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Analysis of Surface Wave Propagation in a Grounded Dielectric Slab Covered by a Resistive Sheet

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Abstract

Both parallel and perpendicular polarized surface waves are known to propagate on lossless and lossy grounded dielectric slabs. This paper considers surface wave propagation on a grounded dielectric slab covered with a resistive sheet. Both parallel and perpendicular polarizations are examined. Transcendental equations are derived for each polarization and are solved using iterative techniques. Attenuation and phase velocity are shown for representative geometries. The results are applicable to both a grounded slab covered with a resistive sheet and an ungrounded slab covered on each side with a resistive sheet.

Symbols

R_s	Surface resistance on the resistive sheet (Ohms/square).
t	Thickness of the resistive sheet.
σ	Conductivity of the resistive sheet.
Ω	Electrical resistance, (ohms).
TM^x	Mode which is transverse magnetic to the $\hat{x}$ direction.
TE^x	Mode which is transverse electric to the $\hat{x}$ direction.
$\vec{H}^I$	Magnetic field vector in region I.
$\vec{H}^{II}$	Magnetic field vector in region II.
$\vec{E}^I$	Electric field vector in region I.
$\vec{E}^{II}$	Electric field vector in region II.
A	Relative amplitude of field in region I.
B	Relative amplitude of field in region II.
h_1	Propagation constant in the $\hat{z}$ direction in region I.
h_2	Propagation constant in the $\hat{z}$ direction in region II.
f	Propagation constant in the $\hat{x}$ direction.
ω	Radian frequency of the electromagnetic field.
μ_I	Permeability in region I.
ε_I	Permittivity in region I.
ε_r	Relative permittivity in region I.
μ_o	Permeability of free space.
ε_o	Permittivity of free space.
k_o	Propagation constant of free space.

d	Thickness of the dielectric slab.
PEC	Perfect electric conductor.
j	$\sqrt{-1}$.
J_s	Surface current in the resistive sheet.
α	Real part of the propagation constant f.
β	Imaginary part of propagation constant f.
α_1	Real part of the propagation constant h_1.
β_1	Imaginary part of propagation constant h_1.
α_2	Real part of the propagation constant h_2.
β_2	Imaginary part of propagation constant h_2.
v/c	Normalized phase velocity.
λ_o	Wavelength in free space.
m	Integer describing the order of each surface wave mode.

I. INTRODUCTION

Surface waves are known to propagate on lossless and lossy dielectric covered ground planes. Surface waves of this type are important in the study of microstrip antennas, travelling wave antennas, and scattering. A good discussion of surface waves on a grounded and an ungrounded lossless dielectric slab has been presented by Balanis [1]. Results are available concerning surface waves on lossy ferrite substrates in a recent article by Richmond, Peters, and Hill [2]. Surface waves on a thin resistive sheet and on a coated substrate were examined by Richmond [3]. Surface waves in a thin lossy dielectric were also examined by Richmond [4]. An abundance of work has been done concerning resistive boundary conditions and scattering from resistive strips [5]-[10].

This paper examines surface wave propagation in a grounded dielectric slab covered with a resistive sheet. Such a geometry is shown in Figure 1(a). For parallel polarized waves only even modes will be considered so that the results will also apply to the geometry in Figure 1(b). Similarly, for perpendicular polarized waves only odd modes will be considered with the results applying to both geometries as well. Note that for these cases the electric field vanishes at z=0. Also, note that the geometry in Figure 1(b) is similar to that examined by Richmond [3] with the addition of a second resistive sheet. The resistive sheet is assumed to have zero thickness and have a surface resistivity in ohms per square $R_s = 1/(\sigma t)$ where t is thickness of the sheet [5]. As t approaches zero, σ is assumed to increase so that R_s remains finite in the limit as t approaches zero [6].

II. THEORY

Figure 1(a) shows the geometry of a dielectric slab backed by a ground plane and covered by a resistive sheet. The region above the resistive sheet, region II, is assumed to be free space. The magnetic field intensity for parallel polarized waves, TM^x, is given by

$$\vec{H}^I = Ae^{-fx} \cosh(h_1 z)\, \hat{y} \tag{1}$$

$$\vec{H}^{II} = Be^{-fx} e^{-h_2 z}\, \hat{y} \tag{2}$$

where the propagation constants f, h_1 , and h_2 are related by

$$h_1^2 + f^2 = -\omega^2 \mu_1 \varepsilon_1 \tag{3}$$

$$h_2^2 + f^2 = -\omega^2 \mu_o \varepsilon_o = -k_o^2. \tag{4}$$

Using Maxwell's equations, the tangential electric field intensity is given by

$$E_x^I = \frac{-Ah_1 e^{-fx} \sinh(h_1 z)}{j\omega\varepsilon_1} \tag{5}$$

$$E_x^{II} = \frac{Bh_2 e^{-fx} e^{-h_2 z}}{j\omega\varepsilon_o}. \tag{6}$$

The boundary conditions at z=d are

$$E_x^I = E_x^{II} \tag{7}$$

$$H_y^I - H_y^{II} = J_s \tag{8}$$

where J_s is the electric surface current supported by the resistive sheet and is given by

$$J_s = \frac{E_x}{R_s} = \frac{Bh_2 e^{-fx} e^{-h_2 d}}{j\omega\varepsilon_o R_s}. \tag{9}$$

