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Measuring the Weibull Modulus of Microscope Slides

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Key Words

Ceramics, fracture test, Weibull statistics, modulus of rupture, glass, bend test.

Prerequisite Knowledge Required

Students should have completed a strength of materials course, or be familiar with the concepts of stress, strain, bending moments, shear force, and free body diagrams. In addition, students should appreciate that ceramics are brittle materials that are fundamentally different than metals.

Objectives

Students will come to understand why a three-point bending test is used for ceramic specimens. They will learn how Weibull statistics are used to measure the strength of brittle materials. They will appreciate the amount of variation in the strength of brittle materials with a low Weibull modulus. They will understand how the modulus of rupture is used to represent the strength of specimens in a three-point bend test. In addition, students will learn that a logarithmic transformation can be used to convert an exponent into the slope of a straight line.

Equipment and Supplies

- Glass microscope slides, 25 mm by 75 mm by 1 mm (thicker slides will also work, thinner slides are unreliable)
- Three-point bending mechanism, which is described in the Instructors Notes
- Materials testing machine with a compression load cell or other method of applying a measured force
- Micrometer or vernier caliper for measuring the microscope slides
- Calculator with natural logarithm function or preferably a computer spreadsheet for reducing the data.

Procedure

Each group of students is given 25 microscope slides. A representative slide is selected and measured to determine the width and thickness; the three-point bending fixture is measured to determine the separation of the outside rollers.

Be sure you are wearing your safety glasses. Place a slide in the test fixture. Place a shield of flexible material around the fixture to catch any flying glass. Gradually increase the load on the slide until the slide breaks. Record the fracture load. Repeat this procedure with the rest of your slides. If you get extremely low or extremely high fracture loads, don't assume them to be in error. This is a common feature of ceramics' fracture strengths.

The raw data has now been collected; the remainder of the lab consists of analyzing the data. Students have found it very helpful to use spreadsheets to complete this analysis, as nearly all the calculations are repeated for each of the slides.

Sort the fracture forces in increasing order. Assign a survival probability to each force. Since all but one of the slides survived the minimum breaking force, the probability of surviving that force is 24/25 or 0.96. Similarly, the probability of surviving the next lowest force is 0.92, and so forth. Since no slides survived the highest breaking force, the experimental probability is 0, but to avoid dividing by zero in future steps, you should use a value of 1×10^{-6} instead.

For each fracture load, calculate the maximum tensile stress in the slide according to equation (1)

$$\sigma_{max} = \frac{3Pd}{2wh^2} \tag{1}$$

where P is the applied load, d the separation between the supports, w the width of the slide, and h the thickness of the slide.

The *modulus of rupture* is defined as the maximum tensile stress in the slide for a survival probability of 50%. The modulus of rupture can now be estimated from the data. Record this answer in your lab book.

The Weibull modulus shows up as the exponent m in equation (2)

$$\ln(1/P_s) = \left(\frac{\sigma_{max}}{\sigma_o}\right)^m \frac{V}{V_o} \frac{1}{2(m+1)^2} \tag{2}$$

where σ_{max} is the maximum tensile stress in the bend specimen, V is the volume of the bend specimen, and σ_o and V_o are constants of integration. To determine the Weibull modulus, we must take a natural logarithm of both sides of equation (2). As we can see in equation (3), if we make a plot with $\ln(\sigma)$ on the horizontal axis and $\ln(\ln(1/P_s))$ on the vertical axis, the data should fall on a straight line with a slope of m. The intercept of the straight line is the term in brackets; this term is not physically useful to us.

$$\ln(\ln(1/P_s)) = m \ln \sigma + \left[\ln\left(\frac{V}{V_o} \frac{1}{2(m+1)^2}\right) - m \ln \sigma_o\right] \tag{3}$$

Transform the stress and survival probability calculated above as suggested by equation (3). To obtain the Weibull modulus, there are a number of ways to proceed. You could plot the transformed data and fit the best straight line through by eye, then measure the slope of that line. You could enter the data into a calculator that performs linear regressions and let it calculate the slope for you For those who are familiar with spreadsheets, most spreadsheets have functions that will automatically perform the regression for you. However you do it, you should generate a plot and measure a slope.

