NASA Contractor Report 189666

ICASE Report No. 92-25

ICASE

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Space Administration

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Abstract. Previously, the stability of a premixed flame in a stagnation flow was discussed for a restricted class of disturbances that are self-similar to the basic undisturbed flow; thus, flame fronts with corrugations only in the cross stream direction were considered. Here, we consider a more general class of three-dimensional flame front perturbations which also permits corrugations in the streamwise direction. It is shown that, because of the stretch experienced by the flame, the hydrodynamic instability is limited only to disturbances of short wavelength. If in addition diffusion effects have a stabilizing influence, as would be the case for mixtures with Lewis number greater than one, a stretched flame could be absolutely stable. Instabilities occur when the Lewis number is below some critical value less than one. Neutral stability boundaries are presented in terms of the Lewis number, the strain rate and the appropriate wavenumbers. Beyond the stability threshold the two-dimensional self-similar modes always grow first. However, if disturbances of long wavelength are excluded, it is possible for the three-dimensional modes to be the least stable one. Accordingly, the pattern that will be observed on the flame front, at the onset of instability, will consist of either ridges in the direction of stretch or the more common three-dimensional cellular structure.

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INTRODUCTION

The response of a premixed flame to stretch is a problem of fundamental importance in combustion theory. Results of studies on the structure, propagation and extinction of stretched flames have been used in experimental and theoretical investigations (Wu and Law 1984; Law et al. 1986; Tien and Matalon 1990; Dixon-Lewis 1991) aimed at understanding the structure and burning characteristics of laminar flames. Results of these studies have been also applied to turbulent combustion, in particular, in the "reaction sheet" regime (Williams, 1985; Peters, 1986) where the turbulent flame is regarded as an ensemble of stretched laminar flamelets. Thus, considerable attention has been given to stretched flames in the literature where a large number of theoretical and experimental papers can be found. In spite of these numerous investigations the effect of stretch on flame front instabilities has not been completely clarified.

Experimental studies (Ishizuka and Law, 1982; Ishizuka, 1988) on the stability limits of premixed flames clearly indicate the importance of flame stretch. Observations were made on propane/air premixed flames in an axisymmetric stagnation point flow. The flow field in such a configuration imposes on the bulk of the flame a well defined strain rate, or stretch, controlled by the mass flow rate of the incoming fresh mixture. Depending on the strain rate, various patterns appear on the flame front for rich mixtures. For a weak strain rate a three dimensional cellular flame evolves, as may be expected; cellular flames are commonly observed in experiments performed in large tubes (Clavin, 1985), namely in the absence of stretch. Upon increasing the strain rate, the flame approaches the stagnation point and a different pattern, characterized by ridges along the radial direction, emerges on the surface of the flame front. For sufficiently large values of

the strain rate, all instabilities are suppressed resulting with a smooth flat flame surface. In contrast, for lean mixtures the observations indicate an absolutely stable flame front for equivalent values of the strain rate. These experiments clearly point out the role of flame stretch, combined with hydrodynamical and diffusional-thermal effects, in suppressing/promoting instabilities.

A complete theoretical study on the effect of stretch on flame stability has not yet been given. Although there is no difficulty in formulating the linearized disturbance equations for a flame in a stagnation-point flow, the resulting problem consists of a coupled system of partial differential equations with variable coefficients. These coefficients, which depend on the unperturbed base flow, can be written explicitly only for a hydrodynamic model, where the flame is treated as a surface of discontinuity (Eteng et al., 1986). Furthermore, since the basic state is a non-parallel flow, with velocities varying normal and tangential to the flame front, the disturbance equations are not separable. Thus, the linear stability problem cannot be resolved using normal modes in both directions that are parallel to the flame front. An exception is a class of disturbances self-similar to the basic state which has been considered by Kim and Matalon (1990). In this case, perturbations have periodic waviness in the cross-stream direction only with no corrugations in the direction of flow divergence. Self similar disturbances were first considered by Görtler (1955) and more recently by Wilson and Gladwell (1978), in analyzing the stability of the incompressible viscous stagnation-point flow without chemical reaction. More general disturbances were later admitted by Spalart (1989) and Brattkus and Davis (1990) in dealing with the fluid dynamical problem. Spalart solved the initial value problem numerically and found that the long term behavior of the solution approaches the spatial structure of the self similar mode. Brattkus

and Davis expressed the disturbances in terms of an infinite series of coupled modes and concluded from their analysis that the least stable mode is that corresponding to a self-similar disturbance. Although these studies provide some justification for the use of self-similar disturbances, perturbations with periodic waviness in the streamwise direction are not of negligible significance, especially for the combustion problem. These modes are undoubtedly important in the nonlinear development of flame disturbances and must be included in any description of the three dimensional cellular pattern which has been observed in experiments.

In addition to the analysis of Kim and Matalon, the only other reported study on the stability of a flame in a stagnation-point flow is that of Sivashinsky et al. (1982). Their analysis, however, is restricted to a very weak stretch rate. Furthermore, they suppressed the hydrodynamic instability by neglecting the effect of thermal expansion, thus deriving results that pertain to the effect of stretch on the diffusional-thermal instability only.

In the present work, we analyze the stability of a premixed flame in a plane stagnation-point and consider a more general class of three-dimensional disturbances which, in particular, allow for flame front corrugations in the streamwise direction. As is well known, the hydrodynamic instability (Landau, 1944; Darrieus, 1945) does not permit the existence of premixed flames that are too flat. Although diffusion often introduces stabilizing influences, as is the case for mixtures with Lewis number greater then one, long wavelength disturbances are hydrodynamically unstable. Our present aim is to determine whether a sufficiently large stretch can stabilize these long wavelength disturbances. We have also incorporated thermo-diffusive effects in our model in order to determine the extent that a flame needs to be stretched to remain stable. For two-dimensional self-similar disturbances, Kim and Matalon (1990) found that stretch indeed stabilizes the flame. In particular, they provided

the modification resulting from stretch, to the neutral stability curve drawn previously for freely propagating flames (Matalon and Matkowsky, 1984; Jackson and Kapila, 1984), thus identifying the critical conditions at the stability threshold. Here, we shall not only extend their results, but also identify the effect that such disturbances have on the pattern that evolves on the flame front and in the induced flow field when an instability develops. It is shown that beyond the stability threshold, the two-dimensional self-similar modes always grow first. However, if disturbances of long wavelength are excluded, it is possible for the three-dimensional modes to be the least stable one. Accordingly, the pattern that will be observed on the flame front at the onset of the instability, will consist of either ridges in the direction of stretch, or the more common three-dimensional cellular structure. These results appear in agreement with the experimental observations summarized above, though a direct comparison requires analyzing the axisymmetric configuration. Having established a procedure for testing self-similar base flows for stability against general disturbances, we shall discuss the stability of a premixed flame in an axisymmetric stagnation-point flow in a future publication.

FORMULATION

The mathematical formulation is identical to that of Kim and Matalon (1990). A combustible mixture impinges toward a flat surface held at $x = 0$ (see Fig. 1). Far upstream, the flow field is the classical stagnation-point flow characterized by the strain rate e. The flame, which separates the burned products from the fresh unburned mixture, is considered thin and is therefore represented by the surface $\Psi(x,y,z,t) = 0$, where $\Psi > 0$ is the burned gas region. The flame structure resolved on the scale of the diffusion length $L_D = D_T/S_f$, where D_T is the mixture thermal diffusivity and S_f the laminar flame speed, yields jump conditions for mass and momentum across the flame, as well as an equation describing the evolution of the flame front (Matalon and Matkowsky, 1983). Since viscosity has only a secondary effect on flame stability, it will be neglected everywhere. Consequently, the flow field on either side of the flame is described to all orders by the inviscid, incompressible Euler equations but with distinct densities in the unburned and burned gas. The jump conditions and the flame speed equation are only correct to $O(\delta)$, where $\delta = L_D/L \ll 1$ is the ratio of the diffusion length to a hydrodynamic length L.

