a qualitative model for the heliospheric termination shock, both of which also require similar levels of feedback among the large scale plasma features, the turbulence, and the energetic particles.

Recent Research Activities and Accomplishments

Transport Theory for MHD Turbulence

One major recent activity is the development of a transport theory for MHD turbulence in a weakly inhomogeneous background. The origin of this work lies in the formative work of Tu and coworkers who wrote down a theory for transport of outward traveling interplanetary Alfvenic fluctuations, consisting of a WKB-like spatial transport operator, along with a simple Kolmogoroff-like model for wavenumber transfer due to strong nonlinear couplings. We began by considering how this theory might be extended to treat both outward and inward Alfvenic fluctuations, or in the language of turbulence, "mixed cross helicities" of the turbulent fluctuations. The need for this kind of extended theory is strongly motivated by observations that show a preponderance of Alfvenic fluctuations int he inner orbit of Helios (0.29 AU), and a distinctive decay of the "Alfvenicity" or cross helicity with increasing heliocentric distance. By 2 AU or so, in the vocabulary of wave theory, the waves appear to be nearly equally of inward and outward types; the cross helicity is almost completely "mixed". In addition, observations also show that the Alfven ratio, or ratio of fluid kinetic energy to magnetic energy in a specified wavenumber interval, generally takes on values somewhat less than unity in outer heliospheric observations. In wave theory, WKB theory and in Tu's theories, both the Alfvenicity and the Alfven ratio remain constant. We realized early on that the Tu models cannot account for these observations,

not only because they do not consider a transport equation for the inward waves, but also because the derivation of the equations ignores the possibility of couplings between inward and outward waves, either through local turbulence effects, or through couplings of the weak background gradients to the cross-correlation of the two wave types. Consequently, there was a need for several a formalism involving several transport equations, involving at least the power spectrum of inward waves, the power spectrum of outward waves, and also an equation for the cross correlation of the inward and outward wave fields.

A theory of this type can be developed through a multiple scale analysis procedure and closely resembles turbulence modeling equations in hydrodynamics, such as the "K-epsilon" models. The most convenient way to represent the theory is using the Elsasser decomposition of MHD, which for the case of locally incompressible turbulence, corresponds to a decomposition into inward and outward type waves. An early result of these theories was the recognition that they admitted the possibility of departures of the Alfven ratio from unity, and also the generation of inward traveling fluctuations from the initially dominant outward fluctuations. The latter phenomenon was called "mixing", and was found to be due to the new linear terms in the transport equations that involve the cross correlation of the Elssaser fields.

A question that raised some interest in the community regarding the new turbulence transport theory was how it can be brought into correspondence with traditional WKB treatments of wave of solar wind fluctuations. We now believe this question has been answered completely. First we considered the behavior of the mixing terms in the spectral transport equations for cases in which nonlinearities can be neglected. We found that the small parameter needed for validity of the turbulence transport formalism, i.e., the ratio of correlation scale to heliocentric distance, is in fact distinct from an additional small parameter needed to recover WKB-type transport. The additional parameter is essentially a measure of the phase mixing between the plus and minus Elsasser fields. For waves obeying a nontrivial dispersion relation, this amounts to a phase mixing term, that becomes small for short wavelength waves. Two cases were identified, within the context of linear MHD, in which the phase mixing term need not be small. These are the case of vanishing large scale Alfven speed, and the case of linear wave fluctuations with wavevectors nearly perpendicular to the large scale magnetic field. In either case the dispersion relation in the plasma frame does

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not distinguish between inward and outward type fluctuations, the phase mixing term is order-one, and the mixing effect remains strong in the scale separated transport theory. More recently we have augmented this perspective by developing a single multiple scale treatment of linearized MHD that gives, alternatively, a theory with strong mixing or WKB equations, depending solely upon the choice of solvability condition imposed on the expansion. The solvability condition leading to WKB theory is allowable for cases when the wave dispersion relation is invoked and is nontrivial, as had been found earlier. However, the solvability condition leading to the mixing type theory is always allowed, provided the scale separation condition is met. Thus, the mixing transport theory is found to have a wider radius of convergence, though it is inherently more complicated than WKB theory since it involves more than one equation.