Using (1)-(6) and the boundary conditions described above the following transcendental equation is obtained for parallel polarized waves:

$$h_1 \sinh(h_1 d)\left[1 + \frac{h_2 \eta_o}{jk_o R_s}\right] + h_2 \varepsilon_r \cosh(h_1 d) = 0 \tag{10}$$

As R_s approaches zero or infinity, (10) reduces to the correct equation for the parallel plate waveguide or the grounded dielectric slab waveguide, respectively.

For waves with perpendicular polarization, TE^x, the electric field intensity is given by

$$\vec{E}^I = Ae^{-fx} \sinh(h_1 z)\, \hat{y} \tag{11}$$

$$\vec{E}^{II} = Be^{-fx} e^{-h_2 z}\, \hat{y} \tag{12}$$

with the tangential magnetic field given by

$$H_x^I = \frac{Ah_1 e^{-fx} \cosh(h_1 z)}{j\omega\mu_1} \tag{13}$$

$$H_x^{II} = \frac{-Bh_2 e^{-fx} e^{-h_2 z}}{j\omega\mu_o}. \tag{14}$$

The boundary conditions for this case are

$$\vec{E}^I = \vec{E}^{II} \tag{15}$$

$$H_x^{II} - H_x^I = J_s \tag{16}$$

and the surface current is given by

$$J_s = \frac{E_y}{R_s} = \frac{Be^{-fx} e^{-h_2 d}}{R_s}. \tag{17}$$

Using (11)-(14) and the boundary conditions the following transcendental equation is obtained for perpendicularly polarized waves:

$$\sinh(h_1 d)\left[h_2 + \frac{jk_o \eta_o}{R_s}\right] + h_1 \cosh(h_1 d) = 0 \tag{18}$$

It is assumed that $\mu_1 = \mu_o$. As before, this equation reduces to the appropriate equation as R_s tends towards zero or infinity and h_1,h_2 and f are related as in (3) and (4).

For both polarizations the propagation constants may be written in complex form as follows:

$$f = \alpha + j\beta \tag{19}$$

$$h_1 = \alpha_1 + j\beta_1 \tag{20}$$

$$h_2 = \alpha_2 + j\beta_2 \tag{21}$$

Using (19) the normalized phase velocity of the wave is given by [2]:

$$\frac{v}{c} = \frac{\omega\sqrt{\mu_o \varepsilon_o}}{\beta} = \frac{k_o}{\beta} \tag{22}$$

where c is the velocity of light in free space. A mode is considered to be cutoff when the normalized phase velocity approaches infinity. For a lossless slab with no resistive covering the mode number m is defined as [2]:

$$m = INT(\beta_1 d/\pi) \tag{23}$$

where the INT function results in the highest integer that is less than the argument of the function. This definition will be used for the grounded slab with a resistive covering also. As stated in references 2 and 3, we will be interested in solutions of equations (10) and (18) for which the x-axis propagation constant f is in the first quadrant of the complex plane and the z-axis propagation constants h_1 and h_2 are in the fourth quadrant of the complex plane. The transcendental equations (10) and (18) can be solved by using (3) and (4) to make each equation a function of f and then using iterative techniques to find the values of f that satisfy each equation.

III. NUMERICAL RESULTS

The transcendental equation derived above for parallel polarized waves has been solved iteratively for the x-axis propagation constant f. The roots of the equation were found by using the IMSL subroutine ZANLY available on the Cray-2 computer at NASA Langley Research Center. Figure 2 shows the attenuation constant, α, and the normalized phase velocity, v/c, as a function of surface resistivity, R_s, for a grounded dielectric slab with thickness d=0.1λ_o and relative permittivity ε_r=4. The attenuation constant is given in dB/wavelength which is calculated as:

$$\alpha \, (dB/\lambda_o) = 8.6859 \times \alpha \, (nepers/m) \times \lambda_o . \tag{24}$$

Only data for the four lowest order propagating modes are shown. As was previously found, [2], when using (23) to calculate the mode number, m, two modes are identified by m=0 for parallel polarization. In Figure 2 note that as R_s increases, mode 1 is eventually cut off. Figure 3 shows the electric field intensity as a function of position for modes 0-3. As expected the tangential electric fields go through zero at z=0, are continuous across the resistive sheet, and decay exponentially above the resistive sheet. Also, note the increase in the number of relative maxima and minima as the mode number increases.

