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Improvement of the Earth's Gravity Field from Terrestrial and Satellite Data

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1. Introduction

This report is the final technical report under NASA Grant NAG 5-1329. This project started in June 1, 1990 in response to a proposal submitted to NASA for the continuation of research studies that were taking place under grant NGR 36-OU8-161. Although the original time frame work of the grant was for one year the work was continued to December 31, 1991 through no cost extensions. This report summarizes the activities that took place under this project.

2. Research Activities

The studies carried out under this project were in two areas. The first was related to the collection of surface gravity data and the second was the development of techniques to connect vertical datums. The following discussion describes the activities in each of these areas.

2.1 Gravity Collection Activities

The determination of the Earth's gravitational potential can be done through the analysis of satellite perturbations, the analysis of surface gravity data, or both. The combination of the two data types yields a solution that combines the strength of each method: the longer wavelength strength in the satellite analysis with the better high frequency information from surface gravity data. Since 1972 Ohio State has carried out activities that provided surface gravity data to a number of organizations who developed combination potential coefficient models that described the Earth's gravitational potential.

The initial collection activities were to lead to $1° \times 1°$ mean free-air gravity anomalies. The primary data collected were the mean values although point gravity values were collected when no mean value estimates were available. Over the years there were a number of $1° \times 1°$ anomaly data sets prepared and documented for the scientific community. One of the most comprehensive reports was that by Kim and Rapp (1990) that described the development of the July 1989 $1° \times 1°$ anomaly set. During this contract period this data file was updated with 1018 $1° \times 1°$ anomalies from a portion of Asia to create the October 1990 $1° \times 1°$ data set. The update process is described in a report, of limited distribution, by Yi and Rapp (1991). The new anomalies were primarily used to replace $1° \times 1°$ anomalies that had been estimated by geophysical correlation techniques and are regarded to be unreliable in terms of today's needs. The location of the 45932 anomalies not estimated through geophysical correlation techniques is shown in Figure 1.

