Single Mode Variable-Sensitivity Fiber Optic Sensors

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ABSTRACT: We review spatially-weighted optical fiber sensors that filter specific vibration modes from one dimensional beams placed in clamped–free and clamped-clamped configurations. The sensitivity of the sensor is varied along the length of the fiber by tapering circular-core dual-mode optical fibers. Selective vibration mode suppression on the order of 10 dB has been obtained. We describe experimental results and propose future extensions to single mode sensor applications.

1. INTRODUCTION: Fiber optic sensors have been developed to respond to a wide variety of physical measurands and, during the past decade, have emerged as viable alternatives to conventional electrical sensing techniques (Dakin and Culshaw 1988, Udd 1991). Specifically, two-mode elliptical-core (e-core) fibers have been applied as structural vibration sensors when operated in the linear region (Murphy *et al* 1990). Although less sensitive than their single-mode counterparts, two-mode fiber sensors present design simplicity and operational robustness for stable performance over long periods of time. In a recent paper, we enhanced e-core sensor performance by including passive optical signal processing through selective sensor placement and proposed the feasibility of applying weighting functions to the sensor to alter its longitudinal sensitivity (Vengsarkar 1991).

The development of piezoelectric spatially weighted sensors has led to research on the advantages of modal sensors and actuators. Modal sensors, which sense the modal coordinate of a particular vibration mode of a structure, can be operated within a control system without extensive computation requirements. In their first description of modal sensors, Lee and Moon (1990) made sensing elements out of polyvinylidene fluoride (PVDF) films shaped in the form of specific modal shapes of a structure.

2. THEORY - REVIEW OF BEAM MECHANICS: The Euler equation for transverse vibration of isotropic beams is given by

$$EI \frac{\partial^4}{\partial x^4} y(x,t) + \rho_\ell \frac{\partial^2}{\partial t^2} y(x,t) = 0\,, \tag{1}$$

where E is Young's Modulus, I(x) is the Moment of Inertia, ρ_ℓ is the linear density and y(x,t) is the transverse displacement of the beam. Using the method of separation of variables, we find a solution of the form

$$y(x,t) = \sum_{n=1}^{\infty} \psi_n(x)\, \eta_n(t), \tag{2}$$

where $\psi_n(x)$ represent the modal shapes and $\eta_n(t)$ represent the associated time varying modal amplitudes of the beam. Substituting Eq. (2) into Eq. (1) allows independent solutions for the

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functions $\psi_n(x)$ and $\eta_n(t)$. The equation for $\psi_n(x)$ has a closed form solution determined by the boundary conditions of the beam (Murphy *et al* 1990). This set of resulting eigenfunctions is also orthogonal, i.e.,

$$\int_0^L \psi_m(x)\,\psi_n(x)\,dx = \delta_{mn} \quad , \tag{4}$$

where δ_{mn} is the Kronecker delta and L is the length of the beam. This property is critical to the design of the fiber-based modal sensors.

TAPERED, CIRCULAR-CORE DUAL-MODE FIBER SENSORS: The dual-mode fiber sensor operates on the principle of differential phase-modulation between the LP_{01} and LP_{11}^{even} modes and consists of three different fiber sections fusion spliced to each other (Kim *et al* 1987). An e-core single-mode fiber is used as the lead-in fiber, a two-mode e-core fiber comprises the sensing section and a circular core multimode fiber acts as the lead-out fiber. At the second fusion splice, the axes of the sensing fiber and the circular core multimode fiber are offset from each other. This allows the lead-out fiber to pick up only one of the lobes of the spatial interference pattern resulting from the interaction between the LP_{01} and LP_{11}^{even} modes in the sensing fiber. The fused lead-out fiber thus acts as a ruggedized, low-profile spatial filter.

In this section we will restrict our analysis to the nature of the output signals and their dependence on the differential propagation constant, $\Delta\beta$ ($\Delta\beta = \beta_{01} - \beta_{11}$). Although the degeneracy of LP_{11}^{even} and LP_{11}^{odd} modes in circular-core fibers makes the operation of such a sensor difficult in practice, practical constraints have limited our experimental results to circular-core fiber sensors. As a result, our analysis in this section will pertain to circular-core fiber theory.