In Figure 4 data are shown for parallel polarized surface waves on a slightly thicker dielectric slab, d=0.125λ_o, with the same relative permittivity, ε_r=4. Modes 2 and 3 maintain similar behavior to that shown in Figure 2. However, in this case it is mode 0 that cuts off as R_s is increased.

The transcendental equation (18) derived above for perpendicular polarization has been solved for the x-axis propagation constant f. Figure 5 shows the attenuation constant and relative phase velocity for these waves in a grounded dielectric slab with thickness $d=0.05\lambda_o$, and relative permittivity $\varepsilon_r=4$. Only the four lowest order propagating modes are shown. As the surface resistivity, R_s, increases all modes are eventually cut off. In Figure 6 similar data are presented for a grounded dielectric slab of thickness $d=0.1\lambda_o$. Note that the attenuation constant has been reduced by approximately one half for each mode when compared to Figure 5. Also, the value of surface resistivity at cutoff in Figure 6 is approximately twice the value shown in Figure 5. Although it is not shown in Figure 6, mode 0 cuts off as R_s increases to approximately $210\Omega/sq$. Electric field intensity for the modes 0-2 described in Figure 6 is shown in Figure 7. As for parallel polarization, the fields go through zero at z=0, are continuous at z=d, and decay exponentially above z=d. Note, however, the increase in the number of minima and maxima for this case when compared to parallel polarization.

IV. SUMMARY

This paper has examined surface wave propagation for both parallel and perpendicular polarization in a grounded dielectric slab covered by a resistive sheet. Results are also valid for the even TM^x and odd TE^x modes for an ungrounded dielectric slab covered on each side by a resistive sheet. Transcendental equations have been derived for both polarizations and solved by using iterative techniques. Attenuation constant and normalized phase velocity are presented for each polarization. Electric field intensity distributions for the propagating modes are also presented.

V. ACKNOWLEDGMENT

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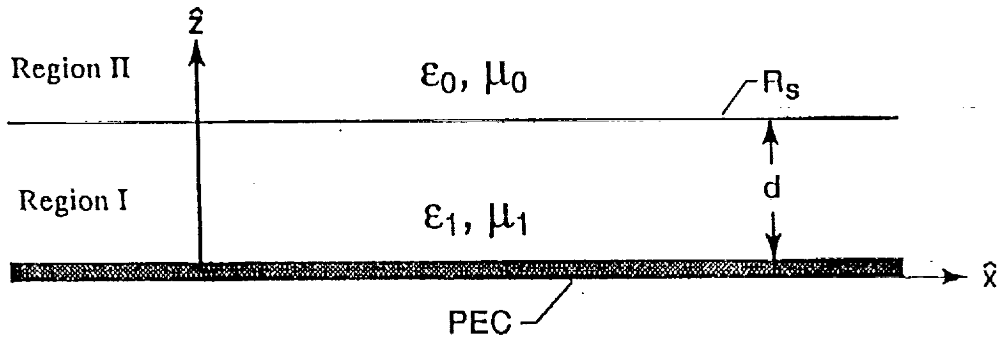
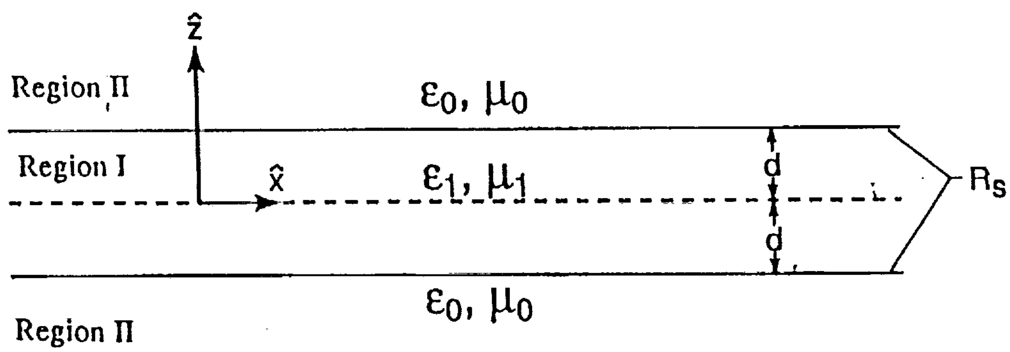


(a)



(b)

Figure 1. Geometry of a grounded dielectric slab covered by a resistive sheet (a), and a dielectric slab covered on both sides by resistive sheets (b).

