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A Procedure for Utilization of a Damage-Dependent Constitutive Model for Laminated Composites

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Abstract

This paper describes the procedure for utilizing a damage dependent constitutive model to predict progressive damage growth in laminated composites. In this model the effects of the internal damage are represented by strain-like second order tensorial damage variables and enter the analysis through damage dependent ply level and laminate level constitutive equations. The growth of matrix cracks due to fatigue loading is predicted by an experimentally based damage evolutionary relationship. This model is incorporated into a computer code called FLAMSTR. This code is capable of predicting the constitutive response and matrix crack damage accumulation in fatigue loaded laminated composites. The structure and usage of FLAMSTR are presented along with sample input and output files to assist the code user.

As an example problem, an analysis of crossply laminates subjected to two stage fatigue loading has been conducted herein and the resulting damage accumulation and stress redistribution have been examined to determine the effect of variations in fatigue load amplitude applied during the first stage of the load history. It is found that the model predicts a significant loading history effect on damage evolution.

Introduction

Laminated continuous fiber composites are increasingly being utilized in engineering applications such as primary load bearing aircraft components. This is in part due to the lower weight and higher specific stiffness obtained by using advanced composite materials. Unfortunately, laminated composite materials are susceptible to the development of microstructural damage when subjected to service loads. This damage includes matrix cracking, delamination, fiber-matrix interface debonding, and fiber fracture. Each microcrack is in itself relatively insignificant since most cracks are arrested at the fibers or adjacent plies. However, the resulting redistribution of load to the surrounding regions creates stress fields favorable to the initiation and propagation of additional damage. Catastrophic failure is triggered when the remaining load paths are no longer able to support the load. In addition, the strength and stiffness of the material are degraded as a result of the load redistribution and the decrease in load paths during the accumulation of the subcritical damage. Since the initiation and accumulation of this subcritical damage are highly dependent on the stress state within the material, analysis of the structural response and service life must account for this stress redistribution.

Due to the multitude of microcracks often observed in laminated composites, it may not be practical to model each flaw explicitly. An alternative approach represents the distributed damage by volume averaged quantities known as internal state variables (ISV). These quantities may describe the average physical attributes of the distributed damage or they might describe the effects of the distributed damage on the material response. In the current approach the damaged material volume is modeled as a continuous domain with altered properties. Thus, although the microcracks in the representative volume element (RVE) are treated as internal boundaries, the global structural problem is treated as simply connected with spatially variable reduced stiffness obtained from a micromechanics solution for the RVE. As a result, the global problem is made more computationally tractable. Even though the ply level stresses obtained from this approach are locally averaged quantities, the model predictions are in qualitative agreement with experimental results. This approach, call damage mechanics, is suitable for damage that is small in size relative to the scale of the structure being analyzed and is spatially homogeneous within the RVE. Although it has been applied to the study of a wide range of phenomena from microcracking to chemical and radiation damage in engineering materials, only recently has it been applied to matrix cracking in laminated composites [1–9].

This report describes how to perform an analysis using this approach. The model is formulated for matrix cracking and delamination damage in laminated polymeric composites with a brittle matrix. At the present time, a damage evolutionary relationship for fatigue induced matrix cracking has been implemented into the lamination computer code, FLAMSTR. This code is used to demonstrate some of the effects that the loading history has on the accumulation of matrix cracking. The evolutionary relationships for delamination damage are currently under development and will be incorporated into the code at a future time. A brief description of this model will be presented. Detailed development of this model can be found in the published literature [5,6,10–14].

Model Description

In the proposed model, the effects of the matrix cracks are introduced into the ply level constitutive equations as follows [15]:

$$\{\sigma_{L_{ij}}\} = [Q]\{\varepsilon_{L_{ij}} - \alpha^M_{L_{ij}}\} \tag{1}$$

where $\sigma_{L_{ij}}$ are the locally averaged components of stress, $[Q]$ is the ply level transformed stiffness matrix, and $\varepsilon_{L_{ij}}$ are the locally averaged components of strain. $\alpha^M_{L_{ij}}$ are the components of the strain-like internal state variable for matrix cracking and are defined by

$$\alpha^M_{L_{ij}} = \frac{1}{V_L}\int_S u_i n_j dS \tag{2}$$

where V_L is the volume of an arbitrarily chosen representative volume of ply thickness which is sufficiently large that $\alpha^M_{L_{ij}}$ do not depend on V_L, u_i are the crack opening displacements, and n_j are the components of the vector normal to the crack face. It is assumed in the current model that α^M_{22}, the internal state variable representing the mode I matrix crack opening, is the only nonzero component. For a uniaxially loaded medium containing alternating 0^o and 90^o plies, α^M_{22}, has been found from a micromechanics solution to be related to the far field normal force and crack spacing as follows [15]:

$$\alpha^M_{22} = \frac{\frac{\rho}{2\tilde{t}}}{\frac{\pi^4}{64\xi} - C_{2222}} \tag{3}$$

where

$$\xi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{C_{2222}(2m-1)^2(2n-1)^2 + C_{1212}(\frac{\tilde{a}}{\tilde{t}})^2(2n-1)^4} \tag{4}$$

ρ is the force per unit length applied normal to the fibers and $2\tilde{t}$ and $2\tilde{a}$ are the layer thickness and crack spacing, respectively. C_{2222} is the modulus in the direction transverse to the fibers and C_{1212} is the inplane shear modulus. Both moduli are the undamaged properties.

Equation (3) requires that the matrix crack spacing be known in each ply of the laminate. Since it is usually necessary to predict the damage accumulation and response for a given loading history, damage evolutionary relationships must be utilized to determine the values of the internal state variables. The authors have used the following relationship for the rate of change of the internal state variable α_{22}^M in each ply during fatigue loading conditions and when the available strain energy release rate is greater than some critical value G_c, [16],

$$d\alpha_{22}^M = \frac{d\alpha_{22}^M}{dS}\tilde{k}G^{\tilde{n}}dN \tag{5}$$

where $\frac{d\alpha_{22}^M}{dS}$ describes the change in the internal state variable for a given change in the crack surface areas, $\tilde{k}$ and $\tilde{n}$ are material parameters, and N is the number of load cycles. G is the *damage dependent* strain energy release rate for the ply of interest and is calculated from the following equation,

$$G = V_L C_{ijkl}(\varepsilon_{L_{ij}} - \alpha_{ij}^M)\frac{d\alpha_{kl}}{dS} \; . \tag{6}$$

where V_L is the local volume. Interactions with the adjacent plies will result in ply strains, $\varepsilon_{L_{ij}}$ which are affected by the strains in adjacent plies. Thus, the energy release rate, G, in each ply will be implicitly reflected in the calculation of the ply level response, so that equation (5) is not restricted to a particular laminate stacking sequence. Utilizing equation (6) in equation (5) and integrating the result in each ply over time thus gives the current damage state in each ply for any fatigue load history.

The ply level strains are defined as follows:

$$\varepsilon_{L_{xx}} = \varepsilon_{L_{xx}}^o - z\kappa_{L_{xx}} \tag{7}$$

$$\varepsilon_{L_{yy}} = \varepsilon_{L_{yy}}^o - z\kappa_{L_{yy}} \tag{8}$$

$$\varepsilon_{L_{zz}} = \varepsilon_{L_{zz}}^o \tag{9}$$

$$\varepsilon_{L_{yz}} = \varepsilon_{L_{yz}}^o - \kappa_{L_{yz}} \tag{10}$$

$$\varepsilon_{L_{xz}} = \varepsilon_{L_{xz}}^o - \kappa_{L_{xz}} \tag{11}$$

$$\varepsilon_{L_{xy}} = \varepsilon_{L_{xy}}^o - \kappa_{L_{xy}} \tag{12}$$

where ε_L^o and κ_L are the midplane strains and curvatures, respectively. The aforementioned ply strains are then substituted into equation (1) to produce the ply level stresses.

