Determining Rules for Closing Customer Service Centers: A Public Utility Company's Fuzzy Decision

RICIS Preface

This research was conducted under auspices of the Research Institute for Computing and Information Systems by Dr. Andre' de Korvin and Dr. Margaret F. Shipley of the University of Houston - Downtown. Dr. A. Glen Houston served as the RICIS research coordinator.

Funding was provided by the Information Systems Directorate, NASA/JSC through Cooperative Agreement NCC 9-16 between the NASA Johnson Space Center and the University of Houston-Clear Lake. The NASA technical monitor for this activity was Robert T. Savely of the Information Technology Division, NASA/JSC.

DETERMINING RULES FOR CLOSING CUSTOMER SERVICE CENTERS: A PUBLIC UTILITY COMPANY'S FUZZY DECISION

André de Korvin

Department of Applied Mathematical Sciences
University of Houston-Downtown

Margaret F. Shipley

Department of Business Management
University of Houston-Downtown
One Main Street
Houston, Texas 77002

(713) 221- 8571
FAX (713) 221- 8632

Running Title: Rules for Fuzzy Decision

Abstract

In the present work, we consider the general problem of knowledge acquisition under uncertainty. Simply stated, the problem becomes: how can we capture the knowledge of an expert when the expert is unable to clearly formulate how he or she arrives at a decision?

A commonly used method is to learn by examples. We observe how the expert solves specific cases and from this infer some rules by which the decision may have been made. Unique to this work is the fuzzy set representation of the conditions or attributes upon which the decision maker may base his fuzzy set decision. From our examples, we infer certain and possible rules containing fuzzy terms.

It should be stressed that the procedure does not determine the quality of the decision, but how closely the expert follows the conditions under consideration in making his decision. We offer two examples pertaining to the possible decision to close a customer service centers by a public utility company. In the first example, the decision maker does not follow too closely the conditions. In the second example, the conditions are much more relevant to the decision of the expert.

1. Introduction

Much effort has recently been devoted to studying the problem of knowledge acquisition under uncertainty. Uncertainty arises in many different situations. It may be caused by the ambiguity in the terms used to describe a specific situation. It may also be caused by skepticism of rules used to describe a course of action or by missing and/or erroneous data. [For a small sample of work done in the area, the reader is referred to (Arciszewski & Ziarko 1986), (Bobrow, et.al. 1986), (Wiederhold, et. al. 1986), (Yager 1984), and (Zadeh 1983).]

To deal with uncertainty, techniques other than classical logic need to be developed. Although, statistics may be the best tool available for handling likelihood, it often requires probabilities to be estimated; sometimes without even the recourse to relative frequencies. Estimates are then typically very inaccurate. [We refer the reader to Mamdani, et. al. (1985) for a study of the limitations of traditional statistical methods.]

Recognizing the limitations of statistics in dealing with uncertainty, the Dempster-Shafer theory of evidence which gives useful measures for the evaluation of subjective certainty has gained in popularity. [For a sample of works using the Dempster-Shafer theory see (Shafer 1976), (de Korvin, et. al. 1990), (Kleyle & de Korvin 1989), (Strat 1990), and (Yager).] Fuzzy set theory is another tool used to deal with uncertainty where ambiguous terms are present. [Articles in (Zadeh 1979, 1981 & 1983) illustrate the numerous works carried out in fuzzy sets.] Other methods include rough sets, the theory of endorsements and nonmonotonic logic. [The work on rough sets is illustrated in (Fibak, et. al. 1986), (Grzymala-Busse 1988), and (Mrozek 1985 & 1987). Also, see (Mrozek 1985) and (Pawlak 1982) for the application of rough sets to medicine and (Arciszewski & Ziarko 1986) and (Pawlak 1981) for applications to industry.] Our work builds on these alternatives to statistics, allowing us to infer knowledge from the uncertainty associated with ambiguous (i.e. fuzzy) terms.

2. Development of the Model

A traditional way to acquire knowledge is based on learning from examples. An effective tool to infer knowledge from examples is rough sets. In Grzymala-Busse's work (1988), the values of attributes are crisp values as in the diagnosis of a particular condition. Possible and certain rules are extracted and a measure of how much the values of attributes determine the diagnosis is established. However, in many situations, the values of the attributes fail to be crisp. The typical cases presented are not "textbook cases" and the values of attributes require some judgment for their determination. The same difficulties reside in the diagnosis.

The diagnosis is often not of "pure type". It is a mixture of several "pure types". Thus, a patient might have a diagnosis of the type $.3/D_A + .6/D_B$ meaning that the physician believes the (fuzzy) symptoms reflect disease D_A with strength .3 and disease D_B with strength .6.