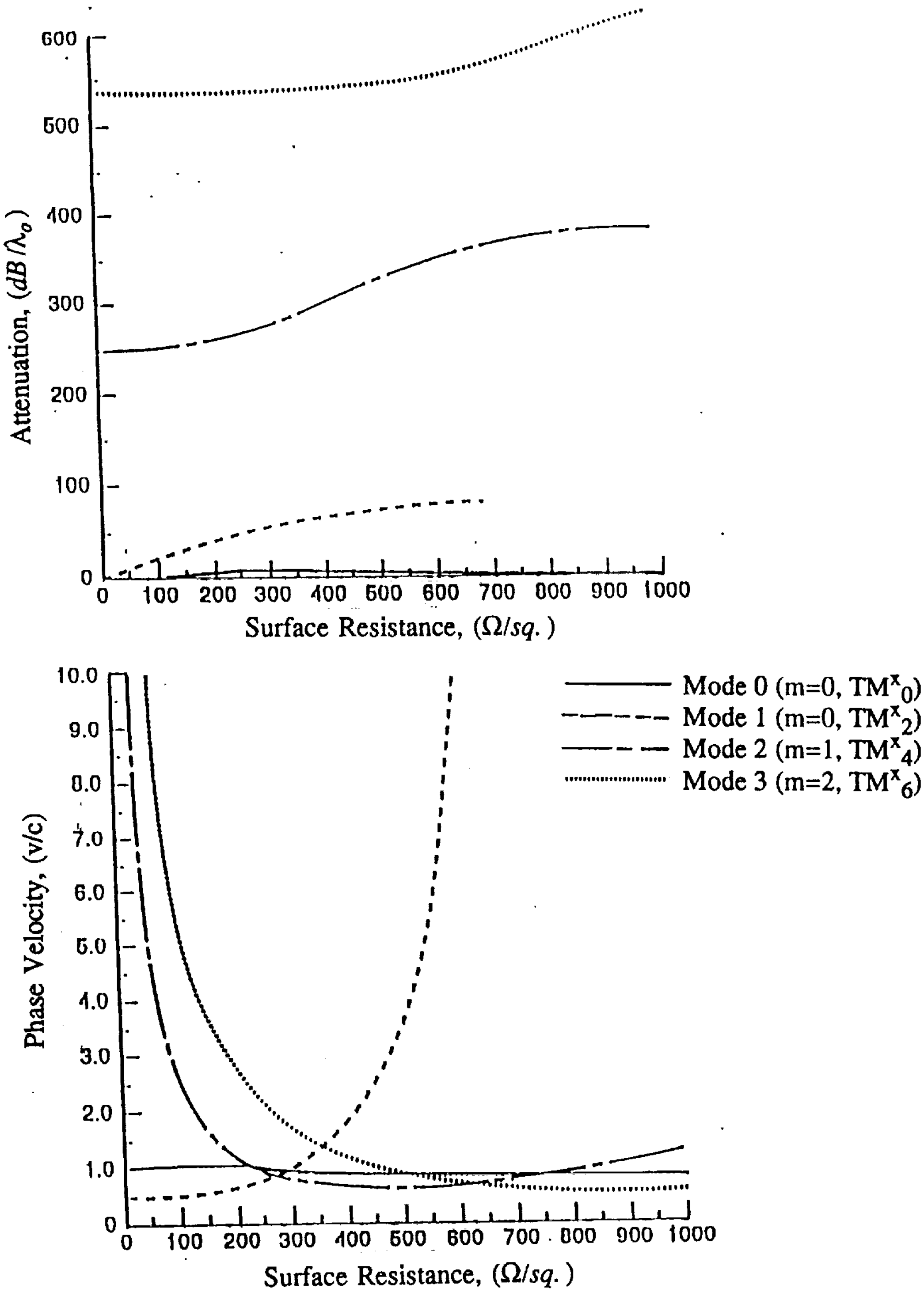


Figure 2. Attenuation and phase velocity for parallel polarized surface waves on a grounded dielectric slab covered by a resistive sheet. (d=0.1λ_o, ε_r=4)