Damage dependent lamination equations are obtained by integrating these ply stresses through the thickness of the laminate [11]. Next, the stiffness matrix in the laminate equation is inverted to produce:

$$\begin{Bmatrix} \varepsilon_L^o \\ \kappa_L \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N - f^M \\ M - g^M \end{Bmatrix} \tag{13}$$

where $[A]$, $[B]$, and $[D]$ are, respectively, the undamaged laminate extensional, coupling, and bending stiffness matrices. They are defined by [17],

$$[A] = \sum_{k=1}^{n} [Q]_k (z_k - z_{k-1}) \tag{14}$$

$$[B] = \sum_{k=1}^{n} [Q]_k ({z^2}_k - {z^2}_{k-1}) \tag{15}$$

$$[D] = \sum_{k=1}^{n} [Q]_k ({z^3}_k - {z^3}_{k-1}) \tag{16}$$

where $[Q]_k$ is the elastic modulus matrix for the k^{th} ply in laminate coordinates. N are the components of the resultant force per unit length and M are the components of the resultant moments per unit length; $\{f^M\}$ and $\{g^M\}$ represent the contribution to the resultant forces and moments from matrix cracking and are calculated from,

$$\{f^M\} = -\sum_{k=1}^{\tilde{n}} [Q]_k (z_k - z_{k-1}) \{\alpha^M\}_k \tag{17}$$

$$\{g^M\} = -\frac{1}{2} \sum_{k=1}^{\tilde{n}} [Q]_k (z_k^2 - z_{k-1}^2) \{\alpha^M\}_k \tag{18}$$

where $\{\alpha^M\}_k$ contains the matrix cracking internal state variables for the k^{th} ply. Thus given the forces, N, and moments, M, as well as the damage variables in each ply, equation (13) can be utilized to calculate the midsurface strains, ε_L^o, and curvature, κ_L.

Program Description

Program Structure

The damage dependent lamination model has been coded into the FORTRAN program FLAMSTR following the algorithm shown in Figure 1. This program enables the analysis of the stress-strain response and accumulation of matrix cracking at a material point in a laminate subjected to fatigue loading. The program begins by reading in the laminate

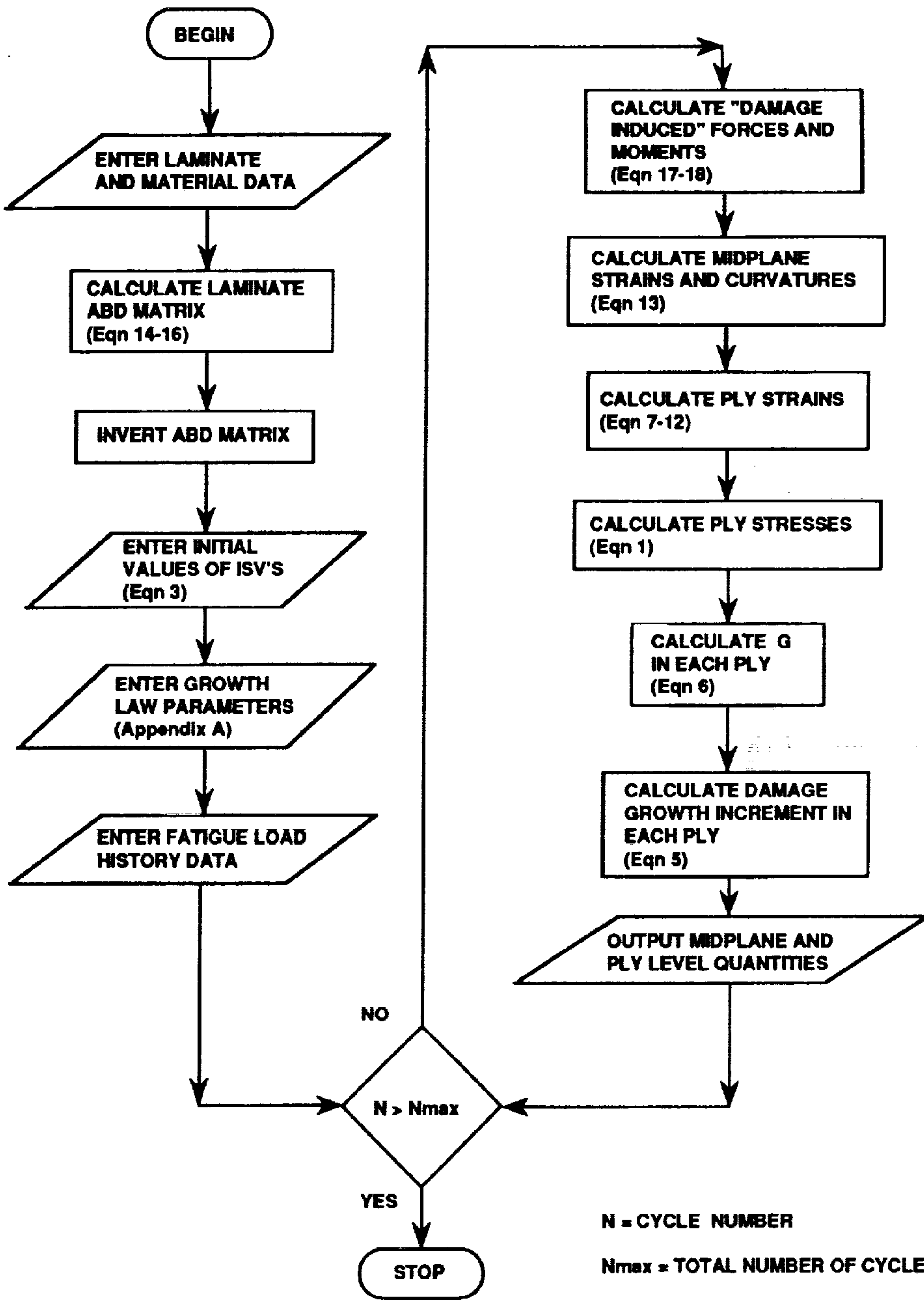

Figure 1. Structure of FLAMSTR used in the analysis of damage accumulation.

and material properties from a data file. The extensional, coupling, and bending stiffness matrices, $[A]$, $[B]$, and $[D]$, respectively are calculated and then inverted for later use. Next, the initial damage states for each ply and the growth law parameters are read into the program. They can be determined if the initial crack spacing is known from equation (3). The procedure for the determination of the damage evolution parameters can be found in Appendix A. Finally, the fatigue loading history is read in. The constitutive portion of the program initiates with the calculation of the "damage induced" forces and moments using equations (17) and (18), respectively. These quantities are combined with the applied forces and moments to calculate the midplane strains and curvatures through equation (13). The strains in each ply are then determined with equations (7) through (12). Equation (1) is then used to calculate the damage dependent stresses in each ply and the strain energy release rate is determined for each ply from equation (6) using the ply stresses. The evolution of α_{22}^{M} in each ply is then obtained from equation (5) and the damage state is updated. The above procedure is repeated for the desired number of load history increments.

The computational algorithm assumes that the rate of damage evolution is small enough that the strain energy release rate can be considered to be constant during each load cycle. Often this condition leads to exponential overflow errors during the execution of the computer code. The high sensitivity of the mode I matrix cracking ISV to the strain energy release rate is due to the power law form of the damage evolutionary relationship. To model this change in the strain energy release rate with the current algorithm, the load cycles experiencing large changes in damage evolution are divided into subincrements for calculation. This enables the strain energy release rate to reflect the damage accumulated during the load cycle. This approach has produced satisfactory results. Subincrementation has also been found to be necessary during the initial cycles of the fatigue load history when the laminate goes from an undamaged to a damaged state and whenever the fatigue load amplitude is increased.

