Effects of Fade Distribution on a Mobile Satellite Downlink and Uplink Performance in a Frequency Reuse Cellular Configuration

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ABSTRACT

In a mobile satellite system with a frequency reuse cellular configuration, significant co-channel interference can be experienced due to the antenna side-lobe level. The signal will be subjected not only to its own fading, but also to the effect of the varying degree of fading on the co-channel interferer, and this interference will behave differently in the up and in the down-link. This paper presents a quantitative evaluation of the combined effects of fades and co-channel interference on a mobile satellite link.

INTRODUCTION

In a mobile satellite system with a frequency reuse cellular configuration, significant co-channel interference can be experienced because of the satellite antenna sidelobe level.

In the downlink case, the satellite can emit simultaneously in the same frequency channel to two mobile stations located in different cells. Some of the signal intended for one station will be diverted to the other through the antenna sidelobe causing thereby some co-channel interference. Both the signal and the interference follow the same path from the satellite to the mobile station and both signal and interference will then fade together.

In the uplink case, a mobile station signal incoming at the satellite receiver can be subjected to co-channel interference from a mobile station located in an other cell. In this case, the signal and the interference fade independently. Thus one can imagine a worst case where the signal is subjected to heavy shadowing and the interference to light shadowing, or, at the contrary, a most favorable case where the signal is subjected to light shadowing and the interference to heavy shadowing.

In this paper, the impact of fades on a mobile satellite link is analyzed quantitatively. To carry on this evaluation, the ratio of signal level on noise added to interference (S/(N+I)) is calculated for an L-band uplink and downlink. The signal and the interference are represented in a probabilistic manner with their PDF's expressed by the model suggested by Loo [1,2] for heavy, light and overall shadowing.

The complementary information required to implement the evaluation of S/(N+I) is derived from estimated link parameters for a mobile satellite system. In the uplink, a C/N of 24 dB and an antenna sidelobe level of-22 dB are assumed. For the downlink case, the assumptions are for a C/N of 20 dB and a sidelobe level of-18 dB, which takes into account an expected degradation of the sidelobe isolation level due to the requirement of flexible power distribution among the different antenna beams. Results are applicable to transmissions in the L band.

It is important to note that, in this paper, the ratios and levels given correspond to power levels when given in dB (i.e.: S/N=24 db, antenna sidelobe level = -22 db), but that they correspond to signal (voltage) ratios or levels when they are given in a linear scale (i.e.: S/N=15.85, antenna sidelobe level = 1/12.58).

THEORY

According to Loo's model [1,2], the amplitude level of a signal transmitted through a land mobile satellite link can be described by the sum of two vectors. The first one represents the slow amplitude variations encountered in the channel and can be statistically described by a lognormal distribution. The second vector amplitude probability density function follows a Rayleigh law and characterizes multipath phenomena. The probability density function (PDF) of the resulting vector can be expressed from those preceding distributions to give:

$$f(r) = \frac{r}{b_o\sqrt{2\pi d_o}} \int_o^{\infty} \frac{1}{z} \exp\left[\frac{-(\ln z - \mu)^2}{2d_o} - \frac{(r^2 + z^2)}{2b_o}\right] I_o\left(\frac{rz}{b_o}\right) dz \qquad (1)$$

where $I_o(u)$ is the modified Bessel function of zeroth order, b_o represents the average scattered power due to multipath, $\sqrt{d_o}$ and μ are the standard deviation and mean respectively of the lognormal distribution forming this model.