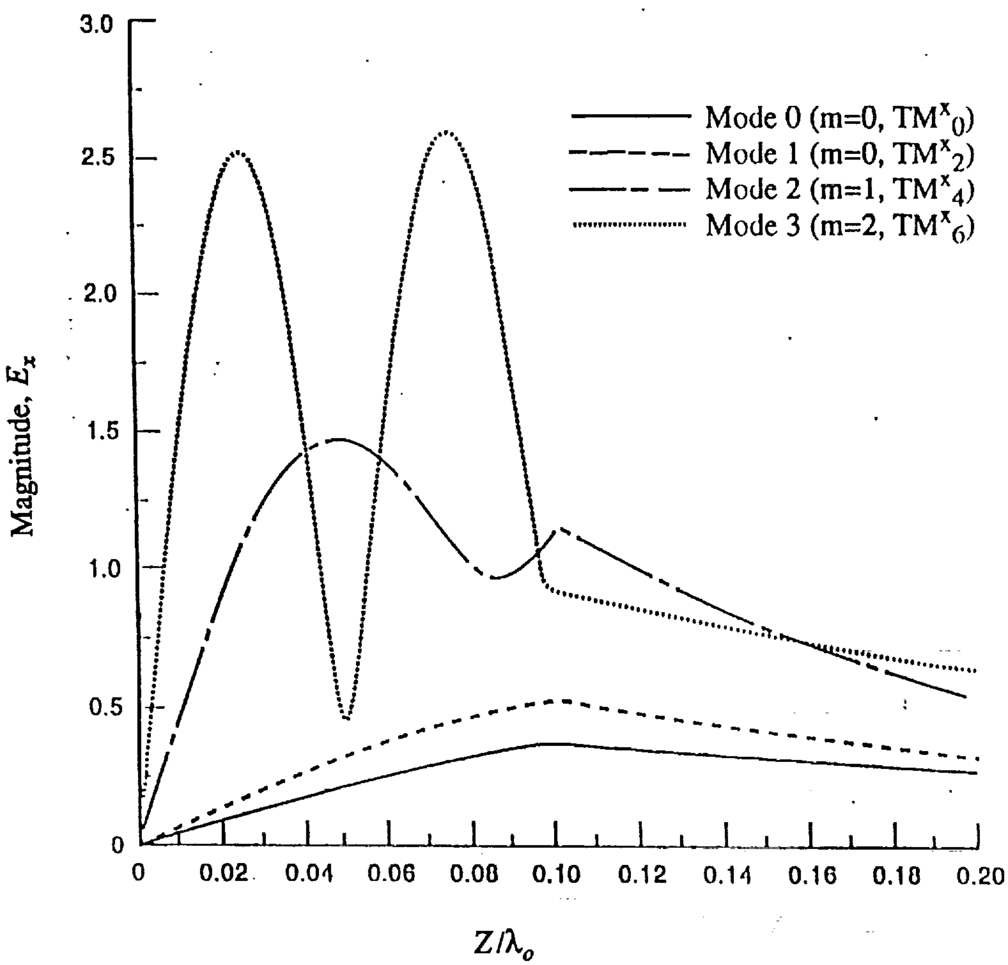


Figure 3. Electric field intensity for parallel polarized surface waves in a grounded dielectric slab covered by a resistive sheet. (d=0.1λ_o, ε_r=4, R_s=300$\Omega/sq.$)