The complexity of the mixing type transport theory goes beyond the need for several linear transport operators. In addition, models are also required for the local nonlinear effects due to triple correlations of the turbulence. This requirement was also present in the Tu theories, but in the present case, nonlinear models are needed for each of the independent turbulence spectra. Though some suggestions have been made for modeling the nonlinear terms, this remains an active area of research, with more input needed from basic turbulence studies. The theory can involve up to 16 independent scalar spectra, each with an associated transport equation and a modeled nonlinear term. It appears to be convenient to adopt assumptions concerning the rotational symmetry of the turbulence to reduce the complexity of the model. Our first discussions of analytical solutions to the transport equations made use of the assumption of isotropy of the fluctuations.

More recently, we have constructed a Chebychev collocation method code to integrate the coupled transport equations, and we have made various assumptions about the fluctuations in our numerical solutions. One set of numerical integrations was performed to show how the theory goes over to WKB results in appropriate limits. For this study we compared transport of two-dimensional (2D) fluctuation spectra with transport of one-dimensional (1D)"slab" fluctuations at varying values of wavenumber and Alfven speed strength. As expected form the analytical work alluded to above, the 2D results never approach WKB theory - the mixing effect is always strong. The 1D problem converges strongly to WKB results, departing only at extremely long wavelengths, as expected. Finally, we used the linearity of the trans-

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port equations, when modeled turbulence effects are neglected, to investigate prospects for explaining solar wind observations. This is motivated by our observational study that suggested solar wind spectra could be viewed as a mixture of slab and 2D fluctuations. We found that reasonable admixtures of isotropic and 2D fluctuations, or slab and 2D fluctuations could account for the observed decrease in cross helicity with heliocentric distance. The latest results were presented at the Solar Wind Seven Conference in 1991.

Quasilinear Theory

Following on Fermi's seminal suggestion that disordered magnetic fields could serve as "scattering centers" of energetic charged particles, much of the early work on particle transport attempted to fit observations to predictions of diffusion theory, thereby obtaining a phenomenological diffusion coefficient. Subsequently, extensive efforts were made to derive the fundamental transport parameters from the principles of plasma kinetic theory. However, these scattering theories have not been able to reproduce, at least not with any predictive power, the phenomenological diffusion coefficients derived from cosmic ray observations.

The problem with quasilinear theory is not that the predicted mean free path is "too small" or has the "wrong" energy dependence, as is sometimes stated. Rather, the problem is that scattering theory is not sufficiently constrained given present understanding of turbulence in space. For example, in earlier work we showed that making seemingly minor technical alterations to the assumed turbulence power spectrum at very small scales (the dissipation range) could change the predicted mean free path from ten times too small to infinitely too large. A major thrust of this grant is to apply concepts of modern turbulence theory to the cosmic ray scattering problem, in order to narrow the allowable range of "predicted" behaviors of the mean free path and make definite, testable predictions as to its magnitude and energy dependence.

Our recent work has emphasized generalizing scattering theory from a magnetostatic to a fully dynamical representation of the turbulence (while still keeping a dissipation range as well as a realistic — i.e., Kolmogoroff — inertial range). Although some earlier models have relaxed the magnetostatic approximation through consideration of wave propagation effects, these still ignore the dynamical couplings between wave modes which are at the heart

of turbulence theory. We feel an approach to scattering theory grounded in turbulence theory is more general, more powerful, and, ultimately, more correct as a description of particle transport in space plasmas.

We derived expressions for the scattering rate (Fokker-Planck coefficient) and spatial mean free path in a dynamical turbulence model which incorporates a Kolmogoroff inertial range, an exponential dissipation range, and a three-dimensional form sufficiently general to encompass slab, isotropic, and some forms of 2D turbulence geometries, as well as intermediate geometries. These rather complicated expressions were then evaluated on the SDSC Cray using turbulence parameters (e.g., correlation length, magnetic field variance) characteristic of interplanetary space. We also studied two different forms of the dynamical correlation function corresponding to weakly and strongly dynamical turbulence.

In brief, we find that the strongly dynamical model yields a mean free path for protons that is too small and has too strong an energy dependence at low to intermediate energies. In contrast, the more weakly dynamical model yields a larger mean free path with a notable flattening of the energy dependence. The slab model is particularly encouraging in this regard: the proton mean free path remains in the range 0.13–0.38 AU as the energy ranges from 0.1 keV to 5 GeV (four decades of rigidity and nearly eight decades of energy), which matches almost precisely the behavior determined from cosmic ray observations.