If your data is ideal, the plot will be linear. However, the very weak and very strong slides usually have incorrect survival probabilities, so the tails of your graph will be nonlinear. It is usually best to calculate the slope based on the middle portion of your data.

Sample Data Sheets

Slide No.	Fracture Load (lbf)	Sorted Loads	Maximum Stress σ_{max} (psi)	Probability of Survival P_s	ln σ_{max}	$\ln(\ln(1/P_s))$
1	4.6	4.3	10178	0.96	9.23	-3.20
2	4.8	4.6	10888	0.92	9.30	-2.48
3	5.9	4.8	11361	0.88	9.34	-2.06
4	7.8	5.1	12071	0.84	9.40	-1.75
5	6.2	5.1	12071	0.8	9.40	-1.50
6	4.3	5.4	12781	0.76	9.46	-1.29
7	7	5.4	12781	0.72	9.46	-1.11
8	7	5.6	13254	0.68	9.49	-0.95
9	7.9	5.9	13964	0.64	9.54	-0.81
10	5.1	6.2	14675	0.6	9.59	-0.67
11	9.1	6.2	14675	0.56	9.59	-0.55
12	7.8	6.5	15385	0.52	9.64	-0.42
13	7	7	16568	0.48	9.72	-0.31
14	6.5	7	16568	0.44	9.72	-0.20
15	8.6	7	16568	0.4	9.72	-0.09
16	5.4	7.3	17278	0.36	9.76	0.02
17	5.4	7.3	17278	0.32	9.76	0.13
18	7.3	7.8	18462	0.28	9.82	0.24
19	6.2	7.8	18462	0.24	9.82	0.36
20	7.3	7.9	18698	0.2	9.84	0.48
21	5.1	8.3	19645	0.16	9.89	0.61
22	9.1	8.6	20355	0.12	9.92	0.75
23	10.5	9.1	21538	0.08	9.978	0.93
24	5.6	9.1	21538	0.04	9.98	1.17
25	8.3	10.5	24852	1E-06	10.12	2.63

