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Practical Quality Control Tools for Curves and Surfaces

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Abstract. Curves and surfaces created by Computer Aided Geometric Design systems in the engineering environment must satisfy two basic quality criteria: the geometric shape must have the desired engineering properties, and the objects must be parameterized in a way which does not cause computational difficulty for geometric processing and engineering analysis. This paper describes interactive techniques in use at the Boeing company to evaluate the quality of aircraft geometry prior to Computational Fluid Dynamic analysis, including newly developed methods for examining surface parameterization and its effects.

Key Words. Computer Aided Geometric Design, Computational Fluid Dynamics, Numerical Grid Generation, Geometric Processing

1. Introduction. Users of Computer Aided Geometric Design (CAGD) systems in Aerodynamic applications are keenly aware of geometric or "shape" properties of curves and surfaces, especially those properties which affect aerodynamic performance. Less attention is paid, however, to the parameterization of geometric objects, even though parametric properties can have a large impact on the cost and accuracy of subsequent Computational Fluid Dynamic (CFD) analysis. This paper focuses on the distinction between geometric and parametric properties, and suggests that considerable cost savings can be achieved by detecting problems in geometry before CFD analysis or other processing is performed.

Section 2 of this paper defines the basic difference between geometric and parametric properties, describes some graphical tools for visualizing specific geometric properties, and suggests an application-driven approach to defining and measuring geometric quality. Section 3 presents techniques similar to those in section 2, but applied to parametric properties, and proposes a qualitative definition of parametric quality which applies across many applications. Section 4 contains observations and opinions regarding implementation of quality control in CAGD, and suggests topics for further research.

All examples shown in this paper were produced using the Aero Grid and Paneling System (AGPS), a Boeing CAGD program used for design and analysis of aircraft geometry. References [3] and [4] describe the AGPS system in more detail.

2. Geometric Properties Affecting Quality. I will define a *geometric property* of a curve $\mathbf{c} : [0,1] \to \mathbf{R}^3$ or surface $\mathbf{s} : [0,1] \times [0,1] \to \mathbf{R}^3$ as any property which is invariant under reparameterization. These are intrinsic properties having to do only with the shape of the curve or surface. Geometric properties are extensively studied in

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the CAGD community; for example, the concept of *geometric continuity* is used to describe continuity of position, tangent, and curvature independently of parameterization. Many tools are available for inspection and manipulation of these properties, but there seem to be no general rules regarding exactly what characteristics are desirable or "good." I claim that this is not surprising, since CAGD systems are used in a wide variety of applications, and quality can only be defined with respect to a particular set of application-dependent requirements. To formalize this concept, I propose the following definition:

> A *geometric quality metric* is a measure of how well the geometric properties of a curve or surface satisfy a particular set of engineering requirements.

The basic goal of Geometric Design in the Engineering environment is the development of curves and surfaces whose geometric properties satisfy a particular set of requirements. It is not unusual for each piece of geometry to be subject to its own unique list of constraints, and the relationships between stated engineering goals and measurable geometric properties can be extremely complicated. In aircraft work, for example, a wing surface might be required to interpolate a large number of fixed data points, possibly with additional tangency and curvature constraints. Structural considerations may limit surface area and enclosed volume, while aerodynamic performance is affected by the distribution of curvature over the entire surface. In this example the interpolation requirements are trivially related to geometric properties, the area and enclosed volume requirements are related through a simple mathematical model, and the aerodynamic characteristics are related in ways which are very difficult to model. Also, many of these requirements will differ from one wing to another. My point is that it is unreasonable to expect a general purpose CAGD system to provide general-purpose geometric quality metrics. Although the same set of geometric properties will always exist, the relationship to quality is totally application dependent. It therefore makes sense to provide a set of flexible tools for interrogating and displaying geometric properties, assisting engineering judgement rather than trying to eliminate it.

2.1. Geometric Properties of Curves. Aerodynamicists are very interested in a class of planar curves known as airfoils, which are the cross-sectional sectional shapes of aircraft wings[1]. These curves are typically defined by a number of discrete points (typically 25 to 100), with the actual parametric curve being constructed by a piecewise polynomial interpolation method. Assuming that the interpolation method produced a reasonable curve with positional and tangential continuity, the designer will be interested in the distribution of curvature along the airfoil, since curvature is directly related to aerodynamic properties. The most common tool for visualizing this type of scalar property is a 2-D plot of the property as a function of the curve's parameter. Illustration 1 shows the type of plot used to examine the curvature of an airfoil. Scalar torsion, and certain types of curvature defined only for curves which lie on surfaces, can also be displayed in this fashion. For a more detailed mathematical description of those properties, see chapter 11 of [1].

The term "fairness" is commonly used to describe the quality of a curve's curvature distribution, and designers may perform a "curve fairing" process where the data and/or interpolation method is adjusted to produce a more desirable curvature plot. Although the details of curve fairing are beyond the scope of this paper (see [7], for example), curvature plots such as the one shown in Illustration 1 are an essential tool in this process.

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[1] See Chapter 13 of [6] for a more detailed description.

2.2. Geometric Properties of Surfaces. Graphical techniques for evaluation of geometric properties related to surface quality have existed in the automotive and aerospace industries for over ten years[2]. In addition to analyzing planar cuts of a surface using methods described in section 2.1, engineers typically examine shaded renderings under various light sources. Color plots showing a surface "painted" according to a scalar curvature property are also very useful. Finally, "bristle" displays are used to illustrate the surface normal or other vector property at many locations simultaneously. The following sections contain some examples of graphical displays showing geometric surface quality.