As noted earlier the collection of $1° \times 1°$ gravity anomalies has taken place since 1992 with numerous updates of the $1° \times 1°$ anomaly data sets. Table 1 shows the number of anomalies in each of the updates. In judging these numbers recall that there are a total of 64800 $1° \times 1°$ on the Earth's surface and that there are now 5667 geophysically predicted anomalies in the anomaly count shown for the October 1990 data set.
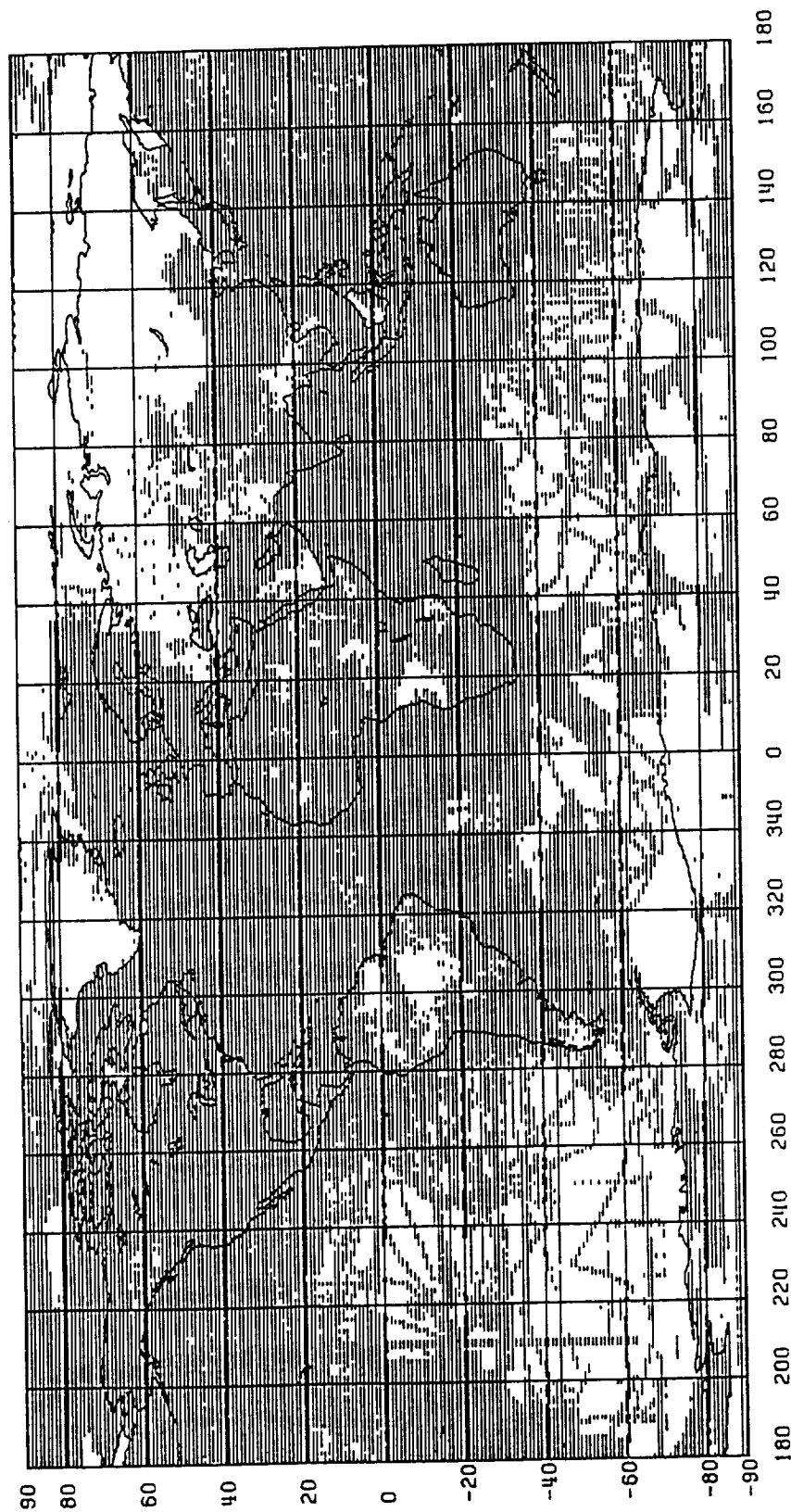
Figure 1

Location of 45932 Non-Geophysically Predicted Anomalies in the October 1990 Data Set

Table 1

Number of Anomalies in Various
1° x 1° Updates at Ohio State University

Date	Number
June 72	23355
Sept 73	29789
July 75	36149
Aug 76	38406
June 78	39405
Oct 79	41973
Jan 83	44513
June 86	48955
July 89	50793
Oct 90	50802

The fundamental data source for these anomalies is the data provided by the Defense Mapping Agency Aerospace Center. This starting data set is improved and increased through data sources available through our collection activities.

In addition to the anomaly estimate a standard deviation is assigned to each anomaly. This information can be used to create maps showing where the coverage has a certain accuracy. Many of the maps have been made for groups who are explaining why we need improved gravity information from space missions.

Examination of Figure 1 and accuracy estimates would indicate the following regions are lacking in gravity information: Greenland, the former USSR, polar regions, and portions of South America, Africa, India, China, Alaska, etc. Coverage in the southern oceans is weak but this is a matter of less urgency than the land areas due to the availability of satellite altimeter data.

Much of the gravity data we have obtained in our collection activities has been through personnel contracts. The most recent example has been the acquistion of 30' x 30' anomaly data for Africa through the African Gravity Project organized by Dr. Derek Fairhead at the University of Leeds in England. This data was first introduced in our July 1989 data file. During this contract period we have discussed the possibility of continuing our cooperation with Professor Fairhead in the South American Gravity Project. Verbal arrangements on getting this new data were made in Vienna at the IUGG meeting in August 1991. A more specific data acquisition procedure was described in a letter written to Fairhead in October 1991. We hope to receive this data in mid 1992 although any analysis will be outside this contract.

Another area in which there is potential for the release of surface gravity data is in the regions of the former USSR. For many years the project Principle Investigator has tried to obtain from this region gravity data for use in our mean anomaly collection activities. Quite recently there is a indication that the contacts may be successful but no resolution of the situation is expected until mid 1992. At that time a data exchange agreement may be implemented. Again any data obtained under such arrangements can not be analyzed under the contract that instituted the data transfer.

The 1° x 1° mean anomaly files have been sent to many organizations and persons. These include: NASA/Goddard Space Flight Center (Frank Lerch); University of Texas at Austin (Byron Tapley); DGFI (Chris Reigher), the National Geophysical Data Center (NGDC), and the International Gravity Bureau (Georges Balmino). The names of other scientists who requested these 1° x 1° anomaly files could be provided if needed. It is of interest to note that the

October 1990 1° x 1° data file has been placed on the Gravity CD that is now being distributed by NGDC.

The 1° x 1° anomalies of the October 1990 data set have been used in several potential coefficient models that have used terrestrial and satellite data. This use has been described by Schwintzer et al. (1991, Section 7), Pavlis (Appendix A) in the GEM-T3 report by Lerch et al. (1992), and in Rapp, Wang, and Pavlis (1991).

In 1986 it was realized that there was a need for 30' x 30' mean gravity anomalies for use in high degree spherical harmonic expansions. Such information was primarily limited to land areas since satellite altimeter data is an excellent source of gravity information at sea except in shallow water and ice areas. In 1989 the original 30' x 30' file was updated to create the July 1989 30' x 30' data file that is described by Kim and Rapp (1990). The file contains data in North and South America, Europe, India, Australia, New Zealand, etc. The number of anomalies on this file is 66990 each with a standard deviation. This data set was not updated in this contact period although the data collection contacts made would enable the file to be updated in the future.

In summary, progress was made in our anomaly collection activities. A new 1° x 1° data set was made available to the scientific community and contacts made for the future improvement of our terrestrial data base.

2.2 Vertical Datum Corrections Through Satellite Analysis

The orthometric height of a point is approximately the height of a point above mean sea land. Unfortunately mean sea level is not an equipotential surface so that a unique origin for a global vertical datum can not be established. In order to provide more precise information on the many vertical datums in the world the datums needed to be connected so that the relative differences between the datums can be determined. One way to make this vertical datum connection is to use satellite data and estimates of the Earth's gravitational potential. Of concern for this contact was the development of mathematical models to carry out the connection. As a follow on to the model development, a set of simulation studies were needed to see if a vertical datum connection, of sufficient accuracy, would be possible in the future. This research was carried out by N.K. Pavlis for his dissertation studies. During this contract time Pavlis completed his dissertation and successfully defended it, receiving his Ph.D. in June 1991. The 158 page dissertation was published as a report of the Department of Geodetic Science and Surveying (see the reference list). The title of the report was "Estimation of Geopotential Differences Over Intercontinental Locations Using Satellite and Terrestrial Measurements.

Instead of writing a summary of the Pavlis report, Chapter V (Summary, Conclusion and Recommendations) is reproduced in the following pages.

Summary, Conclusions and Recommendations

The problem of estimation of geopotential differences over intercontinental locations was re-examined, in order to assess currently achievable accuracies and future anticipated improvements. Accurate estimation of the geopotential differences between points located at different continents, imply the unification of the vertical datums established in them, which at present are defined based on MSL monitoring and thus are (in general) inconsistent due to the presence of the Quasi-stationary Sea Surface Topography.