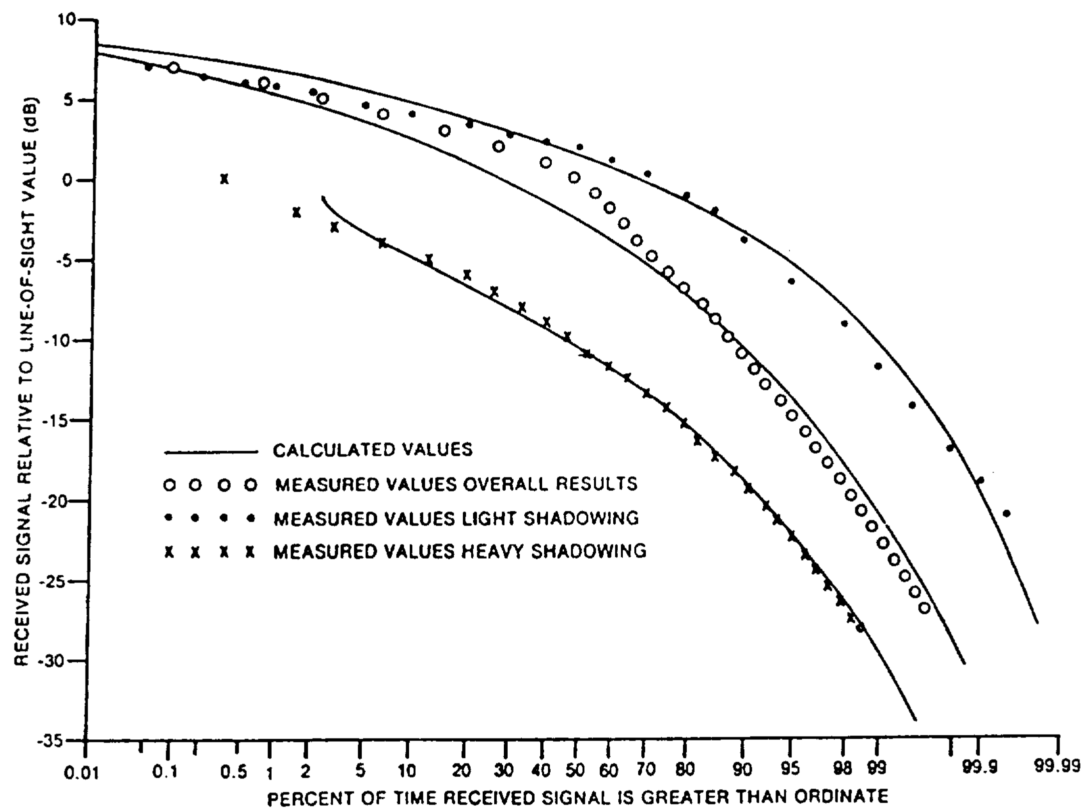

Figure 1: Comparison of measured and calculated values of the probability distribution of signal in a M-SAT channel (from Loo ([1]).

Environment	Standard Deviation $\sqrt{d_o}$	Mean μ	Multipath Power b_o
Infrequent light shadowing	0.0132	0.115	0.158
Frequent heavy shadowing	0.649	-3.91	0.0631
Average shadowing (overall results)	0.053	-0.691	0.251

Table 1: Loo's model parameters

In his work, Loo verified his model with experimental results obtained by Butterworth et al. [3,4] of C.R.C. in Ottawa. Figure 1 [1] gives a comparison of the measured values and of the model with the parameters adjusted as in Table 1 [1]. It represents the cumulative probability distribution of the received signal in a land mobile satellite link for different types of environment at the mobile.

3. UPLINK

In the uplink case, a mobile station signal incoming at the satellite receiver can be subjected to co-channel interference from a mobile station located in another cell. The signal and the interference fade independently.

The analysis below will be carried out considering a S/N ratio of 24 dB and an antenna sidelobe level of -22 dB.