In writing the governing equations the length L is used as a unit of distance, the laminar flame speed S_f as a unit of speed, L/S_f as a unit of time and the state of the fresh mixture ρ_0, p_0, T_0 is used to nondimensionalize the density ρ, pressure p and temperature T, respectively. These equations are:

for $\Psi(x,y,z,t) \gtrless 0$

$$\nabla \cdot \vec{v} = 0 \qquad\qquad (1)$$

$$\rho \frac{D\vec{v}}{Dt} = - \nabla p \quad ; \qquad\qquad (2)$$

across $\Psi(x,y,z,t) = 0$

$$[\rho(\vec{v}\cdot\vec{n}) - V_f] = \delta(\ell n \; \sigma) \; K \qquad\qquad (3)$$

$$[\vec{v} \times \vec{n}] = - \delta\beta\left\{[\nabla\times\vec{v}] + 2(\sigma-1) \; \nabla\times\vec{n}\right\} \qquad\qquad (4)$$

$$[p + \rho(\vec{v}\cdot\vec{n})(\vec{v}\cdot\vec{n} - V_f)] = \delta\left\{\beta[\frac{\partial p}{\partial n}] - (\sigma-1)(1+\beta) \; \nabla\cdot\vec{n} + V_f(\ell n \; \sigma) \; K\right\} \qquad (5)$$

where $[\cdot]$ denotes the jump in the quantity, i.e. $(\cdot)_{burned} - (\cdot)_{unburned}$;

at $\Psi(x,y,z,t) = 0^-$

$$\vec{v}\cdot\vec{n} - V_f = 1 - \delta(\beta+\gamma\ell/2) \; K \quad . \qquad\qquad (6)$$

In the above D/Dt is the convective derivative and the variable p stands for the small pressure deviations from the ambient pressure level (of the order of a squared Mach number). The density is given by

$$\rho = \begin{cases} 1 & \Psi < 0 \\ 1/\sigma & \Psi > 0 \end{cases} \qquad\qquad (7)$$

where σ is the thermal expansion parameter ($\sigma > 1$). The velocity of the flame surface V_f and the unit normal vector $\vec{n}$ are

$$V_f = - \frac{1}{|\nabla\Psi|} \frac{\partial\Psi}{\partial t} \quad , \qquad \vec{n} = \frac{\nabla\Psi}{|\nabla\Psi|} \quad . \qquad\qquad (8)$$

The flame stretch is given by

$$K = V_f \; \nabla\cdot\vec{n} - \vec{n}\cdot \; \nabla\times(\vec{v}\times\vec{n}) \qquad\qquad (9)$$

with the right hand side evaluated at $\Psi(x,y,z,t) = 0^-$. The remaining coefficients are

$$\beta = \frac{\sigma \ln \sigma}{\sigma-1}, \quad \gamma = \int_1^\sigma \frac{\ln \xi}{\xi-1} \, d\xi, \quad \ell = (E/RT_0)(Le-1)/\sigma^2$$

where E is the activation energy of the chemical reaction and R the gas constant. Note that β and γ are both positive quantities which depend solely on the thermal expansion parameter, and ℓ is a scaled Lewis number that measures the deviation of Le from one. Finally the boundary conditions are

$$u = 0 \quad \text{at} \quad x = 0 \tag{10}$$

$$\vec{v} \sim (-\epsilon x, \epsilon y, 0) \quad \text{as} \quad x \to -\infty \tag{11}$$

where $\vec{v} = (u,v,w)$ and $\epsilon = (L/S_f)e$ is the dimensionless strain rate.

The basic state corresponds to a steady planar flame front located at $x = -d$. The velocity field is given by (Eteng et al., 1986)

$$\vec{v} = \{ F(x), -yF'(x), 0 \}, \tag{12}$$

where $F(x) = F_0(x) + \delta F_1(x)$ and prime denotes differentiation with respect to x. Here,

$$F_0(x) = \begin{cases} -\epsilon(x + a_0) & -\infty < x < -d \\ -\sigma(\frac{\sigma}{\sigma-1})^{1/2} \sin qx & -d < x < 0 \end{cases}$$

$$F_1(x) = \begin{cases} -\epsilon a_1 & -\infty < x < -d \\ \epsilon\sigma \frac{\gamma\ell}{2}(\frac{\sigma}{\sigma-1})^{1/2}\left[\sin qx - qx \cos qx\right] & -d < x < 0 \end{cases}$$

where $q = \epsilon\sqrt{\sigma-1}/\sigma$. The pressure field takes the form

$$p = \begin{cases} -\frac{1}{2}\left(F_0^2 + \epsilon^2 y^2\right) - \delta\epsilon^2 a_1(x + a_0) & -\infty < x < d \\ -\frac{1}{2\sigma}\left(F_0^2 + \sigma\epsilon^2 y^2\right) - \frac{\delta}{\sigma}F_0 F_1 - \frac{1}{2}(\sigma-1)(1-\delta\epsilon\gamma\ell) & -d < x < 0. \end{cases} \tag{13}$$

The flame standoff distance d and displacement distance a are expressed as $d = d_0 + \delta d_1$ and $a = a_0 + \delta a_1$, where

$$d_0 = \eta/\epsilon \quad , \qquad d_1 = - \beta - \gamma \ell \eta/2$$

$$a_0 = d_0 - 1/\epsilon \quad , \quad a_1 = d_1 + \beta + \gamma \ell/2 \quad .$$

with $\eta = (\sigma \tan^{-1}\sqrt{\sigma-1})/\sqrt{\sigma-1}$. Note that for a very weak stretch ($\epsilon \ll 1$) the flame recedes back to the nearly uniform flow, far upstream. For O(1) values of ϵ the flame stands at a finite distance from the plate, thus justifying the inviscid approximation assumed earlier. Only when $\epsilon \gg 1$ does viscous effects become important because the flame, in this limit, is pushed into the thin viscous boundary layer adjacent to the plate.

DISTURBANCE EQUATIONS

We introduce small disturbances, superimposed to the basic state (12)-(13), of the form

$$\begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = \begin{pmatrix} \hat{u}(x,y) \\ \hat{v}(x,y) \\ \hat{w}(x,y) \\ \hat{p}(x,y) \end{pmatrix} e^{ikz+\omega t} \quad , \tag{14}$$

and express the perturbed flame front as

$$x = -d + \hat{\psi}(y) e^{ikz+\omega t} \tag{15}$$

where k is the wavenumber associated with the spatial periodicity in the z direction and ω is the growth rate. Note, that using (14)-(15), a solution that assumes normal modes in z and t, but not in y, has been sought. Substituting into the governing equations (1)-(2) and then linearizing about the basic state, one obtains

$$\hat{u}_x + \hat{v}_y + ik\hat{w} = 0 \tag{16a}$$

$$F\hat{u}_x - yF'\hat{u}_y + (\omega + F')\hat{u} = -\hat{p}_x/\rho \tag{16b}$$

$$F\hat{v}_x - yF'\hat{v}_y + (\omega - F')\hat{v} - yF''\hat{u} = -\hat{p}_y/\rho \tag{16c}$$

$$F\hat{w}_x - yF'\hat{w}_y + \omega\hat{w} = -ik\hat{p}/\rho \tag{16d}$$

where, here and thereafter, subscripts denote partial differentiation.

In the absence of vorticity far upstream, and as a result of the density being constant, the flow field up to the flame can be determined from a velocity potential $\varphi(x,y,x,t)$ which satisfies $\nabla^2\varphi = 0$. Writing

$$\varphi = \hat{\varphi}(x,y) \, e^{ikz+\omega t} \quad ,$$

Laplace's equation yields

$$\hat{\varphi}_{xx} + \hat{\varphi}_{yy} - k^2\hat{\varphi} = 0 \quad , \tag{17}$$

from which the velocity field can be determined as

$$\hat{u} = \hat{\varphi}_x \quad , \quad \hat{v} = \hat{\varphi}_y \quad , \quad \hat{w} = ik\hat{\varphi} \quad . \tag{18}$$

The pressure is obtained from any one of the components of the momentum equation, giving

$$\hat{p} = yF'\hat{\varphi}_y - F\hat{\varphi}_x - \omega\hat{\varphi}. \tag{19}$$

Thus, disturbances in the unburned gas are determined in terms of a single variable $\hat{\varphi}$. In the burned gas, however, the flow is no longer potential because the vorticity produced at the flame is transported downstream. Consequently the stability problem reduces to solving (17) in the unburned region and the system (16) in the burned region.