A review of the proposed techniques for the unification of vertical datums, in conjunction with anticipated future satellite missions and in view of the accuracies achievable at present for geocentric positioning, indicated that approaches based on the combination of gravitational information with high accuracy geocentric positioning, are favored at present and in the near

future for practical implementation. In this direction, extending the ideas put forward by Colombo (1980), an observational setup was proposed, whereby gravity disturbance measurements on the Earth's surface, in caps surrounding the estimation points, are combined with corresponding data in caps directly over these points at the altitude of a low orbiting satellite, for the estimation of the geopotential difference between the terrestrial stations. The gravity disturbance data at altitude are inferred from GPS measurements made from the low orbiter to the high-altitude GPS satellites, in a multiple-high-single-low Satellite-to-Satellite Tracking configuration. In the absence of actual measurements, the performance of such an observation/estimation scheme was evaluated by conducting an error analysis study.

The mathematical modeling required to relate the primary observables to the parameters to be estimated, was studied both for the terrestrial data and the data at altitude. Emphasis was placed on the examination of systematic effects and the corresponding reductions that need to be applied to the measurements to avoid systematic errors. For the gravitational accelerations inferred from SST data, it was discovered that the magnitude of a centrifugal acceleration term (δR_{io}) was underestimated by several orders of magnitude in the past as a result of an erroneous derivation. The previous formulation implied a magnitude of δR_{io} about 7×10^5 times smaller than the current corrected formulation. It was shown in this study that in order to keep the systematic effect arising from δR_{io} at the 20 μgal level, a reference geopotential model complete to degree 20 is required (high-low SST configuration). Previous analyses, based on the erroneous formulation, were indicating that a reference model complete to degree 4 is adequate to keep the residual systematic effect of δR_{io} at the 10 μgal level. For a given noise level (0.4 mgal) of the data at altitude, increase of the maximum degree of the reference model, significantly affects the ratio of the residual signal to the noise.

Two different techniques were considered for the estimation of the global mean square error of the geopotential differences. Error propagation using truncation theory, as applied to Hotine's integral formula, and the least-squares collocation using ring averages as input data. Both techniques are applicable in case observations on the Earth's surface only are involved in the geopotential difference estimation, but only lsc can handle efficiently the over-determined case when observations at altitude are added. Alternative formulations related to the sampling (or discretion) and the propagated errors arising in the truncation theory considerations were derived. These are characterized by the same computational requirements as the previous formulation by Christodoulidis (1976), while they provide a more consistent interpretation of the underlying physical principles that give rise to these errors. In an attempt to apply truncation theory principles for the assessment of the contribution of gravitational acceleration data at altitude, to the estimation of geopotential differences on the Earth's surface, recurrence relations for the altitude generalized truncation coefficients implied by Hotine's kernel were developed for the first time.

Both techniques (truncation theory and lsc) require for their implementation a-priori knowledge of the global properties of the signals and the noise involved in the estimation and to this end different covariance models were considered and their spectral characteristics were compared. In addition, an efficient recurrence relation for the degree variances implied by a first-order Gauss-Markov covariance model was developed for the first time in this study.

For the numerical analysis, three global geopotential solutions were considered as reference models. The currently available OSU89B high-degree harmonic expansion, and the global models anticipated to become available from GPS tracking data of the TOPEX/Poseidon and the Gravity Probe B spacecrafts respectively. Augmentations of the latter two models with higher-degree harmonics from OSU89B were also considered. A number of numerical experiments were performed that lead to the following conclusions:

(a) The currently available global geopotential model OSU89B alone is expected to yield geopotential differences between stations separated by 30°, accurate to about 86 kgal cm. The

future models (augmented by OSU89B) can improve this accuracy to about 81 kgal cm (TOPEX/OSU89B) and 74 kgal cm (GPB/OSU89B) respectively.