Modulus of Rupture: 16.0 ksi

Weibull Modulus: 4.5

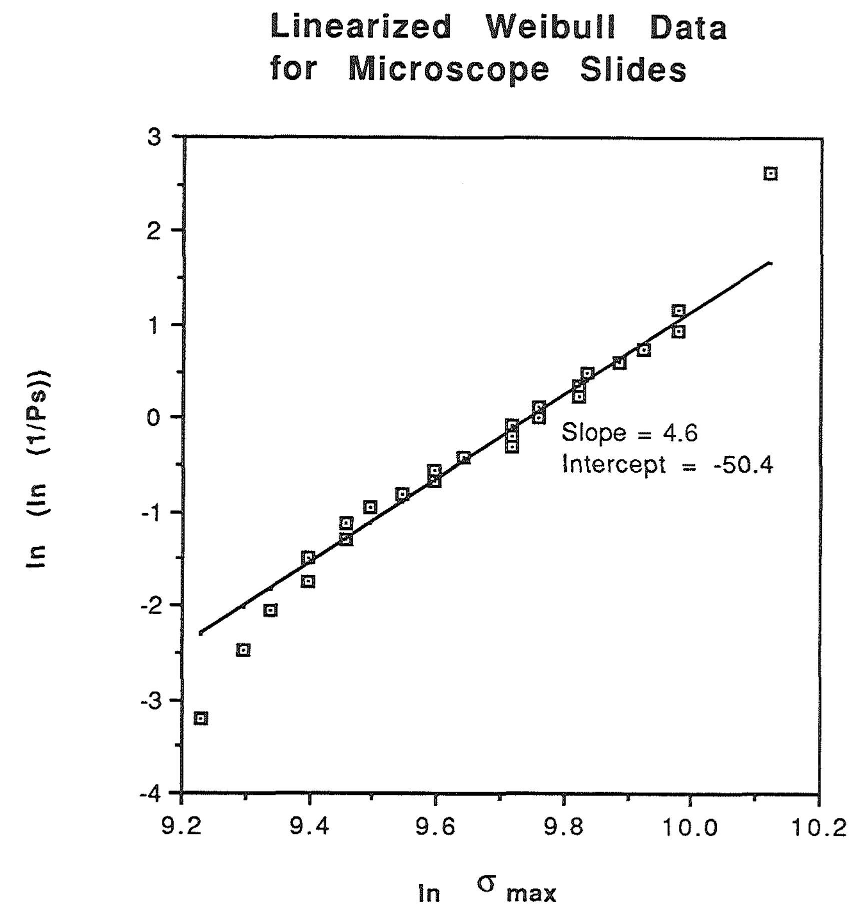

Figure 1: Plot of transformed data. The straight line fits the central portion of the data quite well. The Weibull modulus is the slope of the line.

Instructor Notes

The three-point bend fixture used need not be very strong; the maximum load is less than 15 pounds. However, for good results the support rods must be parallel, so the fixture should be machined, preferably from aluminum or steel. The diameter of the support rods is not critical; we have used rods ranging from 15 mm down to 3 mm with no noticeable effect on the results of the experiment. Cold-rolled steel round stock need only be cut to length to provide adequate supports.

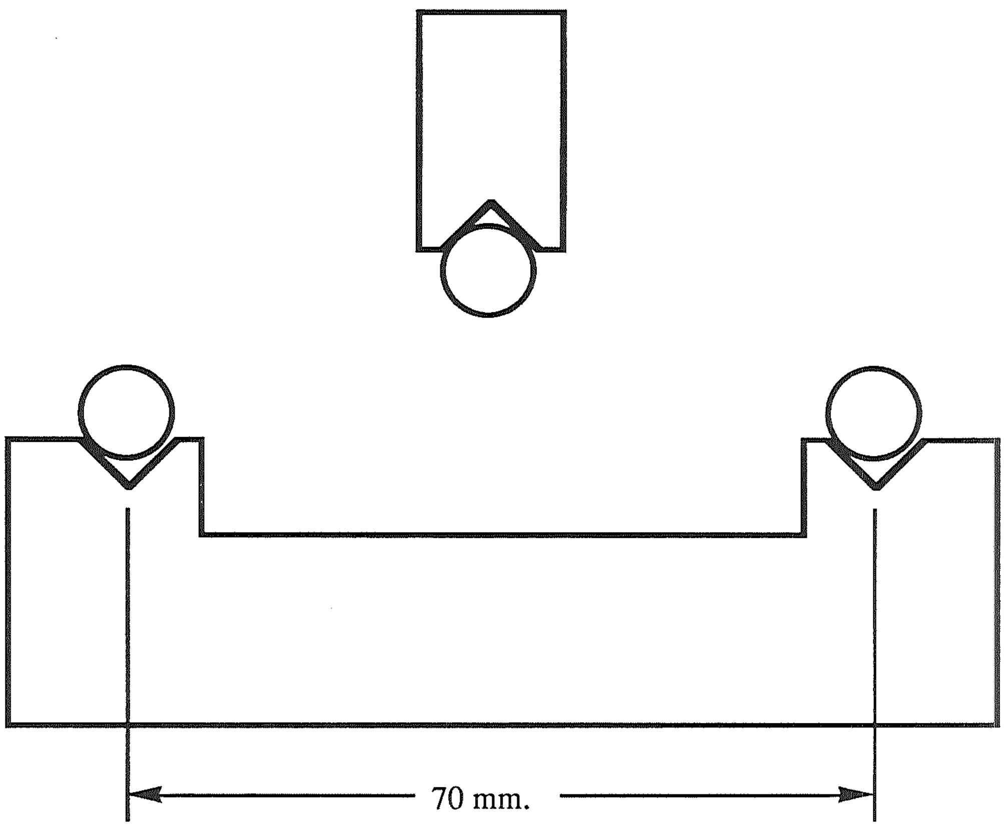

Figure 2: A three-point bend fixture for microscope slides. The support bars may simply be placed in the v-groove; the top force bar should be screwed to its support. The only critical feature of the construction is that the bars must be parallel.

When the slides break, it is not uncommon for glass to fly up to a meter. It is essential that guards be in place to contain the glass fragments. For cleanup, a shop vacuum provides a safe method of collecting the shards.

Few materials textbooks provide any discussion of Weibull statistics; reference 1 is a notable exception. Because this is not in the textbooks, I have found it necessary to hand out the background information below.

BACKGROUND

Metals and most polymers are ductile materials; they readily deform plastically (or yield) under high loads. In contrast, ceramics are brittle materials; the first evidence of permanent deformation is usually fracture of the specimen. This difference causes fundamental differences in the behavior of the tensile strength of the materials.

Tensile strength in ductile materials is limited by bulk properties of the material. This is because the material will yield locally around any microscopic flaws, preventing rapid fracture. In contrast, strength in ceramics is determined by the microscopic flaws found in the specimen, as cracks will run rapidly through the bulk of the material since no local yielding occurs.

When the strength of a material is limited by flaws, standard methods of reporting the strength are inadequate. To obtain accurate predictions of the strength of brittle materials, special statistics known as Weibull statistics are used. These statistics recognize that flaws are likely to be evenly distributed through the specimen and that the probability of having a large flaw depends on the volume of the specimen. Thus, a large specimen is weaker per unit volume than a small specimen.

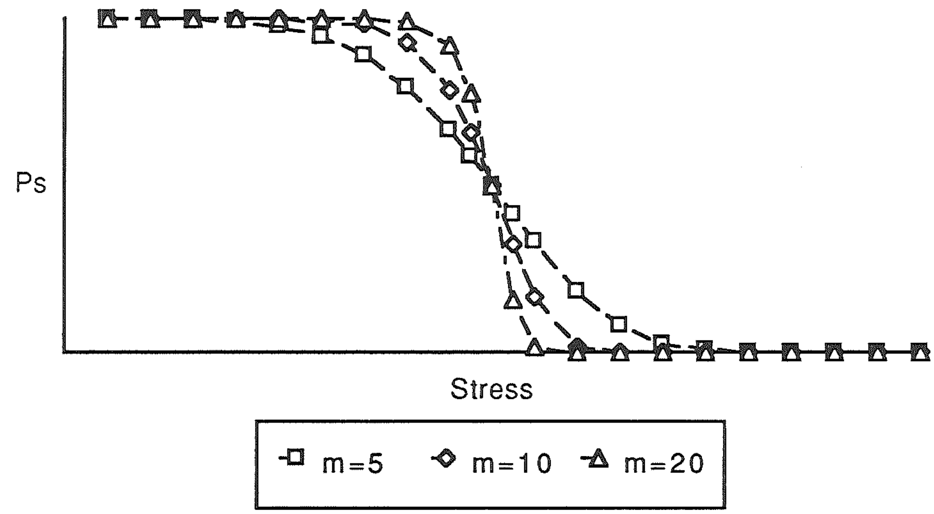

Figure 3: How probability of survival changes with Weibull modulus. Note that for higher values of Weibull modulus, there is less variation in fracture strength from specimen to specimen.

Weibull Statistics

Since each ceramic specimen will have a unique distribution of flaws, each specimen will have a unique strength. Thus, statistics are necessary to describe the strength of an untested specimen; we can only state the probability that a specimen will survive.

The fundamental equation for the probability of survival of a specimen is given by

$$\ln(1/P_s) = \int \left[\frac{\sigma(x,y,z)}{\sigma_o}\right]^m \frac{dV}{V_o} \tag{4}$$

where P_s is the probability of survival of the specimen, σ_o and V_o are constants, $\sigma(x,y,z)$ is the stress at a point (x,y,z) in the specimen, m is a constant known as the Weibull modulus, and the integral is evaluated over the whole specimen. V_o is the volume of a test specimen that is placed under tensile load; σ_o will be evaluated later.

The Weibull modulus of a material is the exponent m in equation 4. Figure 3 shows the effect of m on the survival probability of a specimen. Note that as the Weibull modulus increases, there is less scatter in the breaking strength; we have more confidence that the specimens will survive at loads near the breaking strength.

Uniaxial Tension

Consider a specimen that is loaded uniformly in tension. In this case, the stress is uniform in the specimen, and equation (4) evaluates to

$$\ln(1/P_s) = \left(\frac{\sigma}{\sigma_o}\right)^m \frac{V}{V_o} \tag{5}$$

When we are testing a specimen, V is equal to V_o; for other parts, V will be larger or smaller than V_o. When $V=V_o$, the probability of survival can be determined:

$$P_s = \exp\left[-\left(\frac{\sigma}{\sigma_o}\right)^m\right] \tag{6}$$

We can determine the probability of a specimen surviving a stress of σ_o by substituting σ_o for σ. FOR THE CASE OF UNIAXIAL TENSION,

$$P_s(\sigma=\sigma_o) = 1/e = 0.37 \tag{7}$$

Note that equation (6) describes a distribution with two parameters, σ_o and m. m is known as the Weibull modulus, and describes the scatter inherent in the strength of the material. σ_o determines the position of the distribution, and is related to the strength of the material. We normally do not consider σ_o to be the strength of the material, however. This designation is reserved for the stress that has a survival probability of 50%.

Three-Point Bending

We don't normally test brittle materials in tension. The stress concentration near the jaws of the testing machine causes failure at the jaws, rather than in the gage section. Instead, we load specimens in bending. There are a number of different bending tests; we will use the three-point bend test.

In the three-point bend test, a rectangular coupon is placed on two rollers spaced a known distance d apart. A third roll applies a load midway between the two support rolls; this places the specimen in bending (see Figure 4). In bending, the material above the neutral axis is loaded in compression, while the material below the neutral axis is in tension.

Another commonly used bend test is the four-point bend test, where two rollers separated by a bit less than d are used to apply the load. The four-point bend test creates a more uniform stress distribution in the specimen, and thus is less sensitive to the location of flaws within a single specimen. Therefore, fewer specimens need be tested to obtain the properties of the material. However, the same information is available from the three-point bend test. We will use the three-point test because of its simplicity.

Brittle materials generally have much higher compressive strengths than tensile strengths (because the cracks tend to close in uniaxial compression). Thus, we will consider only the portion of the specimen in tension to be under load in the bend test. (Note that the material outside the bottom rollers is unstressed.) Unlike the tensile specimens, the stress varies with position in the bending specimen. Thus, before we can evaluate the integral in equation (4), we must determine the stress as a function of position.

Consider a free-body diagram of half the specimen, as shown in Figure 5. The external loads and moments are not shown in the figure; this is left as an exercise for the reader.

By cutting a specimen at a distance x from the right-hand roll, we can determine the bending moment M at any position along the length of the specimen.

$$M = xP/2 \tag{8}$$

Stress in the specimen is independent of the width direction but varies in the thickness and length direction according to the equation of stress in pure bending (neglecting the shear force).

$$\sigma = \frac{Mz}{I} \tag{9}$$

where M is the bending moment as given by equation (8), z is the distance from the neutral axis, and I is the moment of inertia of the cross section of the specimen ($bh^3/12$ for a rectangular cross section). If we substitute d/2 for x and h/2 for z in equations (8) and (9), we can obtain an expression for the maximum stress in the specimen.

$$\sigma_{max} = \frac{Pdh}{8I} = \frac{3Pd}{2wh^2} \tag{10}$$

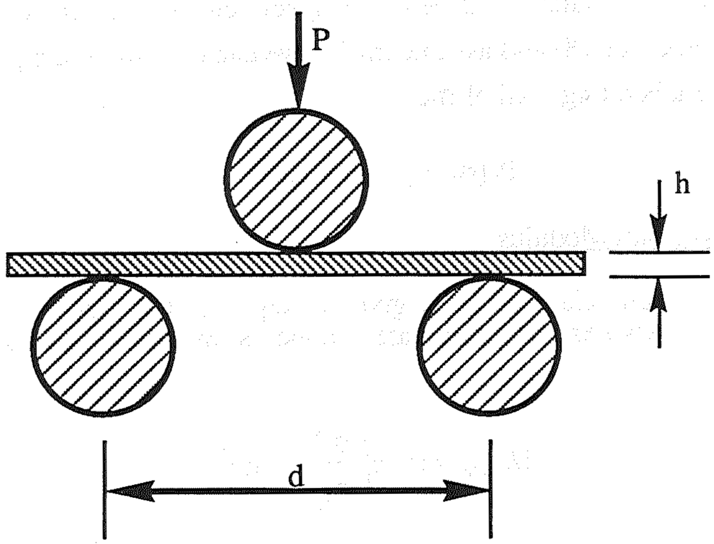

Figure 4: A rectangular specimen under a load P in a three-point bend test. The portion of the specimen under a tensile load has a length d, a thickness h/2, and a width w.

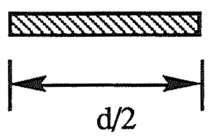

Figure 5: A free-body diagram of the portion of the specimen between the middle roller and the right hand roller in figure 1. The external forces and moments are not shown; this is left as an exercise for the reader.

We can substitute equations (8) and (9) into equation (4) to determine the probability of survival for an applied load P. Solving equation (10) for P and substituting the result into the solved integral gives us the final expression for the probability of survival

$$\ln(1/P_s) = \left(\frac{\sigma}{\sigma_o}\right)^m \frac{V}{V_o} \frac{1}{2(m+1)^2} \tag{11}$$

The stress σ_{max} that will cause the probability of survival in equation (11) to be 50% is known as the modulus of rupture; this is the strength index determined by the three-point bend test.

In uniaxial tension, we found that 37% of the specimens would survive a load of σ_o. If we set σ_{max} to σ_o in equation (8) and assume m=10, we can determine the probability that a specimen will survive a bending load of σ_o.

$$P_s(\sigma=\sigma_o) = 0.9959 \quad (12)$$

Determining the Weibull Modulus

Consider the case of uniaxial loading as given by equation (5). We want to determine m, which appears as an exponent. To eliminate exponents, we commonly take the logarithm of both sides

$$\ln(\ln(1/P_s)) = m \ln\left(\frac{\sigma}{\sigma_o}\right) + \ln\left(\frac{V}{V_o}\right) \quad (13)$$

or

$$\ln(\ln(1/P_s)) = m \ln(\sigma) + \ln\left(\frac{V}{V_o}\right) - m \ln(\sigma_o) \quad (14)$$

If we make a plot with $\ln(\sigma)$ on the horizontal axis and $\ln(\ln(1/P_s))$ on the vertical axis, the data should fall on a straight line with a slope of m. This allows us to readily determine the Weibull modulus. The same transformation can be applied to the bending specimen; the slope will remain m but the intercept will change (try it yourself to see).

References

1. Ashby, M.F. and D.R.H. Jones, *Engineering Materials 2: An Introduction to Microstructures, Processing, and Design*, Pergamon, 1986, pp.169-177.

Source of Supplies

Microscope slides can be obtained from any scientific equipment supplier. Micrometers or vernier calipers are available at machinist or industrial supply houses. Three-point bend fixtures are available from the manufacturers of material testing machines.