Program Inputs

The execution of FLAMSTR requires the creation of two input data files. One of these files is labeled **datfil.d** and contains two entries. The first entry is the name of the other input file from which all the information required in the calculations are retrieved. On the next line is the name of the data file in which the output will be stored. This output file is created by the program during execution. The second input file contains information about the stacking sequence, material properties, initial damage state, and the loading history. The variables, listed in the order in which they should appear in the data file, along with their description are presented in the following list:

nplies	Number of plies in laminate.
Q_{11}, Q_{12}, Q_{22}	Components of the transformed ply stiffness matrix.
Q_{13}, Q_{33}, Q_{66}	Components of the transformed ply stiffness matrix.
iflag	Damage condition: 0 = no damage, 1 = matrix cracks.
N_x, N_y, N_{xy}	Applied forces.
M_x, M_y, M_{xy}	Applied moments.
$t(i)$, $theta(i)$	Thickness and orientation (deg) of ply i. For i = 1, nplies.
$alpham(i,2)$, $alpham(i,8)$	Initial values of the Mode I and Mode II matrix cracking ISV for ply i. For i = 1, nplies.
dpara, $\hat{k}$, $\hat{n}$	Slope of the relationship between the far field normal stress and $\frac{d\alpha}{dS}$ and growth law parameters.
nci, *ncf*, *ninc*	Initial cycle number; final cycle number; number of increments taken to go from *nci* to *ncf*; $1 \le ninc \le (ncf - nci)$.
iprnum, *nsubic*	Output results to datafile every *iprnum* increments. Number of subincrements employed during a change in the maximum load.
njump, *xfac*	Cycle number at which the maximum applied load is changed; maximum applied load factor. ($nci \le njump \le ncf$, set *xfac* = 1 if applied load remains constant)

Note that each line is read in a format free manner. A sample set of input files can be found in Appendix B.

Program Outputs

The output data file contains a listing of the input variables plus the program generated results. Due to the number of load history increments involved in the analysis of fatigue, only the results at preselected increments are stored in the output file. The frequency at which the data is recorded is set by the variable *iprnum* in the input files. Information stored consists of the load cycle number, the values of the matrix cracking ISV's in each ply, the midplane strains and curvatures, and the damage dependent strains and stresses at the outer fiber of each ply. The output file corresponding to the aforementioned sample input files can also be found in Appendix B.

Sample Calculations

The model has been employed to simulate the damage dependent stress redistribution and the accumulation of matrix cracks in a $[0/90_2/0]_s$ laminate subjected to the uniaxial loading histories shown in Figure 2. Each load case consists of two load segments with the latter at a maximum load of 600 lb/in. A fatigue load ratio of 0.1 has been assumed for both load segments. Material properties for AS4/3501-6 Graphite/Epoxy, as listed in Table 1, have been used in the calculations. The sample input and output FLAMSTR files found in Appendix B correspond to load case I.

Figure 3 illustrates the dependence of the accumulation of matrix cracks on the maximum load amplitude. The damage caused by loading initially at a lower load amplitude, cases I and II, causes only a minor effect on the final damage state as compared to load case III in which a maximum load of 600 lb/in is maintained during the entire load history. However, when the initial fatigue load level is 800 lb/in (case IV) much of the damage occurs during the initial fatigue cycles. Furthermore, even though only a minute amount of damage accumulates during the second load segment, the amount of damage at the completion of this fatigue load history is almost twice as much as the other three cases. The decelerated growth of matrix cracks during the second stage occurs when the spacing between the matrix cracks no longer enables the transfer through shear of sufficient load back to the ply to create additional matrix cracking. On the other hand, the negligible accumulation of damage during the first stage of load in case I, when the maximum load is 200 lb/in, indicates that the applied load is not sufficient to produce an appreciable amount of damage. Thus, even though a critical value for damage growth has not been specified in the calculations, the damage evolution relationship shown by equation (5) behaves as if a threshold exists for damage growth. The accuracy of this predicted behavior, however, requires further investigation.

The average axial stresses in the 90^o plies of the crossply laminate are shown in Figure 4. The plies experience a sharp decrease in axial stress upon the initiation of matrix cracking. This is followed by a period of gradual decrease as further damage accumulates. The axial ply stress then abruptly changes when the maximum load is changed to 600 lb/in at the initiation of the second load segment. Since each case has the same maximum fatigue load the corresponding axial stress serves as an indication of the relative load carrying capability remaining in the 90^o plies. The laminate retains a

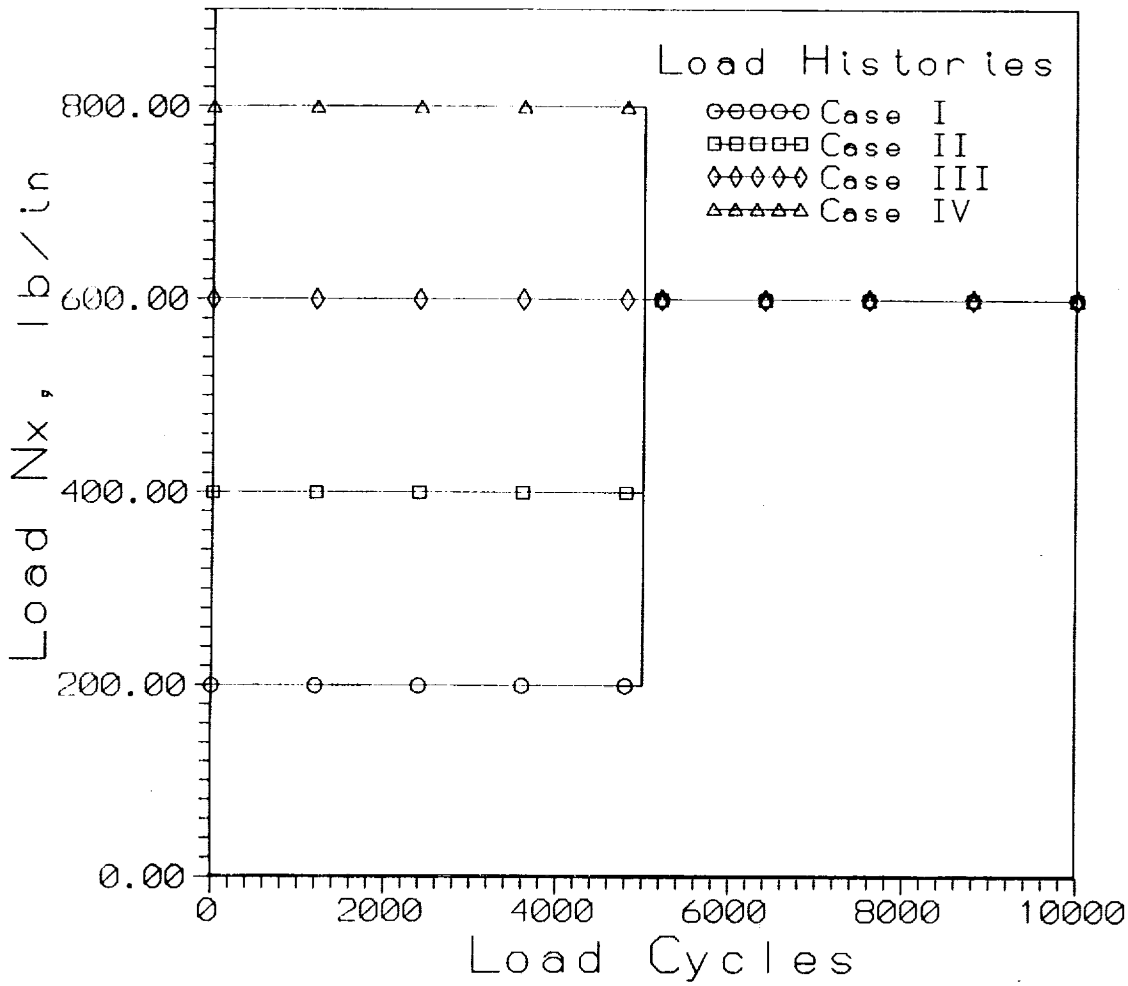

Figure 2. Fatigue load amplitude applied to crossply laminates.