(b) Introduction of gravity disturbance measurements in terrestrial 2° caps reduces the previous error estimates to the following: 23 kgal cm (OSU89B), 17 kgal cm (TOPEX/OSU89B) and 12 kgal cm (GPB/OSU89B), when pessimistic error estimates are used for the gravity disturbance measurements ($m_0 = 2$ mgal). With $m_0 = 1$ mgal the case GPB/OSU89B yields an error of about 9 kgal cm for 30° station separation. The error estimates for these cases were computed using both truncation theory and lsc (ring averages) and the results from the two techniques were compared. It was found that the lsc error estimates are always smaller than the ones obtained from truncation theory, as mandated from theory. The largest difference between the two error estimates was found to be about 21%.

(c) When gravity disturbance data at the altitude of GP-B (about 600 km) were introduced, a moderate (7%) improvement in accuracy, over the corresponding case without such data, was found. In both cases, the reference geopotential model used was complete to degree 45, obtained from the analysis of the GPS tracking data on GP-B. However, gravity disturbance data at this altitude are unable to resolve medium and high frequency variations of the gravity field and thus the result in this case is inferior by about 5 kgal cm to the result obtained from the combined GPB/OSU89B high-degree model (complete to degree 250).

To enrich the data at altitude with more high-frequency information, it is recommended here that additional measurements of a higher-order gradient of the disturbing potential made from a lower flying spacecraft, be incorporated in the estimation. In this direction, the gradiometer data from ARISTOTELES mission can provide a significant contribution. In addition, it should be emphasized that the error estimates reported here correspond to a "worst-case" scenario where only one pair of benchmarks is considered for the intercontinental connection. Additional benchmarks on each continent, connected with leveling lines, can provide a better network configuration and yield an improvement on the accuracy of the intercontinental connections up to 25%, as the study by Hajela (1983) has indicated.

Finally, the results reported here are promising enough to warrant an actual testing of the technique. For this purpose, stations whose geocentric coordinates are accurately known (e.g. SLR sites or VLBI stations connected to a geocentric system using GPS) and between which the geopotential difference has been estimated independently using spirit leveling and gravimetry can be used as test sites. At present (1991), collection of gravity disturbance measurements (using relative GPS positioning and gravimetry) in caps surrounding these test sites, will enable testing of the procedure described in section 4.2. As it was discussed in that section, a cap size of 2° and an approximate spacing of 6' between the points where the gravity disturbances are determined, are optimum parameters for the observational setup, provided a state-of-the-art high-degree global geopotential model (e.g. OSU89B) is used as reference. In the actual implementation, least-squares collocation using the original gravity disturbance data (as opposed to ring-averages) should be used to maintain highest computational rigor. In addition, detailed elevation information around the test sites should be used for the consideration of the terrain effects by means of analytical continuation, as it was discussed in section 2.3.

In a more future time frame (1995), the availability of the data from the anticipated satellite missions (TOPEX/Poseidon, Gravity Probe-B, ARISTOTELES), will enable to improve the above scheme in two ways. First, by the use of global geopotential models with more accurately estimated lower-degree harmonics than those of OSU89B, as such models will become available from the analysis of the global sets of observations collected by these missions, and second by the use of gravitational information in caps at altitude (GP-B, ARISTOTELES), as discussed in section 4.3.

We have concluded that Pavlis has developed a workable methods for vertical datum connection. However its success lies with the determination of an improved model of the Earth's gravitational potential.

2.3 Other Activities

During this contract the principal investigator participated in a meeting in Capri, Italy related to the ARISTOTELES gravity field/magnetic field mapping mission. The meeting, in September 1991, summarized the reasons for needing the mission, a proposal for the mission, and recommendations for future activities. For this conference Rapp (1991) prepared a status report on the determination of the Earth's gravitational field.

3. Personnel

The principal investigator of this project was Richard H. Rapp, Professor. Mr. N.K. Pavlis and Mr. Yuchan Yi carried out studies under this project as Graduate Research Associates. Ms. Melanie Hennell was a Student Intern Assistant under this project.

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