The main purpose of the present work is to study the general situation described above where the decision maker is faced with uncertain (i.e. fuzzy) conditions and makes a fuzzy decision which might be strongly or weakly based on these conditions. In this situation, fuzzy rules will be extracted. Fuzzy rules are naturally present in descriptions, crisp rules are the exceptions. Also, fewer fuzzy rules are needed than crisp ones to build an expert system.

In the first part of this work, we develop a methodology to extract such rules from fuzzy conditions and fuzzy decisions. In fact, we will extract two sets of rules; certain and possible rules as well as a measure of how much we believe these rules. A related problem is to define the decision in terms of the conditions. We give the basic notations and results necessary to understand the rest of the paper. [Most of these concepts are discussed in (Grzymala-Busse 1988), and (Pawlak 1981, 1982 & 1985) as they relate to crisp sets.]

Basic Notations and Concepts

Let U be the universe. Let R be an equivalence relation on U. Let X be any subset of U. If [x] denotes the equivalence class of x relative to R, then we define

$\underline{R}(X) = \{x \in U/[x] \subset X\}$ and

$\overline{R}(X) = \{x \in U/[x] \cap X \neq \emptyset\}$.

$\underline{R}(X)$ is called the lower approximation of X and $\overline{R}(X)$ is called an upper approximation of X. Then $\underline{R}(X) \subset X \subset \overline{R}(X)$. If $\underline{R}(X) = X = \overline{R}(X)$, then X is called definable.

An information system is a quadruple (U,Q,V,τ) where U is the universe and Q is a subset of $C \cup D$ where $C \cap D = \emptyset$. The set C is called the set of conditions; D is called the set of decisions. We assume here that Q = C. The set V stands for value and τ is a function from UxQ into V where $\tau(u,q)$ denotes the value of attribute q for element u. The set C induces naturally an equivalence on U by partitioning U into sets over which all attributes are constant. The set X is called roughly C-definable if

$\underline{R}(X) \neq \emptyset$ and $\overline{R}(X) \neq U$.

It will be called internally C-undefinable if

$\underline{R}(X) = \emptyset$ and $\overline{R}(X) \neq U$.

It will be called externally C-undefinable if

$\underline{R}(X) \neq \emptyset$ and $\overline{R}(X) = U$.

Unfortunately, uncertainty is all too often present in the conditions and the decisions. The conditions and the decisions fail to partition the universe into well defined classes and some overlap is present. For example, there are no sharp boundaries between conditions defined to represent large and those defined to represent small objects. The best we can hope is that condition definitions of large and small "sort of partition the universe" by not overlapping "too much". In the next section we will deal with this issue of transferring rough set theory to fuzzy sets.

As background for this transformation, we recall that a fuzzy subset A of U is defined by a characteristic function $\mu_A: U \to [0,1]$. The notation $\Sigma\, \alpha_i/x_i$ $(0 \le \alpha_i \le 1)$ denotes a fuzzy subset whose characteristic function at x_i is α_i. Finally, we recall that if A and B are fuzzy subsets, $A \cap B$, $A \cup B$, and $\neg A$ are defined by $\mathrm{Min}\,\{\mu_A(x), \mu_B(x)\}$, $\mathrm{Max}\,\{\mu_A(x), \mu_B(x)\}$, and $1 - \mu_A(x)$, respectively. The implication $A \to B$ is defined by $\neg A \cup B$. The corresponding characteristic function is $\mathrm{Max}\,\{1 - A(x),\ B(x)\}$.

Rough Set Notation Applied to Fuzzy Sets

We now define two functions of pairs of fuzzy sets that will be used to determine rules for closing a utility company's customer service centers (CSCs). We define.

$$I(A \subset B) = \inf_x \mathrm{Max}\,\{1 - A(x),\ B(x)\} \tag{1}$$

$$J(A \# B) = \mathrm{Max}_x\, \mathrm{Min}\,\{A(x),\ B(x)\}. \tag{2}$$

Here A and B denote fuzzy subsets of the same universe. The function $I(A \subset B)$ measures the degree to which A is included in B and $J(A \# B)$ measures the degree to which A intersects B. Indeed, if A and B are crisp sets it is easy to establish that $I(A \subset B) = 1$ if and only if $A \subset B$; otherwise it is zero. Also, in the case of crisp sets $J(A \# B) = 1$ if and only if $A \cap B \ne \emptyset$; otherwise it is zero. It is also clear that I and

J can be expressed as

$$I(A \subset B) = \inf_x (A \rightarrow B) \tag{3}$$

$$J(A \# B) = \operatorname{Max}_x (A \cap B). \tag{4}$$

In addition, the following relation holds:

$$I(A \subset B) = 1 - J(A \# \neg B). \tag{5}$$

Indeed, the right-hand side of (5) is

$$\inf_x (1 - \operatorname{Min}(A(x), 1 - B(x)) =$$

$$\inf_x \operatorname{Max}(1 - A(x), 1 - (1 - B(x))) =$$

$$\inf_x \operatorname{Max}(1 - A(x), B(x)).$$

This last expression is the left-hand side of (5).