The output signal of a two-mode fiber sensor is sinusoidal and can be expressed

$$I(t) = I_0 + I_{ac} \cos\left[\phi(t)\right] , \tag{4}$$

where ϕ is the phase difference between the LP_{01} and the LP_{11}^{even} modes and can be written as

$$\phi(t) = \int_a^b \Delta\beta(x)\,\varepsilon(x,t)\,dx, \tag{5}$$

where ε is the strain experienced by the fiber, $\Delta\beta$ is the difference in the propagation constants of the LP_{01} and the LP_{11}^{even} modes, x denotes the longitudinal direction along the fiber axis, and a and b denote the end-points of the two-mode sensing region of the fiber.

SYSTEM ANALYSIS: In order to evaluate the vibration modes of the beam, we express strain as

$$\varepsilon(x,t) = \frac{\partial^2 y\,(x,t)}{\partial x^2}, \tag{6}$$

where y(x,t) denotes the deflection of the beam away from its equilibrium point. It is possible to weight the information actually present in the structure by using a priori knowledge of the mode shapes of the structure. Substituting Eq. (6) into (5) and integrating by parts leads to the equation

$$\phi(t) = \eta_n(t)\left(Q(a,b) + \int_a^b \Delta\beta''(x)\, \psi_n(x)\, dx \right), \tag{7a}$$

where

$$Q(a,b) = \left[\Delta\beta(x)\, \psi'_n(x)\right]_a^b - \left[\Delta\beta'(x)\, \psi_n(x)\right]_a^b \tag{7b}$$

and the primes indicate spatial derivatives with respect to x. Comparing Eqs. (7a) and (4) leads one to choose a possible weighting function given by $\Delta\beta''(x) = \psi_m(x)$. The functional dependence of $\Delta\beta''$ on V, the normalized frequency, is given below in Fig. 1 where V is defined as

$$V = \frac{2\pi}{\lambda} \rho \sqrt{n_1^2 - n_2^2} = V(\rho), \tag{8}$$

and ρ is the core radius. Because we are varying the value of ρ as a function of x, by construction, $\Delta\beta''$ becomes a function of x. Except for the contribution of Q(a,b), $\phi(t)$ would filter out all but the mth mode for a fiber sensor spanning the entire length of the beam.

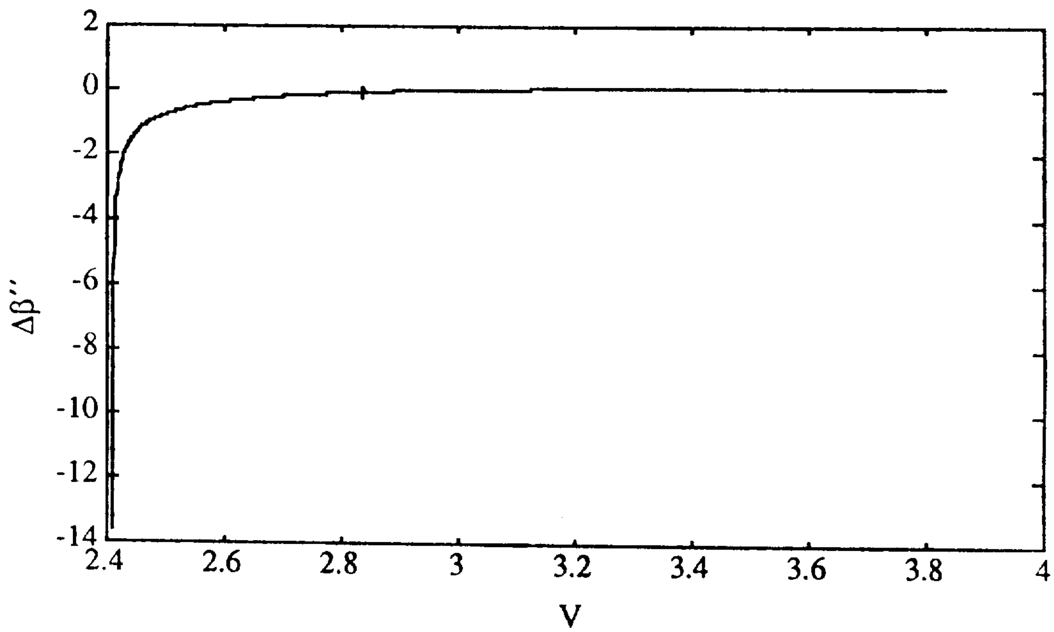

Figure 1: $\Delta\beta''(x)$ over the full range of dual-mode operation.

