OPTIMUM DATA ANALYSIS PROCEDURES FOR TITAN IV AND SPACE SHUTTLE PAYLOAD ACOUSTIC MEASUREMENTS DURING LIFT-OFF

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by

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ABSTRACT

Analytical expressions have been derived to describe the mean square error in the estimation of the maximum rms value computed from a step-wise (or running) time average of a nonstationary random signal. These analytical expressions have been applied to the problem of selecting the optimum averaging times that will minimize the total mean square errors in estimates of the maximum sound pressure levels measured inside the Titan IV payload fairing (PLF) and the Space Shuttle payload bay (PLB) during lift-off. Based on evaluations of typical Titan IV and Space Shuttle launch data, it has been determined that the optimum averaging times for computing the maximum levels are (a) $T_o = 1.14$ sec for the maximum overall level, and $T_{oi} = 4.88 \, f_i^{-0.2}$ sec for the maximum 1/3 octave band levels inside the Titan IV PLF, and (b) $T_o = 1.65$ sec for the maximum overall level, and $T_{oi} = 7.10 \, f_i^{-0.2}$ sec for the maximum 1/3 octave band levels inside the Space Shuttle PLB, where f_i is the 1/3 octave band center frequency. However, the results for both vehicles indicate that the total rms error in the maximum level estimates will be within 25% of the minimum error for all averaging times within $\pm 50\%$ of the optimum averaging time, so a precise selection of the exact optimum averaging time is not critical. Based on these results, the following linear averaging times T are recommended for computing the maximum sound pressure levels during lift-off:

Titan IV - T = 1 sec for the overall and all 1/3 octave bands above 250 Hz;
T = 2 sec for all 1/3 octave bands at or below 250 Hz.

Space Shuttle - T = 1.5 sec for the overall and all 1/3 octave bands above 250 Hz;
T = 3 sec for all 1/3 octave bands at or below 250 Hz.

If an exponentially weighted average (RC lowpass filter) is used to compute the levels, the RC averaging time constant K should be one-half the recommended linear averaging time T (i.e., K = T/2).

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1. INTRODUCTION

The Jet Propulsion Laboratory (JPL), with the support of the Piersol Engineering Company, is preparing a proposed Military Handbook (MIL-HDBK) on "Guidelines for Dynamic Data Acquisition and Analysis" [1]. This Handbook includes a separate appendix that covers recommended procedures for the spectral analysis of the nonstationary aeroacoustic and vibration data routinely measured during the launch of space vehicles. The spectral analysis procedures recommended in [1] are designed to yield accurate time-averaged estimates of the "maximax" spectra for the aeroacoustic and vibration data measured during those launch events that produce the maximum high frequency dynamic loads (lift-off, transonic flight, and maximum dynamic pressure flight). This report is concerned with the development of the procedures in [1] for the analysis of the acoustic levels measured inside the Titan IV payload fairing (PLF) and the Space Shuttle orbiter payload bay (PLB) during lift-off, which usually produce the highest aeroacoustic loads experienced by Titan IV and Space Shuttle payloads during launch. The analysis of vibration measurements during key launch events will be covered in a separate report.

2. BACKGROUND

The launch acoustic environment for the payloads of all launch vehicles, including Titan IV and Space Shuttle, is stochastic and nonstationary in character due to a sequence of time-varying aeroacoustic events that occur during the launch phase. The most important of these events and the nonstationary random excitations they produce are

(a)   the acoustic noise from the rocket motors during lift-off,
(b)   the aerodynamic shock wave-boundary layer interactions during transonic flight, and
(d)   the turbulent aerodynamic boundary layer during flight through maximum dynamic pressure.

Of course, these aeroacoustic loads are applied on the exterior of the launch vehicle structure, and reach the payload either as structureborne noise (mechanical vibrations) transmitted to the payload through its attachment points, or as PLF transmitted acoustic noise radiated into the payload enclosure and impinging directly on the payload surfaces. Experience suggests that the acoustic levels inside the payload enclosure are the dominant source of the payload dynamic loads at frequencies above about 50 Hz. Since the aerodynamic excitations during transonic and maximum dynamic pressure flight occur at relatively high altitudes where the air density is low, the acoustic loading on the payload usually reaches a maximum during lift-off.

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The description of acoustic signals in terms of sound pressure levels (SPLs) in 1/3 octave bands with the center frequencies and bandwidths detailed in Table 1 has become an internationally recognized standard [2]. The 1/3 octave band spectrum for a stationary signal x(t) is defined as

$$L_x(f_i) = 10 \log_{10} \left[\frac{\psi_x(f_i)}{\psi_{ref}} \right]^2 \quad ; i = 1, 2, ... \quad (1)$$

where

f_i = 1/3 octave band center frequency, in Hz (see Table 1)

$L_x(f_i)$ = SPL (in dB) in 1/3 octave band centered at f_i

$\psi_x(f_i)$ = rms value of acoustic pressure (in Pa or psi) in 1/3 octave band centered at f_i
(1 Pa = 1.45×10^{-4} psi)

ψ_{ref} = standard reference SPL = 20 μPa = 2.90×10^{-9} psi

A similar relationship is defined for the overall SPL, L_x, and the overall rms pressure, ψ_x. The subscript x on L and ψ will be omitted henceforth for convenience.