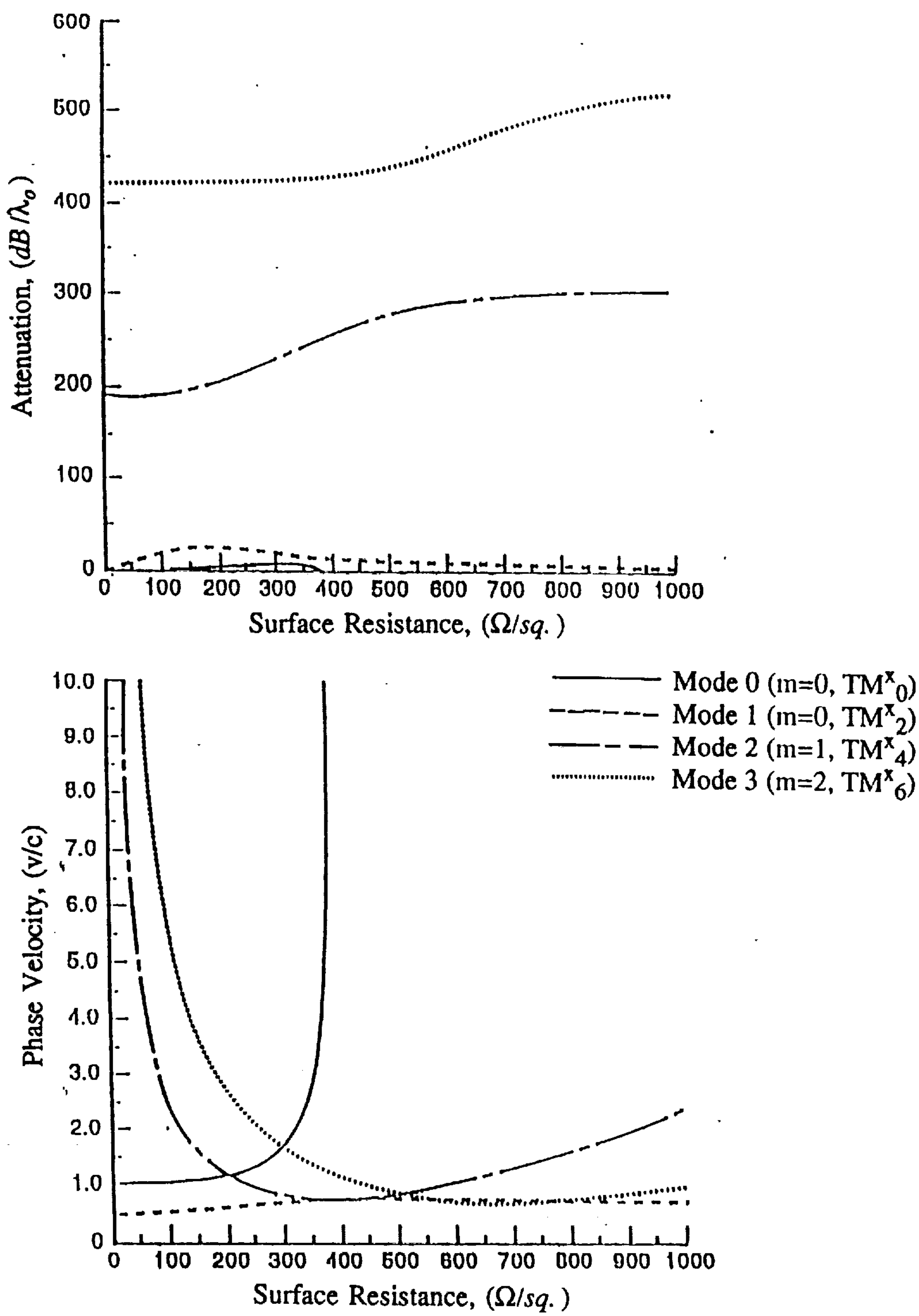


Figure 4. Attenuation and phase velocity for parallel polarized surface waves on a grounded dielectric slab covered by a resistive sheet. ($d=0.125\lambda_o$, $\varepsilon_r=4$)