Table 1. Material Properties for AS4/3501-6

E_{11}	20.04 Msi
E_{22}	1.60 Msi
G_{12}	0.70 Msi
ν_{12}	0.26
t_{ply}	0.00505 $in.$
transformed ply stiffness components:	
Q_{11}	20.29 Msi
Q_{12}	0.43 Msi
Q_{22}	1.61 Msi
Q_{13}	0.43 Msi
Q_{33}	1.61 Msi
Q_{66}	0.70 Msi
parameter for $\frac{d\alpha}{dS}$:	
$dpara$	6.90×10^{-6}
growth law parameters:	
$\check{k}$	4.42
$\check{n}$	6.39

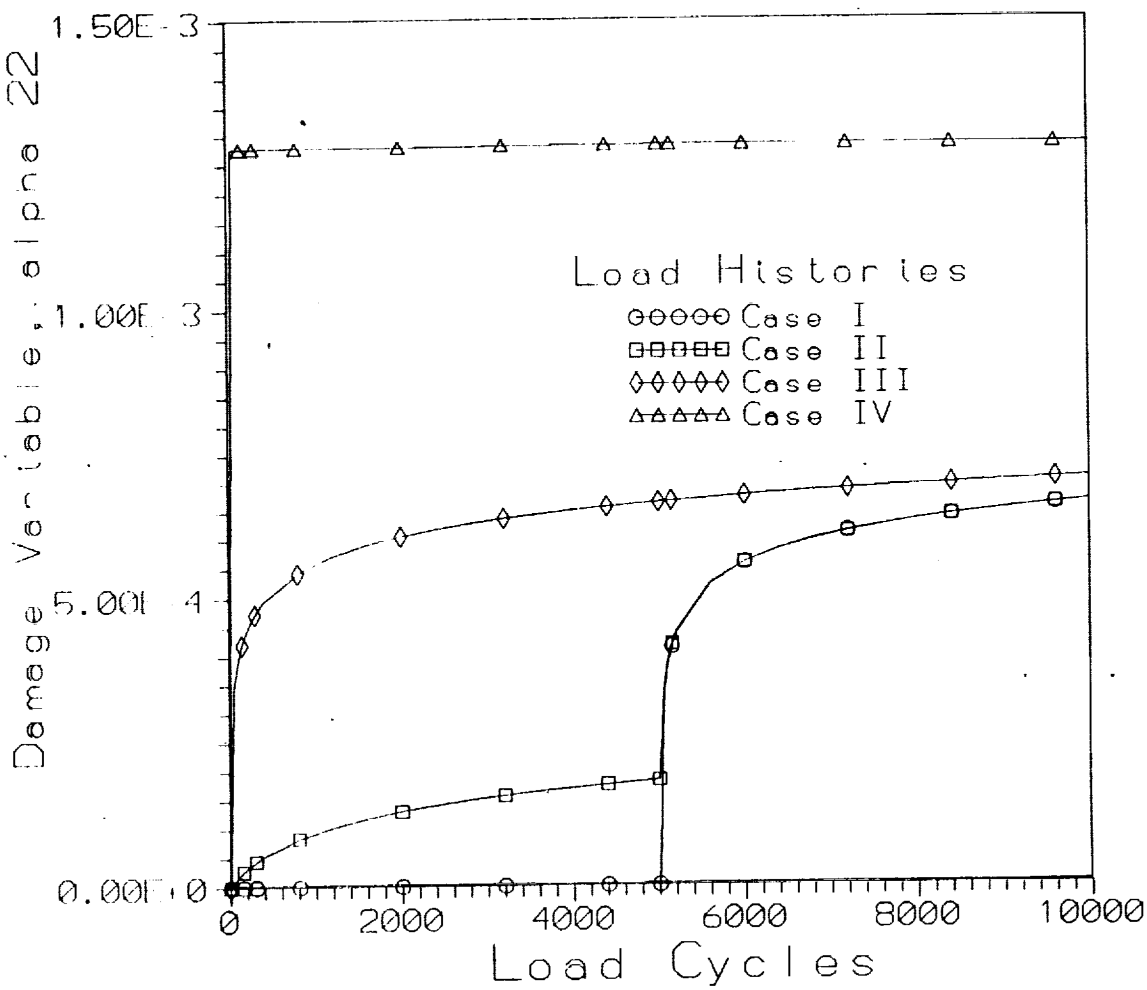

Figure 3. Mode I matrix cracking ISV in the 90° plies.

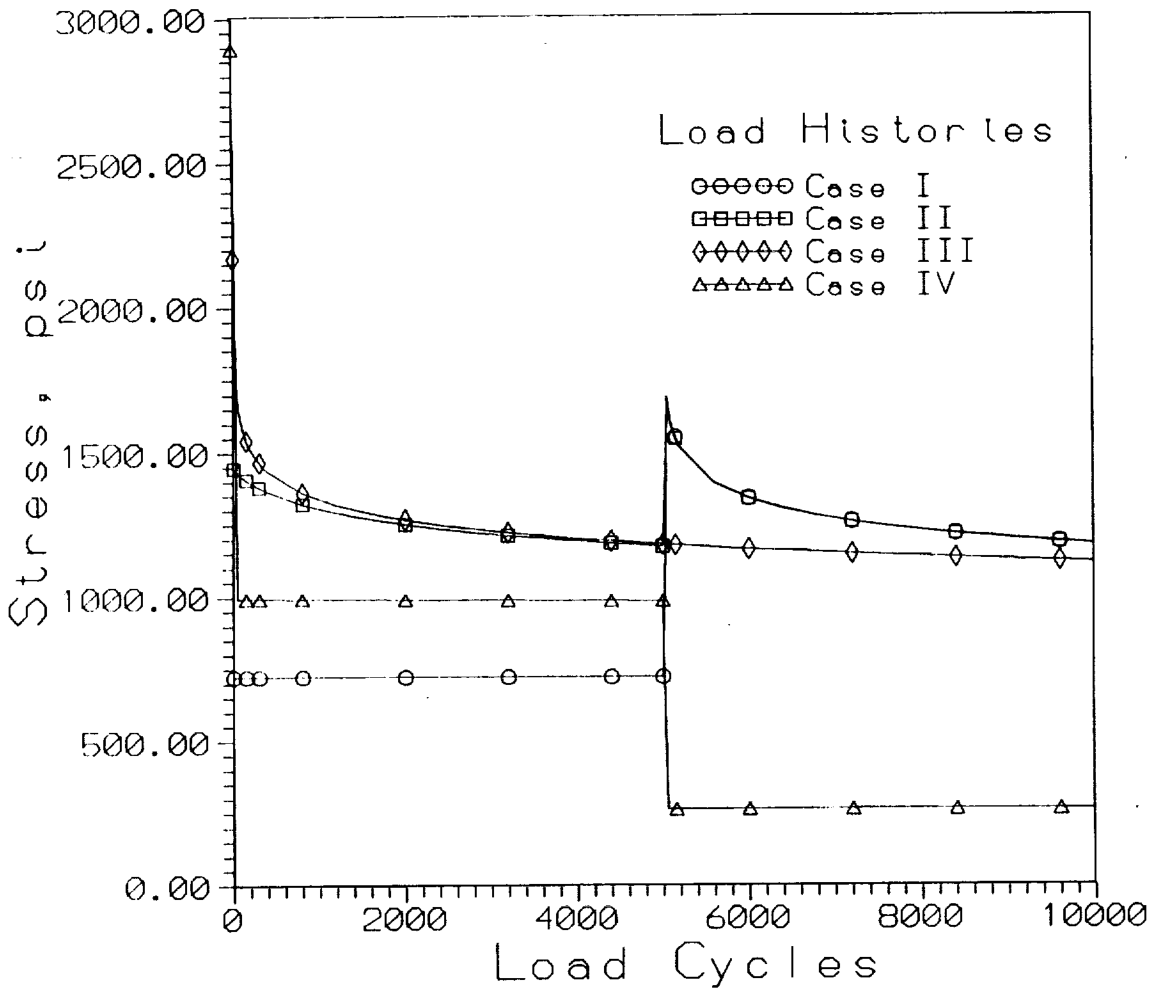

Figure 4. Axial stress in the 90° plies.

similar amount of load carrying capability for the first three load cases. However, the load carrying capability for load case IV is substantially reduced compared to the other cases. This is in accordance with the amount of damage sustained for this load case.