From the Fig. 1 and Eq. 8, it is clear that one can form a spatially varying value of $\Delta\beta''$ by choosing appropriate values for ρ. Notice that the function has useful behavior near the lower V numbers and quite flat behavior elsewhere. We have provided two specific examples. For a fiber with a linear taper ($\rho(x)$ is a linear function), $\Delta\beta''(x)$ is plotted in Fig. 2a superimposed with the first mode shape of a clamped-free beam. A similar plot for a fiber with an exponential profile superimposed with the first mode shape of a clamped-clamped beam is shown in Fig. 2b. By applying the orthogonality property of the structural modes along with Eq. (7), we are be able to tailor fiber profiles that will lead to vibration mode-selective sensor behaviors.

3. EXPERIMENTS: To test the validity of shaping $\Delta\beta''$, we constructed a weighted sensor. The sensor had an insensitive, lead-in, single-mode e-core fiber and an offset-spliced, lead-out multimode fiber that would spatially filter the contribution from only one of the two lobes at the output of the sensing region. A weighted fiber sensor with a taper that matches the $\Delta\beta''(x)$ function to the first mode of the clamped-free beam (Fig. 1a) was fabricated. The sensing fiber was pulled on a draw tower by varying the preform-feed and fiber-pull speeds as well as controlling the temperature of the furnace while the fiber was being drawn. The tapers could also be made on a coupler station used conventionally for fabricating fused-biconical-tapered couplers. The weighted fiber was attached to a clamped-free beam with a piezoelectric patch providing a benchmark. Results obtained from the weighted fiber sensor are shown in Fig. 3. The FFT's of the fiber sensor and the piezoelectric patch show that the second mode has been suppressed by 7 dB and the third mode by 12 dB in comparison to the piezoelectric sensor output.

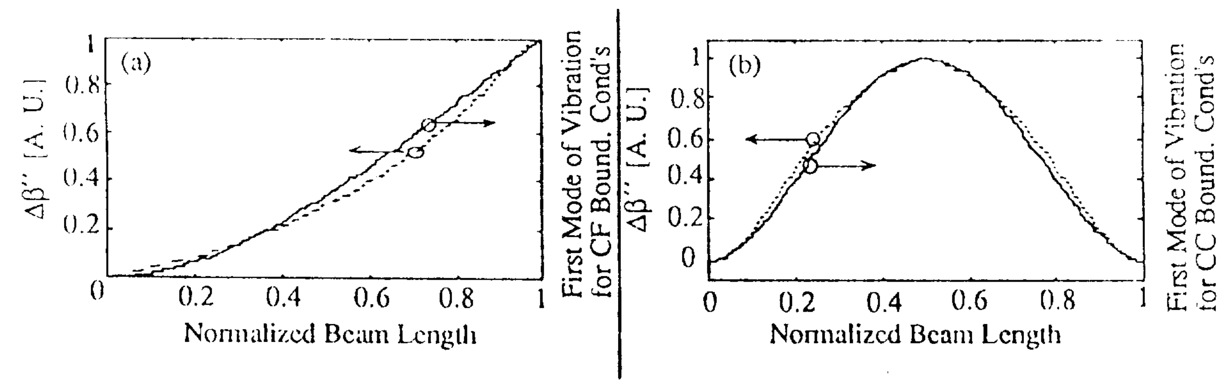

Figure 2: Examples of useful taper profiles (a) Linear and (b) Exponential.

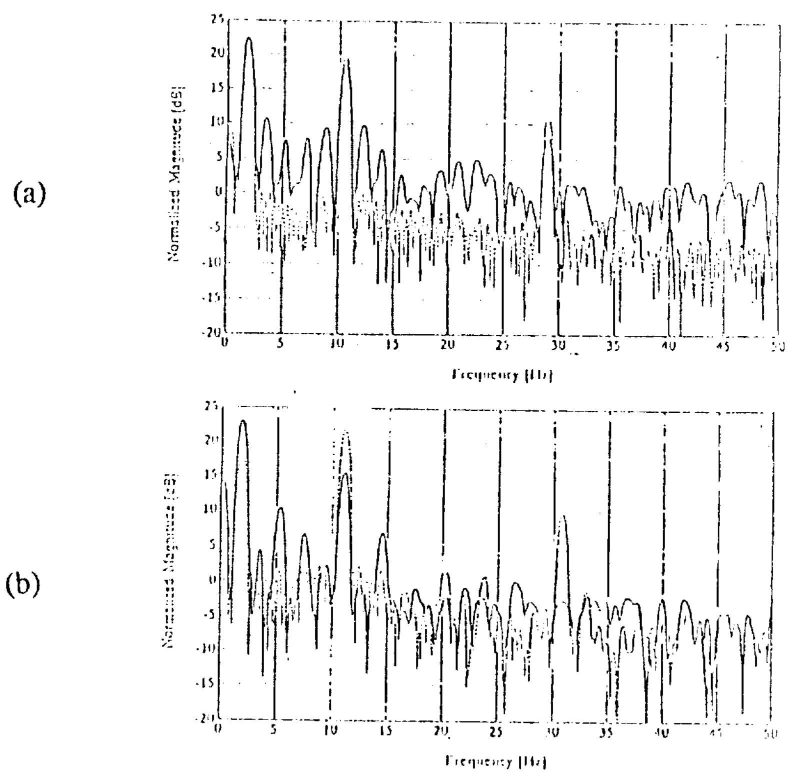

Fig. 3. Fast Fourier transform of output signals for clamped-free beam. (a) Conventional e-core fiber sensor and piezoelectric sensor comparison.(b) Tapered fiber sensor and piezoelectric sensor comparison.

4. **EXTENSION TO SINGLE MODE OPTICAL FIBER SENSORS:** As mentioned in the introduction, by choosing to investigate dual-mode optical fiber sensors, one gains robustness at the expense of sensitivity. However, there is no reason to expect that the benefits of weighted sensitivity cannot be applied to general single mode sensors. Analogous to dual-mode sensors, single mode fiber sensors have an integral dependence that describes their phase variation, namely,

$$\phi(t) = \int_a^b \beta(x)\, \varepsilon(x,t)\, dx, \tag{9}$$

where β is the propagation constant associated with the optical fiber. Substituting Eq. (9) into (5) and, again, integrating by parts leads to the equations

$$\phi(t) = \eta_n(t)\left(Q(a,b) + \int_a^b \beta''(x)\, \psi_n(x)\, dx \right), \tag{10a}$$

where

$$Q(a,b) = \left[\beta(x)\, \psi'_n(x)\right]_a^b - \left[\beta'(x)\, \psi_n(x)\right]_a^b . \tag{11b}$$

Therefore we are interested in the second spatial derivative of the modal propagation constant to achieve the weighted sensitivity. Fig. 4 shows this functional dependence.

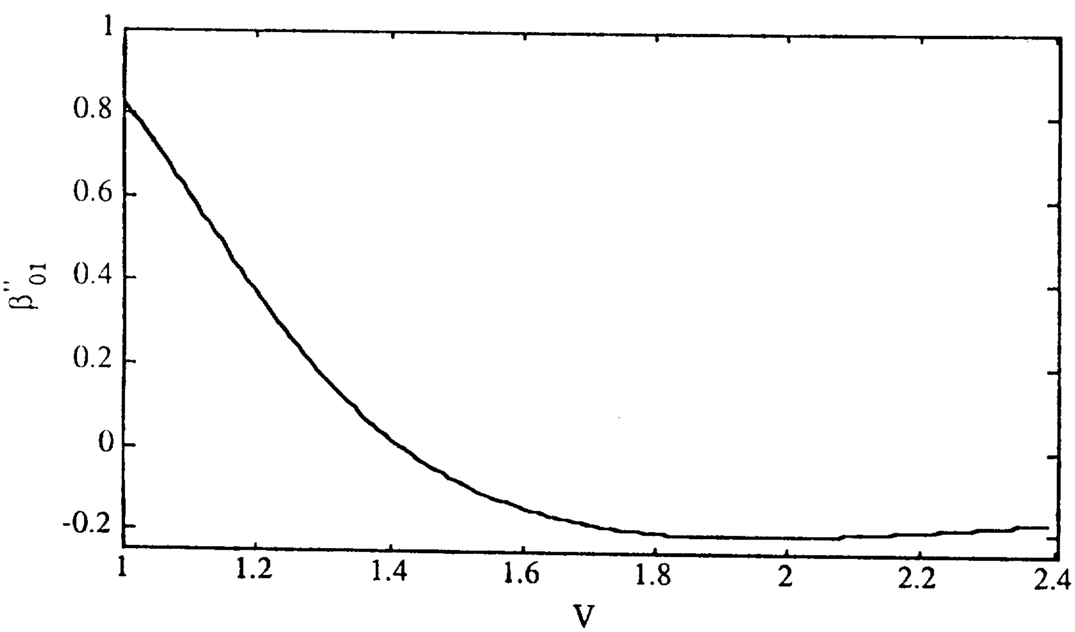

Figure 4: $\beta''(x)$ over the full range of single mode operation.

This relationship offers some interesting possibilities. In contrast to the dual-mode case, this curve is much richer in behavior and hence allows easier visualization of the shapes one is trying to ultimately match. Notice that a decreasing followed by an increasing linear taper will produce a first mode shape of the clamped-clamped beam for values of V near 1.2. Referring back to the previous section, a more complicated exponential taper was required to form a first mode shape with clamped-clamped boundary conditions in the dual-mode scheme. Also notice that the larger values of V seem to form a shape very close to that of the second mode with clamped-free boundary conditions. The possibilities seem only limited by the imagination of the person designing the taper.

5. **CONCLUSIONS:** The results presented in this paper demonstrate the use of spatially distributed fiber optic sensors with intrinsic weighting functions for selective vibration modal analysis. Modal suppressions of 7 and 12 dB were obtained for the second and third modes of vibration, respectively, for a clamped-free beam; in comparison, the PVDF-based modal sensors developed by Lee and Moon (1990) have resulted in a 7.3 dB suppression of the second mode for the fundamental mode sensor and in a 15.45 dB suppression of the fundamental vibration mode for a sensor shaped to enhance the second mode sensitivity. For fiber optic sensors, a precision-controlled fabrication station is required for the development of such weighting functions and we are currently addressing manufacturing issues. Although only the first mode of vibration was filtered with tapered two-mode fibers, the approach is equally suited for higher-order modal sensing and control using exotic shapes for the fiber tapers. However, the dual-mode configuration suffers from a rather bland shape of the $\Delta\beta''$ curve and hence complex shapes may be difficult to produce effectivelyx. This is not the case for single mode optical fiber sensors. Because the functional shape is quite rich, a wide range of complex shapes can be produced from concatenating simple linear tapers.

REFERENCES

Dakin J and Culshaw B 1988 *Optical Fiber Sensors: Principles and Components* (Boston: Artech House)

Kim B Y, Blake J N, Huang S Y, and Shaw H J 1987 *Opt. Lett* **12** p 729

Lee C-K and Moon F C 1990 *J. Appl. Mechanics* **57** pp 434-41

Murphy K A, Miller M S, Vengsarkar A M, and Claus R O 1990 *J. Lightwave Technol.* **8** pp 1688-96

Udd E 1991 *Fiber Optic Sensors: An Introduction for Engineers and Scientists* (New York: Wiley-Interscience)

Vengsarkar A M, Fogg B R, Miller W V , Murphy K A , and Claus R O 1991 *Electron. Lett.* **27** pp 931-32