The goal is to define the fuzzy terms involved in the decision as a function of the terms used in the conditions. This is accomplished as a function of how much the decision follows the conditions. Let $\{B_i\}$ be a finite family of fuzzy sets. Let A be a fuzzy set. By a lower approximation of A through $\{B_i\}$, we mean the fuzzy set

$$\underline{R}(A) = \bigcup_i I(B_i \subset A) B_i \tag{6}$$

The decision making process may be simplified by disregarding all sets B_i if $I(B_i \subset A)$ is less than some threshold α. Then,

$$\underline{R}(A)_\alpha = \bigcup_i I(B_i \subset A) B_i \tag{7}$$

over all B_i for which $I(B_i \subset A) \geq \alpha$.

Similarly, we can define the upper approximation of A through $\{B_i\}$ as

$$\overline{R}(A)_\alpha = \bigcup_i J(B_i \# A) B_i \tag{8}$$

over all B_i for which $J(B_i \# A) \geq \alpha$.

The operators I and J will yield two possible sets of rules: the certain rules and the possible rules. The data given for the Customer Service Centers (CSCs) will be converted to fuzzy diagnosis of the attributes and we will be able to extract fuzzy rules from the raw data. Each rule for the decision to close a CSC will have some measure of belief associated with it. The primary objective is to see to what degree a combination of attributes is a subset of the decision (certain rules) or intersects the decision (possible rules) to close a customer service center. In addition, fuzzy terms involved in the decision have a lower and an upper approximation so that we have a measure of the minimum degree to which the lower approximation implies the decision and the minimum degree to which that decision satisfies the upper approximation. The specific computations are in the Application section.

It is important to realize that the present methodology does not give any indication of the quality of the decision we have. What is determined is how closely the decision maker seems to depend on the values of the selected set of attributes. If the decisions seem to follow consistently these values and if we trust the decision maker, we then have acquired knowledge, in terms of these attributes, as to how decisions are made.

3. Application

Houston Lighting & Power Company is the largest investor-owned electric utility in the Southwest. HL&P is responsible for generating and distributing electricity throughout twelve counties surrounding Houston. Even though it is a private company, its operations are regulated in Texas by the Public Utility Commission (PUC).

In November 1988, HL&P filed a request with the Public Utility Commission for a $432 million rate increase. The public's perception of HL&P's stability and sound judgment in the daily management of its operations was critical to the outcome of the rate case. HL&P needed to show that its decisions and operating procedures were initiated with total consideration given to effectively serving its customers.

The Customer Relations Group within Houston Lighting & Power was of primary interest to HL&P's case preparation since it was responsible for all company activities that primarily involved customer contact. Customer Relations Group employees served as company liaisons to handle diverse customer inquiries and requests in order to establish, monitor and support continuous and reliable electric service.

The District Operations Division of Customer Relations considered a plan for closing one customer service center. A Customer Service Center (CSC) handled walk-in customer traffic for payment of bills and general customer inquiries and was the service portion of a district office. The company felt that in order to reduce expenses in the event that the rate request before the PUC was denied one or more CSCs would have to be closed. These customer service centers were operated for the public's convenience and were not considered necessary for the company's operation. Still, with the rate increase request before the PUC, HL&P had to carefully analyze the CSC closing decision. The main consideration for HL&P was the public's reaction. Although a decision to close a site would potentially impact only a few customers, there might be those who challenged the PUC rate hike request on the grounds of paying more for less service.

HL&P investigated all relevant factors in making its decision. The difference in relative operating expenses of CSCs was negligible according to the company's operating and maintenance budget. Therefore, operating cost could not be regarded as a major consideration in the elimination of one of the CSCs. Four factors could be considered in this decision: the total number of customers in a district, the increase or decrease in a district's population, the number of customers utilizing the CSC in relation to the district's population, and the distance that customers would have to travel to an alternate CSC in the event their local CSC was closed. This data is given in Table 1.