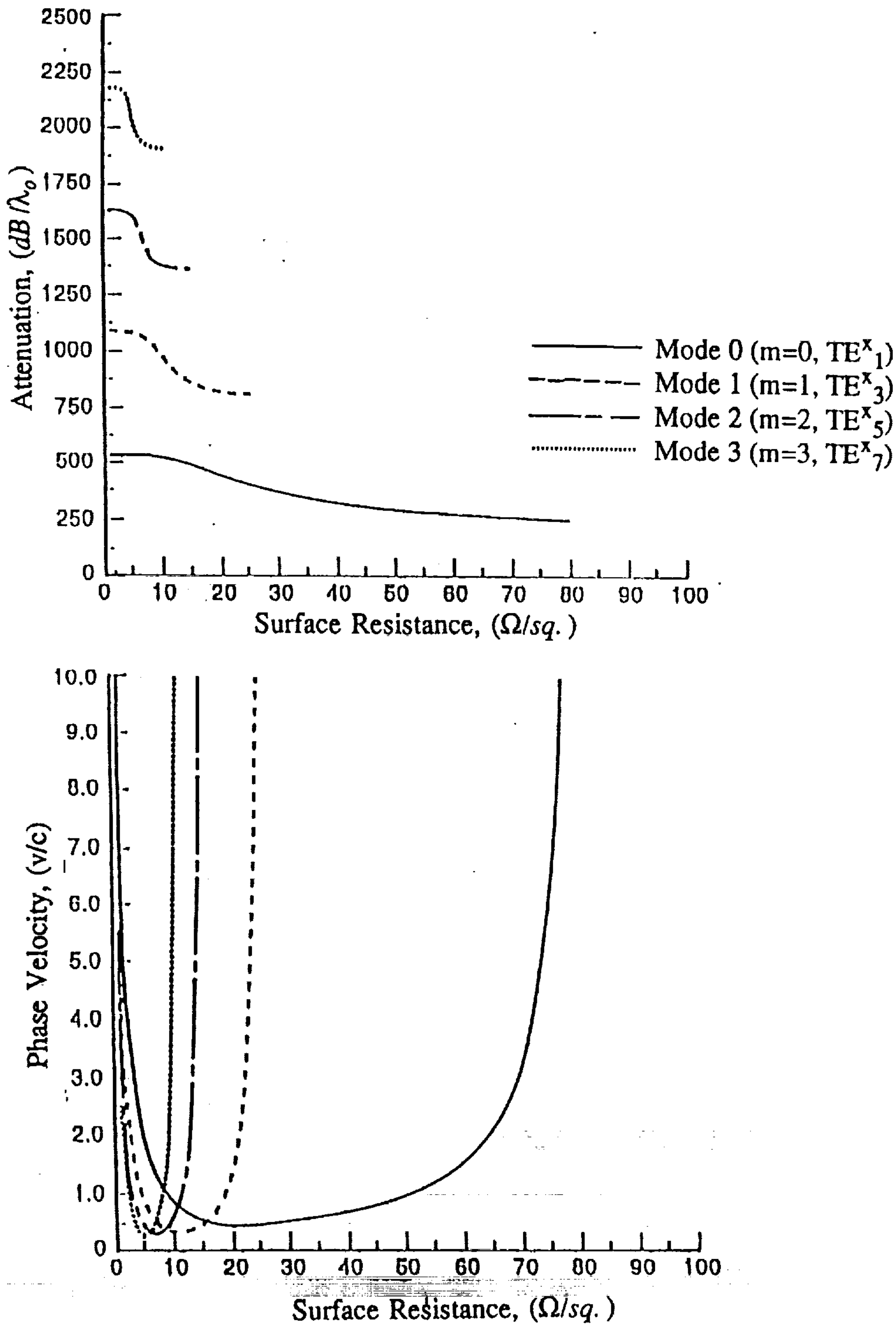


Figure 5. Attenuation and phase velocity for perpendicular polarized surface waves on a grounded dielectric slab covered by a resistive sheet. (d=0.05λ_o, ϵ_r=4)