With a reference level normalized to one, equation 1 can be used directly to represent the PDF of the signal level. S is then considered as a random variable following Loo distribution with the parameters b_o, d_o and μ fixed according to the channel or, more precisely, to the environment of the mobile.

$$f(S) = \frac{S}{b_o\sqrt{2\pi d_o}} \int_o^{\infty} \frac{1}{z} \exp\left[\frac{-(\ln z - \mu)^2}{2d_o} - \frac{(S^2+z^2)}{2b_o}\right] I_o\left(\frac{Sz}{b_o}\right) dz \quad (2)$$

In the same way, the interference coming from neighboring cells follows a path that imposes the same type of PDF to its amplitude. But one must take into account the relative attenuation caused by the fact that this interference is picked up by the antenna sidelobe. The interference can be expressed in term of the normalized reference level which gives, on a voltage scale, 1/12.58 (-22 dB).

By applying the rules for functions of random variables, the PDF of the interference level in relation to the normalized reference level is obtained.

Considering that I = S/12.58, then

$$f(I) = \frac{12.58\, I}{b_o\sqrt{2\pi d_o}} \int_o^{\infty} \frac{1}{z} \exp\left[\frac{-(\ln z - \mu)^2}{2d_o} - \frac{((12.58\, I^2) + z^2)}{2b_o}\right] I_o\left(\frac{12.58 Iz}{b_o}\right) dz \quad (3)$$

It is important to note here that the parameters b_0, d_0 and b_0 shown above are not related to those given in equation 2 since, in the uplink case, the signal and the interference follow two different paths. It can happen however that these parameters take the same value for the signal and for the interference, for instance in the particular case where both the mobile station and the interference station are in a similar environment.

The last value to define in order to be able to implement the evaluation of the ratio of interest is the thermal noise. Considering for the uplink a S/N ratio of 24 dB, then, relative to a signal level of unity, N will be given by 1/15.85.

The ratio S/(N+I) to be evaluated is in fact a function of random variables. If we pose Z= S/(N+I), our work will then be equivalent to finding $F_Z(Z)$. The CPF should be expressed in term of the joint PDF of the random variables S and I:

$$F(Z) = \{z \leq Z\} = P\{(S, I) \in D_z\} = \int\int_{Dz} f(S, I)\, dS\, dI \quad (4)$$

To determine F(Z), one has to find the region D_Z for every Z and to evaluate the above integral.

From our previous discussion we know that S and I are independant, thus:

$$f(S, I) = f(S)\, f(I) \quad (5)$$

With further observations we can sort out the appropriate region of integration and define:

$$F\left(Z \leq \frac{S}{N+I}\right) = \int_0^{\infty}\int_0^{Z(N+I)} f(S)\, f(I)\, dS\, dI \quad (6)$$

with f(S) and f(I) defined by equations (2) and (3) respectively. For ease of interpretation, it is preferable to evaluate the percentage of time when the ratio S/(N+I) is larger than a certain value. Thus we will calculate:

$$P\left(Z \geq \frac{S}{N+1}\right) = 1 - P\left(Z \leq \frac{S}{N+1}\right) \quad (7)$$

4. DOWNLINK

For the downlink, the evaluation is carried out considering a S/N ratio of 20 dB and an antenna sidelobe level of -18dB. Thus the thermal noise level N with reference to a signal level S of unity is 0.1, and the PDF of the random variable S is again defined in term of Loo's model by equations (1) and (2).

But for the downlink, the signal and the interference are transmitted through the same channel. The fluctuations that one signal will encounter will affect the other in the same way. Thus S and I cannot be considered as independent random variables anymore.

In fact they should be viewed as "dependent" events in that sense that the value of one variable can be directly computed when the value of the other one is known. The joint PDF of S and I will be expressed in term of their conditional PDF:

$$f(S, I) = f(I/S)\, f(S) \quad (8)$$

The relation linking these two variables is simply the downlink sidelobe level. It states that the interference level will always be 7.94 times lower (18 dB) than the signal level. The conditional PDF takes the form:

$$f(I/S) = \delta(I - S/7.94) \quad (9)$$

so we can define:

$$f(S, I) = \delta(I - S/7.94)\, f(S) \quad (10)$$

F(Z) is evaluated with the same integral equation as in the previous section.