TABLE 1: Customer Service Center Data

	Customers in District (Avg.)	% Change in Customers	Usage/ Population	Rerouting Distance (Miles)
Centers				
Bayshore	38,510	5.1	4.64	15
Baytown	36,360	-1.4	21.5	15
Brazoria	20,689	3.4	14.07	20
Brazosport	21,976	.4	8.51	20
Cypress	44,074	8.3	1.87	17
Fort Bend	39,145	5.3	15.5	18
Galveston	31,263	- .1	36.44	20
Humble	55,911	1.0	12.44	15
Katy/Sealy	26,760	2.4	18.54	17
Wharton	8,707	- .74	39.43	18

NOTE: All of the above is based on 1985-1987 data.

Assuming that the total operating revenue generated by each CSC would be the overriding decision factor for closing a center, these authors attempted to determine how much a decision made essentially by looking at the total operating revenue would conform to the values associated with the four attributes: Customers in District, Percent Change in Customers, Usage/Population, and Rerouting Distance. Based upon the data, one of the authors served as a decision maker in specifying a value indicative of a high number of customers in the district and a low number in the district; a great and a small percent change in usage; a high and a low percentage of customers utilizing the center; and a large and small rerouting distance. A high number of customers was 60,000 and a low number of customers was 5000. A great percent change was

± 9.00 and a small percent change was ± 0.1. A high usage population ratio was 40.00 percent and a low usage was 1.00 percent. A large rerouting distance was 20 miles and a small distance was 10 miles. The degree to which each site satisfied the definition of high, low; great, small; high, low; and large, small is given by the ratios of the actual data and the defined values. (See Table 2)

TABLE 2: Values for Fuzzy Sets of Conditions

	Customers in District		% Change in Customers		Usage/ Population		Rerouting Distance	
	HIGH	LOW	GREAT	SMALL	HIGH	LOW	LARGE	SMALL
Centers								
Bayshore	.640	.130	.567	.020	.116	.216	.75	.667
Baytown	.606	.138	.156	.071	.538	.047	.75	.667
Brazoria	.345	.242	.378	.029	.352	.071	1.00	.500
Brazosport	.366	.228	.044	.250	.213	.118	1.00	.500
Cypress	.735	.113	.922	.012	.047	.535	.85	.588
Fort Bend	.652	.128	.589	.019	.388	.065	.90	.556
Galveston	.521	.160	.011	1.000	.911	.027	1.00	.500
Humble	.932	.089	.111	.100	.311	.080	.75	.667
Katy/Sealy	.446	.187	.267	.042	.464	.054	.85	.588
Wharton	.145	.574	.082	.135	.986	.025	.90	.556

The total operating revenue generated for each service center was to be used to determine whether or not a center should be closed. The decision maker determined that if revenue was less that 1% of the total generated from all centers, the CSC would be closed. Conversely, the center would not be closed if revenue exceeded 10% of the total. The raw data and the reflective valuation of each center for closing and not closing are given in Table 3.

TABLE 3: Revenue of each CSC & Closing Weight

	Total Revenue (Dollars)	Close	Do Not Close
Centers			
Bayshore	270,411,636	.039	1.000
Baytown	142,262,298	.075	1.000
Brazoria	44,464,243	.239	.419
Brazosport	144,290,786	.074	1.000
Cypress	92,178,304	.115	.869
Fort Bend	88,498,221	.120	.834
Galveston	89,125,871	.119	.840
Humble	120,219,083	.088	1.000
Katy/Sealy	53,675,510	.198	.506
Wharton	15,660,308	.677	.148
	1,060,786,260		

Of course, no one at HL&P would specifically state exactly how the decision to close a CSC would be determined. Since most businesses define profitability in terms of revenue generated and since HL&P representatives had obtained this information, we have assumed that the total operating revenue would be the major factor affecting the decision to close a CSC. In reality, many factors, some of them even unknown to the decision maker himself, may go into the decision of closing a Customer Service Center. However, we are interested in learning by examples how much the decision can be attributed to the attributes for which HL&P had accumulated data for each CSC.

Example 1

In the first example we selected two attributes: Usage/Population and Rerouting Distance.

First, we let x_i denote the customer service centers, such that x_1 = Bayshore, x_2 = Baytown,..., x_{10} = Wharton. Then D_A = Close the CSC, and D_B = Do Not Close the CSC. The decision to close the facility can be evaluated as:

$$D_A = .039/X_1 + .075/x_2 + .239/x_3 + .074/X_4 + .115/x_5 + .120/x_6 + .119/x_7 + .088/x_8 + .198/x_9 + .677/x_{10}$$

This indicates that based upon revenue generated, Wharton is a fairly good example of a CSC to be closed, while Bayshore is not a good example of D_A.