The results presented illustrate the damage accumulation and stress redistribution occurring within uniaxially fatigue loaded crossply laminates. As the load carrying capability diminishes in the 90^o plies, the surrounding 0^o plies must assume a greater portion of the applied load. The load transfer continues until the axial stress in the 0^o plies exceeds the strength of a fiber. Since the 0^o plies are the primary load bearing member for crossply layups, laminate failure is probably triggered by this event. Although this example is a relatively simplistic representation of the progressive failure process found in laminated composites, it does demonstrate how this model can be employed to analyze this process. The ability to model the damage dependent stress redistribution and damage accumulation can be advantageous in the design and maintenance of laminated composites. By simulating the progressive failure of a laminate, potential damage modes and their locations can be identified. The laminate can thus be redesigned to suppress or minimize the effects of such damage.

The introduction of multiaxial loading and/or angle plies will promote the development of mixed mode matrix cracking and delamination during the application of fatigue loads. In addition, fiber breakage in one ply may not cause the unstable fracture of the laminate. While this model possesses the capability to account for mixed mode matrix cracking and delamination, relationships describing their evolution are necessary to analyze their contribution in the progressive failure process. The present version of FLAMSTR calculates only the mode I matrix crack contribution in each ply. This factor must be considered in the interpretation of the predicted results until this code is updated to account for these conditions. To analyze laminated composite structures with spatially varying stress fields this constitutive code has been incorporated into a finite element structural analysis algorithm [18]. This finite element analysis code has been successfully used to predict spatial variations of damage evolution in structural components such as a plate with a circular cutout [19].

Summary

A computer code called FLAMSTR has been developed for the analysis of the progressive failure process in fatigue loaded laminated composites. This code utilizes volume averaged internal state variables to describe the kinematics of matrix cracks. The effects of the damage enter the analysis via the damage dependent laminate equations. Ply level stresses obtained from the results of these equations are then used to predict the amount

of mode I matrix crack growth. This procedure is repeated for each load cycle to simulate the damage accumulation process.

This report serves as a user's guide for the constitutive code. Sample input and output datafiles are enclosed to assist in the usage of this computer code. A simulation of crossply laminates subjected to two stage uniaxial fatigue load histories has been conducted to illustrate the ability of the model to predict the path dependent damage accumulation and stress redistribution in the laminate. Since the information generated from this analysis can be used to construct the sequence of events leading to the failure of the laminate, it has potential uses in the design of composite structures.

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Appendix A. Determination of Growth Law Parameters

The damage evolution parameters, $\frac{d\alpha_{22}^M}{dS}$, $\bar{k}$, and $\bar{n}$, are unique to each material system and must be determined prior to the analysis. The term $\frac{d\alpha_{22}^M}{dS}$ reflects the changes to the internal state variable with respect to changes in the crack surfaces. If it is assumed that the matrix crack surfaces are flat and aligned perpendicular to the plane formed by the ply, thereby permitting the description of the total crack surface area in terms of the crack spacing, then $\frac{d\alpha_{22}^M}{dS}$ can be calculated analytically form equation (3). It was found, for typical brittle graphite/epoxy material properties, that α_{22} exhibits an almost linear relationship to the crack spacing at each far field load. Thus this analysis assumes the slope, $\frac{d\alpha_{22}^M}{dS}$, to be constant for each far field load. Furthermore, equation (3) indicates that $\frac{d\alpha_{22}^M}{dS}$ varies linearly with the far field load. It is the slope of the relationship between $\frac{d\alpha_{22}^M}{dS}$ and the far field load that is required by FLAMSTR. This variable is stored in the input file under the named *dpara*.

The constants $\bar{k}$ and $\bar{n}$ in equation (5) must be determined from experimental data. Since $\bar{k}$ and $\bar{n}$ are assumed to be parameters unique to each material system, the values determined from one laminate stacking sequence and one loading condition should be valid for all other cases. This has been shown for uniaxially fatigue loaded crossply laminates with varying number of consecutive transverse plies and maximum fatigue stresses [20]. To evaluate $\bar{k}$ and $\bar{n}$, a curve of the quantity $\frac{d\alpha_{22}^M}{dN}\frac{dS}{d\alpha_{22}^M}$ versus the strain energy release rate, G, must be generated for a particular stacking sequence and maximum fatigue stress. Experimental data from uniaxially fatigue loaded crossply laminate are used because it can be assumed that the matrix crack opening mode to be essentially mode I. Thus the ISV, α_{22}^M, is sufficient in describing the damage state and the contributions from the other two opening mode do not have to be considered in the calculations. In addition, the transverse plies should be grouped together in the laminate so that the damage evolution at a single layer needs to be considered. Since most damage accumulation data are reported as crack spacing or density at a particular point in the loading history, equation (3) is utilized to convert this data to the form of the ISV, α_{22}^M. The resulting α_{22}^M versus load cycle, N, curve serves as the starting point in this procedure. The following steps describe this process:

(1) $\frac{d\alpha_{22}^M}{dN}$ at a point in the loading history is determined by taking the slope of the α_{22}^M vs. N curve. This task can be facilitated if the α_{22}^M vs. N curve is fitted numerically and the first derivative taken.

(2) Equation (3) is employed to calculate $\frac{d\alpha_{22}^M}{dS}$ at each data point. These will be used in the determination of the strain energy release rate and growth law parameters.

(3) The strain energy release rate, G, is calculated using equation (6) where the height of the local volume, V_L, is assumed to be equal to the thickness of the damaged layer. Since α_{22}^M and the maximum applied fatigue stress are known, $\varepsilon_{L_{ij}}$ in equation (6) can be determined via equations (7) through (12). This is performed for each data point.

(4) Now $\frac{d\alpha_{22}^M}{dN}$ is divided by the corresponding $\frac{d\alpha_{22}^M}{dS}$ so that the damage evolution law,

equation (9), becomes

$$\frac{d\alpha_{22}^{M}}{dN}\frac{dS}{d\alpha_{22}^{M}} = \tilde{k}G^{\tilde{n}} \tag{A1}$$

With the left hand side of equation (A1) and the corresponding values of the strain energy release rate, G, calculated at various points in the fatigue loading history, $\tilde{k}$ and $\tilde{n}$ can be determined by taking the natural log of equation (A1) and then employing a linear regression procedure.

For the present model, $\tilde{k}$ and $\tilde{n}$ of AS4/3501-6 graphite/epoxy are determined from damage accumulation data published by Chou, et al. [21]. The data employed is for $[0_2/90_2]_s$ AS4/3501-6 laminates fatigue loaded at the maximum fatigue stress of 43 ksi and a stress ratio of 0.1 . The parameters are determined to be

$$\tilde{k} = 4.42, \qquad \tilde{n} = 6.39\,. \tag{A2}$$

This values are then used in the calculations involving the AS4/3501-6 graphite/epoxy system.