Expressing the jump conditions (3)-(5) and the flame speed relation (6) at the mean position $x = -d_0$, then linearizing about the basic state and using the solution expressed in terms of $\hat{\varphi}$ for the unburned region, one obtains the following conditions:

At $x = - d_0$:

$$\hat{u} = \hat{\varphi}_x - \delta(\sigma-1)\left\{\left[\beta+(1+\eta)\frac{\gamma\ell}{2}\right]\left[\hat{\psi}_{yy} - k^2\hat{\psi}\right] + \frac{\gamma\ell}{2\sigma}\epsilon^2\eta(y\hat{\psi})_y + \frac{\gamma\ell}{2}\left[\hat{\varphi}_{yy} - k^2\hat{\varphi}\right]\right\} \tag{20a}$$

$$\hat{v} = \hat{\varphi}_y - \frac{\sigma-1}{\sigma}\left[\sigma\hat{\psi}_y + \epsilon^2 y\hat{\psi}\right] - \delta\frac{\sigma-1}{\sigma}\frac{\gamma\ell}{2}\eta\left\{\frac{\epsilon^3}{\sigma}(1+\sigma/\eta)y\hat{\psi} + [\omega+\epsilon-\epsilon\sigma/\eta]\hat{\psi}_y\right.$$
$$\left. + \epsilon y\hat{\psi}_{yy} + (\omega+\epsilon)\hat{\varphi}_y + \epsilon y\hat{\varphi}_y\right\} \tag{20b}$$

$$\hat{w} = ik\hat{\varphi} - ik(\sigma-1)\hat{\psi} - ik\delta\frac{\sigma-1}{\sigma}\frac{\gamma\ell}{2}\eta\left\{(\omega-\epsilon\sigma/\eta)\hat{\psi} + \epsilon y\hat{\psi}_y + \omega\hat{\varphi} + \epsilon y\hat{\varphi}_y\right\} \tag{20c}$$

$$\hat{p} = -\epsilon y\hat{\varphi}_y - \left[1-\delta\epsilon(1-\eta)\frac{\gamma\ell}{2}\right]\hat{\varphi}_x - \omega\hat{\varphi} + \delta(\sigma-1)\left\{\left[1+\beta+\frac{\gamma\ell}{2}(\eta+2)\right]\left[\hat{\psi}_{yy} - k^2\hat{\psi}\gamma\ell\right]\right.$$
$$\left. + \gamma\ell\left[\hat{\varphi}_{yy} - k^2\hat{\varphi}\right] + \frac{\gamma\ell}{2\sigma}\eta\left[(2\epsilon^2-\omega^2)\hat{\psi} - 2\epsilon\omega\, y\hat{\psi} - \epsilon^2 y^2\hat{\psi}_{yy}\right]\right\} \tag{20d}$$

$$\hat{\varphi}_x = (\omega+\epsilon)\hat{\psi} + \epsilon y\hat{\psi}_y + \delta\left\{-(\beta+\frac{\gamma\ell}{2})\left[\hat{\psi}_{yy} - k^2\hat{\psi}\right] + \frac{\gamma\ell}{2}(\eta-1)\left[\hat{\varphi}_{yy} - k^2\hat{\varphi}\right]\right\} \tag{20e}$$

Far upstream, i.e. as x → -∞, all velocities must vanish and therefore $\hat{\varphi}$ must be bounded. At the plate, only the axial velocity must vanish, i.e. $\hat{u}$ = 0 at x = 0, as appropriate for inviscid flows.

The system (16) for x > -d together with equation (17) for x < -d, the relations (20a-e) at the flame front and the boundary conditions at x = -∞ and at x = 0, constitute an eigenvalue problem for the determination of the growth rate ω. The eigenvalues ω and the associated eigenfunctions depend on the following parameters: the thermal expansion σ, the strain rate ϵ, the (reduced) Lewis number ℓ, the flame thickness δ and the form of the disturbance represented by k, the wavenumber associated with the z-direction, and an appropriate parameter that remains to be determined, identifying the form of the corrugations in the y-direction.

FORM OF THE DISTURBANCES

As already pointed out in the introduction, the eigenvalue problem we have just obtained is not separable so that normal modes in y are not admissible solutions. An exception is the self similar form

$$\{ \hat{u}, \hat{v}, \hat{w}, \hat{p} \} = \{ U(x), yV(x), W(x), P(x) \} \qquad (21)$$

with $\hat{p}$ = const., considered by Kim and Matalon (1990). In this case, all variables are functions of the axial coordinate x, with the exception of the streamwise velocity component which is linearly proportional to y. Clearly, this follows from the fact that the stagnation-point flow is a local solution valid near the axis (y = 0). It is plausible, therefore, to expect that more general disturbances will include higher powers of y as well.

Following Brattkus and Davis (1990), further insight in choosing the allowable disturbances can be obtained by examining the vorticity equation

$$\omega\hat{\Omega} + F \hat{\Omega}_x - yF' \hat{\Omega}_y = -ikyF'' \hat{w} + F'' \hat{v} + yF''' \hat{u} \quad , \qquad (22)$$

obtained by cross-differentiating equations (16b) and (16c), where $\hat{\Omega} = \hat{v}_x - \hat{u}_y$ is the z-component of the vorticity perturbation. Note that the base flow contains vorticity in the burned gas $\Omega_s = -yF''$, which gives rise to the nonzero terms on the right hand side of equation (22). If one ignores Ω_s and considers a potential flow F = -x, the right hand side of equation (22) vanishes. The simplified equation, for $-d < x \leq 0$, $-\infty < y < \infty$, admits separable solutions of the form $\hat{\Omega} \sim x^{-n+\omega} y^n$. Depending on n, and on whether the disturbances grow or decay, the vorticity may either vanish or amplify as $x \to 0$; in the latter case the vorticity decays in the boundary layer near the

wall due to viscous action. In any case $\hat{\Omega}$ increases with distance from the axis as y^n.

In the presence of vorticity in the base flow, equation (21) is no longer separable and therefore the disturbances are not expressible in a simple form as in (22). Nevertheless, we may write

$$\hat{u} = \sum_{n=0}^{\infty} U_n(x)y^n, \qquad \hat{v} = \sum_{n=0}^{\infty} V_n(x)y^n, \qquad \hat{w} = \sum_{n=0}^{\infty} ikW_n(x)y^n$$

$$\hat{\varphi} = \sum_{n=0}^{\infty} \Phi_n(x)y^n, \qquad \hat{\psi} = \sum_{n=0}^{\infty} A_n y^n \tag{23}$$

expecting that now the various modes, corresponding to different indices n, are all coupled. Brattkus and Davis (1990) have also expressed the disturbances as an infinite series, but in terms of Hermite polynomials which arise naturally when viscous effects are incorporated in the vorticity equation. Finally, it should be pointed out that unlike the self-similar disturbances (21), which are essentially two dimensional with corrugations only in the z-direction, the disturbances considered here are three dimensional.