Scattering in dynamical turbulence has some novel characteristics, which may provide a suitable basis for testing the model observationally and, indirectly, for deriving new information on the dynamical properties of turbulence in space. For example, the mean free path exhibits a rather strong sensitivity to the Alfvén speed, a dependence that is completely lacking in standard magnetostatic models. Also, the model predicts larger mean free paths for electrons than for protons of the same rigidity, a feature which finds some support (previously unexplained) in cosmic ray observations.

Test Particle Acceleration

Magnetic reconnection has long been studied as a mechanism for converting magnetic energy into kinetic energy in solar flares and magnetospheric substorms. Most of the theoretical models for magnetic reconnection treat it as a steady-state process with magnetic fields that are essentially laminar. The

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reason for this is that MHD processes are commonly believed to be dominated by long-wavelength, low-frequency effects. From a theoretical standpoint, the major difficulty with these models is that the time spent by particles in the diffusion region, where the induced electric fields are greatest, is too short to account for significant energization of particles. The introduction of a turbulent component of the magnetic field in reconnection dramatically alters the physics of the above models. Recent test particle studies[1-3] have shown that the effect of MHD turbulence in reconnection alters the particle dynamics by trapping particles and retaining them in the diffusion region. We are studying this process as a possible mechanism for cosmic ray acceleration, in the context of homogeneous MHD turbulence, wherein reconnection plays an important role in the inverse cascade from small- to large-scale magnetic field structures. The preliminary results presented here show that this process can result in significant particle "heating," (i.e. an increase of the average test particle energy). to cosmic ray energies.

The technique we are using is test particle calculations with electric and magnetic fields (e.g. A with $E = -\partial A/\partial t$) obtained from simulations of reconnection in homogeneous turbulence using a two dimensional, incompressible, MHD code[4]. The fields thus generated are used to advance the equations of motion for large numbers (typically 10^5) of test particles with various choices of initial velocities and spatial distributions. From the time history of the particle positions and velocities estimates of the distribution function, and its moments, may be obtained, as well as a dynamical picture of its evolution. Previously, this technique has been used to study effects of turbulence on particle acceleration by reconnection in sheet pinch geometry[2]. In these studies, simple scaling argument were used to obtain estimates for maximum and mean particle energy which fit reasonably well with simulation results. Our initial results of test particle studies on acceleration by reconnection in homogeneous turbulence show that power law distributions can be generated from initially Maxwellian distributions by the reconnection process in a few characteristic times. The mean and maximum test particle energy scale according to the estimates developed for acceleration by reconnection[2], suggesting a simple statistical description of the process. The results also demonstrate that reconnection associated with the merging of magnetic islands in MHD turbulence is an effective mechanism for accelerating particles. It remains to be determined whether the scaling demonstrated above extends to the extremely large values of α and the associated high particle energies

expected in the cosmic ray parameter regime. We are currently extending these calculations to systems that more closely model the case for cosmic rays, an effort which will require larger two dimensional simulations, three-dimensional simulations, relativistic equations of motions and a supporting theoretical kinetic model.

Particle acceleration and shock structure

One of the most interesting aspects of the SPTP project which draws together intimately each of the three components in the title concerns the development of models which describe the acceleration of energetic particles at shock waves. It is by now widely recognized that particles are energized by shocks throughout the Heliosphere, thus leading to the formation of power-law tails on particle distribution functions. Rather often, e.g., at interplanetary travelling shocks, cometary shocks, the Heliospheric termination shock, the Jovian bow shock, and at SNR shocks, the pressure contribution from the energetic particles can modify the dynamics of the background flow and gas shock significantly. Particles, especially at quasi-parallel shocks, gain energy through scatterings across the shock, the scattering centers being resonant (often self-excited Alfvén) waves. The particles execute a random walk in space (and energy), which can be described in terms of an appropriate transport theory. The various diffusion coefficients that arise in such a transport theory depend critically on the nature and properties of the underlying turbulence. In particular, for example, since the energized (shock accelerated) particles are highly mobile, a very extended foreshock region can exist upstream of the shock, for which one requires a turbulence description appropriate to inhomogeneous flows. Thus, at this level, we have a close tie-in of the physics of turbulence in the inhomogeneous solar wind to that of particle acceleration at shock waves.

