Performance of Concatenated Reed-Solomon Trellis-Coded Modulation over Rician Fading Channels

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ABSTRACT. In this paper a concatenated coding scheme for providing very reliable data over mobile-satellite channels at power levels similar to those used for vocoded speech is described. The outer code is a shortened Reed-Solomon code which provides error detection as well as error correction capabilities. The inner code is a one-dimensional 8-state trellis code applied independently to both the inphase and quadrature channels. To achieve the full error correction potential of this inner code, the code symbols are multiplexed with a pilot sequence which is used to provide dynamic channel estimation and coherent detection. The implementation structure of this scheme is discussed and its performance is estimated.

1.0 INTRODUCTION

There are several considerations to be made when designing modulation and coding strategies for reliable communications in the mobile-satellite environment. One of the major technical considerations is the fading of the received signal due to vehicle motion. Conventional modulation and detection schemes, such as differentially detected MSK and BPSK, provide poor performance over moderately and severely faded channels without the use of coding. Furthermore, current mobile-satellite systems are typically power-limited but it is expected that future systems will be bandwidth-limited which implies that bandwidth conservation is an important consideration when choosing a coding scheme. In addition, mobile-satellite systems are concerned about minimizing the mobile terminal complexity and cost while maintaining its flexibility.

One way of satisfying these constraints and to meet the requirement of providing highly reliable data is to use a concatenated coding scheme where the inner codec is chosen to be bandwidth efficient and to provide good performance by itself, performance acceptable for vocoded data, while the outer codec would be a completely independent codec to be used only when very reliable data is required.

2.0 THE CODES

The Inner Code

Trellis coded modulation techniques are strong candidates for the inner code because they include low complexity codes which provide significant coding gain and diversity without increasing the transmission bandwidth. The drawback of applying such techniques to fading channels is that they require coherent detection to achieve their full potential. However, this drawback can be overcome by interleaving a known pilot sequence with the trellis coded (TCM) sequence in an scheme known as TCMP [1],[2]. In TCMP, the inclusion of a pilot sequence, at the expense of a fractional increase in power and bandwidth, allows the receiver to dynamically estimate the channel and perform a form of coherent detection. The bit error rate performance of TCMP with the optimum two-term 8-state one-dimensional code [3] applied to both the inphase and quadrature channels is shown in Figure 1. The scheme clearly provides very robust performance under Rician fading conditions.

The Outer Code

The factors to consider when choosing candidates for the outer code include the tendency of the inner code to produce short error bursts because of its
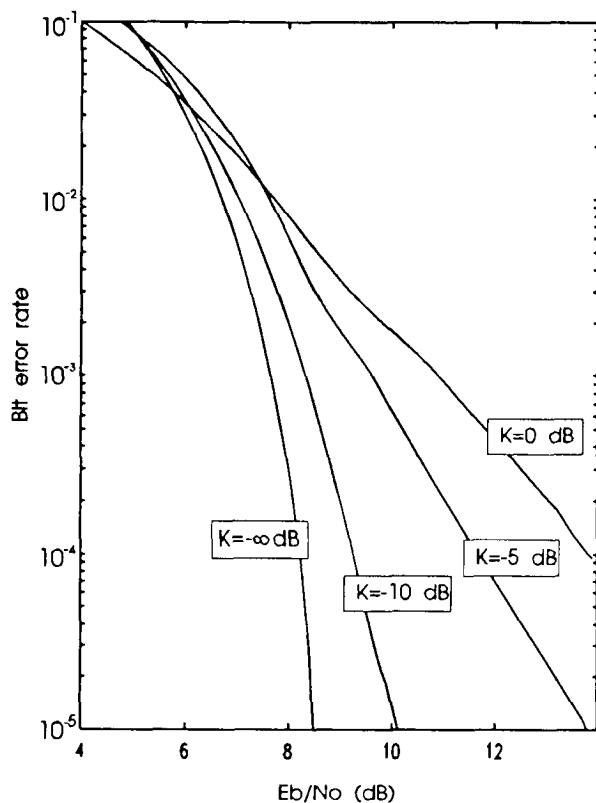
Figure 1. Performance of TCMP modulation and coding strategy over Rician fading channels.
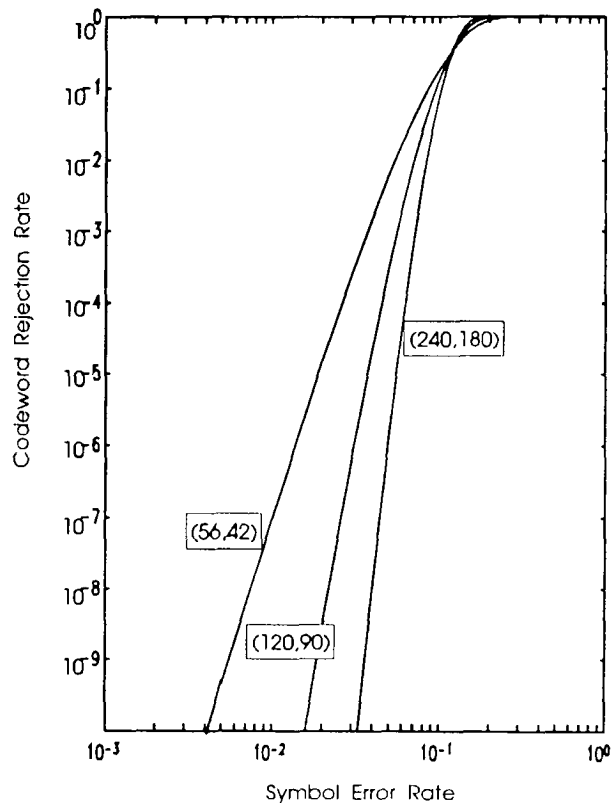