Appendix B. Sample Input and Output Files

Input datafile: **datfil.d***

finp.dat	*iname*
ft200.d	*outname*

Input datafile: **finp.dat***

6	*nplies*
20.29e6 0.433e6 1.61e6	Q_{11}, Q_{12}, Q_{22}
0.433e6 1.61e6 0.695e6	Q_{13}, Q_{33}, Q_{66}
1	*iflag*
200.0 0.0 0.0	N_x, N_y, N_{xy}
0.0 0.0 0.0	M_x, M_y, M_{xy}
0.00505 0.0	$t(1)$, $theta(1)$
0.01010 90.0	$t(2)$, $theta(2)$
0.00505 0.0	$t(3)$, $theta(3)$
0.00505 0.0	$t(4)$, $theta(4)$
0.01010 90.0	$t(5)$, $theta(5)$
0.00505 0.0	$t(6)$, $theta(6)$
0.0 0.0	$alpham(1,2)$, $alpham(1,8)$
0.0 0.0	$alpham(2,2)$, $alpham(2,8)$
0.0 0.0	$alpham(3,2)$, $alpham(3,8)$
0.0 0.0	$alpham(4,2)$, $alpham(4,8)$
0.0 0.0	$alpham(5,2)$, $alpham(5,8)$
0.0 0.0	$alpham(6,2)$, $alpham(6,8)$
6.9017e-6 4.41613 6.388592	$dpara$, $\bar{k}$, $\bar{n}$
0.0 10000.0 10000.0	nci, ncf, $ninc$
500 200.0	$iprnum$, $nsubic$
5000.0 3.000	$njump$, $xfac$

* Italicized items are descriptions of the input variables and are not part of the data file.

Output datafile: **ft200.d**

```
OUTPUT FOR     6  PLIES

THE C MATRIX IS

       0.2029000E+08  0.4330000E+06  0.1610000E+07
       0.4330000E+06  0.1610000E+07  0.6950000E+06


IFLAG =     1

THE APPLIED FORCE Ni ARE
  0.2000000E+03  0.0000000E+00  0.0000000E+00  0.0000000E+00
       0.0000000E+00  0.0000000E+00


THE APPLIED MOMENT Mi ARE
  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00
       0.0000000E+00  0.0000000E+00


PLY NO.                  T                   THETA

     1        0.5050000E-02        0.0000000E+00
     2        0.1010000E-01        0.9000000E+02
     3        0.5050000E-02        0.0000000E+00
     4        0.5050000E-02        0.0000000E+00
     5        0.1010000E-01        0.9000000E+02
     6        0.5050000E-02        0.0000000E+00

DPARA=   0.6901700E-05     XK1=   0.4416130E+01     XN1=   0.6388592E+01


INITIAL CYCLE:   0.0000000E+00          FINAL CYCLE:          0.1000000E+05

CYCLE INCREMENT:   0.1000000E+01

PRINT OUTPUT INCREMENT:     500

SUBINCREMENT CYCLES (RAMP UP):   0.2000000E+03


LOAD JUMP AT CYCLE:   0.5000000E+04          LOAD FACTOR:          0.3000000E+01


CYCLE NUMBER        0.4990000E+03

PLY NO.          ALPHAM2          ALPHAM8
      1        0.1828431D-18       0.0000000D+00
      2        0.7381833D-08      -0.3226702D-15
      3        0.1828431D-18       0.0000000D+00
      4        0.1828431D-18       0.0000000D+00
      5        0.7381833D-08      -0.3226702D-15
      6        0.1828431D-18       0.0000000D+00

EPSO(1) - EPSO(6)

       0.4539266D-03       -0.1678741D-04       -0.5878301D-04
       0.0000000D+00        0.0000000D+00       -0.6708493D-11

KAPPA(1) - KAPPA(6)

      -0.2951981D-17        0.1070011D-18        0.1293403D-18
       0.0000000D+00        0.0000000D+00        0.4801262D-25

THE STRAINS AT THE OUTER FIBER ARE
PLY NO.          E(X)          E(Y)          E(Z)          E(XY)

    1   0.45393D-03  -0.16787D-04  -0.58783D-04  -0.67085D-11
    2   0.45393D-03  -0.16787D-04  -0.58783D-04  -0.67085D-11
    3   0.45393D-03  -0.16787D-04  -0.58783D-04  -0.67085D-11
    4   0.45393D-03  -0.16787D-04  -0.58783D-04  -0.67085D-11
```

5	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67085D-11
6	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67085D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.45	169.52	101.91	0.00
2	-169.52	723.54	-101.91	0.00
3	9177.45	169.52	101.91	0.00
4	9177.45	169.52	101.91	0.00
5	-169.52	723.54	-101.91	0.00
6	9177.45	169.52	101.91	0.00

CYCLE NUMBER 0.9990000E+03

PLY NO.	ALPHAM2	ALPHAM8
1	0.3610451D-18	0.0000000D+00
2	0.1457456D-07	-0.6370742D-15
3	0.3610451D-18	0.0000000D+00
4	0.3610451D-18	0.0000000D+00
5	0.1457456D-07	-0.6370742D-15
6	0.3610451D-18	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539271D-03	-0.1678728D-04	-0.5878309D-04
0.0000000D+00	0.0000000D+00	-0.6708418D-11

KAPPA(1) - KAPPA(6)

-0.2952008D-17	0.1069916D-18	0.1293454D-18
0.0000000D+00	0.0000000D+00	0.4800801D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11
2	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11
3	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11
4	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11
5	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11
6	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67084D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.46	169.52	101.91	0.00
2	-169.52	723.53	-101.91	0.00
3	9177.46	169.52	101.91	0.00
4	9177.46	169.52	101.91	0.00
5	-169.52	723.53	-101.91	0.00
6	9177.46	169.52	101.91	0.00

CYCLE NUMBER 0.1499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.5392533D-18	0.0000000D+00
2	0.2176582D-07	-0.9514142D-15
3	0.5392533D-18	0.0000000D+00
4	0.5392533D-18	0.0000000D+00
5	0.2176582D-07	-0.9514142D-15
6	0.5392533D-18	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539277D-03	-0.1678716D-04	-0.5878318D-04
0.0000000D+00	0.0000000D+00	-0.6708343D-11

KAPPA(1) - KAPPA(6)

-0.2952028D-17	0.1069888D-18	0.1293488D-18
0.0000000D+00	0.0000000D+00	0.4800658D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67083D-11
2	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67083D-11
3	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67083D-11
4	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67083D-11

5	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67082D-11
6	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67082D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.49	169.52	101.91	0.00
2	-169.52	723.50	-101.91	0.00
3	9177.49	169.52	101.91	0.00
4	9177.49	169.52	101.91	0.00
5	-169.52	723.50	-101.91	0.00
6	9177.49	169.52	101.91	0.00

CYCLE NUMBER 0.2999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1073915D-17	0.0000000D+00
2	0.4333084D-07	-0.1894051D-14
3	0.1073915D-17	0.0000000D+00
4	0.1073915D-17	0.0000000D+00
5	0.4333084D-07	-0.1894051D-14
6	0.1073915D-17	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539292D-03	-0.1678679D-04	-0.5878344D-04
0.0000000D+00	0.0000000D+00	-0.6708118D-11

KAPPA(1) - KAPPA(6)

-0.2952133D-17	0.1069506D-18	0.1293694D-18
0.0000000D+00	0.0000000D+00	0.4798803D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11
2	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11
3	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11
4	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11
5	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11
6	0.45393D-03	-0.16787D-04	-0.58783D-04	-0.67081D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.50	169.52	101.91	0.00
2	-169.52	723.49	-101.91	0.00
3	9177.50	169.52	101.91	0.00
4	9177.50	169.52	101.91	0.00
5	-169.52	723.49	-101.91	0.00
6	9177.50	169.52	101.91	0.00

CYCLE NUMBER 0.3499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1252148D-17	0.0000000D+00
2	0.5051625D-07	-0.2208136D-14
3	0.1252148D-17	0.0000000D+00
4	0.1252148D-17	0.0000000D+00
5	0.5051625D-07	-0.2208136D-14
6	0.1252148D-17	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539298D-03	-0.1678667D-04	-0.5878353D-04
0.0000000D+00	0.0000000D+00	-0.6708043D-11

KAPPA(1) - KAPPA(6)

-0.2952218D-17	0.1069260D-18	0.1293855D-18
0.0000000D+00	0.0000000D+00	0.4797596D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
2	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
3	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
4	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11

5	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
6	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.51	169.53	101.91	0.00
2	-169.53	723.48	-101.91	0.00
3	9177.51	169.53	101.91	0.00
4	9177.51	169.53	101.91	0.00
5	-169.53	723.48	-101.91	0.00
6	9177.51	169.53	101.91	0.00

CYCLE NUMBER 0.3999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1430387D-17	0.0000000D+00
2	0.5770021D-07	-0.2522156D-14
3	0.1430387D-17	0.0000000D+00
4	0.1430387D-17	0.0000000D+00
5	0.5770021D-07	-0.2522156D-14
6	0.1430387D-17	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539303D-03	-0.1678655D-04	-0.5878362D-04
0.0000000D+00	0.0000000D+00	-0.6707968D-11

KAPPA(1) - KAPPA(6)

-0.2952227D-17	0.1069154D-18	0.1293880D-18
0.0000000D+00	0.0000000D+00	0.4797095D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
2	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
3	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
4	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
5	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11
6	0.45393D-03	-0.16787D-04	-0.58784D-04	-0.67080D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.52	169.53	101.91	0.00
2	-169.53	723.47	-101.91	0.00
3	9177.52	169.53	101.91	0.00
4	9177.52	169.53	101.91	0.00
5	-169.53	723.47	-101.91	0.00
6	9177.52	169.53	101.91	0.00

CYCLE NUMBER 0.4499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1608632D-17	0.0000000D+00
2	0.6488271D-07	-0.2836113D-14
3	0.1608632D-17	0.0000000D+00
4	0.1608632D-17	0.0000000D+00
5	0.6488271D-07	-0.2836113D-14
6	0.1608632D-17	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539308D-03	-0.1678642D-04	-0.5878370D-04
0.0000000D+00	0.0000000D+00	-0.6707893D-11

KAPPA(1) - KAPPA(6)

-0.2952305D-17	0.1069002D-18	0.1294020D-18
0.0000000D+00	0.0000000D+00	0.4796336D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11
2	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11
3	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11
4	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11

5	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11
6	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67079D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.53	169.53	101.91	0.00
2	-169.53	723.46	-101.91	0.00
3	9177.53	169.53	101.91	0.00
4	9177.53	169.53	101.91	0.00
5	-169.53	723.46	-101.91	0.00
6	9177.53	169.53	101.91	0.00

CYCLE NUMBER 0.4999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1786883D-17	0.0000000D+00
2	0.7206375D-07	-0.3150007D-14
3	0.1786883D-17	0.0000000D+00
4	0.1786883D-17	0.0000000D+00
5	0.7206375D-07	-0.3150007D-14
6	0.1786883D-17	0.0000000D+00

EPSO(1) - EPSO(6)

0.4539313D-03	-0.1678630D-04	-0.5878379D-04
0.0000000D+00	0.0000000D+00	-0.6707818D-11

KAPPA(1) - KAPPA(6)

-0.2952428D-17	0.1068708D-18	0.1294247D-18
0.0000000D+00	0.0000000D+00	0.4794883D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11
2	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11
3	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11
4	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11
5	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11
6	0.45393D-03	-0.16786D-04	-0.58784D-04	-0.67078D-11

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	9177.55	169.53	101.91	0.00
2	-169.53	723.44	-101.91	0.00
3	9177.55	169.53	101.91	0.00
4	9177.55	169.53	101.91	0.00
5	-169.53	723.44	-101.91	0.00
6	9177.55	169.53	101.91	0.00

CYCLE NUMBER 0.5499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1313856D-11	0.0000000D+00
2	0.5086256D-03	-0.2223273D-10
3	0.1313856D-11	0.0000000D+00
4	0.1313856D-11	0.0000000D+00
5	0.5086256D-03	-0.2223273D-10
6	0.1313856D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1398948D-02	-0.4165405D-04	-0.1825181D-03
0.0000000D+00	0.0000000D+00	-0.1481940D-10

KAPPA(1) - KAPPA(6)

-0.9910421D-17	0.5265107D-19	0.5767951D-18
0.0000000D+00	0.0000000D+00	0.1228763D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10
2	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10
3	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10
4	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10

5	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10
6	0.13989D-02	-0.41654D-04	-0.18252D-03	-0.14819D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28287.59	538.68	311.89	0.00
2	-538.68	1415.38	-311.89	0.00
3	28287.59	538.68	311.89	0.00
4	28287.59	538.68	311.89	0.00
5	-538.68	1415.38	-311.89	0.00
6	28287.59	538.68	311.89	0.00

CYCLE NUMBER 0.5999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.2852440D-11	0.0000000D+00
2	0.5588768D-03	-0.2442928D-10
3	0.2852440D-11	0.0000000D+00
4	0.2852440D-11	0.0000000D+00
5	0.5588768D-03	-0.2442928D-10
6	0.2852440D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1402620D-02	-0.4079366D-04	-0.1831276D-03
0.0000000D+00	0.0000000D+00	-0.1429515D-10

KAPPA(1) - KAPPA(6)

-0.9590869D-17	0.5540902D-19	0.5245836D-18
0.0000000D+00	0.0000000D+00	0.1538825D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10
2	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10
3	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10
4	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10
5	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10
6	0.14026D-02	-0.40794D-04	-0.18313D-03	-0.14295D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28362.21	541.66	312.50	0.00
2	-541.66	1340.76	-312.50	0.00
3	28362.21	541.66	312.50	0.00
4	28362.21	541.66	312.50	0.00
5	-541.66	1340.76	-312.50	0.00
6	28362.21	541.66	312.50	0.00

CYCLE NUMBER 0.6499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.4479322D-11	0.0000000D+00
2	0.5870540D-03	-0.2566094D-10
3	0.4479322D-11	0.0000000D+00
4	0.4479322D-11	0.0000000D+00
5	0.5870540D-03	-0.2566094D-10
6	0.4479322D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1404679D-02	-0.4031122D-04	-0.1834694D-03
0.0000000D+00	0.0000000D+00	-0.1400119D-10

KAPPA(1) - KAPPA(6)

-0.9763569D-17	0.1096710D-18	0.5457380D-18
0.0000000D+00	0.0000000D+00	0.4008539D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10
2	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10
3	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10
4	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10

5	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10
6	0.14047D-02	-0.40311D-04	-0.18347D-03	-0.14001D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28404.05	543.33	312.84	0.00
2	-543.33	1298.92	-312.84	0.00
3	28404.05	543.33	312.84	0.00
4	28404.05	543.33	312.84	0.00
5	-543.33	1298.92	-312.84	0.00
6	28404.05	543.33	312.84	0.00

CYCLE NUMBER 0.6999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.6164068D-11	0.0000000D+00
2	0.6065163D-03	-0.2651167D-10
3	0.6164068D-11	0.0000000D+00
4	0.6164068D-11	0.0000000D+00
5	0.6065163D-03	-0.2651167D-10
6	0.6164068D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1406102D-02	-0.3997799D-04	-0.1837055D-03
0.0000000D+00	0.0000000D+00	-0.1379814D-10

KAPPA(1) - KAPPA(6)

-0.1000685D-16	0.1477874D-18	0.5800803D-18
0.0000000D+00	0.0000000D+00	0.5676971D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10
2	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10
3	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10
4	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10
5	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10
6	0.14061D-02	-0.39978D-04	-0.18371D-03	-0.13798D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28432.95	544.48	313.08	0.00
2	-544.48	1270.02	-313.08	0.00
3	28432.95	544.48	313.08	0.00
4	28432.95	544.48	313.08	0.00
5	-544.48	1270.02	-313.08	0.00
6	28432.95	544.48	313.08	0.00