THE EIGENVALUE PROBLEM

Upon substituting (23) into the governing equations (16)-(17) and the boundary conditions (20), the following system results:

for $x < -d_0$:

$$\Phi_n'' - k^2\Phi_n = -(n+1)(n+2)\Phi_{n+2} \tag{24}$$

for $-d_0 < x < 0$:

$$U_n' + (n+1)V_{n+1} - k^2 W_n = 0 \tag{25a}$$

$$FU_n' + [\omega - (n-1)F']U_n = -\sigma P_n' \tag{25b}$$

$$FV_n' + [\omega - (n+1)F']V_n = -\sigma(n+1)P_{n+1} + F''U_{n-1} \tag{25c}$$

$$FW_n' + [\omega - nF']W_n = -\sigma P_n \tag{25d}$$

at $x = -d_0$:

$$U_n = \Phi_n' - \delta(\sigma-1)\left\{\left[\beta+(1+\eta)\frac{\gamma\ell}{2}\right]\left[(n+1)(n+2)A_{n+2} - k^2 A_n\right] + \frac{\gamma\ell}{2\sigma}\epsilon^2\eta(n+1)A_n \right.$$
$$\left. + \frac{\gamma\ell}{2}\left[(n+1)(n+2)\Phi_{n+2} - k^2\Phi_n\right]\right\} \tag{26a}$$

$$V_n = (n+1)\Phi_{n+1} - \frac{\sigma-1}{\sigma}\left[\sigma(n+1)A_{n+1} + \epsilon^2 A_{n-1}\right] - \delta\frac{\sigma-1}{\sigma}\frac{\gamma\ell}{2}\eta\left\{\frac{\epsilon^3}{\sigma}(1+\sigma/\eta)A_{n-1}\right.$$
$$\left. + [\omega+\epsilon(n+1)-\epsilon\sigma/\eta)](n+1)A_{n+1} + [\omega+(n+1)\epsilon](n+1)\Phi_{n+1}\right\} \tag{26b}$$

$$W_n = \Phi_n - (\sigma-1)A_n - \delta\frac{\sigma-1}{\sigma}\frac{\gamma\ell}{2}\eta\left\{[\omega+\epsilon n-\epsilon\sigma/\eta)]A_n + (\omega+n\epsilon)\Phi_n\right\} \tag{26c}$$

$$P_n = -(\omega+n\epsilon)\Phi_n - \left[1-\delta\epsilon(1-\eta)\frac{\gamma\ell}{2}\right]\Phi_n' + \delta(\sigma-1)\left\{\gamma\ell\left[(n+1)(n+2)\Phi_{n+2} - k^2\Phi_n\right]\right.$$
$$+ \left[1+\beta+\frac{\gamma\ell}{2}(\eta+2)\right]\left[(n+1)(n+2)A_{n+2} - k^2 A_n\right]$$
$$\left. + \frac{\gamma\ell}{2\sigma}\eta\left[2\epsilon^2-\omega^2-2n\epsilon\omega-\epsilon^2 n(n-1)\right]A_n\right\} \tag{26d}$$

$$\Phi'_n = [\omega + \epsilon(n+1)]A_n + \delta\left\{-\left[\beta + \frac{\gamma\ell}{2}\right]\left[(n+1)(n+2)A_{n+2} - k^2 A_n\right]\right.$$

$$\left. + \frac{\gamma\ell}{2}(\eta-1)\left[(n+1)(n+2)\Phi_{n+2} - k^2\Phi_n\right]\right\} \quad (26e)$$

at x = 0

$$U_n = 0 \quad (27a)$$

at x = → -∞

$$\Phi_n \text{ are bounded} \quad . \quad (27b)$$

We note that the problem for the nth order mode is coupled to the systems corresponding to the n-1, n+1 and n+2 modes. The mode coupling in the equations appears only on the right hand side of equation (25c) and is a result of downstream pressure gradients, and of vorticity produced by the base flow at the flame front. Mode coupling in the conditions (26) is of two origins: The first, which is a result of vorticity generation at the perturbed flame front, appears in the O(1) terms on the right hand side of equation (26b). The second, which is a result of flame front curvature and flame stretch (effects that are of the same order as the flame thickness), appears in the $O(\delta)$ terms of these relations.

A direct observation suggests that the eigenfunctions of the system (24)-(27) fall into two distinct classes, with the velocity $\hat{v}$ being either symmetric or anti-symmetric about the axis y = 0. Each of these two classes has its own set of eigenvalues. We refer to the even or odd solution as the series containing the sequence $\left\{ A_{2n}, U_{2n}, V_{2n-1}, W_{2n} \mid n = 0,1,2\ldots \right\}$ or, $\left\{ A_{2n-1}, U_{2n-1}, V_{2n}, W_{2n-1} \mid n = 0,1,2\ldots \right\}$, respectively. The classification is made according to whether the disturbed flame front $\hat{\phi}$ and hence $\hat{u}$ and $\hat{w}$ are even or odd functions of y; $\hat{v}$ always has the opposite parity. We determine these solutions by truncating the series at n = N and finding the eigenvalues

at any given level of truncation. For a given N, one of the two following system of equations results for the determination of the eigenvalues:

System I

$$U'_{N-1} + NV_N - k^2W_{N-1} = 0 \tag{28a}$$

$$FU'_{N-1} + [\omega - (N-2)F']U_{N-1} = -\sigma P'_{N-1} \tag{28b}$$

$$FV'_N + [\omega - (N+1)F']V_N = F''U_{N-1} \tag{28c}$$

$$FW'_{N-1} + [\omega - (N-1)F']W_{N-1} = -\sigma P_{N-1} \tag{28d}$$

System II

$$U'_N - k^2W_N = 0 \tag{29a}$$

$$FU'_N + [\omega - (N-1)F']U_N = -\sigma P'_N \tag{29b}$$

$$FV'_{N-1} + [\omega - NF']V_{N-1} = -\sigma NP_N + F''U_{N-2} \tag{29c}$$

$$FW'_N + [\omega - NF']W_N = -\sigma P_N \tag{29d}$$

The remaining equations, for the variables with lower indices, are only needed for the determination of the eigenfunctions. We note parenthetically that equation (29c) is uncoupled from the remaining equations in system II and is therefore not needed for the determination of ω. Each of these two systems is to be supplemented by the conditions (26a-d), truncated appropriately. The latter requires the determination of Φ_N and Φ'_N; however, from equation (24) one notes that the solution that decays as $x \to -\infty$ always satisfies $\Phi'_N = k\Phi_N$ which, when substituted in (26e) determines Φ_N.

As N increases from $N = 0$, we first find the solution $V_0 = 0$; for $N = 1$ we find the first even mode $(U_0; V_1; W_0)$ which is the self similar solution

(see equation 21) and the first odd mode $(U_1; V_0; W_1)$; for $N = 2$ we find the second odd and even modes $(U_1; V_2, V_0; W_1)$ and $(U_2, U_0; V_1; W_2, W_0)$ respectively, etc.... . In general, when N is odd system I determines the eigenvalues of the N-even mode and system II determines the eigenvalues for the N-odd mode; when N is even system I determines the eigenvalues of the N-odd mode and system II determines the eigenvalues for the N-even mode.

STABILITY OF A STRETCHED FLAME

The mathematical formulation presented above is valid for $\delta \ll 1$, so that the solution must be expressed in power series of δ. Here we only follow the first two terms in this expansion and, in particular, express the dispersion relation as

$$\omega = \omega_0(\sigma, \epsilon; k, N) + \delta\omega_1(\sigma, \epsilon, \ell; k, N) \quad . \tag{30}$$

For a given N, the eigenvalues for the even and odd modes are obtained by solving equations (28) or (29) with their appropriate boundary conditions. The leading problem, obtained by setting $\delta = 0$, yields ω_0. The next term ω_1 is obtained as a solvability condition from a system with the same differential operator as that of the leading problem but with nonhomogeneous terms that depend on the leading order solution. The solutions presented below were obtained by numerically solving these systems using the boundary value solver COLSYS (Ascher et al., 1981).

The dependence of the solution on the strain rate ϵ can be scaled out in the whole problem, as shown by Kim and Matalon (1990). Consequently, the dispersion relation (30) can be rewritten as

$$\omega = \epsilon\alpha_0(\sigma; \kappa, N) + \delta\epsilon^2\alpha_1(\sigma, \ell; \kappa, N) \tag{31}$$

where $\kappa = k/\epsilon$.

