ISOTROPIC THIN-WALLED PRESSURE VESSEL EXPERIMENT

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KEY WORDS: (thin-walled) pressure vessel, cylinder, principal stress, brass, Poisson's Ratio, Poisson effect, axial stress, combined loading, superposition, strain rosette, elastic material.

PREREQUISITE KNOWLEDGE: This material could be taught as presented to a materials science or mechanics student at the college level. The student should be familiar with the concepts of principal stress/strain and the Poisson effect, which is an indication of the material strain transverse to applied load. Use of the linear regression function on a scientific calculator is expected. Previous exposure to thin walled pressure vessel theory is helpful, but not required.

OBJECTIVES: To investigate the stress and strain distributions on the surface of a thin walled cylinder subject to internal pressure and/or axial load. To relate stress and strain distributions to material properties and cylinder geometry.

EQUIPMENT AND SUPPLIES: 1) Brass tube of known Young's modulus and internal length, with pressure port and end treatment to facilitate internal pressure and axial loading, with strain rosette installed, as shown in Figure 1; 2) electromechanical test machine (ETM); 3) two digital strain indicators and switching unit; 4) load cell with greater than 9 kN capacity (minimum of 20 kN capacity recommended); 5) hydraulic pump with connections to tube and regulating valve, as shown in Figure 2; 6) hydraulic fluid; 7) pressure gage, preferably digital; 8) dial calipers; 9) scrap piece of same brass tubing, approximately 50 mm long; 10) calculator with linear regression capability.

PROCEDURE: Preparation: Set up hydraulic system and suspend brass tube with strain rosette in advance, leaving enough time to work air out of the hydraulic fluid (two days minimum), as shown in Figure 2. Present or review thin walled pressure vessel (PV) theory, including calculation and direction of hoop (circumferential) stress and longitudinal stress, and discuss assumptions needed for a PV to be considered thin walled. (If necessary, review axial stress). Review equations for calculation of principal strain from strain gage rosette data and conversion of principal strain to principal stress. Tell students the Young's modulus and internal length of the tube and the bulk modulus of elasticity of the hydraulic fluid.

Experiment: Students begin by calibrating the load cell and one strain indicator to read load in kiloNewtons. Next, they will complete and calibrate the strain gage rosette circuitry as shown in Figure 2 to permit reading of each of the three strains in units of microstrain for each loading. A quarter Wheatstone bridge will be used. To start the test, students should verify that the brass PV is affixed at top and bottom to the ETM, then apply a tensile axial load in increments of 4 kN up to a maximum of 16 kN. At each increment, the load and three strains will be recorded. Following the 16 kN measurements, all axial load must be removed. The students will next apply internal pressure in increments of 0.7 MPa up to a maximum pressure of 2.8 MPa, and obtain pressure and strain readings at each increment. Again, the loading should be removed. For the final loading variation, the students will apply an internal pressure of 2.8 MPa, then apply tensile axial load in increments of 4 kN up to 12 kN. At each increment, the axial load, strains, AND pressure readings are recorded. As axial loading increases, the instructor should point out two significant phenomena. The more obvious of these is that the principal direction changes from hoop (horizontal) to longitudinal (vertical). A second, subtle observation is that the elongation of the tube from the axial loading actually reduces the internal pressure of the tube by increasing the tube's volume. The students next remove all loading and disconnect all wiring. The scrap of tube is then measured to obtain internal diameter, external diameter, and wall thickness. (All should be measured at several locations to establish the range of actual values).

Data Reduction: Theoretical values required include Poisson's Ratio for the brass, and principal stresses and/or strains for each loading condition. (Principal directions will be assumed to be vertical and horizontal). The brass material properties and bulk modulus of elasticity of the hydraulic fluid should be provided by the instructor or obtained from standard material references. The principal stresses should be calculated from equations for axial stress, hoop and longitudinal stress, and their combination. (Equations and symbols are listed in Appendix A).

Extensive calculations are needed from the measured strains. For all loadings, principal strains must be calculated from the measured strain values. This requires initial calculation of shear strains. Principal strains can then be determined directly, using a reference axis through gage one of the rosette, or the measured strains can first be transformed to vertical and horizontal positions, then principal strains found. (Hopefully the transformed strains and principal strains will agree!) At least one principal direction per loading type should be determined to confirm that vertical and horizontal are indeed the principal directions.