CYCLE NUMBER 0.7499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.7892060D-11	0.0000000D+00
2	0.6213159D-03	-0.2715858D-10
3	0.7892060D-11	0.0000000D+00
4	0.7892060D-11	0.0000000D+00
5	0.6213159D-03	-0.2715858D-10
6	0.7892060D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1407183D-02	-0.3972460D-04	-0.1838850D-03
0.0000000D+00	0.0000000D+00	-0.1364374D-10

KAPPA(1) - KAPPA(6)

-0.9869425D-17	0.9974921D-19	0.5631240D-18
0.0000000D+00	0.0000000D+00	0.3486876D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10
2	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10
3	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10
4	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10

5	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10
6	0.14072D-02	-0.39725D-04	-0.18388D-03	-0.13644D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28454.92	545.35	313.26	0.00
2	-545.35	1248.05	-313.26	0.00
3	28454.92	545.35	313.26	0.00
4	28454.92	545.35	313.26	0.00
5	-545.35	1248.05	-313.26	0.00
6	28454.92	545.35	313.26	0.00

CYCLE NUMBER 0.7999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.9654631D-11	0.0000000D+00
2	0.6332189D-03	-0.2767888D-10
3	0.9654631D-11	0.0000000D+00
4	0.9654631D-11	0.0000000D+00
5	0.6332189D-03	-0.2767888D-10
6	0.9654631D-11	0.0000000D+00

EPSO(1) - EPSO(6)

0.1408053D-02	-0.3952080D-04	-0.1840293D-03
0.0000000D+00	0.0000000D+00	-0.1351956D-10

KAPPA(1) - KAPPA(6)

-0.9686882D-17	0.6966505D-19	0.5370526D-18
0.0000000D+00	0.0000000D+00	0.2168140D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10
2	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10
3	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10
4	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10
5	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10
6	0.14081D-02	-0.39521D-04	-0.18403D-03	-0.13520D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28472.60	546.06	313.40	0.00
2	-546.06	1230.37	-313.40	0.00
3	28472.60	546.06	313.40	0.00
4	28472.60	546.06	313.40	0.00
5	-546.06	1230.37	-313.40	0.00
6	28472.60	546.06	313.40	0.00

CYCLE NUMBER 0.8499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1144603D-10	0.0000000D+00
2	0.6431517D-03	-0.2811306D-10
3	0.1144603D-10	0.0000000D+00
4	0.1144603D-10	0.0000000D+00
5	0.6431517D-03	-0.2811306D-10
6	0.1144603D-10	0.0000000D+00

EPSO(1) - EPSO(6)

0.1408779D-02	-0.3935073D-04	-0.1841498D-03
0.0000000D+00	0.0000000D+00	-0.1341594D-10

KAPPA(1) - KAPPA(6)

-0.9947443D-17	0.8218032D-19	0.5771187D-18
0.0000000D+00	0.0000000D+00	0.2618052D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10
2	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10
3	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10
4	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10

5	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10
6	0.14088D-02	-0.39351D-04	-0.18415D-03	-0.13416D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28487.35	546.65	313.52	0.00
2	-546.65	1215.62	-313.52	0.00
3	28487.35	546.65	313.52	0.00
4	28487.35	546.65	313.52	0.00
5	-546.65	1215.62	-313.52	0.00
6	28487.35	546.65	313.52	0.00

CYCLE NUMBER 0.8999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1326214D-10	0.0000000D+00
2	0.6516601D-03	-0.2848497D-10
3	0.1326214D-10	0.0000000D+00
4	0.1326214D-10	0.0000000D+00
5	0.6516601D-03	-0.2848497D-10
6	0.1326214D-10	0.0000000D+00

EPSO(1) - EPSO(6)

0.1409401D-02	-0.3920505D-04	-0.1842530D-03
0.0000000D+00	0.0000000D+00	-0.1332717D-10

KAPPA(1) - KAPPA(6)

-0.9824341D-17	0.7020157D-19	0.5586203D-18
0.0000000D+00	0.0000000D+00	0.2120855D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10
2	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10
3	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10
4	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10
5	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10
6	0.14094D-02	-0.39205D-04	-0.18425D-03	-0.13327D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28499.98	547.15	313.62	0.00
2	-547.15	1202.99	-313.62	0.00
3	28499.98	547.15	313.62	0.00
4	28499.98	547.15	313.62	0.00
5	-547.15	1202.99	-313.62	0.00
6	28499.98	547.15	313.62	0.00

CYCLE NUMBER 0.9499000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1509990D-10	0.0000000D+00
2	0.6590917D-03	-0.2880982D-10
3	0.1509990D-10	0.0000000D+00
4	0.1509990D-10	0.0000000D+00
5	0.6590917D-03	-0.2880982D-10
6	0.1509990D-10	0.0000000D+00

EPSO(1) - EPSO(6)

0.1409944D-02	-0.3907781D-04	-0.1843432D-03
0.0000000D+00	0.0000000D+00	-0.1324964D-10

KAPPA(1) - KAPPA(6)

-0.9833615D-17	0.7886193D-19	0.5590246D-18
0.0000000D+00	0.0000000D+00	0.2525333D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10
2	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10
3	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10
4	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10

5	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10
6	0.14099D-02	-0.39078D-04	-0.18434D-03	-0.13250D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28511.02	547.59	313.71	0.00
2	-547.59	1191.95	-313.71	0.00
3	28511.02	547.59	313.71	0.00
4	28511.02	547.59	313.71	0.00
5	-547.59	1191.95	-313.71	0.00
6	28511.02	547.59	313.71	0.00

CYCLE NUMBER 0.9999000E+04

PLY NO.	ALPHAM2	ALPHAM8
1	0.1695690D-10	0.0000000D+00
2	0.6656820D-03	-0.2909789D-10
3	0.1695690D-10	0.0000000D+00
4	0.1695690D-10	0.0000000D+00
5	0.6656820D-03	-0.2909789D-10
6	0.1695690D-10	0.0000000D+00

EPSO(1) - EPSO(6)

0.1410425D-02	-0.3896497D-04	-0.1844231D-03
0.0000000D+00	0.0000000D+00	-0.1318089D-10

KAPPA(1) - KAPPA(6)

-0.1006521D-16	-0.1896032D-19	0.6065785D-18
0.0000000D+00	0.0000000D+00	-0.2215612D-25

THE STRAINS AT THE OUTER FIBER ARE

PLY NO.	E(X)	E(Y)	E(Z)	E(XY)
1	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10
2	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10
3	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10
4	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10
5	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10
6	0.14104D-02	-0.38965D-04	-0.18442D-03	-0.13181D-10

THE STRESSES AT THE OUTER FIBER ARE

PLY NO.	S(1)	S(2)	S(3)	S(6)
1	28520.81	547.98	313.79	0.00
2	-547.98	1182.17	-313.79	0.00
3	28520.81	547.98	313.79	0.00
4	28520.81	547.98	313.79	0.00
5	-547.98	1182.17	-313.79	0.00
6	28520.81	547.98	313.79	0.00

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13. ABSTRACT *(Maximum 200 words)*

This paper describes the procedure for utilizing a damage dependent constitutive model to predict progressive damage growth in laminated composites. In this model, the effects of the internal damage are represented by strain-like second order tensorial damage variables and enter the analysis through damage dependent ply level and laminate level constitutive equations. The growth of matrix cracks due to fatigue loading is predicted by an experimentally based damage evolutionary relationship. This model is incorporated into a computer code called FLAMSTR. This code is capable of predicting the constitutive response and matrix crack damage accumulation in fatigue loaded laminated composites. The structure and usage of FLAMSTR are presented along with sample input and output files to assist the code user.

As an example problem, an analysis of crossply laminates subjected to two stage fatigue loading has been conducted herein and the resulting damage accumulation and stress redistribution have been examined to determine the effect of variations in fatigue load amplitude applied during the first stage of the load history. It is found that the model predicts a significant loading history effect on damage evolution.

14. SUBJECT TERMS

Composites; Laminates; Damage progression; Computer code; Graphite epoxy

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