Likewise, we can indicate the degree of membership of each CSC for each fuzzy-defined condition/attribute; High (H) Usage/Population, Low (L) Usage/Population, Large (G) Rerouting Distance, and Small (S) Rerouting Distance. Thus, we define the following fuzzy sets:

$$H = .116/x_1 + .538/x_2 + .352/x_3 + .213/x_4 + .047/x_5 + .388/x_6 + .911/x_7 + .311/x_8 + .464/x_9 + .986/x_{10}$$

$$L = .216/x_1 + .047/x_2 + .071/x_3 +.118/x_4 + .535/x_5 +.065/x_6 + .027/x_7 + .080/x_8 + .054/x_9 + .025/x_{10}$$

$$G = .75/x_1 + .75/x_2 + 1.00/x_3 + 1.00/x_4 + .85/x_5 + .90/x_6 + 1.00/x_7 + .75/x_8 + .85/x_9 + .90/x_{10}$$

$$S = .667/x_1 + .667/x_2 + .50/x_3 + .50/x_4 + .558/x_5 + .556/x_6 + .50/x_7 + .667/x_8 + .588/x_9 + .556/x_{10}$$

We compute the minimum degree to which possible combinations of conditions/attributes are related to decision D_A. Thus,

$I (H \subset D_A) = .119$ $\quad$ $I (H \cap G \subset D_A) = .119$

$I (L \subset D_A) = .465$ $\quad$ $I (H \cap S \subset D_A) = .462$

$I (G \subset D_A) = .074$ $\quad$ $I (L \cap G \subset D_A) = .465$

$I (S \subset D_A) = .333$ $\quad$ $I (L \cap S \subset D_A) = .465$

With a threshold of $\alpha = 0.40$, the rules for closing a CSC are:

1. If usage/population percentage is low (i.e. 1% or less of the customers in the district utilizing the CSC), then the CSC should be closed. (D_A is present .465 or Belief = .465)
2. If the usage/population percent is high (approximately 40% of the customers in the district utilize the CSC) and the rerouting distance is small (approximately 10 miles), then the CSC should be closed. (Belief = .462)
3. If the usage/population percent is low and the rerouting distance is high (20 miles), then the CSC should be closed. (Belief = .465)
4. If the usage/population is low and the rerouting distance is low, the CSC should be closed. (Belief = .465)

Since no new information is provided by rules 3 and 4, the extracted rules for closing are:

1. If usage/population percentage is low then the CSC should be closed. [The belief is .465.]
2. If usage/population is high and the rerouting distance is small then the CSC should be closed. [The belief is .462.]

Rule 1 is certainly reasonable. Rule 2 sounds less reasonable.

It is generated by the decision maker deciding fairly strongly in favor of Wharton to be closed, although its usage/population was definitely high and its rerouting distance was over .5 small. From such examples, we learn that for high usage and relatively low rerouting distance a CSC can be closed. Note that from the data, we do not feel that strongly about these rules. The extracted rules would not be sufficient to infer closing from past experience.

We now measure the degree to which the fuzzy sets intersect D_A as:

$$J\,(\,H \# D_A\,) = .677 \qquad J\,(\,H \cap G \# D_A\,) = .677$$
$$J\,(\,L \# D_A\,) = .115 \qquad J\,(\,H \cap S \# D_A\,) = .556$$
$$J\,(\,G \# D_A\,) = .677 \qquad J\,(\,L \cap G \# D_A\,) = .115$$
$$J\,(\,S \# D_A\,) = .556 \qquad J\,(\,L \cap S \# D_A\,) = .115$$

With $\alpha = 0.60$, the acceptable rules are:

5. If usage/population percent is high, then closing is possible .667.
6. If rerouting distance is great, then closing is possible .677.
7. If usage/population is high and rerouting distance is great, then closing is possible .677.

The extracted rule would be Rule 7. The possibility of closing if usage/population is high and rerouting distance is great can't be discounted. Brazoria was recommended to be closed with strength .239 versus not closing with strength .419. Nevertheless, the rerouting distance was definitely high and the usage/population was rated .352 high versus .071 low.

We determine the lower approximation of D_A, using α =.40, as:

$$\underline{R}\ (D_A) = .465\ L \cup .465\ (L \cap G) \cup .465\ (L \cap S) \cup .462\ (H \cap S)$$
$$= .465\ L \cup .462\ (H \cap S)$$

Note that this result shows Rule 3 and Rule 4 to be superfluous to Rule 1 and unnecessary for the calculation of $\underline{R}\ (D_A)$.

We can also show that Rules 5 & 6 should not be accepted since the upper approximation of D_A for α = .60, results in Rule 7:

$$\overline{R}\ (D_A) = .677\ H \cup .677\ G \cup .677\ (H \cap G)$$
$$= .677\ H \cup .677\ G$$

It can be observed that $\underline{R}\ (D_A) \not\subset \overline{R}\ (D_A)$. Therefore, $\underline{R}\ (D_A)$ and $\overline{R}\ (D_A)$ would be the "lower and upper approximations" to the set of closed CSCs. Note that $\underline{R}\ (D_A)$ and $\overline{R}\ (D_A)$ are both expressed in terms of attributes but D_A is not. We can compute $I\ [\underline{R}\ (D_A) \subset D_A] = .751$ and $I\ [D_A \subset \overline{R}\ (D_A)] = .667$ to show that a relatively strong containment exists for both the lower and upper approximation of the decision to close a Customer Service Center.

Although, Rule 1 appears to be the most logical rule to accept, it eliminates Wharton as the primary candidate for closing. It should be noted that Wharton's valuative scores based on high customer utilization (.986) and relatively large as well as relatively small rerouting values (.90 and .556, respectively) are influencing the second and third decision rules. This example is an excellent illustration of the necessity for the attributes to properly reflect the decision criteria. In this example, the decision to close a center was to be based solely on revenue generated. This means that HL&P would select a center which generated the lowest revenue as that to be closed and the one which generated the highest revenue becomes that least likely to be closed. This suggests that Wharton is our best site to close. However, the usage/population percentage at Wharton is high leading one to the conclusion that, in general, those centers with high customer usage should be closed.

Example 2

A second example is given to show that a closer relationship between the decision and the attributes selected will lead to seemingly more logical rules being determined. For this illustration, we used the size of the customer base with the percent usage which suggests that although the percent usage may be high, there may be many fewer customers at the center generating much less revenue and thus being candidates for closing.

Using the values of the fuzzy sets High (NH) and Low (NL) for the number of customers, and High (UH) and Low (UL) for the usage/population percentages given in Table 2:

$I (NH \subset D_A) = .088$ $\quad$ $I (NH \cap UH \subset D_A) = .463$

$I (NL \subset D_A) = .677$ $\quad$ $I (NH \cap UL \subset D_A) = .465$

$I (UH \subset D_A) = .119$ $\quad$ $I (NL \cap UH \subset D_A) = .677$

$I (UL \subset D_A) = .465$ $\quad$ $I (NL \cap UL \subset D_A) = .87$

With $\alpha = .60$, the following rules would be determined:

1. If the number of customers is low, the belief that the CSC should be closed is .677.
2. If the number of customers is low and the usage/population is low, the CSC should be closed .87.
3. If the number of customers is low and the usage/population is high, the CSC should be closed .677.

Rule 3 is redundant and we would keep Rules 1 and 2.

Also using $\alpha = .60$, we can determine the following rules from:

$J (NH \# D_A) = .239$ $\quad$ $J (NH \cap UH \# D_A) = .239$

$J (NL \# D_A) = .574$ $\quad$ $J (NH \cap UL \# D_A) = .115$

$J (UH \# D_A) = .677$ $\quad$ $J (NL \cap UH \# D_A) = .574$

$J (UL \# D_A) = .116$ $\quad$ $J (NL \cap UL \# D_A) = .113$

4. If the number of customers in the district is low, closing is possible .574.
5. If the usage/population is high, closing is possible .677.
6. If the number of customers in the district is low and the usage/population is high, closing is possible .574.

From these rules, we select Rule 5.

Computing the upper and lower approximations based on

α = .60, we have:

$\underline{R}\ (D_A) = .677\ NL \cup .87\ (NL \cap UL) \cup .677\ (NL \cap UH)$ and

$\overline{R}\ (D_A) = .677\ UH$ such that:

$I\ [\ \underline{R}\ (D_A) \subset D_A\ ] = .677$ and $I\ [\ D_A \subset \overline{R}\ (D_A)\ ] = .667$

Again, these values indicate a relatively strong containment of the lower and upper approximation of the decision to close a CSC.

Thus, the acceptable rules where Rule 1 and Rule 2 come from certainty and Rule 3 come from possibility are:

1. If the number of customers is low and usage/population is low, the CSC should be closed. [Belief is .87.]
2. If the number of customers is low, the CSC should be closed. [Belief is .677.]
3. If the usage/population is high, the CSC can be closed. [Plausibility is .677.]

If strictly ordering the CSCs to be closed based upon Rule 2, Wharton would be the decision maker's first choice for closing (followed by Brazoria and Brazosport). Although Rule 3 appears to be illogical, if strictly ordering a center to be closed based upon this rule, Wharton would be selected (followed by Galveston and Baytown). If using the more logical Rule 1, Wharton would not be considered first. Brazoria, ranking second in having the lowest number of customers and fifth in having a low usage/population ratio would be one possible choice for a CSC to be closed. Brazosport with the third lowest number of customers and the third lowest usage/population ratio would also be a good choice for closure. Notice that these were the second choices if strictly ordering by Rule 2, based upon the number of customers in the district. Since the number of customers in the district would directly relate to the revenue generating power of a CSC, this example provides a more realistic result and supports the need to have well chosen attributes, reflecting the decisions made.

4. Contribution

One of the advantages of this process is that the decision maker does not have to specify arbitrarily determined or pairwise-comparison determined relative importance weights for each attribute or condition, or subjectively evaluate each alternative according to these attributes as would be necessary using a weighted scoring approach. Our process allows the user to learn and determine rules based on the examples available. Of course, the quality of the learning depends on how relevant are the chosen attributes to the decision to be made.

The process allows rules to be determined through incorporation of raw data for each condition or attribute over all available alternatives for which a decision must be made. The decision maker assigns values he considers to be high, low, great, small, etc. based on the given data. The DM's values for high, low, etc. are translated into the degree to which each combination of alternatives is a member of the fuzzy set defined by the decision maker. The procedure can also determine a value for medium size. The decision maker can specify a value he considers to be medium and we can calculate the degree of membership of each alternative in the fuzzy set "Medium" by interpolating between the values of the fuzzy sets, Large and Small. For example, if the decision maker specifies 40,000 customers as a Medium amount of customers in the district, we can interpolate between the degree of membership of Bayshore in the fuzzy set High (.640) and in the fuzzy set Low (.130); where High was defined as 60,000 customers and Low was 5000 customers. Thus, the degree of membership of Bayshore in the fuzzy set Medium is .455. Similarly, Relatively High can be defined as 55,000 customers in the district and the degree of membership of Bayshore in the fuzzy set Relatively High is .594. Through such an interpolation process all possible fuzzy sets between High and Low, Large and Small, etc. can be determined.

Ranges of values can be specified as we did for the decision to close a customer service center. In the example, the decision to close a center was primarily based on total operating revenue generated by each CSC. The decision maker specified a dollar amount at or below which the CSC should be closed, and another at or above which the CSC should not be closed. Values at or below some lower bound have zero membership in the fuzzy set, and those at or above the upper bound have total membership (1.00) in the fuzzy set.

Importantly, the process can actually be initiated without the DM's judgment if we use the highest value and the lowest value, the largest and smallest, etc. as those given in the raw data file for the specific conditions the decision maker wants to consider. This can be extended if a value for medium size is defined to be some convex combination of the ratios determined using the largest and smallest data values. It can also be extended to the decision membership set, although some range would have to be defined for the lower and upper bound.

5. Conclusion

The rough sets formulation that forms the basis for determining the decision rules is easily performed through maximization and minimization of combinations of the fuzzy set values. The process is not computationally intensive, although it does become more labor intensive beyond the two attribute with one decision case presented in this paper. The authors hope to have a computer program available in the near future to handle large-scale problems.

The methodology described in previous sections extends to many problems and need not be limited to the problem of closing customer service centers. In our setting, the decision maker is faced with uncertain (i.e. fuzzy) conditions and makes fuzzy decisions which might be strongly or weakly based on the conditions. Fuzzy rules are extracted. such rules are naturally present in descriptions, crisp rules are the exceptions. Also, fewer fuzzy rules are needed than crisp rules to build an expert system. Since the crisp set is a limiting case of the fuzzy setting, expected benefits that arise from our fuzzy set based method are a more realistic and general approach to knowledge acquisition. Acquisition of knowledge through examples, which is particularly of interest when the decision maker is unable to articulate how he arrives at a decision, is a very natural approach to learning.

Again, we stress that the proposed method does not give an answer to: "are the decisions made, good decisions?". It is assumed that the expert is knowledgeable about the conditions under which the decision will be made. Our methodology gives an answer to "how closely does the expert follow the attributes under consideration in making his decision?". If the decisions seem to closely follow the values of the attributes, then strong rules can be acquired through examples and the expert's knowledge can be put into machine representable form.