To achieve our goal of an almost fully self-consistent treatment of particle acceleration at shock waves such that it can find application to the Heliospheric termination shock and SNR shocks, still requires some considerable effort. To some extent, this is because numerous open questions need to be solved along the way, all of which is reflected in the spate of papers produced by the various investigators in this project. To achieve a detailed understanding of the physics of particle acceleration at quasi-parallel requires therefore the wedding together of a variety of disciplines, something to which a critical

mass project such as SPTP is ideally suited.

In investigating shocks modified by an energetic particle component, we have considered both the general problem and the more specific problem of cometary shocks. The latter problem introduces the additional complication of ion pick-up and thus mass-loading of the solar wind, but has the virtue of being well observed and a wealth of observations exist.

Cometary shocks

The one-dimensional magnetohydrodynamics of shocked flows subjected to significant mass-loading are considered. Recent observations at comets Giacobini-Zinner and Halley suggest that simple non-reacting MHD is an inappropriate description for active cometary bow shocks. The thickness of the observed cometary shock implies that mass-loading represents an important dynamical process within the shock itself, thereby requiring that the Rankine-Hugoniot condition for the mass flux possess a source term. In a formal sense, this renders mass-loading shocks qualitatively similar to combustion shocks, except that mass-loading induces the shocked flow to shear. Nevertheless, a large class of stable shocks exist, identified by means of the Lax conditions appropriate to MHD. Thus, mass-loading shocks represent a new and interesting class of shocks which, although found frequently in the solar system, both at the head of comets and, under suitable conditions, upstream of weakly- and non-magnetized planets, has not been discussed in any detail. Owing to the shearing of the flow, mass-loading shocks can behave like switch-on shocks regardless of the magnitude of the plasma beta. Thus, the behaviour of the magnetic field in mass-loading shocks is significantly different from that occurring in non-reacting classical MHD shocks. It is demonstrated that there exist two types of mass-loading fronts for which no classical MHD analogue exists, these being the fast and slow compound mass-loading shocks. These shocks are characterized by an initial deceleration of the fluid flow to either the fast or slow magnetosonic speed followed by an isentropic expansion to the final decelerated downstream state. Thus, these transitions take the flow from a supersonic to a supersonic, although decelerated, downstream state, unlike shocks which occur in classical MHD or gas dynamics. It is possible that such structures have been observed during the Giotto-Halley encounter, and a brief discussion of the appropriate Halley parameters is therefore given, together with a short discussion on the determination of the shock normal

from observations. A further interesting new form of mass-loading shock is the "slow-intermediate" shock, a stable shock which possesses many of the properties of intermediate MHD shocks yet which propagates like a slow mode MHD shock. An important property of mass-loading shocks is the large parameter regime (compared with classical MHD) which does not admit simple or stable transitions from a given upstream to downstream state. This suggests that it is often necessary to construct compound structures consisting of shocks, slip waves, rarefactions and fast and slow compound waves in order to connect given upstream and downstream states. Thus the Riemann problem is significantly different from that of classical MHD.

An invited talk was presented by G.P. Zank at the annual APS Plasma Physics Meeting in Tampa, Fl on this subject (November, 1991) and he has been invited to review "The Physics of the Cometary Shock" at the "Critical Problems in the Plasma Environments of Comets and other Non-Magnetized and other weakly Magnetized Bodies" meeting at the 4th COSPAR Colloquium in August 1992.

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Shock structure with injection

More general investigations of the structure of cosmic-ray-modified shocks have continued. It is believed that cosmic ray acceleration can occur where

energetic particles are diffusively scattered by (presumably turbulent) magnetic inhomogeneities in a shocked background flow. Little is known, however, about the structure and dynamics of such a shock and the mechanism for "injecting" particles from the thermal background gas into the energetic particle distribution. We have considered this injection problem by developing a new, self-consistent model of cosmic ray hydrodynamics within the context of a non-linear, two-fluid description. The injection is shown to be a form of "thermal leakage" between the background gas and cosmic ray fluid which plays a significant role in determining the structure and dynamics of the shock. Previously developed steady-state shock structure models are shown to be modified by the injection. In addition, we find an entirely new class of compound shock transitions which do not exist in the non-injection models. The results also demonstrate that the efficiency of particle acceleration is affected by the injection process, as the shock tends to dynamically regulate the injection. It is of interest to observe that many of the techniques developed to handle cometary shocks proved to be effective tools in developing the theory of injection fronts.

In addition to the steady-state results, we have also developed a one- dimensional finite-difference code for time-dependent solutions to the two-fluid equations, including particle injection. The code has been used to investigate the time-dependent evolution of cosmic-ray-modified shocks with various initial conditions. For example, we have studied the transition to a steady-state, and find the calculated downstream states do not always agree with earlier analytic solutions. There exists a sensitive dependence on the diffusion rate which is not a part of the steady-state models. Currently, we are using the time-dependent code to investigate the structure of the solar wind termination shock, including possible source models and the dynamic effect of the anomolous cosmic ray component.

Multiple scales perturbation techniques have been used to study weakly multi-dimensional, long wavelength cosmic ray modified shocks. The canonical equation governing the shock structure in the limit of substantial cosmic ray pressures is the 1+3D Burgers' equation which includes the effects of cosmic ray diffusion, non-linearity and wave diffraction. For very low cosmic ray pressures, the balance between cosmic ray diffusion, Hall current dispersion and non-linearity yields the 1+3D Korteweg-de Vries-Burgers' equation for the shock structure. The effects of diffraction have been investigated in terms of Green's function solutions of the linearized 1+3D Burgers and

KdVB equations, and also in terms of solutions with singular Dirac-delta initial distributions. This shows that the curvature of the wave front surfaces decreases monotonically with increasing time owing to the effects of wave diffraction. The shape of the wave surface can also be discussed in terms of the wave eikonal equation and the characteristic effects of dispersion and dissipation on fast and slow magnetosonic waves have been discussed.

Finally, the question of the origin of the very highest energy cosmic rays (in excess of 10^{14}eV) is currently being addressed by a group consisting of a plasma physicist and high energy particle astrophysicists, again demonstrating the unique opportunities that are possible within a critical mass project such as the SPTP project. A study of the gross features of elemental composition at energies above 10^{14}eV has been made to distinguish among different classes of models of cosmic ray origin. The importance of this region is that estimates indicate that supernova remnants can accelerate cosmic rays up to energies of about 10^{14}eV/nucleon. Thus regions such as the "knee" and "ankle" of the cosmic ray spectrum are of particular interest because they may signify the necessity for either a multi-stage shock acceleration mechanism or even a completely new source for the very highest energy cosmic rays. Since air shower experiments classify events approximately by total energy, the upper limit of a cosmic ray accelerator should show up as an abrupt change of composition in such an experiment.

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Nearly incompressible fluid dynamics and turbulence

The theory of nearly incompressible (NI) fluid dynamics developed previously for hydrodynamics has been extended to magnetohydrodynamics (MHD). On the basis of a singular expansion technique, modified systems of fluid equations are derived for which the effects of compressibility are admitted only weakly in terms of the different possible incompressible solutions (hence the phrase "nearly incompressible MHD"). The models of NI MHD are of fundamental importance to the general theory of (subsonic) magnetofluids, representing as it does the interface between the compressible and incompressible descriptions. The theory developed here does not hold in the presence of very large thermal, gravitational or field gradients. It is found that there exist three distinct NI descriptions corresponding to each of the three possible plasma beta ($\beta \equiv$ the ratio of thermal to magnetic pressures) regimes ($\beta \ll 1$, $\beta \sim 1$, $\beta \gg 1$). In the $\beta \gg 1$ regime, the compressible MHD description converges in the low Mach number limit to the equations of classical incompressible 3D MHD. However, for the remaining plasma beta regimes, the imposition of a large DC magnetic field forces the equations of fully compressible 3D MHD to converge to the equations of 2D incompressible MHD in the low Mach number limit. The "collapse in dimensionality" corresponding to the different plasma beta regimes clarifies the distinction between the 3D and 2D incompressible MHD descriptions (and also that of $2\frac{1}{2}$D incompressible MHD) and demonstrates, for example, that one cannot investigate the effects of a large DC magnetic field on an incompressible magnetofluid by simply considering the classical 3D incompressible MHD equations in the presence of a large applied magnetic field. This distinction has not been adequately appreciated in the past. The collapse in dimensionality that occurs as a result of a decreased plasma beta can carry over to the weakly compressible corrections. For a $\beta \sim 1$ plasma, Alfvén waves propagate parallel to the applied magnetic field (reminiscent of reduced MHD), while for a $\beta \ll 1$ magnetofluid, quasi-1D long wavelength acoustic modes propagate parallel

to the applied magnetic field. The detailed theory of weakly compressible corrections to the various incompressible MHD descriptions is presented and the implications for the solar wind emphasized. The NI theory of MHD can account for observed Kolmogorov-like density fluctuation spectra, anisotropic magnetic and density fluctuation spectra, temperature fluctuation spectra as well as many other observed features in the solar wind in a very natural way. However, besides space plasmas, the NI theory is likely to be of value whenever an incompressible or reduced MHD description is thought to be a reasonable approximation.

It appears that significant observational evidence exists to support the general framework of nearly incompressible fluid dynamics and that this represents a new and developing field, rich in possibilities, both theoretical and observational.

Invited talks on this and related subjects have been presented by G.P. Zank at the Solar Wind 7 Meeting (September, 1991) and at the Sherwood International Plasma Theory Meeting (April, 1992).

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Basic Turbulence and Numerical Methods Studies

Turbulent Spectral Transfer

One of the major questions in turbulence research involves the nature of spectral transfer of various ideal invariants. In incompressible turbulence some quantities are known to cascade directly to shorter scales and yet some other inverse cascade to larger scales. In 3D hydrodynamic Kolmogoroff type turbulence, energy cascades to smaller scales and dissipate. The direction of cascade may alter in the presence of large scale flows or magnetic fields. A particular progress made recently involves a 3D magnetohydrodynamic (MHD) turbulence at large kinetic and low magnetic Reynolds numbers. Turbulence is assumed to evolve in the presence of a strong uniform magnetic field. In this situation the leading order dynamics is reduced to simple hydrodynamics plus a body force of magnetic origin expressed in terms of the flows. It is shown (Hossain 1991a) by numerical simulation of a model of MHD that the energy inverse cascades to longer length scales when the interaction parameter is large. (The interaction parameter is essentially the ratio of the square of the external magnetic field to the diffusivity). As a result of the interaction of the flows with the magnetic field, the flow organizes itself into large scale motion. While the steady state dynamics of this driven problem is three-dimensional in character, the behavior has resemblance to two-dimensional hydrodynamics. These results have implications both in the turbulence theory and the planetary dynamos.

The next step is to consider the full 3D MHD so that no restriction on the magnetic Reynolds number will be imposed. Eventually, compressible turbulence will be considered.

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Subgrid Modeling

Direct numerical simulation has become a major tool in the study of complicated non-linear problems. Although simulations have provided us with valuable insights and revealed new phenomena, it has been limited to a small subset of parameter range we are interested in. Turbulent flows with moderately high Reynolds number, particularly flows in three spatial dimensions have not been possible to simulate directly. The number of grid points required to resolve all scales ranging from large energy containing eddies down to smallest scales representing dissipation (which scales as $Re^{9/4}$ in three dimensions) is simply impossible. A practical solution to this closure problem is to simulate directly the large eddies and model the effects of small unresolved scales on them. This approach, also known as large eddy simulation (LES), led to various eddy viscosity models. Most popular sub-grid models provide an eddy viscosity. If eddy viscosity is adequate to model the effects of sub-grid scale motion on the large eddies, then it would be possible to simulate large Reynolds number turbulence using only a modest number of grid points by solving the Navier-Stokes equation with eddy viscosity replacing the molecular viscosity. The purpose is to simulate fluids in the presence of *driving* and dissipation. Freely decaying turbulence may exhibit Kolmogoroff spectra as a transient state in a simulation which starts from a nearby state. Long time driven simulations are required to achieve statistical steady states to validate any serious sub-grid model which may be used in LES with confidence. We have examined the utility of a class of eddy viscosity models in LES numerically (Hossain 1991b). It is shown that the simple eddy viscosity models do not represent spectral transfer accurately enough. Then, a nondiffusive sub-grid model is introduced where modes with wavenumbers greater than and near the cutoff are retained, as a part of closure, to provide stochastic sub-grid interactions of non-diffusive nature for the large scale modes. Low resolution driven numerical simulations using this sub-grid model produce Kolmogoroff power law spectra. The non-diffusive sub-grid model seems promising, although it is computationally expensive. Taking this success as a guide, more research needs to be done to seek efficient sub-grid model keeping the main features of the current model intact. It is also desirable to extend this model to include compressible turbulence.

Spectral Methods for Compressible Turbulence

Turbulence theory has increasingly relied on direct numerical simulation. The complications of turbulence, particularly the involvement in the dynamics of a wide range of spatial and temporal scales, places stringent requirements on computational algorithms. With the advent of computers capable of direct fluid simulations, spectral and pseudospectral algorithms have emerged as the methods of choice for the turbulence theorist. Moreover, basic turbulence questions are often posed for incompressible, homogeneous flows, and so periodic boundary conditions can be used. In these instances spectral methods based on Fourier series are particularly accurate and effective. There has been considerable recent interest in extending theoretical models of turbulence to compressible fluids and magnetofluids. This is especially true in MHD studies of astrophysical plasmas, for which there have often been questions regarding the applicability of the incompressible approximation. One is led then to consider whether the mathematical attractiveness and computational advantages of spectral methods can be carried over to compressible turbulence. Operating largely in analogy to the incompressible case, a number of useful spectral method compressible turbulence studies have begun to appear, both in hydrodynamics and MHD. There is a need to measure compressible turbulence algorithms against the same yardstick that establishes the robustness of certain incompressible algorithms: how well and for how long do these algorithms conserve certain dynamical invariants in the ideal limit?

We have recently reported (Hossain et al. 1992; Ghosh et al. 1992) the progress made in addressing various issues in compressible spectral methods. In Ghosh et al. we have presented results of some systematic studies of pseudospectral and Galerkin spectral algorithms for simulation of compressible MHD turbulence, and its simpler relative, compressible Navier Stokes turbulence. Our efforts have been geared towards formulating compressible algorithms that possess as many as possible of the advantageous features of spectral method algorithms for incompressible MHD and fluid turbulence. Thus, we are led to emphasize the importance of "exact" conservation laws for certain global invariants, especially the energy. The influence of aliasing errors has also been emphasized in discussing properties of pseudospectral algorithms. We have compared the energy conservation properties of com-

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pressible pseudospectral and Galerkin spectral approximations. It is generally not possible to "ruggedly" conserve energy in the compressible cases using Galerkin methods, owing principally to the fact that the energy is a cubically-nonlinear quantity. In contrast, it is possible to construct pseudospectral algorithms in algebraic forms that obey certain adjoint properties, thus securing essentially exact compressible energy conservation. Numerical examples have been given for these effects.

Organizational Activities and Personnel

During the second year, the project hired a postdoctoral fellow, Dr. Denis Donohue, and a Research Associate, Dr. Duane Pontius. Dr. Donohue is working mainly with Co-I Prof. Gary Zank on shock structure and acceleration of energetic particles at shocks, including studies related to particle injection and the physics of the heliospheric termination shock. Dr. Pontius is working closely with Graduate Student Sean Oughton on transport theories for solar wind turbulence, and along with W. Matthaeus, is gearing up for assembly of our preliminary attempt at a solar wind model with modeled and transported dynamically evolving MHD turbulence.

During this year of the project Dr. Gary Zank began his faculty appointment, and he continies to be a major contributor to the SPTP goals.

A number of Bartol investigators were on the organizing for the Conference entitled "Particle Acceleration In Cosmic Plasmas" held at Bartol late last year. In addition about a dozen papers were presented at this cnference on research supported by the SPTP. The conference was well attended by international leaders in cosmic ray physics, and the proceedings will be published by the AIP.

References and Additioal Publications Supported by the Grant

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2. Zank, G. P., G. M. Webb and D. J. Donohue, Particle Injection and the Structure of Cosmic-Ray-Modified Shocks, Particle Acceleration in Cosmic Plasmas, ed. G. Zank and T. Gaisser, 1992.

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4. Donohue, D. J. and G. P. Zank, Time-Dependent Cosmic Ray Shocks With Injection: A Progress Report, Particle Acceleration in Cosmic Plasmas, ed. G. Zank and T. Gaisser, 1992.

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6. Ghosh, S., Hossain. M., and Matthaeus, W. H., *Computer Phys. Commun.*, submitted.

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9. Relaxation Processes in a Low Order Three Dimensional Magnetohydrodynamics Model, T. Stribling and W. Matthaeus, Phys. Fluids B, 3, 1848 (1991).

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