Using principal strains from the tensile axial load only, an empirical Poisson's Ratio may be obtained by performing a linear regression of the minimum and maximum principal strains. The resulting slope is the empirical ratio desired. This should be compared to the theoretical Poisson's Ratio for verification. The empirical value should be used for all remaining calculations.

Next, all principal strains must be converted to principal stresses, using the theoretical Young's modulus and the empirical Poisson's Ratio. At least one pair of principal stresses per loading should be compared to theoretical values for verification. A linear regression of maximum principal stress and pressure from the pressure only loading can be performed to determine an equation relating hoop stress and internal pressure. When the theoretical hoop stress is substituted into this equation, an empirical value for wall thickness is obtained, which can be compared to the measured thickness.

For the combined pressure and axial loading, calculate the change in pressure due to the change in volume caused by the axial load. Compare calculated pressure change to measured pressure change. Note that the theoretical calculations for principal stresses for the combined load case assumed zero pressure drop; superposition assumes that the strains or stresses from the acting loads are additive, so this should allow discussion of the accuracy of the assumption used for the theoretical calculations. A second theoretical calculation of the principal stresses for the combined loading case could be completed based on the anticipated pressure drop due to the increase in volume.

SAMPLE DATA SHEETS: Self-evident.

INSTRUCTOR NOTES: This experiment was designed for students enrolled in an advanced strength of materials course. For additional study, a filament-wound composite cylinder could be included to investigate the effect of wind angle on stress/strain. Modeling of the vessels prior to testing could also be incorporated.

Conversely, the experiment could be modified to be conducted as a demonstration for beginning materials or strength of materials students. Additional study could include common polyvinylchoride piping. To simplify the experiment and its data reduction, a two gage rectangular strain rosette should be mounted along principal directions (vertical and horizontal), thus eliminating all principal strain and direction calculations.

Since published values for the bulk modulus of elasticity are difficult to find in standard reference material, and will vary widely if air is trapped in the fluid, it is advisable that the lab instructor should experimentally determine the bulk modulus prior to the lab experiment.

WARNING: Before conducting the experiment, test the PV at maximum loadings to confirm that the joints are sufficiently strong. Use of plastic sheeting or other means to catch any potential leaking hydraulic fluid is advisable.

REFERENCES: Beer, F. P., & Johnston, E. R. Jr. (1981). Mechanics of Materials. New York: McGraw-Hill Book Company, (pp. 62-70, 293-926, 325-342). Beckwith, T. G., Buck, N. L., & Marangoni, R. D. (1987). Mechanical Measurements (3rd ed.). Reading, MA: Addison-Wesley Publishing Company, (pp. 353-373). Fox & McDonald (1978) Introduction to Fluid Mechanics (2nd ed) New York, NY: Wiley & Sons, Inc. (pp. 610) Murdock, James W. PE (1976) Fluid Mechanics and its Applications USA: Houghton Mifflin Company (pp. 29-31)

SOURCES OF SUPPLIES: Brass tubing and caps - any metal distributor. Strain gages and rosettes can be obtained at substantially reduced cost for educational purposes from Measurements Group, P.O. Box 27777, Raleigh, NC, 27611, (919) 365-3800. New pumps can cost several hundred dollars, so investigate local industry and/or scrap dealers as potential sources (our pump was part of scrapped airplane landing gear). Hydraulic system (tubing, pressure ports, pressure valve) can be purchased from any local plumbing supply store.

ACKNOWLEDGEMENT: The thin walled pressure vessel experiment would not exist without the initial development work of George Stinchcombe, former Visiting Professor and current engineer with British Aerospace Corporation, and Jim Osborne, laboratory technician. Their efforts are greatly appreciated.

APPENDIX A: SYMBOL LIST (in order of appearance on Equation List)

t = wall thickness of tube

d = internal diameter

p = internal pressure

σ_H = hoop (circumferential) stress

σ_L = longitudinal stress

σ_R = radial stress

σ_A = axial stress

F = axial load

A = cross-sectional area (of tube)

Σ_1, Σ_2, Σ_3 = strain gage rosette data

Σ_x, Σ_y = normal strains along X&Y reference axes (X-axis through gage 1; Y-axis through gage 3)

γxy = shear strain with respect to X & Y reference axes

Σ_{max} and Σ_{min} = principal strains

γ_{max} = max shearing strain

Θp = principal direction(s)

μ = Poisson's Ratio

E = Young's Modulus

V = internal volume of the tube

L = internal length of the tube

B = bulk modulus of elasticity (for hydraulic fluid)

Σ_H = hoop strain

ΣL = longitudinal strain

EQUATION LIST (in order of appearance in the text)

1. Thin PV: If $t \leq \frac{d}{20}$

Pressure-Induced $\sigma_H \simeq \frac{pd}{2t}$ (1a)

Stresses:

$\sigma_L \simeq \frac{pd}{4t}$ (1b)

$\sigma_R \simeq 0$ (zero) (1c)

2. Axial Stress $\sigma_A = \frac{F}{A}$

3. Principal Strains from Rectangular Strain Gage Rosette Data

$$\Sigma_x = \Sigma_1 \quad (3a)$$

$$\Sigma_y = \Sigma_3 \quad (3b)$$

$$\frac{\gamma_{xy}}{2} = \Sigma_2 - \frac{1}{2}(\Sigma_1 + \Sigma_3) \quad (3c)$$

$$\gamma_{max} = \sqrt{(\Sigma_x - \Sigma_y)^2 + \gamma_{xy}^2} \quad (3d)$$

$$\Sigma_{max} = \frac{\Sigma_x + \Sigma_y}{2} + \frac{\gamma_{max}}{2} \quad (3e)$$

$$\Sigma_{min} = \frac{\Sigma_x + \Sigma_y}{2} - \frac{\gamma_{max}}{2} \quad (3f)$$

4. Principal Directions with respect to reference axes on rosette

$$\tan \Theta_p = \frac{\gamma_{xy}}{\Sigma_x - \Sigma_y} \quad (4)$$

EQUATION LIST (in order of appearance in the text) CONTINUED

5. Poisson Ratio (for Uniaxial Stress only)

$$\mu = -\frac{\Sigma_{min}}{\Sigma_{max}} \qquad (5)$$

6. Principal Stress from Principal Strain

$$\sigma_{max} = \left(\frac{\Sigma_{max} + \Sigma_{min}}{2}\right)\left(\frac{E}{1-\mu}\right) + \left(\frac{\gamma_{max}}{2}\right)\left(\frac{E}{1+\mu}\right) \qquad (6a)$$

$$\sigma_{min} = \left(\frac{\Sigma_{max} + \Sigma_{min}}{2}\right)\left(\frac{E}{1-\mu}\right) - \left(\frac{\gamma_{max}}{2}\right)\left(\frac{E}{1+\mu}\right) \qquad (6b)$$

7. Pressure Change Due to Volume Change

$$\Delta p = B\left(\frac{\Delta V}{V}\right) \qquad (7a)$$

$$V_{initial} = \frac{\pi}{4} d^2L \qquad (7b)$$

$$V_{final} = \frac{\pi}{4} d^2L \ (1+\Sigma_H)^2 \ (1+\Sigma_L) \qquad (7c)$$

$$B = \left(\frac{\Delta p}{\Delta V}\right) V \qquad (7d)$$

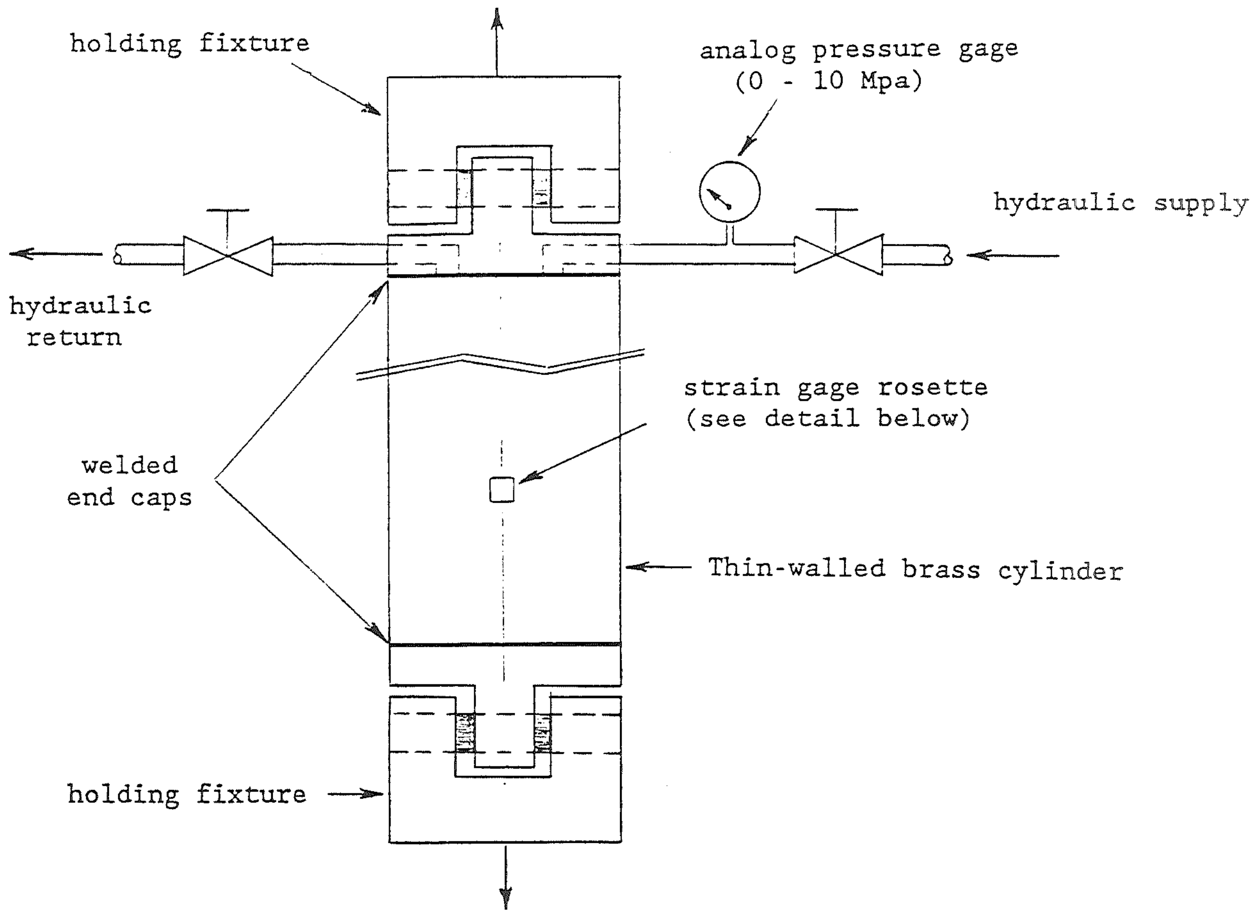

Figure 1.(a) Thin-walled brass pressure vessel

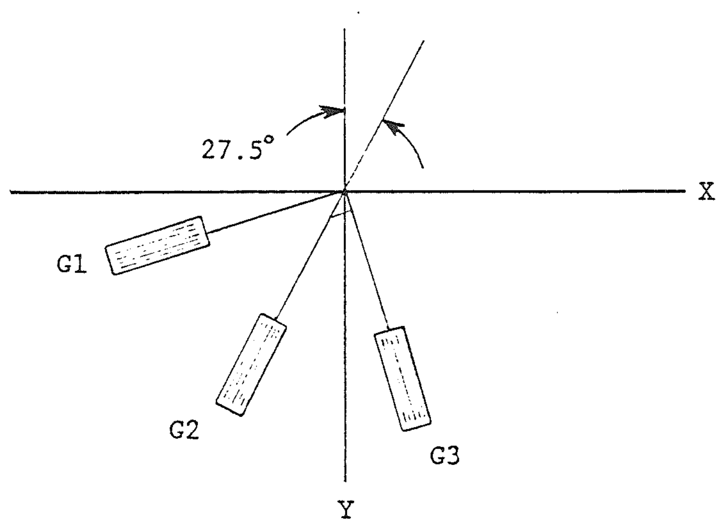

Figure 1.(b) Orientation of 45 degree strain gage rosette

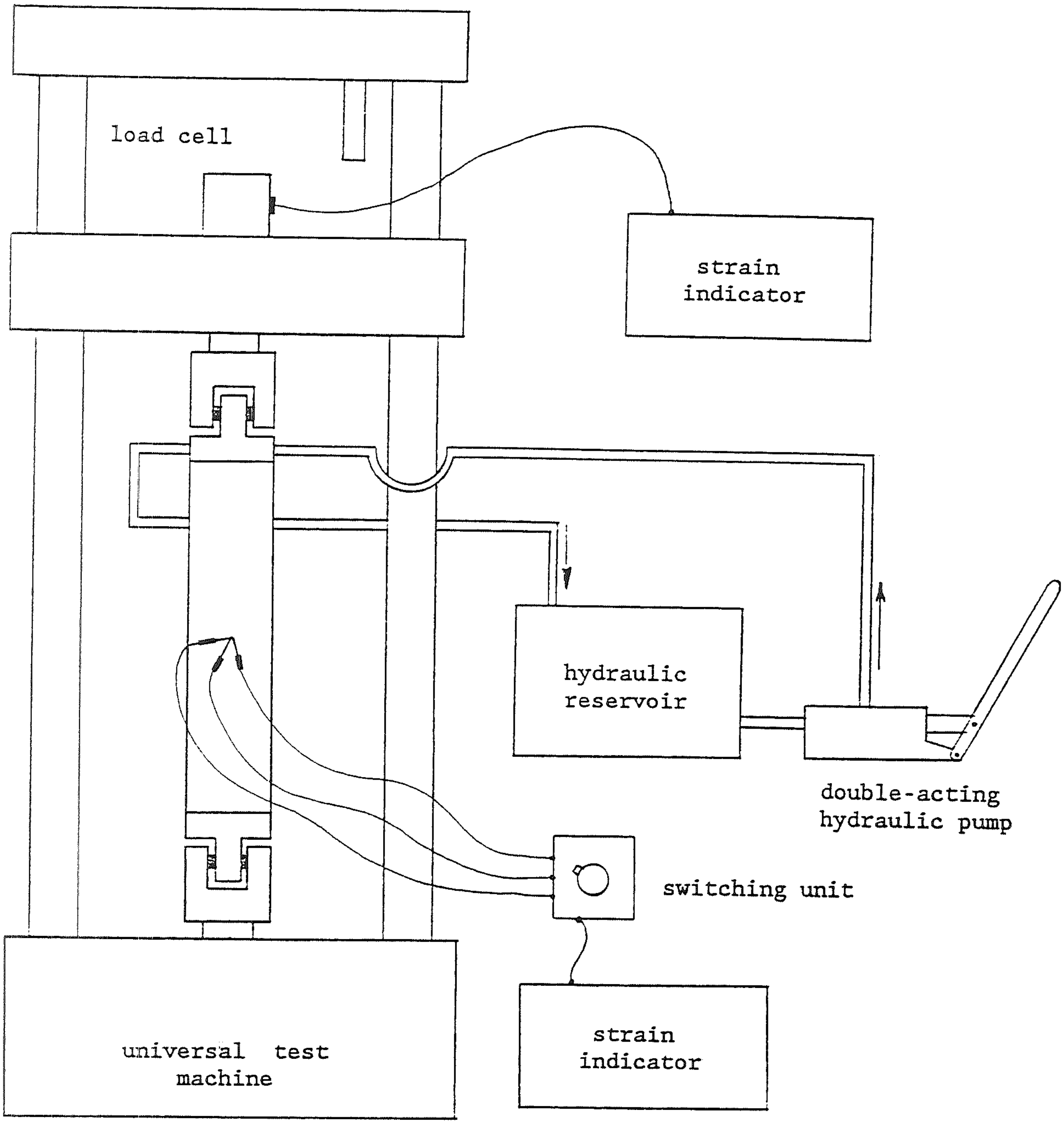

Figure 2. Schematic of test equipment