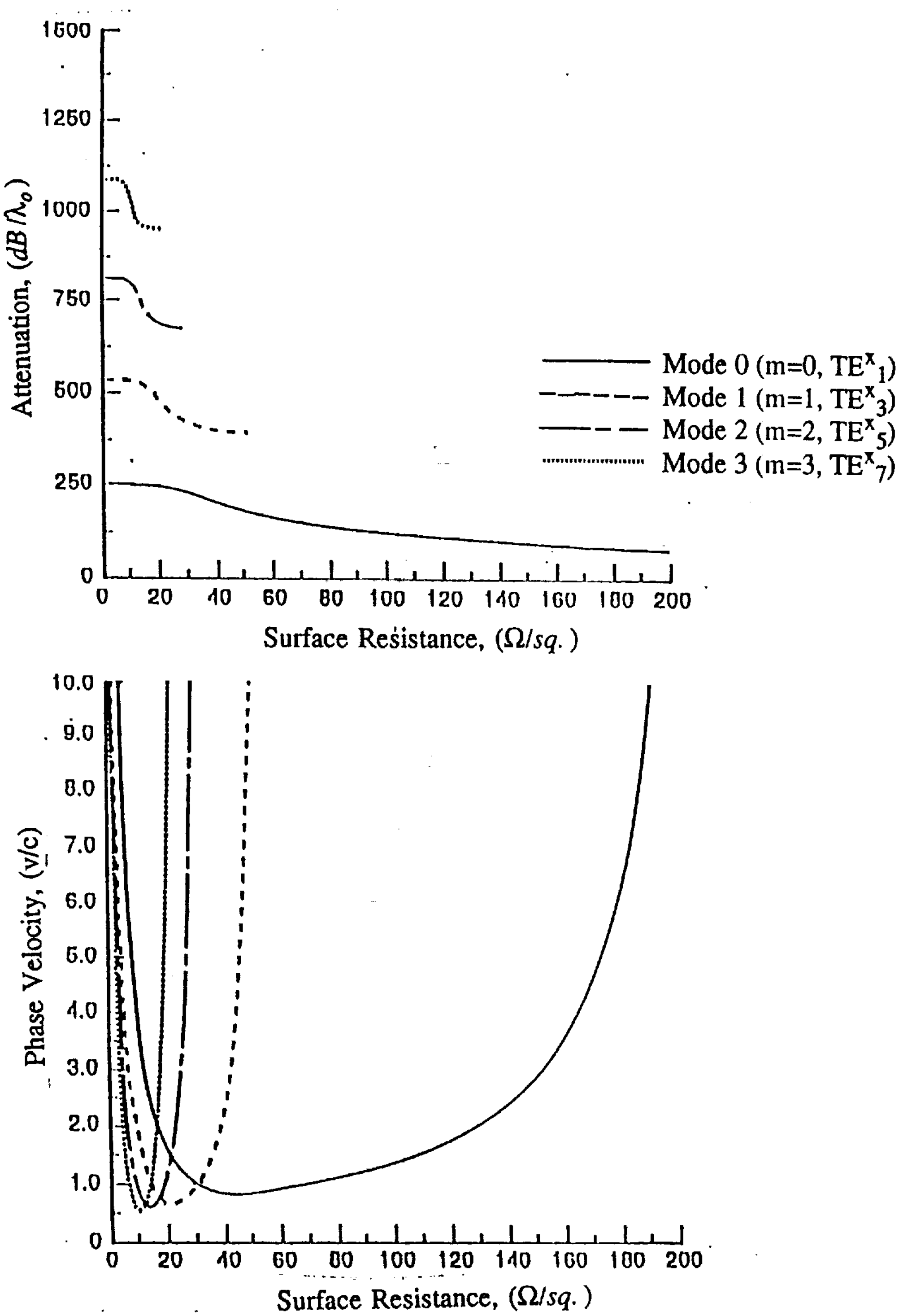


Figure 6. Attenuation and phase velocity for perpendicular polarized surface waves on a grounded dielectric slab covered by a resistive sheet. ($d=0.1\lambda_o$, $\varepsilon_r=4$)

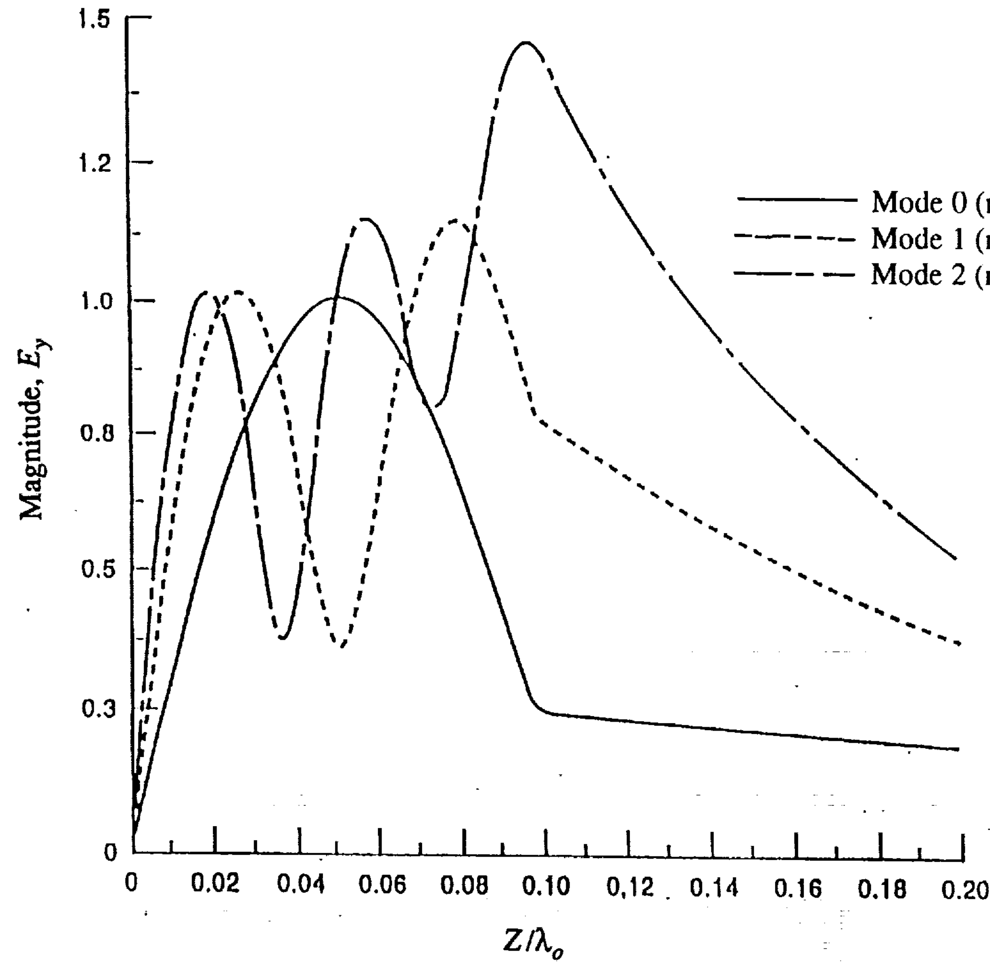


Figure 7. Electric field intensity for perpendicular polarized surface waves in a grounded dielectric slab covered by a resistive sheet. (d=0.1λ_o, ε_r=4, R_s=15Ω/sq.)

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