The leading order term in the dispersion relation corresponds to the modification of the Darrieus-Landau instability resulting from flame stretch. For very small ϵ, the flame is located far upstream in a nearly uniform flow. Thus, the hydrodynamic disturbances grow in the infinitely long downstream region of burned gas according to

$$\omega_0^{(0)} = \frac{\sigma}{\sigma+1}\left\{\left[\frac{\sigma^2+\sigma-1}{\sigma}\right]^{1/2} - 1\right\} k = \hat{\alpha}_0 \ k \tag{32}$$

Landau, 1944; Darrieus, 1945). For moderate values of ϵ, the flame is located at the mean position x = -d. Consequently the hydrodynamic disturbances which are now confined to the region -d < x < 0 of burned gas, must vanish at a finite distance from the flame front. This damping effect is one source of stabilization, the only one experienced by the two dimensional self-similar disturbances. An additional source of stabilization comes from the stretching that three dimensional disturbances experience in the direction of flow divergence. These two effects modify the growth rate (32), as shown in Fig. 2, where the leading order term of the dispersion relation is plotted against κ for a selected number of even and odd modes corresponding to various N. [Here, and in all following figures, solid curves result from solutions of system I while dashed curves result from solutions of system II; the notation associated with the different modes consists of a digit which stands for the value N and the letters E for even and O for odd]. We note from Fig. 2a that disturbances with long wavelength in the z-direction (i.e. with small k) are now damped out. However, for a given k, disturbances with short wavelength in the y-direction (i.e. corresponding to large N) are the more stable one. For a fixed ϵ there exists therefore a value $k^-(N)$ such that the only growing modes are those with $k > k^-(N)$, with k^- increasing with N. For comparison, we have plotted in Fig. 2b the relative growth rate ω_0 compared to that of an unstretched flame $\omega_0^{(0)}$. For a fixed k this graph identifies the dependence of the growth rate on $1/\epsilon$. Note that the ratio $\omega_0/\omega_0^{(0)}$ is less then one for $\epsilon > 0$ and approaches one as $\epsilon \rightarrow 0$.

The effects of diffusion appear in the $O(\delta)$ term of the dispersion relation (30). For an unstretched flame, the correction to the growth rate is of the form

$$\omega_1^{(0)} = - \left\{ \left[\frac{1}{2} \frac{\sigma(\sigma-1)(1+\beta) + 2\sigma\beta(1+\hat{\alpha}_0)}{\sigma+(\sigma+1)\hat{\alpha}_0} \right] + \left[\frac{1}{2}\gamma\ell \frac{\sigma(1+\hat{\alpha}_0)(\sigma+\hat{\alpha}_0)}{\sigma+(\sigma+1)\hat{\alpha}_0} \right] \right\} k^2 \quad (33)$$

(Pelcé and Clavin, 1982; Matalon and Matkowsky, 1983). In the presence of stretch it is clear from the boundary conditions (26) that the solution also depends linearly on ℓ so that one can write

$$\alpha_1 = \alpha_{11}(\sigma;\kappa,N) + \ell\alpha_{12}(\sigma;\kappa,N) \quad . \quad (32)$$

Our calculations indicate that, similar to the case $\epsilon = 0$, the coefficients α_{11} and α_{12} are both negative (see Fig. 3). This implies that diffusion has always a stabilizing influence when the Lewis number Le is greater than one (i.e. $\ell > 0$). Diffusion is a source of instability only when Le < 1; the precise determination of the critical ℓ will be discussed later. Note that the coefficients α_{11} and α_{12} are quite insensitive to variations in N; for N as large as 5 the variations in α_{11} remain indistinguishable while those in α_{12} are less than 10%.

The combined effects of hydrodynamic, including stretch, and diffusion can be understood by considering the expansion (30) as an approximation to the exact dispersion relation. This procedure was found to provide qualitatively correct results for the case $\epsilon = 0$ (Matalon and Matkowsky, 1984) which were also quantitatively in agreement with the numerical results of Jackson and Kapila (1984) for small k. The results for a stretched flame are shown in Fig. 4 where the growth rate ω is plotted versus κ for a selected number of modes N. The stabilization effect of diffusion is clearly seen when compared to Fig. 2. First, disturbances with short wavelength in the z-direction are damped out so that, for a given ϵ, growing modes are limited to $k^- < k < k^+$. Second, both k^- and k^+ decrease with increasing N so that, for sufficiently large N, all modes are damped out (i.e. $\omega < 0$ for all k and N) resulting with

a stable flame front. We note from Fig. 4a, which corresponds to $\ell = -0.53$, that the most dangerous disturbance is 1E, namely the two-dimensional self-similar disturbance, with $0.5 < \kappa < 3$. However, for $\ell = -0.54$ (see Fig. 4b), three dimensional disturbances with $N \leq 2$ and relatively much larger κ, grow at a faster rate than mode 1E does. As ℓ decreases further, more of the three-dimensional disturbances grow faster than 1E; for example, when $\ell = -0.6$ all five modes shown in Fig. 4b have growth rates that are larger than that of 1E. The implication of this observation will be discussed next.

The neutral stability boundary is obtained by setting $\omega = 0$ in the dispersion relation (31). The results, plotted in Fig. 5, are presented in the κ-ℓ plane for several values of the parameter $\epsilon\delta$. The various curves correspond to different values of N. The region to the right of these curves is where ω is negative and the flame is stable. The stability boundary is the envelope of these curves and the critical ℓ_c is defined as the maximum value of ℓ taken over all κ and N. Note that $\ell_c < 0$ so that a stretched flame in a premixed combustible mixture with Lewis number Le > 1, such as a lean hydrocarbon/air mixture, is stable. Upon decreasing ℓ, for example by making the mixture richer, an instability develops. From Fig. 5 one sees that at the stability threshold the two-dimensional self-similar disturbances grow first and that the transition occurs at a larger value of ℓ, the smaller $\epsilon\delta$ is. The pattern that will be observed on the flame front, in this case, is one with ridges in the z-direction only. If the finite width of the apparatus in the laboratory exclude disturbances with long wavelength, it is possible for a three dimensional mode to be the least stable one and, consequently, for a cellular flame to be observed at the onset of instability. For example, if disturbances with $\kappa \lesssim 3$ and $N \lesssim N^0$ are excluded for $\epsilon\delta = 2$, one of the dashed curves characterized by some κ^* and $N^* > N^0$ will be crossed first upon

decreasing ℓ. Thus, depending on the strain rate, two or three dimensional patterns could result on the flame front of rich hydrocarbon/air mixtures. The structure of the flow field at the onset of instability is quite different in these two cases, as shown in Fig. 6. In the figure, streamlines of the perturbed flow field on either side of the flame are plotted in the y plane. Case (a) corresponds to the 1E solution derived from system I (eqs. 28) associated with the right most point of the solid curve in Fig. 5; case (b) corresponds to the 10 solution derived from system II (eqs. 29) associated with the right most point of the dashed curve in Fig. 5. We note that in the latter case the flow pattern consists of several arrays of vortices stacked in the upstream direction and periodically distributed in the spanwise direction z. This whole structure is convected along the diverging streamlines of the mean flow.

CONCLUSIONS

We have considered the stability of a premixed flame in a stagnation-point flow against a class of three dimensional flame front perturbations. It has been concluded that the least stable mode corresponds to disturbances which are self-similar to the basic state, in accord with the known stability results for a viscous incompressible stagnation-point flow without chemical reactions. If disturbances of long wavelength are excluded, however, it is possible for a three-dimensioanl mode to grow first. This of course determines the pattern that is likely to be observed on the flame front at the onset of instability. We have also determine in this study the stability criteria in terms of the strain rate, the Lewis number and the thermal expansion parameter.