Figure 2. Performance of three shortened Reed-Solomon codes as a function of symbol error rate.

trellis-like structure, and the requirement for both error correction and error detection capabilities to provide highly reliable data. These factors suggest using Reed-Solomon block codes, which are among the best linear block codes for correcting bursts of errors up to a given length. The data to be transmitted will generally be in a byte format thus a symbol size of 8 bits would be the most convenient. This suggests a primitive Reed-Solomon (RS) code (255,k) where k is the number of information bytes in a codeword but it is also of interest to have the code rate of the outer code as a low order rational.

A code satisfying these requirements is the shortened RS code (240,180), shortened from (255,195). This is a rate 3/4 code but it also has the advantage that the same codeword length can be used for rate 7/8 and 15/16 codes. The RS(240,180) code can correct a codeword having up to 30 bytes in error, and its codeword error rate performance as a function of the input symbol error rate, assuming independent symbol errors, is shown in Figure 2.

The other performance criterion is the probability of decoding error. Since the non-shortened RS codes

are maximum distance codes, their probability of a decoding error can be determined analytically [4]. These probabilities of decoding error for the full length code provide upper bounds on the error detection capabilities of the shortened RS codes, and are shown in Table 1. These results indicate that the probability of decoding error for the (240,180) code is less than 10^{-33} which, for practical purposes, is zero.

Prob. of Symbol Error	Probability of Decoding Error			
	(31,23)	(63,47)	(127,97)	(255,195)
.6	$2E-2$	$7E-6$	$2E-13$	$3E-34$
.5	$1E-2$	$6E-6$	$1E-13$	$3E-34$
.4	$1E-2$	$6E-6$	$1E-13$	$2E-34$
.2	$6E-3$	$3E-6$	$7E-14$	$1E-34$
.1	$1E-3$	$4E-7$	$7E-15$	$6E-36$
.01	$7E-8$	$2E-14$	$3E-27$	$3E-59$

Table 1. Probability of decoding error versus probability of input symbol error for various Reed-Solomon codes.

For comparison purposes, the performance of the two shortened RS codes (120,90) and (56,42), which are shortened from (127,97) and (63,49) RS codes, respectively, are also shown in Figure 2 and Table 1. These also are rate 3/4 codes, and even with these shorter codes, the performance and probability of decoding error are still very good.