At this time, HL&P has not made a decision to close either of the customer service centers. Management has relied on reducing the operating costs at each of the centers by moving to the company's downtown Houston location, the CSC employees who generally dealt with telephone contact with district customers. A complete evaluation of the data from Tables 1, 2, and 3 is to be performed and submitted to HL&P as soon as the prototype computer program is completed.

Although relatively small, the two examples presented in this paper are realistic and illustrate the underlying rough set theory. Through the examples, we can see how the process presented will generate a decision making rule based upon a minimal amount of subjective judgment by the decision maker. Indeed, the decision maker has merely to indicate that the maximum, minimum, or a range of actual values are to be used and the process will generate rules relating the attributes to the selected decision criteria. Importantly, however, the two examples show that the DM must choose carefully the attributes upon which he will make the decision.

REFERENCES

Arciszewski, T. and Ziarko, W. 1986. "Adaptive expert system for preliminary engineering design," Proceedings 6th International Workshop on Expert Systems and their Applications. Avignon, France, 1, 696-712 .

Bobrow, D.G., Mittal, S. and Stefik, M.J. 1986. "Expert systems perils and promises" Communications of the ACM, 29, 880-894.

Cheeseman, P. 1986. "Induction of models under uncertainty," Proceedings of ACM SIGART International Symposium on Methodologies for Intelligent Systems, Knoxville, Tennessee, 130-144.

Fibak, J., Slowinski, K. and Slowinski, R. 1986. "The application of rough set theory to the verification of indications for treatment of duodenal ulcers by HSV," Proceedings 6th International Workshop on Expert Systems and their Applications, Avignon, France, 1, 587-594.

Grzymala-Busse, J.W. 1988. "Knowledge acquisition under uncertainty: a rough set approach," Journal of Intelligent and Robotic Systems 1, 3-16.

Kleyle, R. and de Korvin, A. 1989. "A unified model for data acquisition and decision making," The Journal of the American Society for Information Science 15, 149-161.

de Korvin, A., Kleyle, R. and Lea, R. 1990. "An evidential approach to problem solving when a large number of knowledge systems are available," The International

Journal of Intelligent Systems, 5, 293-306.

Mamdani, A., Efstathiou, J. and Pang, D. 1985. "Inference under uncertain expert systems 85," Proceedings Fifth Technical Conference British Computer Society, Specialist Group on Expert Systems, 181-194.

Mrozek, A. "Information systems and control algorithms", 1985. Bulletin Polish Academy of Science, Technical Science, 33, 195-204.

Mrozek, A. 1987. "Rough sets and some aspects of expert systems realization," Proceedings 7th International Workshop on Expert Systems and their Applications, Avignon, France, 597-611.

Pawlak, Z. 1981. "Rough sets. Basic Notions," Institute Computer Science, Polish Academy of Science Report No. 431, Warsaw.

Pawlak, Z. 1981. "Classification of objects by means of attributes," Institute Computer Science Polish Academy Science Report No. 429, Warsaw.

Pawlak, Z. 1982. "Rough sets," International Journal of Information Computer Science ,11, 341-356.

Pawlak, Z. 1983. "Rough classifications," International Journal of Man-Machine Studies, 20, 469-483.

Pawlak, Z. 1985. "Rough sets and fuzzy sets," Fuzzy Sets and Systems, 17, 99-102.

Shafer, G. 1976. Mathematical Theory of Evidence, Princeton University Press.

Strat, T.M. 1990. "Decision analysis using belief functions," International Journal of Approximate Reasoning, 4, 391-417.

Wiederhold, G.C., Walker, M., Blum, R., and Downs, S. 1986. "Acquisition of knowledge from data," Proceedings ACM SIGART International Symposium on Methodologies for Intelligent Systems, Knoxville, Tennessee, 78-84.

Yager, R.R. 1984. "Approximate reasoning as a basis for rule based expert systems," IEEE Transactions on Systems, Man and Cybernetics, 14, 636-643.

Yager, R.R. "Decision making under Dempster-Shafer uncertainties," Iona College, Machine Intelligence Institute Tech. Report MII-915.

Zadeh, L.A. 1983. "The rule of fuzzy logic in the management of uncertainty in expert systems," Fuzzy Sets and Systems 11, 119-227.

Zadeh, L.A. 1979. "Fuzzy sets and information granularity," Advances in Fuzzy Set Theory and Applications, 3-18.

Zadeh, L.A. 1981. "Possibility theory and soft data analysis," Mathematical Frontiers of the Social and Policy Sciences. Eds. L. Cobb and R.M. Thrall, 69-129. Westview Press, Boulder, Colorado.