ACKNOWLEDGMENT

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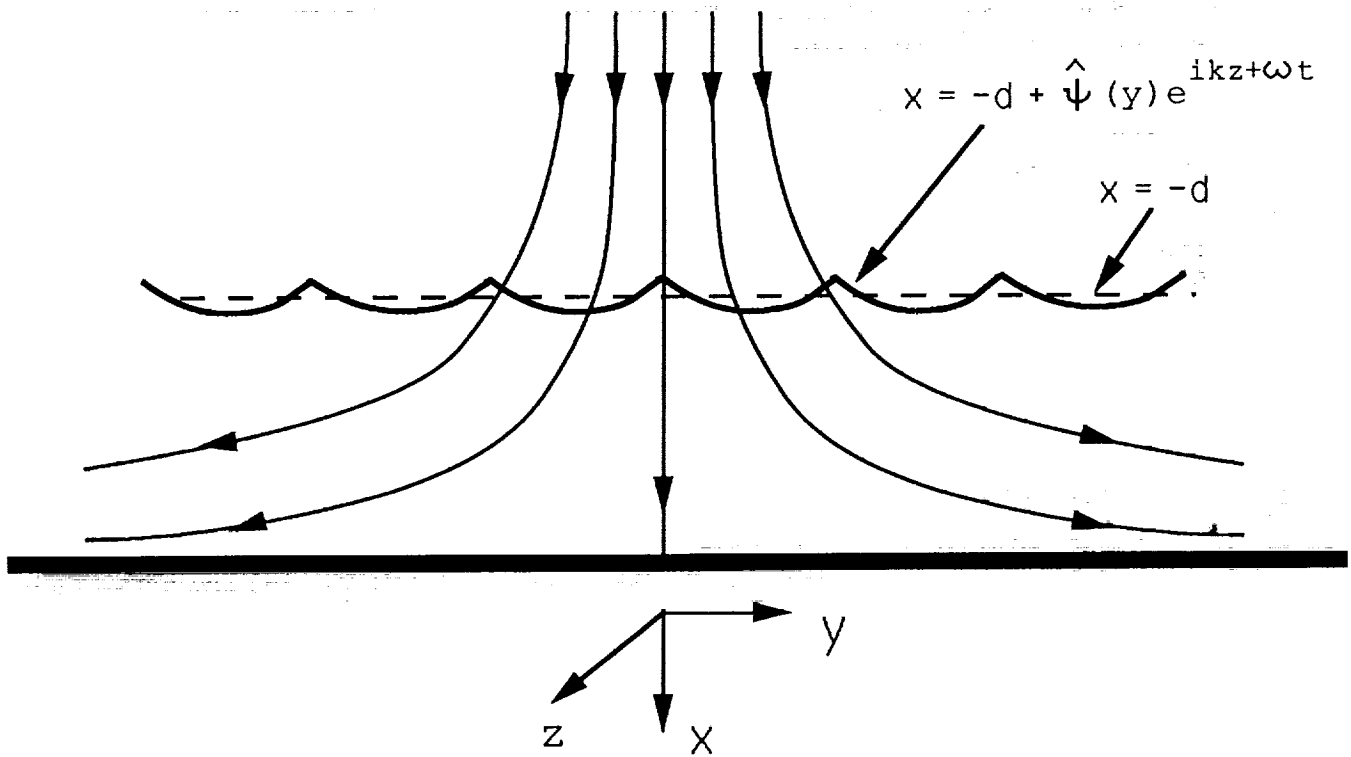
Fig. 1:   Schematic representation of a flame in a stagnation-point flow.
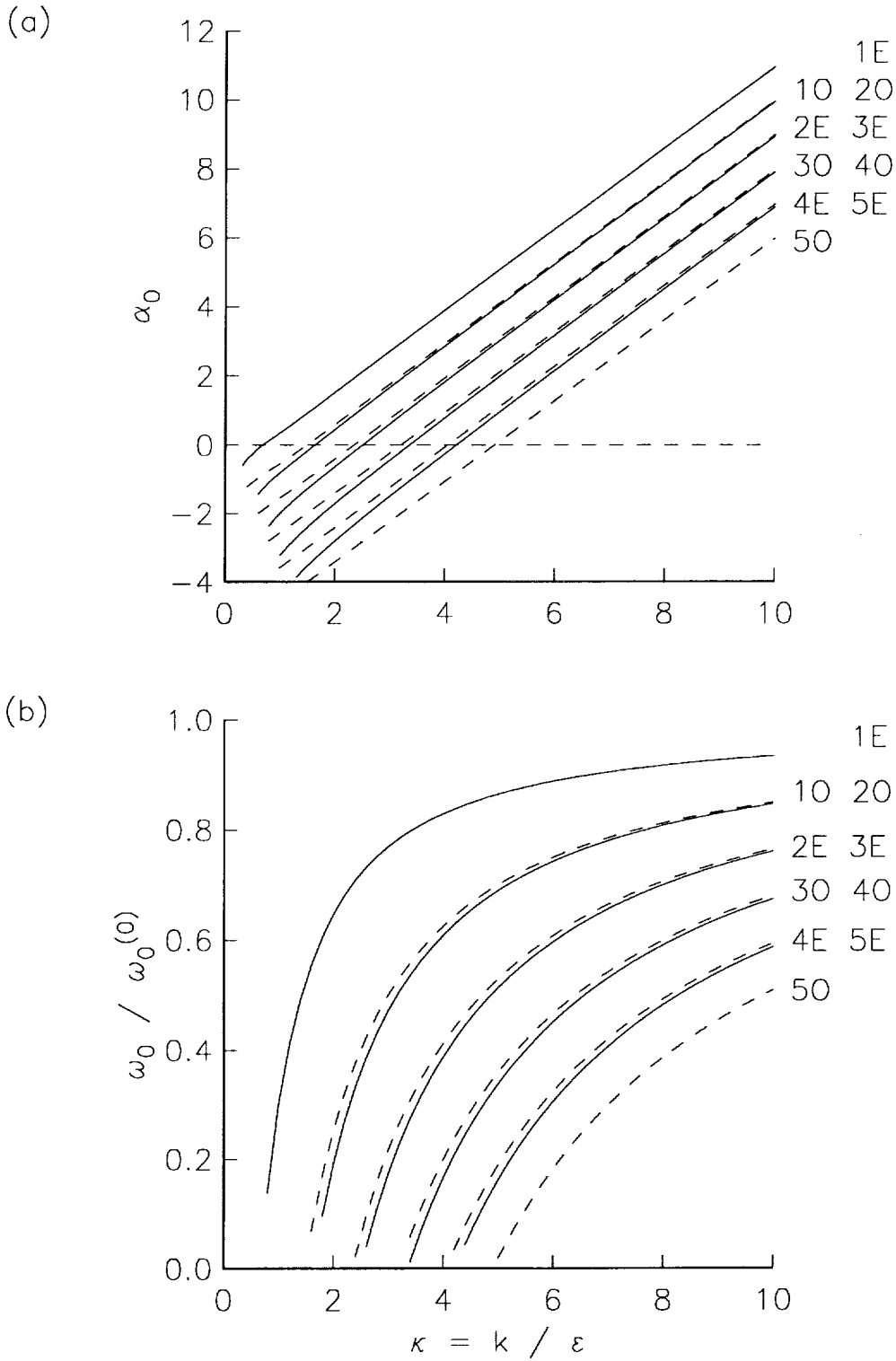
(a)

(b)