2.2.1 Shaded Images. Many techniques for generating realistic, aesthetically pleasing shaded renderings of surfaces have been developed in the Computer Graphics industry.[3] In geometric quality control applications, however, the goal is almost the opposite: we want to make unwanted surface features <u>more</u> obvious. Gross defects such as positional gaps may show up clearly when a standard shading technique is used, but specialized "non-realistic" renderings are sometimes more useful.

One of the most useful methods for checking the behavior of a surface's normal vector is to take a "realistic" grey-shaded rendering and replace adjacent shades of grey with contrasting colors[4]. This trick produces apparent contour curves much like the isophote method described in [5], and can be used to verify planarity or geometric continuity. Illustration 2 shows this technique applied to an aircraft wing.

2.2.2 Color Displays of Scalar Properties. Several different scalar curvature quantities can be defined at a point on a parametric surface[5]. Boeing's AGPS system, for example, can display mean or Gaussian curvature on a surface, as well as the curvature in each of the parametric directions. The user may request that the curvature be sampled on a uniform grid of parametric values, or that an adaptive sampling technique be used. This type of display gives important information about the surface, especially in aerodynamic applications, but their interpretation requires some sophistication and experience. Illustration 3 shows curvature on the surface of an aircraft wing in the span-wise direction. The obvious curvature discontinuity exists because the wing consists of two different surfaces assembled into a single "composite" surface.

2.2.3 Displays of Vector Properties. One way to investigate the behavior of a surface's normal vector is to simply draw a representation of this vector at various points on the surface. For example, one may draw surface normals at specific points on a wing, as shown in Illustration 4. Rather than drawing surface normals at many points simultaneously, it is often more useful to allow the user to interactively select and change the location at which a single vector is displayed. In this way, one may study the behavior of the surface normal in regions of interest.

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[2]For example, [8] describes color graphic curvature inspection tools in use at General Motors in the early 1980's.

[3]Standard Computer Graphics textbooks such as [9] discuss Illumination and Shading at great length.

[4]This can be done easily in many CAGD systems by manipulating the entries in a color look-up table.

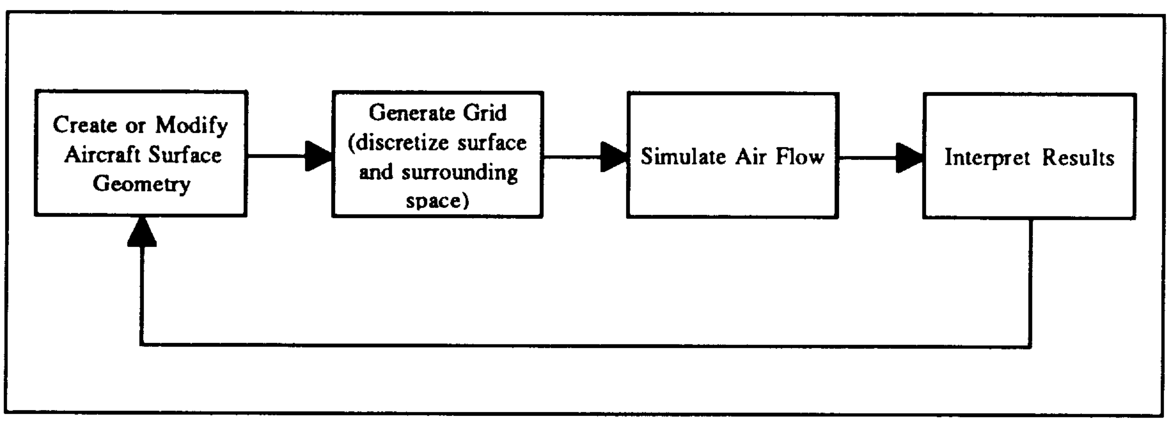
[5]For a more detailed discussion of surface curvature, see a Differential Geometry or Geometric Design text such as chapter 5 of [10].

3. Parametric Properties Affecting Quality. Given curves and surfaces whose geometric properties satisfy the relevant engineering requirements, why does it matter how the geometry is parameterized? It seems reasonable to focus on geometric properties when modeling a physical object in a CAGD system, since parameterization is only a mathematical artifact which ideally should have no physical manifestation. The main message of the following sections, however, is that parametric properties are critical when the CAGD model is subjected to geometric processing or computational analysis, and that in some cases, choices made regarding the paramaterization of a surface can have a drastic effect on the surface's geometric shape. Paralleling section 2, I propose the following definition:

> A *parametric quality metric* is a measure of how well a curve or surface's parameterization satisfies the requirements of the geometric processing, grid generation, and engineering analysis algorithms to be used.