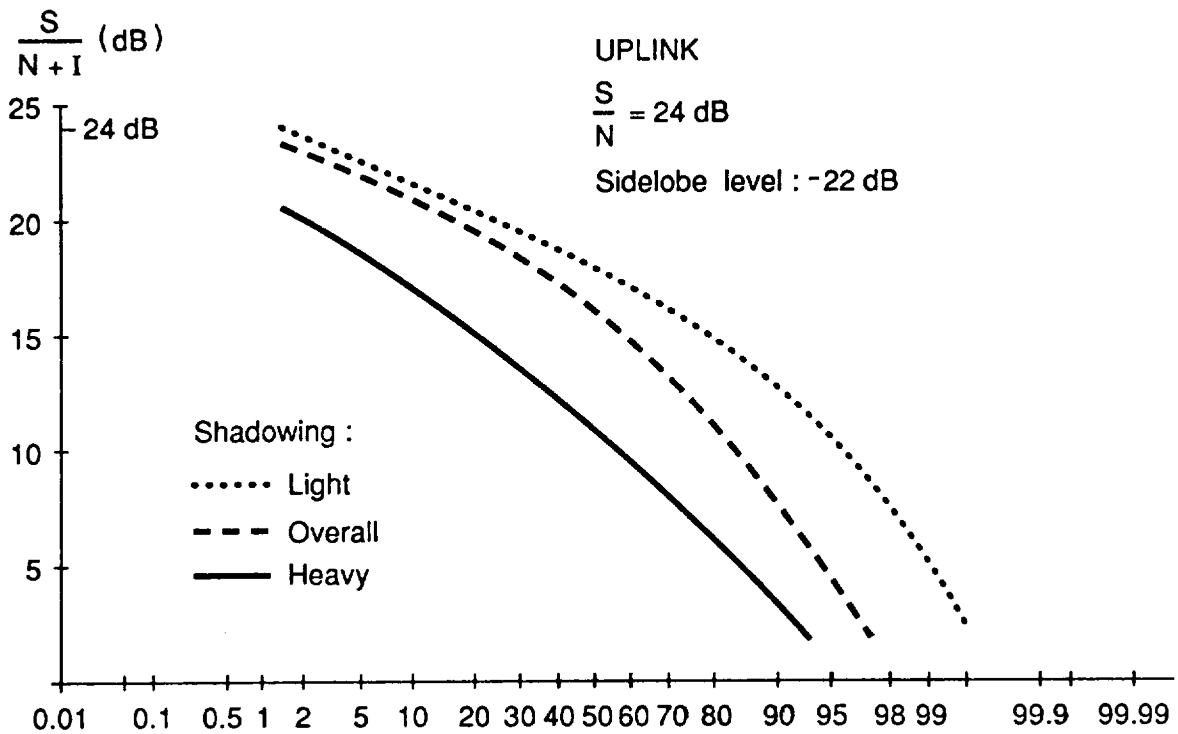

Fig. 2 - % OF TIME $\frac{S}{N+I}$ IS >THAN THE ORDINATE FOR THE UPLINK

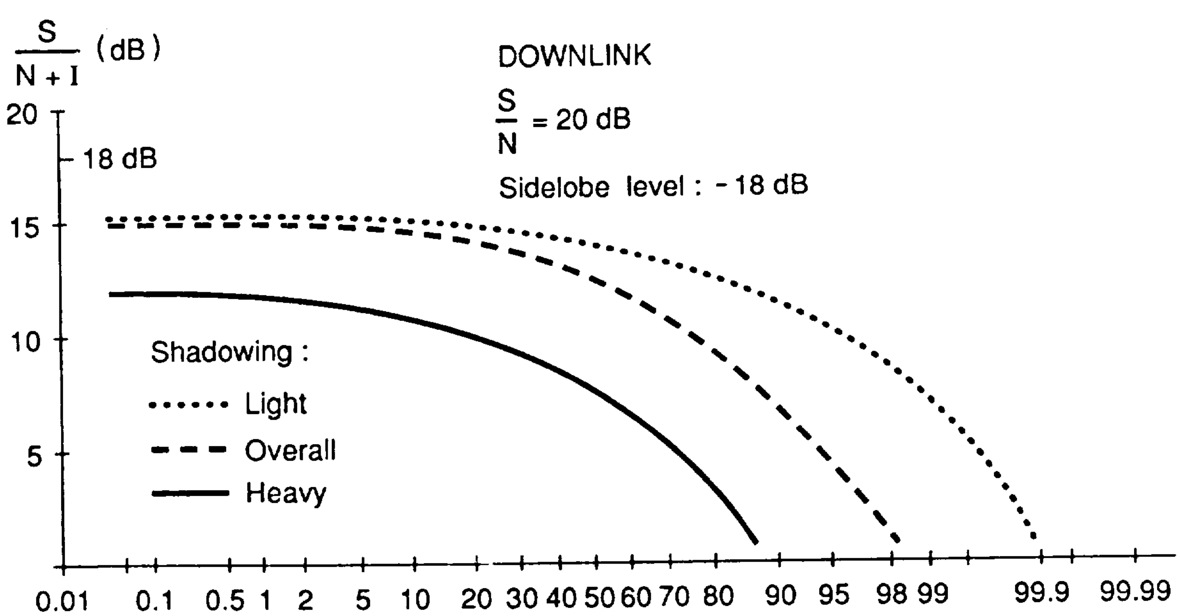

Fig. 3 - % OF TIME $\frac{S}{N+I}$ IS >THAN THE ORDINATE FOR THE DOWNLINK

$$F(Z) = \int_0^{\infty}\int_0^{Zx(N+I)} f(S,I)\,dS\,dI \qquad (11)$$

But with f(S,I) given by (10), this integral equation will be simplified to the following form:

$$F(Z) = \int_0^{\frac{N}{1/z-1/7.94}} f(S)\,dS \qquad (12)$$

The final step will be to seek the cumulative distribution of:

$$P\left(Z \geq \frac{S}{N+1}\right) = 1 - F(Z) \qquad (13)$$

RESULTS AND CONCLUSION

The results for the uplink case are illustrated in figure 2. The three curves give the percentage of time the signal to noise plus interference ratio will be exceeded for the cases where both signal and interference are subjected either to light, or to average or to heavy shadowing. Estimations are for a S/N of 24 dB (for the main signal) and a sidelobe level of -22 dB (for the interfering signal). As can be seen on the figure, for light and for average shadowing, the S/(N+I) ratio would be higher than 20 or 19 dB 50% of the time, and larger than 14 or 10 dB 80% of the time.

Figure 3 illustrates the results for the downlink. The three curves give the percentage of time a given signal to noise plus interference ratio will be exceeded for the cases where the mobile unit is submitted to light, average or heavy shadowing.Estimations are made for a S/N of 24 dB and a sidelobe level of -18 dB. As can be seen on the figure, for light and for average shadowing, the S/(N+I) ratio would be higher than 13 or 11 dB 50% of the time, and larger than 12 and 9 dB 80% of the time.

Those definitions of shadowing come from the experimental work of Butterworth [3] and give a fair representation of the kind of environment that would be imposed to the transmission channel of a canadian customer. Light shadowing and heavy shadowing correspond to a surrounding of sparse and dense tree cover respectively. The representation called average shadowing comes from the combination of the light and heavy shadowing results in Butterworth's experiment.

In this study, the performance of mobile satellite links affected by fades has been analyzed. This was achieved by evaluating the S/(N+I) ratio for an L-band uplink and downlink. The statistical representation of the signal and fade fluctuations has been assumed to correspond to Loo's model. Performance curves have been obtained for various types of channel environment and with estimated link parameters of 24 dB C/N and -22 dB antenna sidelobe level for the uplink with 20 dB C/N and -18 dB antenna sidelobe level for the downlink.

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