Fig. 2: The modification of the Darrieus-Landau instability resulting from flame stretch: (a) the growth rate for a stretched flame; (b) the relative growth rate compared to an unstretched flame. The solid curves correspond to solutions of system I with the mode classification given in the second column; the dashed curves correspond to solutions of system II with the mode classification given in the first column;
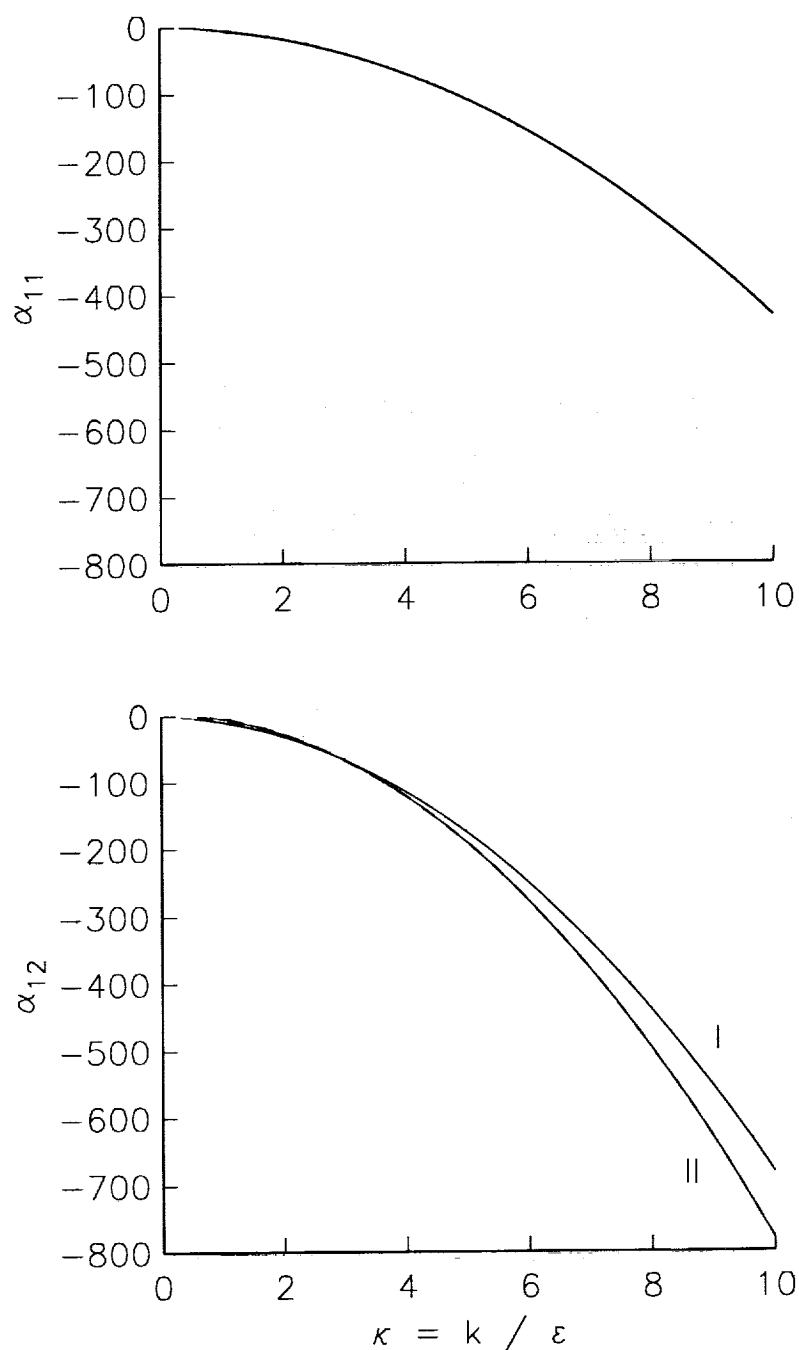
Fig. 3: The coefficients α_{11} and α_{12} calculated for even and odd modes with $N = 1,2,3,4,5$ and $\sigma = 5$. Note that α_{11} is practically independent of N and that α_{12} varies slightly with N depending on whether the eigenvalues result from system I or II.
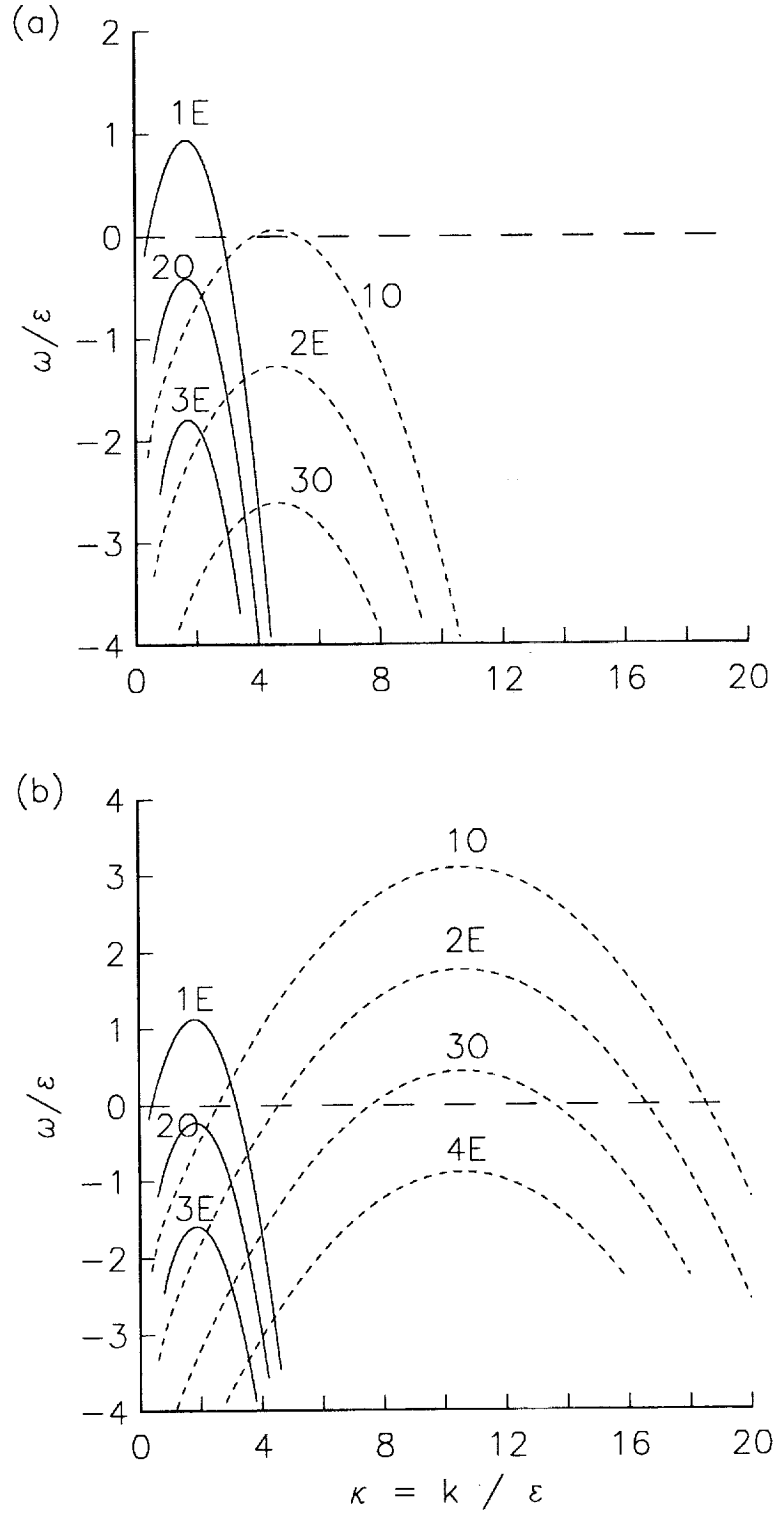
Fig. 4: The dependence of the growth rate ω on the wavenumbers κ and N showing the combined effects of hydrodynamics and diffusion; calculated for $\epsilon\delta = 0.8$ and $\sigma = 5$. (a) $\ell = -0.53$, (b) $\ell = -0.54$.
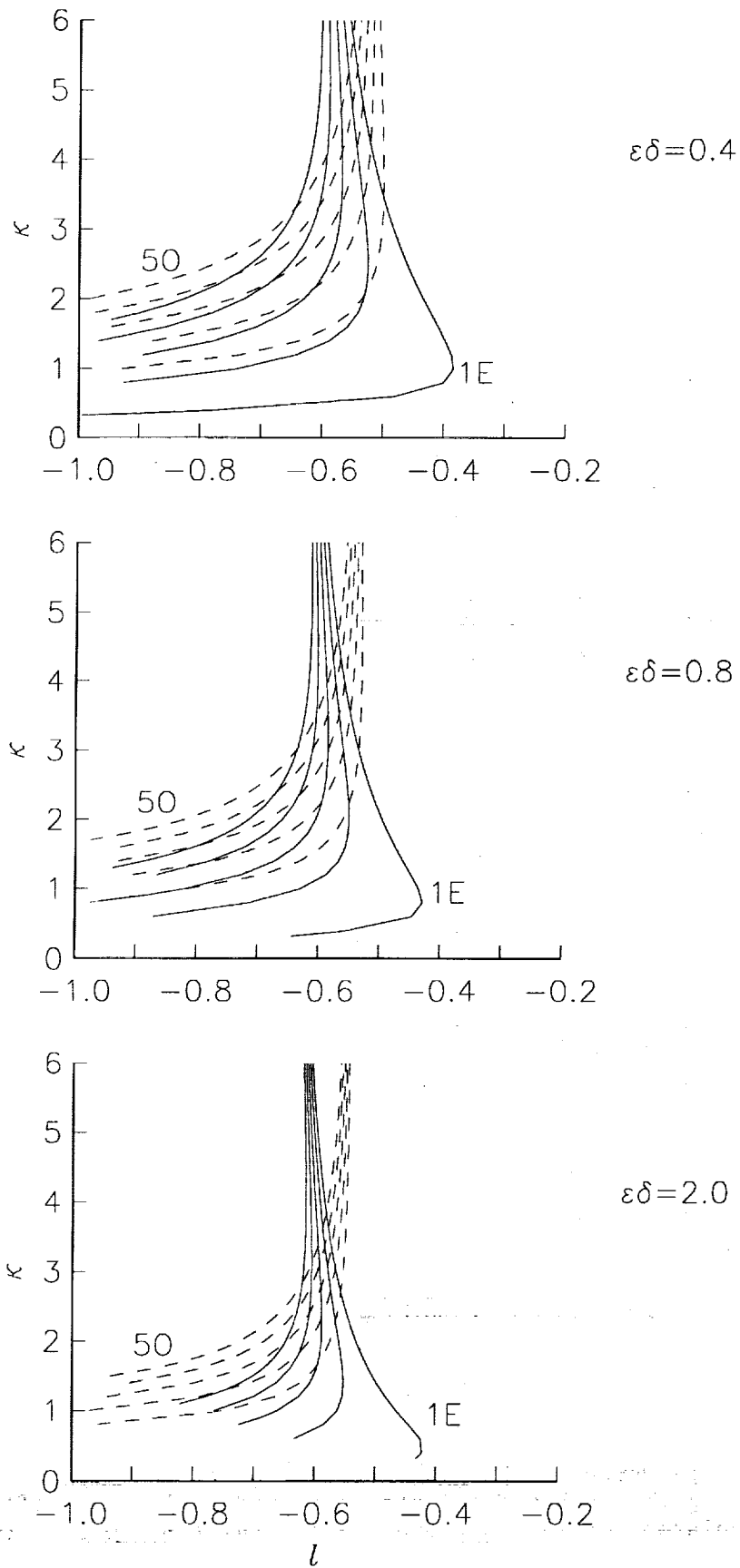
Fig. 5: Neutral stability curves for the stretched flame; calculated for
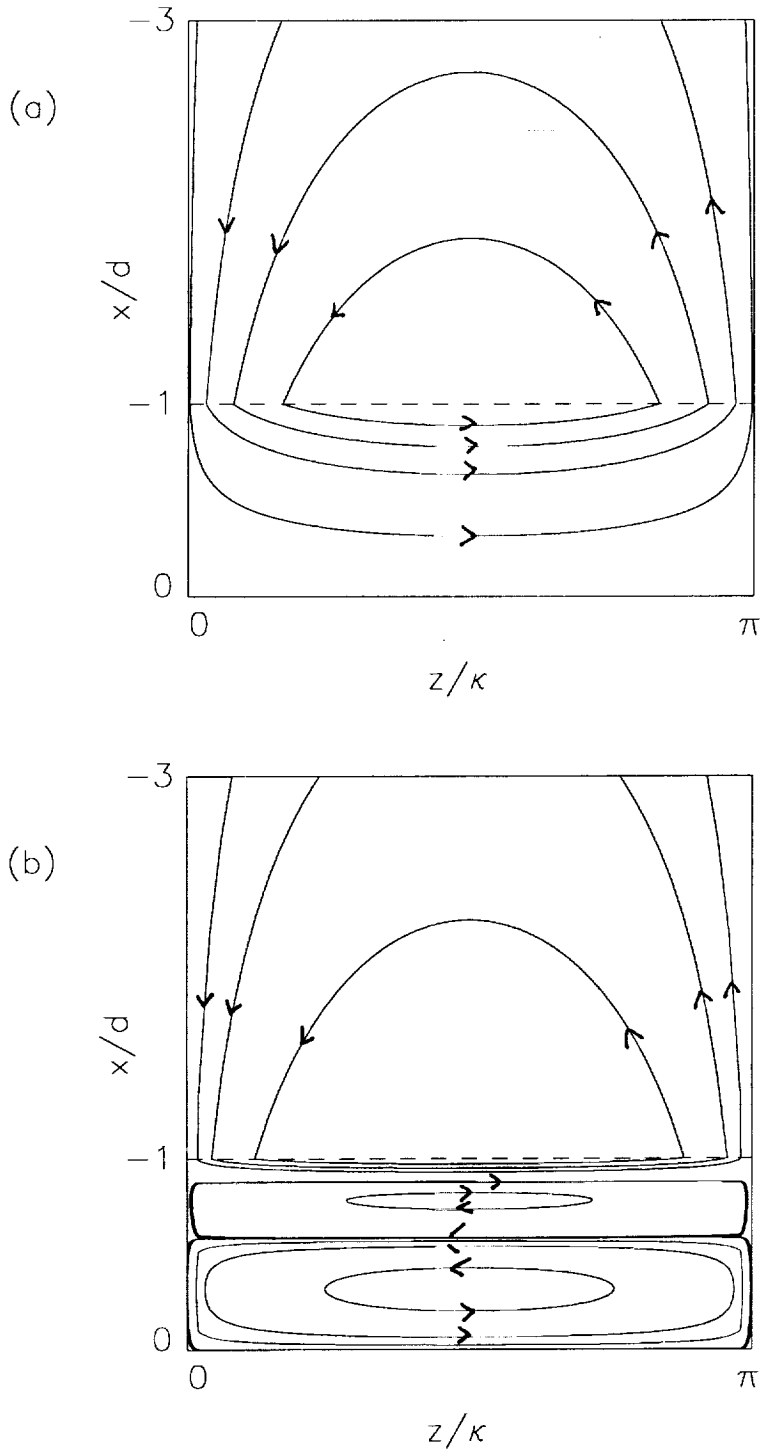$\sigma = 5$. The mode classification is as in Fig. 2.