3.0 MODEM-CODEC STRUCTURE

To maximize the benefits obtained from a concatenated coding strategy, the two codes clearly must be combined in a manner which emphasizes the strengths of each. Trellis decoders are designed with the basic premise that errors in the input data sequence are independent. Consequently, to effectively use these codes on a slow fading channel interleaving must be performed to break up the error bursts caused by the channel and to create a sequence of errors which are approximately independent. In Figure 3 the interleaving of the trellis code is performed using a multiplexed codec approach which allows simple insertion of the pilot sequence.

When errors occur in a trellis codec, they tend to occur in bursts. To take advantage of the burst error-correcting potential of the RS code, the input to the multiplexed codecs were interleaved on a subsymbol (RS) basis as shown in Figure 3. Thus the bursts of bit errors produced by a trellis decoder tend to be combined into a smaller number of RS symbols.

In the simulation of the concatenated coding scheme described in Figure 3, an information rate of 3600 bits per second was assumed. This incoming data was buffered and coded using the rate $\frac{3}{4}$ RS code to a rate of 4800 bits per second. Each of the trellis codecs consisted of two one-dimensional
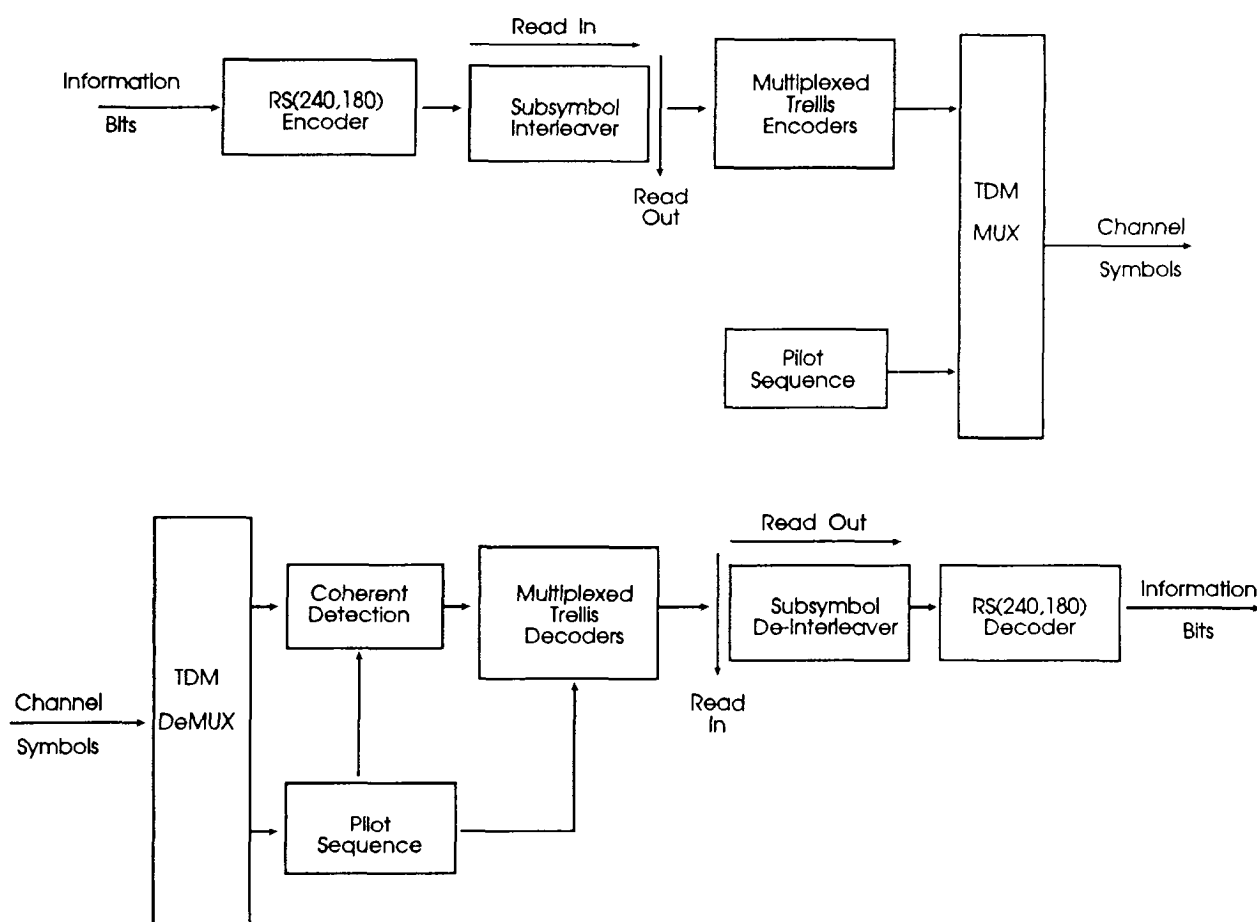

Figure 3. Illustration of concatenated coding scheme with subsymbol interleaving matched to the interleaving depth of the trellis code.

encoders whose outputs are combined in quadrature to produce a 16-QAM constellation. The output symbols of the RS code were allocated sequentially to the parallel trellis encoders. The output symbols of the multiplexed codecs — at a combined rate of 2400 symbols per second — were multiplexed with a pilot sequence of one quarter that rate, filtered to obtain the desired square root raised cosine spectral shape having a 50% rolloff factor, and transmitted at a rate of 3000 symbols per second.

The detection strategy for TCMP described in [1] used the pilot symbols to estimate the phase and gain of the channel and bypassed the need for explicit carrier recovery and short-term gain control. The output of each of the multiplexed trellis codecs was stored in an 8-bit buffer; which served as the symbol inputs to the RS decoding algorithm.

4.0 CONCATENATED PERFORMANCE

If the symbol errors are independent then the performance of the outer RS code can be characterized by the binomial distribution with parameter, θ, the symbol error rate. However, unless the interleaving strategy is perfect, the errors in the input symbol stream to the RS decoder will be correlated due to the combination of the trellis inner code and the slow fading channel. Also note that the average symbol error rate is not a simple function of the output bit error rate of the inner trellis decoder, since the correlation properties of these latter errors are a function of the channel conditions. However, a lower bound on performance can be obtained by noting that in the ideal case, the symbol error rate and the bit error rate will be the same (eight bit errors are mapped into a single symbol error), and the symbol errors are uncorrelated. An upper bound is provided by the case when the bit errors are totally uncorrelated and the symbol error rate is eight times the bit error rate. These bounds are shown in Figure 4 as a function of the input bit error rate, together with some simulation results. These simulated results are also reported in Table 2 as a function of the channel fading and noise parameters. The channel conditions simulated were those of a Rician fading channel with a fading bandwidth of 120 Hz and with additive white gaussian noise (AWGN). The fading conditions were varied from a static channel (K= −∞ dB) to a severely faded channel (K=0 dB). Over the range of E_b/N_o simulated the percentage of blocks rejected ranges from 0 to 100% and the knee

E_b/N_o (dB)	K-factor (dB)				cdwd. sim.
	−∞	−10	−5	0	
6	.94	.96	.96	.47	168
7	.012	.06	.13	.04	336
8	.0	.0	.002	.002	504
9	.0	.0	.0	.0	1472

Table 2. Fraction of RS(240,180) codewords rejected in a 120 Hz Rician fading channel.

of the performance curve lies between 7 and 8 dB in E_b/N_o. Correlating this with the output error rates of the trellis codec, given in Table 3, it is noted that the performance of the Reed-Solomon codec starts to degrade at an average input bit error rate of 1×10^{-2}. To achieve a codeword rejection ratio of 1 in 10^7, the lower bound in Figure 4 together with the worst case performance of the TCMP strategy (K = 0 dB in Figure 1) indicates that an E_b/N_o of 9.6 dB is required. (An additional 1.2 dB is added to the performance shown in Figure 1 to account for power loss due to the rate $\frac{3}{4}$ RS code.)