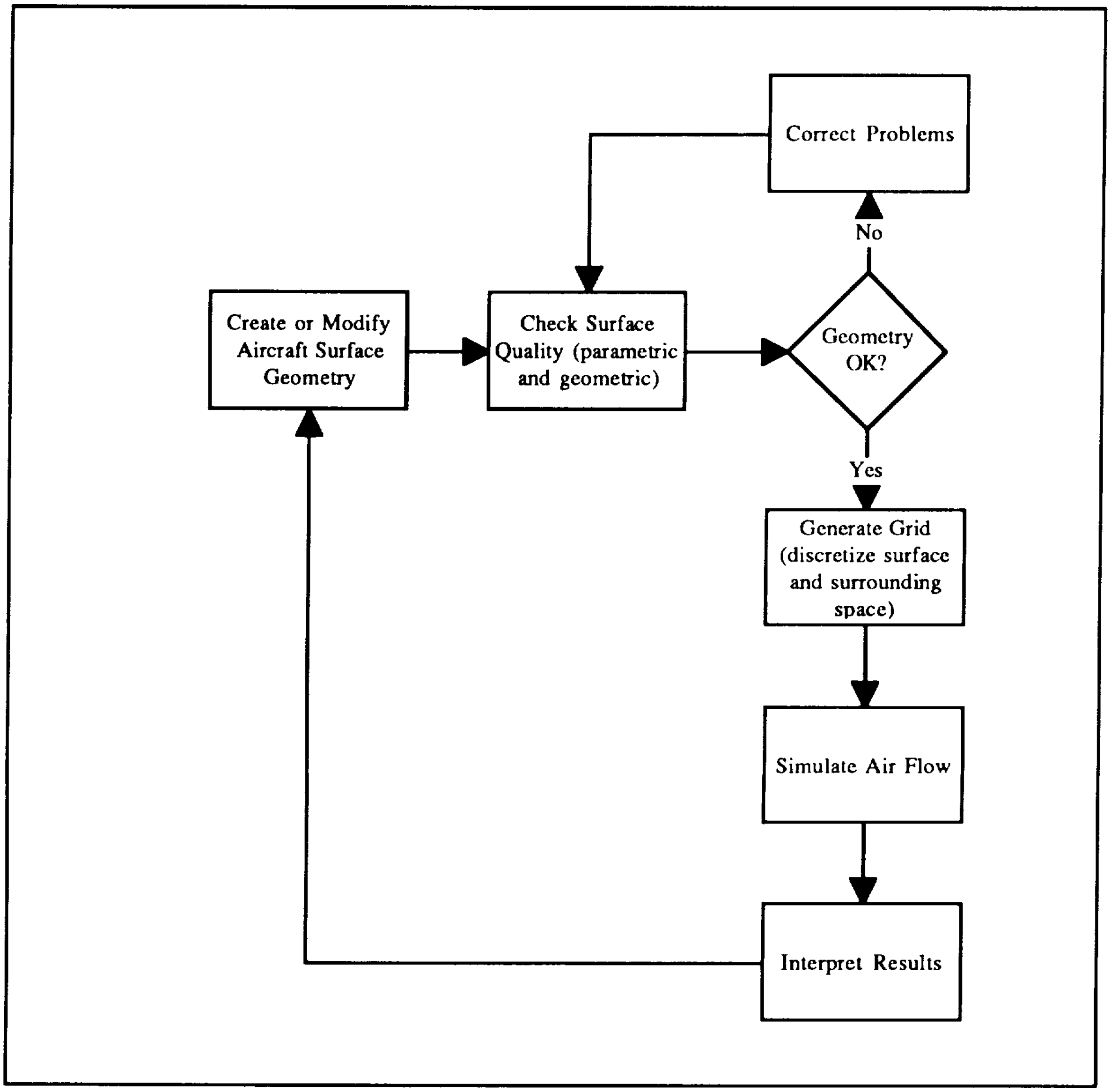
Assuming that parameterization does matter, users would like their CAGD system to automatically choose a good parameterization as objects are created, or at least to repair any problems which developed. Although automatic quality assurance is an important research topic, the current state of the art depends heavily on the user to detect problems, and repair techniques are usually ad hoc and difficult. Section 3 of this paper focuses on graphic tools which make the job of parameterization checking easier.

In the Computational Fluid Dynamics (CFD) environment, many proposed aircraft configurations are modeled in a CAGD system and analyzed by a separate computer program which simulates air flow around the surfaces of interest. The results of the simulation are then examined using a visualization tool (which in our case is also AGPS), and the configuration model is modified and analyzed again, as shown in the following diagram:



The preceding diagram may be viewed as a manufacturing process, where the product being produced is an aircraft surface definition, and the tools used are those provided by the CAGD system. In order to optimize a process, it is necessary to measure the quality of its output. Also, if a defect is introduced into a process, the resulting cost is generally lower if the defect can be detected earlier. Thus it makes sense to provide low-cost inspection tools in a CAGD system, and to encourage the engineers to use them throughout the geometry creation process.

Some issues related to Numerical Grid Generation provide a good illustration of the benefits of inserting quality control steps into a Geometric Design process. Most fluid flow simulation programs require a discretized representation of the aircraft surface as input; that is, the CAGD system is required to produce a "grid" of discrete points which, when interpolated in some way, represent the surface to a specified accuracy. Also, the engineer will want to increase the grid density at specific locations to capture certain features of the flow. Grid generation is a complex issue by itself[6], but the important point here is that most grid generation techniques are sensitive to curve and surface parameterization. The mathematical details remain to be shown, but the point is that a significant amount of frustration can be avoided by inserting a "quality control" step into the process as shown in the following modified diagram:



[6]Textbooks such as [11], and several yearly conferences, are devoted just to grid generation.

Geometric processing methods are also highly sensitive to curve and surface parameterization. Intersecting two geometric objects is a fundamental operation in all CAGD systems, and the iterative methods commonly used depend upon parametric continuity[7]. Again, "bad" parameterization can cause slow performance, inaccuracy, and even total failure in this type of operation. In the following sections, I will give some explanation of why this happens, and show some tools which can help the user detect this type of problem early in the design process.

3.1. Parametric properties of curves. For a curve $\mathbf{c} : [0,1] \to \mathbf{R}^3$ which maps a parametric variable u to points ($x(u)$, $y(u)$, $z(u)$), the most important scalar parametric property is the *parametric velocity*[8] $PV(u)$ defined by

$$PV(u) = \sqrt{(x'(u))^2 + (y'(u))^2 + (z'(u))^2} \ ,$$

where the prime indicates differentiation with respect to u. To see why parametric velocity is of interest, note that $PV(u)$ is the magnitude of the curve's tangent vector at the parameter value u, and that the arc length of the curve between the parameter values u_0 and u_1 is given by

$$\mathrm{ARCLEN}(u_0,u_1) = \int_{u_0}^{u_1} PV(u)du$$

Since this integral generally cannot be computed in closed form for non-linear curves, a numerical method[9] must be used, and the convergence of this method can be very sensitive to the smoothness of the integrand. Jump discontinuities in a curve's PV have the worst effect on arc length convergence, while a smooth, slowly varying (or constant) PV is best. In a typical grid generation application, where a number of points are to be distributed along a curve according to some specified arc length distribution, the majority of the CPU time is spent in the multiple evaluations of $PV(u)$ required by the numerical integration routine. Curves with well-behaved PV can clearly be gridded more accurately at lower cost.

The basic tool for evaluation of a curve's parametric velocity is a plot of $PV(u)$ such as the one shown in Illustration 5. Users would like an automated, quantitative method for evaluating the quality of a curve's parameterization, but it is difficult to find any applicable mathematical results. The main source of difficulty is the "black box" paradigm used in many CAGD systems, wherein geometry can only be evaluated at individual parameter values. Without an analytic description or any other global information, we can only work with discrete samples of properties such as $PV(u)$. An obvious approach to estimating the cost of arc length calculation is simply to approximate a curve's arc length with an adaptive quadrature method, and count the number of evaluations of $PV(u)$. This technique measures the computational cost associated with curve parameterization, but it cannot

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[7]See the article [12] or the textbook [10] for descriptions of typical surface-surface intersection algorithms.

[8]The term "parametric velocity" has historically been used for this scalar quantity, although "parametric speed" would be more correct.

[9]The AGPS system uses an adaptive Romberg integration method, with recursive subdivision in difficult cases.

evaluate the accuracy of the computations. Several people have suggested the application of discrete Fourier transform techniques to a sample of $PV(u)$, but it is not clear how to relate the results to computational cost or accuracy. The area of quantitative parametric quality metrics seems to be wide open for research.

3.2. Parametric Properties of Surfaces. For a surface $s : [0,1] \times [0,1] \rightarrow \mathbf{R}^3$ which maps the parametric variables u and v to points ($x(u,v)$, $y(u,v)$, $z(u,v)$), one may evaluate the parametric velocity of various curves passing through a given point on the surface. That is, one may evaluate the surface's parametric velocity in any desired parametric direction. The most obvious choices are simply the u and v directions, and the AGPS system provides color graphic display of the properties PV_u and PV_v defined by

$$PV_u(u,v) = \sqrt{(x_u(u,v))^2 + (y_u(u,v))^2 + (z_u(u,v))^2} \quad \text{and}$$

$$PV_v(u,v) = \sqrt{(x_v(u,v))^2 + (y_v(u,v))^2 + (z_v(u,v))^2} \quad ,$$

where the subscript u and v inside the radical denote differentiation of the coordinate functions with respect to one parameter.

In addition to the parametric velocity in a particular direction on a surface, one may compute an "area expansion factor" AEF defined by

$$AEF(u,v) = \sqrt{E(u,v)\; G(u,v) - F(u,v)^2} \quad ,$$

where

$$\begin{aligned} E(u,v) &= (x_u(u,v))^2 + (y_u(u,v))^2 + (z_u(u,v))^2 \\ F(u,v) &= x_u(u,v)x_v(u,v) + y_u(u,v)y_v(u,v) + z_u(u,v)z_v(u,v) \\ G(u,v) &= (x_v(u,v))^2 + (y_v(u,v))^2 + (z_v(u,v))^2 \end{aligned}$$

To visualize this property, consider a small rectangle with area $du * dv$ in parameter space, located approximately at the point (u,v). This parametric area element is mapped onto a surface element whose area is approximately

$$AEF(u,v)\; du\; dv$$

In the formal language of differential geometry, AEF is the *discriminant* of the *first fundamental form* on the surface. Intuitively, it measures the "average parametric velocity" in all directions.

A plot of AEF on a surface is useful in checking for parametric discontinuities between surface patches. As a simple example, consider a surface composed of bicubic patches of differing size. The CAGD system must construct an overall parameterization for the whole surface, and this is typically done by dividing the unit square into rectangles, each of which is mapped onto one surface patch. A simpleminded approach is to divide the overall parameter space into rectangles of uniform size, but this generally produces parametric discontinuities at patch boundaries if the patches are not identical. Illustration 6 shows the what can happen to the area expansion factor when a CAGD system does not divide the surface's overall parameter space intelligently among the surface patches. This situation occurs surprisingly often in practice, especially when geometry is imported from another CAGD system, since mathematical translations may be performed which do not preserve parametric continuity.

The motivation for checking the quality of a surface's parameterization is essentially the same as for curves: geometric processing techniques are sensitive to the continuity and smoothness of surface parameterization. For example, suppose a surface is "cut" with planes at several locations, and that a grid is generated by distributing points by arc length along each of the cut curves. Each cut curve will inherit its parameterization from the parent surface, and by the arguments made in section 3.1, the gridding process will be more accurate and less expensive if the surface is smoothly parameterized. Again, further research is needed to produce automated, quantitative parametric quality metrics.

3.3. Surfaces Which Cannot be Properly Parameterized. As described in section 17.5 of [1], the method commonly used to construct bicubic spline surfaces interpolating an array of data points only works when the data points are arranged in a "nice" rectangular grid. This is because one set of parameter breaks is used for all isoparametric curves in the u direction, and another set is used for all curves in the v direction. If the data points are arranged in a way which demands a different parameterization on the various isoparametric curves, there is no hope of constructing a surface without unwanted folds or ripples. These defects are obvious in extreme cases such as the one shown in Illustration 7, but more subtle ripples can be hard to find.

Since small geometric features can affect the solutions produced by fluid flow simulation codes, it is important to provide the designer with a means of detecting them. A very effective detection tool was recently implemented in the AGPS system, based on this observation: where an unwanted surface feature exists, the arc length on the surface along an isoparametric curve is different from the arc length on a curve fit through the single "row" or "column" of data points. In other words, the surface is either too "loose" or too "tight," due to the compromise made when selecting the overall u and v parameter breaks. Since a single curve through a subset of the data points needs no such compromise, it is a good standard for comparison.

Using the AGPS system's geometric programming language, for example, it was not difficult to write a procedure which takes an arbitrary surface and checks the arc length of its isoparametric curves. A graphic display is produced showing the surface "painted" with colors corresponding to the degree of discrepancy found, making unwanted features stand out clearly. Illustration 8 shows this technique applied to the gross example used in Illustration 7, and Illustration 9 shows an aircraft body surface containing a subtle defect which is difficult to find using any other technique.

4. Summary and Conclusions. I have taken the viewpoint that Computer Aided Geometric Design is a process whose output is curves and surfaces, and expressed the need to measure the quality of these products throughout the process. Emphasizing the distinction between geometric and parametric properties, I claim that geometric quality must be the user's responsibility, but that CAGD system designers should take responsibility for parametric quality. Experienced engineers develop an understanding of the relationship between geometric properties and quality as defined in their application, and several types of graphical tools can be very helpful in focusing engineering judgement. Parametric quality, although not as commonly appreciated, can also be efficiently evaluated using graphical techniques. Since geometric processing and engineering analysis are relatively expensive, a large cost saving can be realized by early detection of defects which could cause subsequent computations to fail; thus I recommend addition of a quality-control step to the geometric design process.

Automated inspection and repair of geometry is certainly a worthwhile goal, but the current state of the art is based upon engineering judgement. The inspection process requires the user to make qualitative judgements from color graphic images, and the repair process is entirely ad hoc. I believe that a larger payoff will come from automating the inspection process rather than the repair process, since many curves and surfaces need to be inspected, but relatively few will need repair. Automated, quantitative geometric inspection remains an important topic for research.

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Curvature Plot for an Airfoil

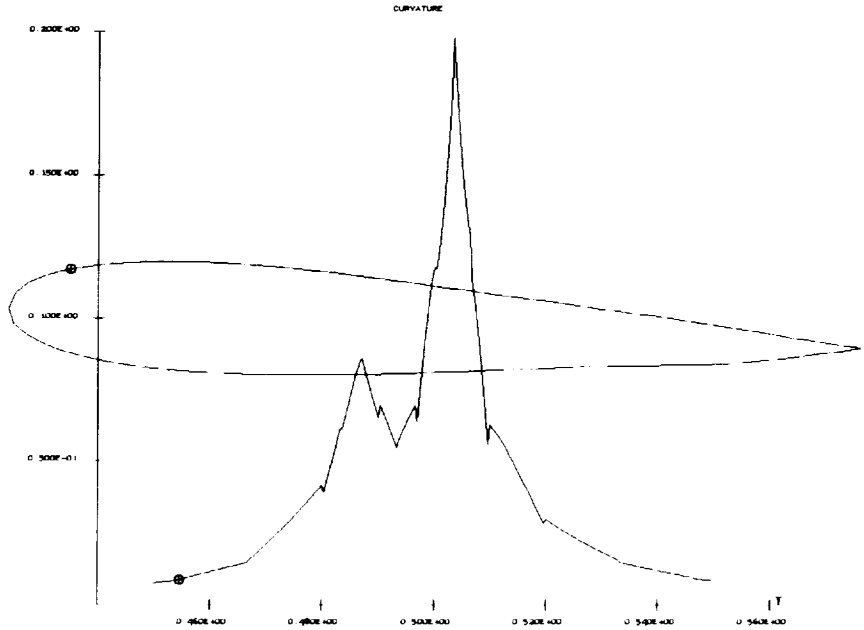


Illustration 1

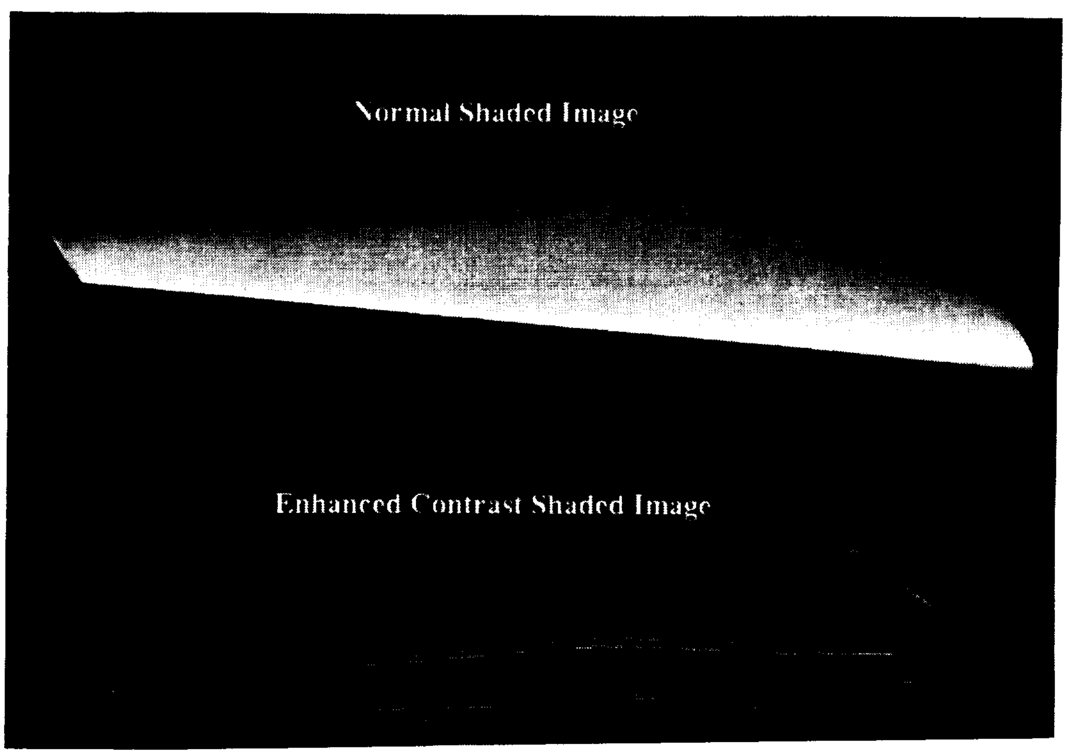
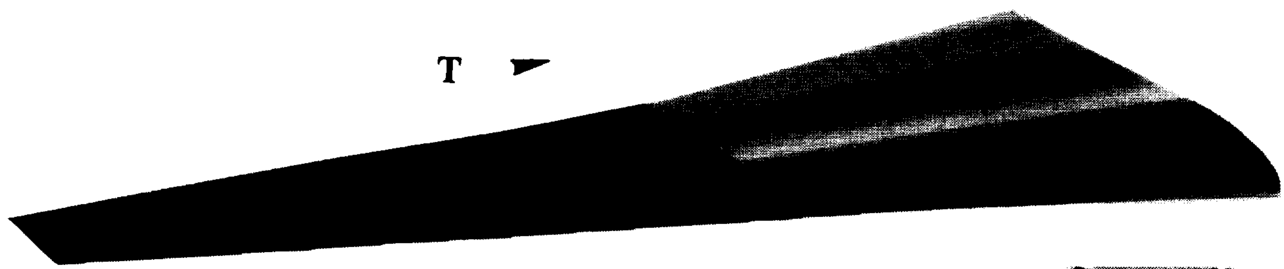


Illustration 2

Curvature in the T Direction on a Wing Surface



(discontinuity due to composite surface)

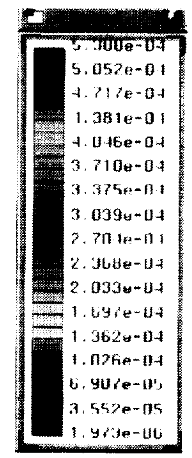


Illustration 3

Normal Vectors on a Wing Surface

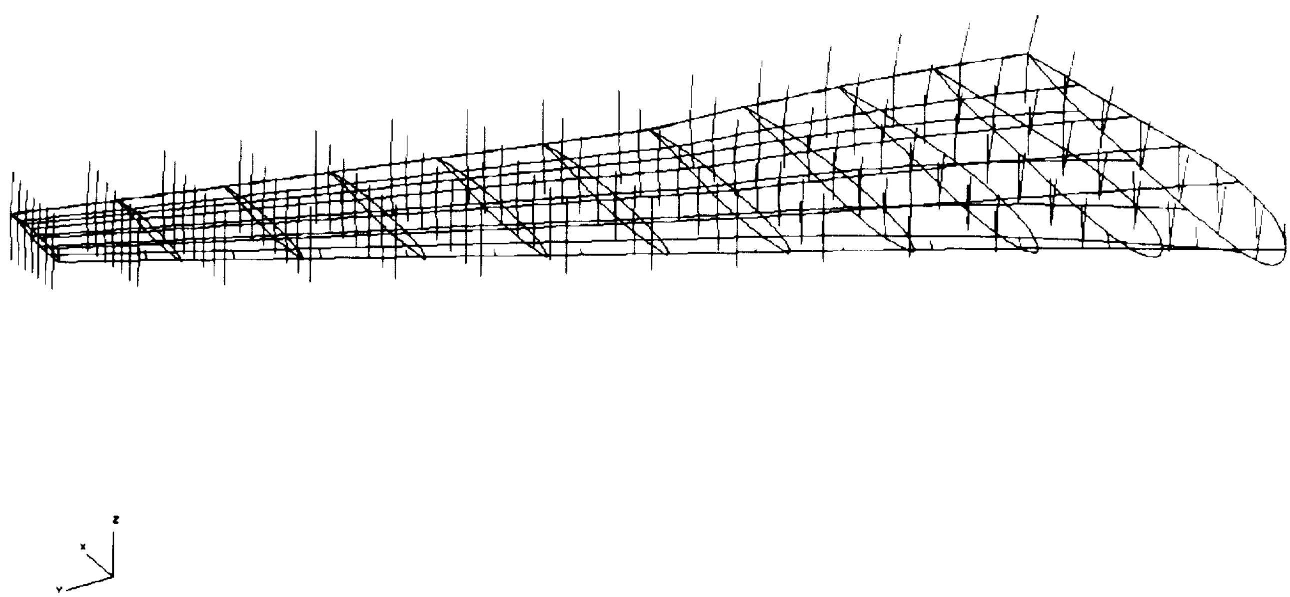


Illustration 4

Parametric Velocity Plot for Airfoil

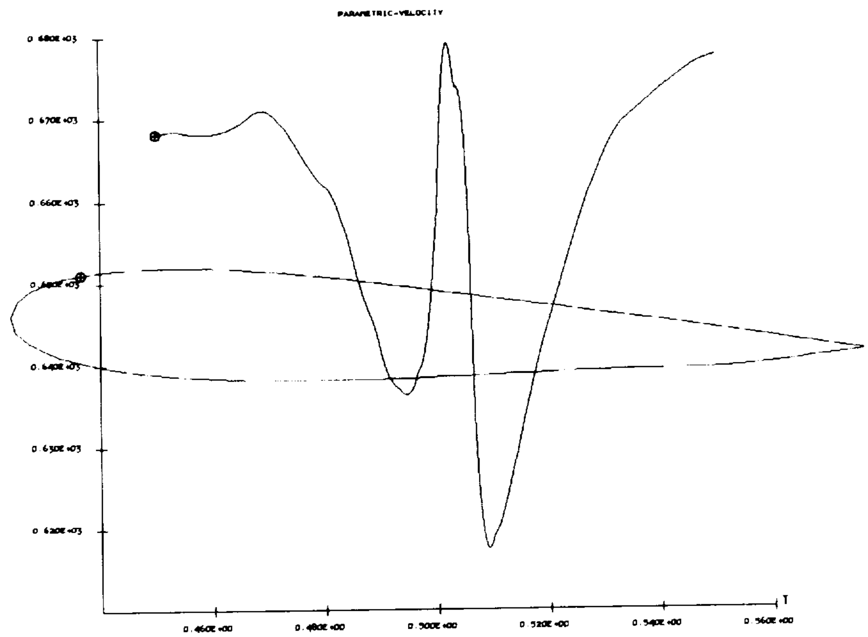


Illustration 5