Fig. 6: Streamlines of the perturbed flow field, on either side of the flame located at x/d = -1, at the onset of instability; (a) from the solution 1E derived from system I for $\sigma = 5$, $\epsilon\delta = 0.8$, $\ell = -0.43$ and $\kappa = 0.8$; (b) from the solution 10 derived from system II for $\sigma = 5$, $\epsilon\delta = 0.8$, $\ell = -0.53$ and $\kappa = 4.4$.

REPORT DOCUMENTATION PAGE

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1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE June 1992	3. REPORT TYPE AND DATES COVERED Contractor Report

4. TITLE AND SUBTITLE STABILITY OF A PREMIXED FLAME IN STAGNATION-POINT FLOW AGAINST GENERAL DISTURBANCES	5. FUNDING NUMBERS C NAS1-18605 C NAS1-19480
6. AUTHOR(S) Thomas L. Jackson Moshe Matalon	WU 505-90-52-01

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225	8. PERFORMING ORGANIZATION REPORT NUMBER ICASE Report No. 92-25

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225	10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA CR-189666 ICASE Report No. 92-25

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Michael F. Card   Submitted to Journal -- Final Report   Combustion Science Technology

12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category 34, 59	12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)* Previously, the stability of a premixed flame in a stagnation flow was discussed for a restricted class of disturbances that are self-similar to the basic undisturbed flow; thus, flame fronts with corrugations only in the cross stream direction were considered. Here, we consider a more general class of three-dimensional flame front perturbations which also permits corrugations in the streamwise direction. It is shown that, because of the stretch experienced by the flame, the hydrodynamic instability is limited only to disturbances of short wavelength. If in addition diffusion effects have a stabilizing influence, as would be the case for mixtures with Lewis number greater than one, a stretched flame could be absolutely stable. Instabilities occur when the Lewis number is below some critical value less than one. Neutral stability boundaries are presented in terms of the Lewis number, the strain rate and the appropriate wavenumbers. Beyond the stability threshold the two-dimensional self-similar modes always grow first. However, if disturbances of long wavelengh are excluded, it is possible for the three-dimensional modes to be the least stable one. Accordingly, the pattern that will be observed on the flame front, at the onset of instability, will consist of either ridges in the direction of stretch or the more common three-dimensional cellular structure.

14. SUBJECT TERMS stability; flame; stagnation-point	15. NUMBER OF PAGES 33
	16. PRICE CODE A03

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT