STRESS-STRAIN CHARACTERISTICS OF RUBBER-LIKE MATERIALS: EXPERIMENT AND ANALYSIS

David J. Allen

School of Engineering Technology
Northeastern University
120 Snell Engineering Center
Boston, Massachusetts 02115

Telephone 617-437-2500

N92-30285

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School of Engineering Technology
Northeastern University, Boston, Massachusetts

KEY WORDS: rubber, neoprene, stress, strain, elasticity, tensile test, power equation.

PREREQUISITE KNOWLEDGE: The student should understand the concepts of stress and strain either previously or concurrently presented with experiment.

OBJECTIVES: To demonstrated tensile testing of materials and the application of the concepts of stress and strain. To introduce a method of analysis of curvilinear data which will yield a mathematical relationship between stress and strain for many artificial rubbers and plastics.

EQUIPMENT AND SUPPLIES: (1) No. 32 or No. 64 rubber (neoprene) bands; (2) Two S hooks approximately 5 cm long; (3) One screw eye; (4) Lumber as required; (5) mm scale; (6) micrometer; (7) Calibrated weights; (8) Log-log paper; (9) calculator.

PROCEDURE: The students may either design their own testing apparatus or make one similar to the one shown in figure 1. If they design their own, design factors such as the amount of space required should be discussed.

Students make a pair of gage marks near the middles of each side of a rubber band and measure the length between each pair of marks. These measurements are the initial unstrained length, L_o. The thickness and width at each gage section is also made which will yield the initial cross sectional area, A_o. Two sets of data are made from each test.

The test is conducted by placing the specimen in the testing apparatus and adding incrementally calibrated weights. The length between the gage marks are measured for each load. Optionally, the thickness and the width of the rubber band may be measured at each loading. The test is conducted to failure.

Once the data is collected the students begin their analysis. The engineering stress may be computed from

$$\sigma = \frac{P}{A_o} \tag{1}$$

where σ is the engineering stress and P is the load on the section. It is important not to forget that P is one half of the applied load since there are two sides of the rubber band. If the actual thickness, t_{ACTUAL}, and width, w_{ACTUAL}, are measured for each loading, the true stress can be determined from

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$$\sigma_{TRUE} = \frac{P}{A_{ACTUAL}} \tag{2}$$

where the actual cross sectional area is

$$A_{ACTUAL} = t_{ACTUAL} \cdot w_{ACTUAL} \tag{3}$$

Strain is computed from

$$\epsilon = \frac{\Delta L}{L_o} \tag{4}$$

where ΔL is the change in length from the original undeformed length to the one being computed.

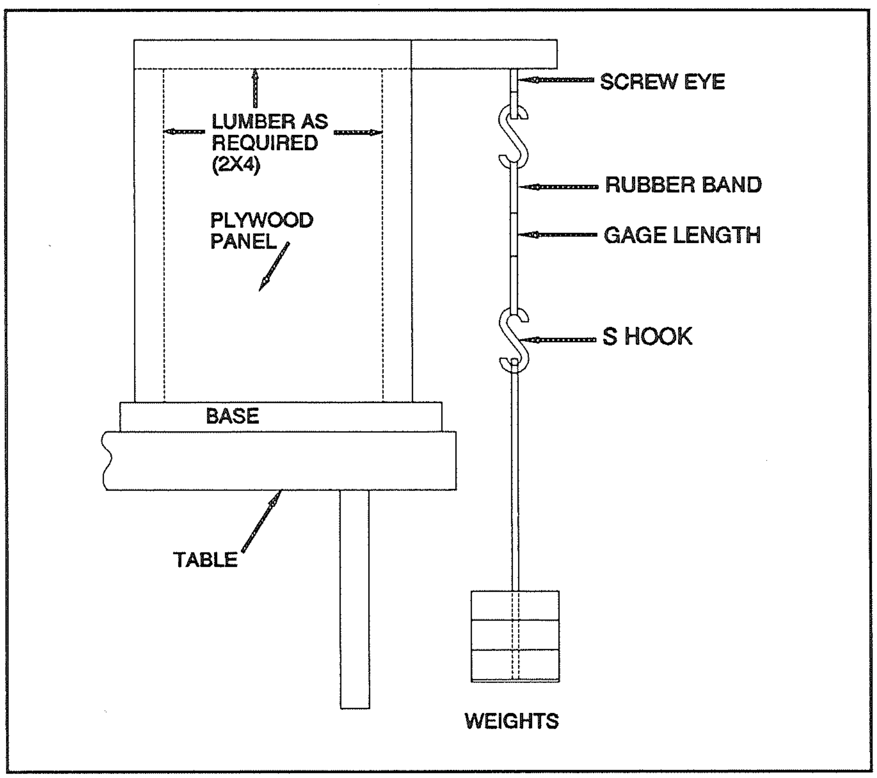

Figure 1

The stress and strain values are plotted on an arithmetic graph and a smoothed best fit curve is drawn. Typical results are shown in figure 2. Note that the curve is nonlinear.

It would be useful to obtain a mathematical relationship between stress and strain but one is not readily apparent from figure 2. However, if the data are plotted on a log-log plot as shown in figure 3, the best-fit curve is obviously a straight line.

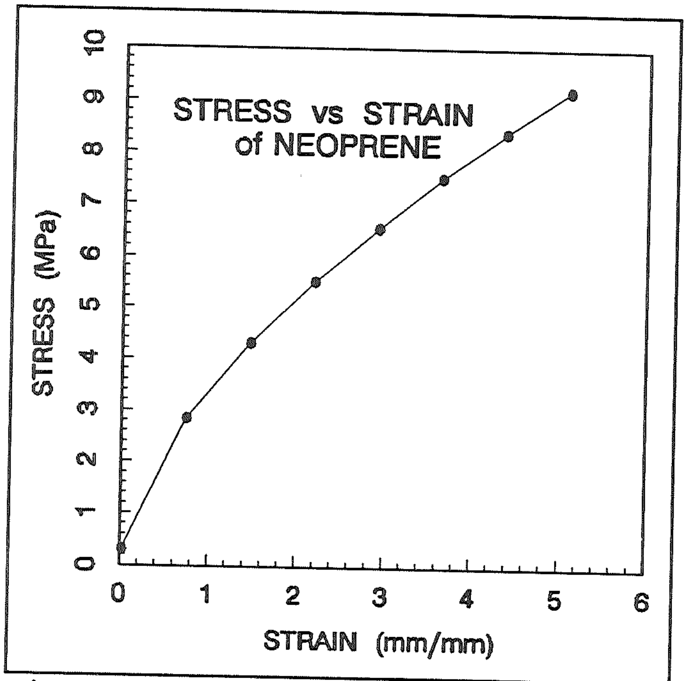

Figure 2

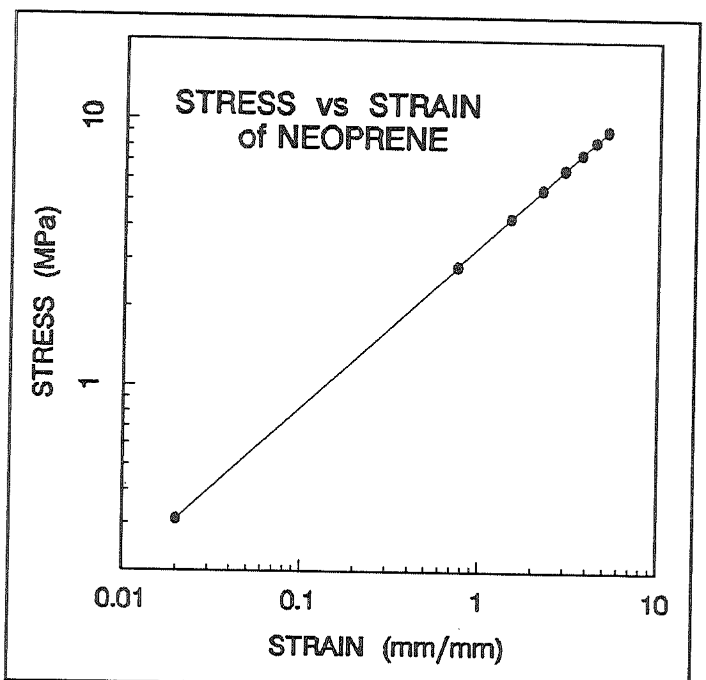

Figure 3

The equation of a straight line takes the form of

$$y = m \cdot x + b \tag{5}$$

but since the graph is log-log the equation needs to be modified to

$$\ln y = m \cdot \ln x + \ln b \tag{6}$$

or in terms of the stress-strain relationship

$$\ln \sigma = m \cdot \ln \epsilon + \ln b \tag{7}$$

which only leaves the determination of parameters m and b once the best-fit line is drawn.

The slope, m, of the curve can be determined by choosing two arbitrary coordinates on the best-fit curve, say (ϵ_1, σ_1) and (ϵ_2, σ_2). The slope can now be defined as

$$m = \frac{\ln\sigma_2 - \ln\sigma_1}{\ln\epsilon_2 - \ln\epsilon_1} = \frac{\ln\left(\frac{\sigma_2}{\sigma_1}\right)}{\ln\left(\frac{\epsilon_2}{\epsilon_1}\right)} \tag{8}$$

Once the parameter m has been computed from equation 8 the parameter b can be determined. Reorganizing equation 7 results in

$$\ln b = \ln \sigma - m \cdot \ln \epsilon \tag{9}$$

Substituting the arbitrary coordinates, (ϵ_1, σ_1) and (ϵ_2, σ_2), into equation 9 allows b to be computed from

$$\ln b = \ln \sigma_1 - m \cdot \ln \epsilon_1 = \ln \sigma_2 - m \cdot \ln \epsilon_2 \tag{10}$$

The value of lnb can be evaluated two different ways in equation 10. This gives a check of the computations. Utilizing the laws of logarithms equation 7 can be written as

$$\ln \sigma = \ln \epsilon^m + \ln b \tag{11}$$

or

$$\ln \sigma = \ln(b \cdot \epsilon^m) \tag{12}$$

or

$$\sigma = b \cdot \epsilon^m . \tag{13}$$

This is the form called the power equation. Data whose plots show a straight line best-fit relationship on log-log graphs can be represented mathematically by equation 13. Using the above method on the data shown in figure 3 yields

$$\sigma = 3.404 \cdot \epsilon^{0.6105}. \qquad (14)$$

Many materials and systems will follow a power equation and it is worthwhile to make a log-log plot of data points following a curvilinear trend on an arithmetic plot. Allen and Phalen[2] extend this approach to the artificial rubbers used in single-ply roof systems.

For a more thorough treatment true stress utilizing equation 2 versus strain may be analyzed. Arithmetic and log-log plots and the corresponding best-fit curves and equations for true stress vs. strain are compared with the engineering stress analysis given above.

SAMPLE DATA SHEETS: Self-Evident.

INSTRUCTOR NOTES: Having the students design and build their own testing apparatus is a good project. The student's interest and creativity is harnessed which results in a better appreciation of experimentation. A thorough discussion of factors involved in experimental design should be made without dampening the student's creativity. One student used rolls of pennies as weights and obtained good results. Ideas should be encouraged even though they may lead to unsatisfactory results. This provides a fertile ground for discussion.

Some computer graphing and statistical programs provide linear regression analysis. They may use the term "geometric" equation rather than "power" equation. Even if computer programs are available it is recommended that the student do the graphing and analysis by hand computations to reinforce the concepts introduced. The computer can be used as a check.

This experiment also gives the students a better understanding and a physical "feel" of the concepts of stress and strain than that obtained by using a steel specimen in a testing machine. The large strains and applied loads are readily apparent.

REFERENCES:

(1) Giesecke, F.E., et.al., Engineering Graphics, Third ed., 1981, New York

(2) Allen, D.J., and T.E. Phalen Jr.,"Stress-Strain Characteristics for EPDM, CSPE, and PVC for the Development of Stresses in Membranes Utilized as Single-Ply Roof Systems", Proceeding of the 1991 International Symposium on Roofing Technology, 1991

SOURCES OF SUPPLY: A well stocked hardware store or home center can supply the materials needed for the testing apparatus.