Area Expansion Factor

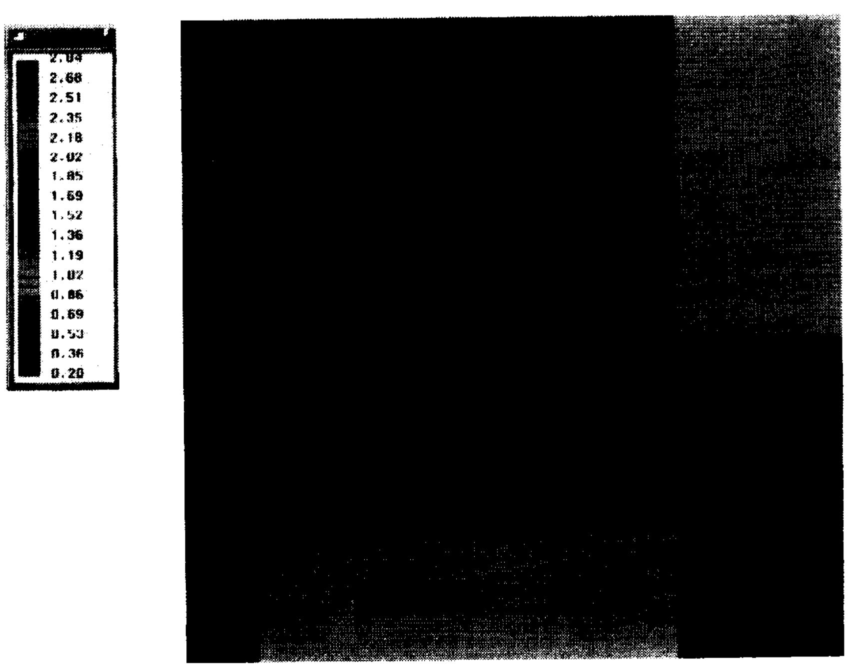


Illustration 6

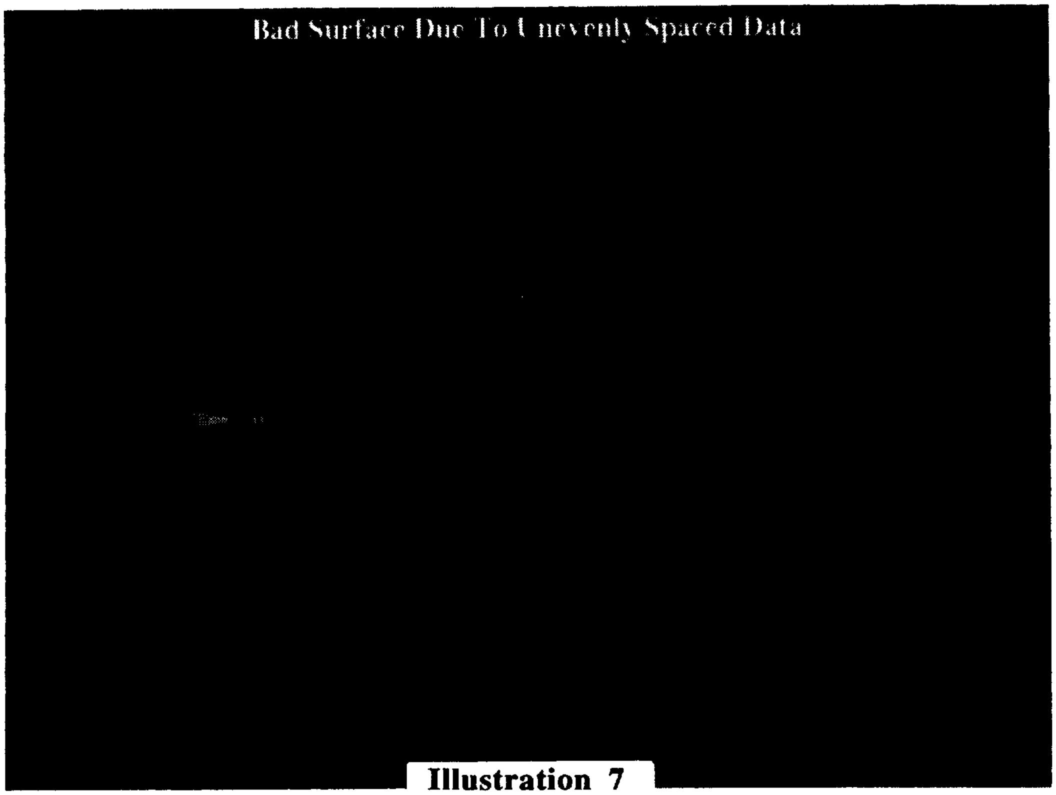


Illustration 7

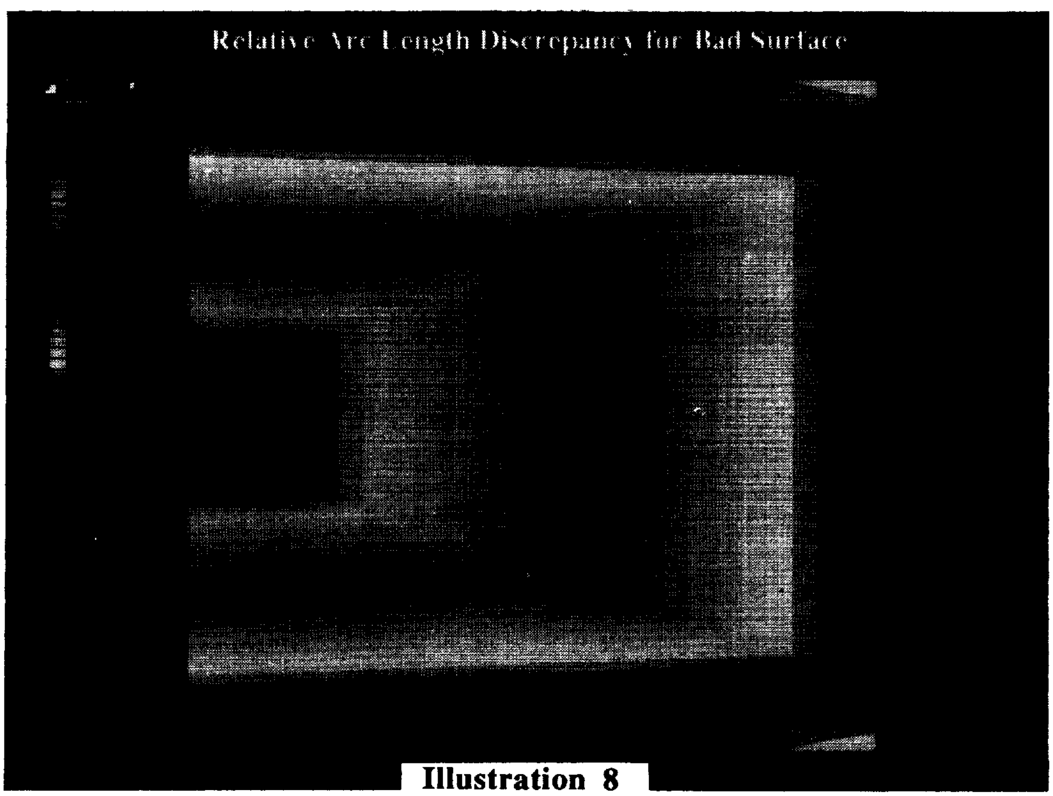


Illustration 8

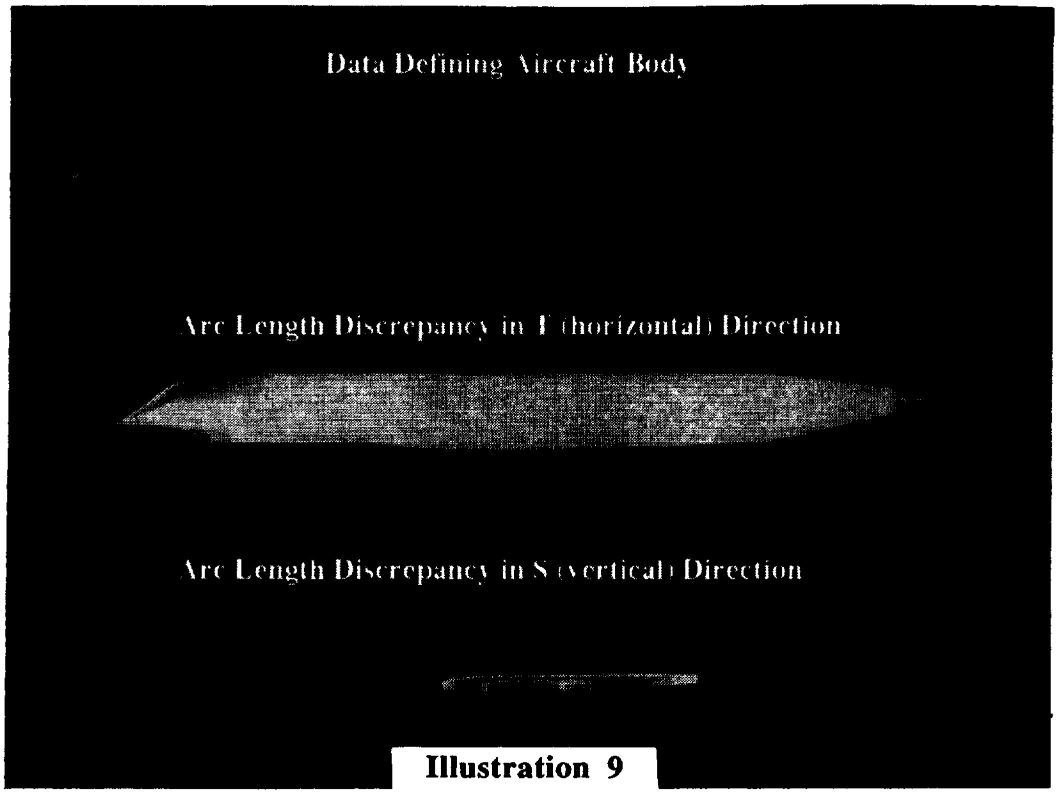


Illustration 9