At first glance, this level of performance may not appear to be outstanding until one realizes that it is obtained in a severely faded channel. Under AWGN
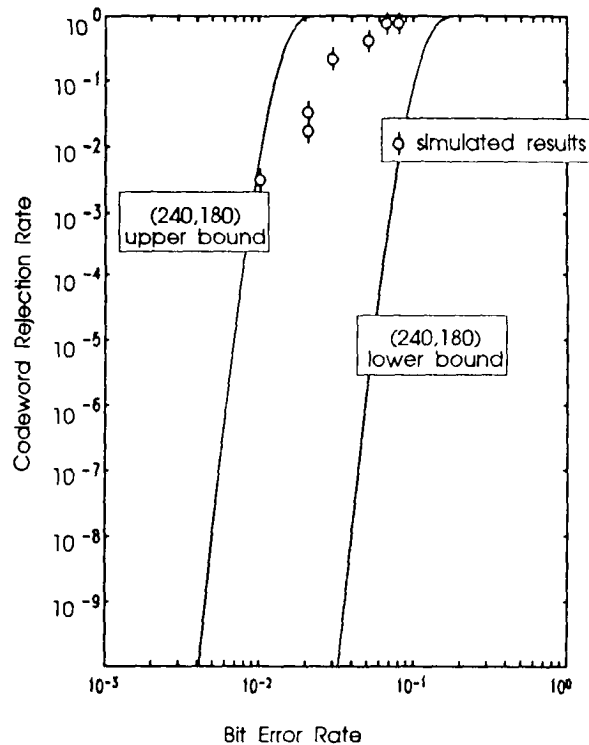

Figure 4. Bounds of performance of the (240,180) code as a function of the bit error rate.

E_b/N_o (dB)	K-factor (dB)			
	$-\infty$	-10	-5	0
6	8E−2	8E−2	7E−2	5E−2
7	2E−2	3E−2	3E−2	2E−2
8	5E−2	8E−3	1E−2	1E−2
9	6E−4	2E−3	3E−3	5E−3

Table 3. Average output error rates of the inner trellis code which produced the results in Table 1.

conditions it can be shown using Figures 1 and 4 that a codeword rejection rate of 10^{-7} can be obtained with an E_b/N_o of between 6 and 7 dB. Furthermore, in AWGN the pilot sequence, which is crucial to the performance under fading conditions, could be removed for an approximate 2 dB [1] improvement in performance.

In Tables 4 and 5, the codeword rejection rates for the other rate 3/4 codes, RS (120,90) and RS (56,42), are shown. It may be surprising that the simulated performance of these codes is almost as good as, and in some cases better than, the longer code. However, if one considers that the number of bit errors in a received codeword will have greater variation with shorter block lengths then the results can be explained. At high input error rates, shorter codewords have a greater probability of having a correctable number of errors. Similarly, at low input error rates, shorter codewords have a greater probability of having an uncorrectable number of errors which would result in either a decoding failure or a decoding error. The E_b/N_o range for the simulation results happens to be the range in which the performance curves of the codes of different blocklengths crossover, as illustrated by Figure 2.

E_b/N_o (dB)	K-factor (dB)				cdwd. sim.
	$-\infty$	-10	-5	0	
6	.78	.78	.73	.33	96
7	.05	.11	.12	.12	144
8	.0	.0	.005	.008	384
9	.0	.0	.0	.0	576

Table 4. Fraction of RS(120,90) codewords rejected in a 120 Hz Rician fading channel.

E_b/N_o (dB)	K-factor (dB)				cdwd. sim.
	$-\infty$	-10	-5	0	
6	.67	.69	.61	.32	360
7	.067	.11	.18	.10	720
8	.0	.007	.010	.016	1432
9	.0	.0	.001	.003	2348

Table 5. Fraction of RS(56,42) codewords rejected in a 120 Hz Rician fading channel.

5.0 CONCLUSIONS

In this paper a concatenated coding scheme for providing very reliable data over Rician fading channels has been described. The inner code is a simple 8-state trellis coded modulation scheme with an interleaved pilot sequence. This code takes full advantage of soft decision and channel state information to provide robust performance with significant coding gain and little bandwidth expansion. The outer code is a shortened Reed-Solomon code which provides significant error correction capabilities as well as virtually perfect error detection performance. This concatenated scheme can provide very reliable data at power levels similar to those required for vocoded speech.

References

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