During a space vehicle lift-off, the SPLs in all 1/3 octave bands are varying continuously with time. It is the maximum SPLs measured in the various 1/3 octave bands, independent of when they occur, that are of primary interest. This is true because payload failures due to dynamic loads tend to be highly frequency dependent. A plot of the maximum SPLs in the various 1/3 octave bands, independent of when they occur, is called a "maximax spectrum". This maximax spectrum (with some margin added to assure a conservatism) is commonly used as the criterion for a stationary acoustic test designed to simulate the maximum high frequency (above 50 Hz) dynamic loads experienced by a payload during a space vehicle lift-off. Hence, the accurate estimation of the maximum SPLs in the various 1/3 octave bands during lift-off is an important issue.

Table 1. Center Frequencies and Bandwidths for 1/3 Octave Bands.

Center Freq. (Hz)	Band-width (Hz)	Center Freq. (Hz)	Band-width (Hz)	Center Freq. (Hz)	Band-width (Hz)	Center Freq. (Hz)	Band-width (Hz)
20	4.5	80	18.5	315	75	1250	300
25	5.7	100	22.5	400	92	1600	360
31.5	7.5	125	30	500	113	2000	450
40	9.2	160	36	630	150	2500	570
50	11.3	200	45	800	185	3150	750
63	15.0	250	57	1000	225	4000	920

3. AVERAGING PROCEDURES

The maximum SPL in each 1/3 octave band during a spacecraft lift-off is usually determined by computing a time dependent rms value of the acoustic signal x(t) in each 1/3 octave band during the lift-off event using a "running average". Ideally, the running linear average would be computed using an analog integration device to obtain a continuous rms value estimate for the acoustic pressure given by

$$\widehat{\psi}(t) = \left[\frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x^2(t)\,dt \right]^{\frac{1}{2}} \tag{1}$$

where the hat ($\wedge$) denotes "estimate of", and T is the linear averaging time. Note that the rms value is identified with the time at the middle of the averaging interval. On the other hand, if the linear averaging operation is accomplished by digital techniques, where $x(t) = x(n\Delta t)$; $n = 1, 2, 3, ...,$ then the rms value estimate is computed by a linear average over N data values, as follows:

$$\widehat{\psi}\{[ik+(N/2)]\,\Delta t\} = \left[\frac{1}{N} \sum_{n=ik+1}^{ik+N} x^2(n\Delta t) \right]^{\frac{1}{2}} \tag{2}$$

where $i = 0, 1, 2, ... , (n_s - 1)$, n_s = number of steps, and k = size of the step (the number of data values between each step). For example, if k = 1, a new average is initiated for every new data point (i.e., every Δt sec), producing the closest approximation to a continuous average. If k = N, then a new average is initiated at the end of the previous average (i.e., every $N\Delta t$ sec), producing average values over contiguous segments of the signal. Most data processing is presently accomplished using digital procedures, so the linear averaging procedure defined in Equation (2) is the more commonly used.

It should be mentioned that analog devices often compute an exponentially-weighted average using a simple series resistance (R) - shunt capacitance (C) lowpass filter, which produces a continuous rms value estimate for the acoustic pressure given by [3]

$$\widehat{\psi}(t) = \left[\frac{1}{K} \int_0^t x^2(\tau)\,e^{-(t-\tau)/K}\,d\tau \right]^{\frac{1}{2}} \tag{3}$$

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where K = RC is the averaging time constant. The optimum averaging time derived in the next section is based on a linear average, and will be different if an exponentially-weighted (or any other nonlinear) averaging operation is used.

4. DERIVATION OF OPTIMUM AVERAGING TIME

The problem in practice is to determine an "optimum" averaging time for the computation of the 1/3 octave band levels from which a maximax spectrum will be determined. To evaluate this problem, consider the rms value estimate in Equation (1), which involves two types of errors [4]:

(1) A statistical sampling (random) error due to the finite averaging time T of the analysis. This statistical sampling error would be zero if the signal were deterministic.

(2) A time resolution bias error due to the smoothing of the time dependent characteristics of the signal over the averaging time interval T.

For the case of autospectral density estimates of narrowband random signals (e.g., those measured inside a payload enclosure where there are well-defined acoustic modes), there is a third error, namely, a frequency resolution bias error due to the smoothing of the frequency dependent characteristics of the signal over the frequency resolution bandwidth B [4]. However, for the 1/3 octave band analysis of acoustic signals, the frequency resolution bias error is not relevant since the resolution bandwidth of the analysis is fixed by an accepted standard [2].

The statistical sampling (random) error in the estimate of the rms value of a random signal passed through a 1/3 octave band filter with a bandwidth B_i centered on frequency f_i is given in terms of a normalized standard deviation (coefficient of variation) by [4]

$$\varepsilon_r\left[\hat{\psi}(f_i,t)\right] = \frac{\sigma[\hat{\psi}(f_i,t)]}{\psi(f_i,t)} \approx \frac{1}{2\sqrt{B_iT}} \tag{4}$$

where

$\hat{\psi}(f_i,t)$ = estimate of $\psi(f_i,t)$

$\sigma\left[\hat{\psi}(f_i,t)\right]$ = standard deviation of estimate $\hat{\psi}(f_i,t)$

B_i = bandwidth (in Hz) of 1/3 octave band centered at f_i (see Table 1)

T = averaging time of analysis (in sec)

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It is clear from Equation (4) that the rms value estimates will be least accurate in the lowest 1/3 octave band, since it has the smallest bandwidth B_i.

If the acoustic signal being analyzed is stationary, all 1/3 octave band levels can be computed with any desired degree of accuracy by simply increasing the averaging time T. On the other hand, if the signal is nonstationary, as is true of space vehicle lift-off acoustic data, then increasing T will introduce a time resolution bias error approximated in normalized terms by (see Appendix)

$$\varepsilon_b\left[\widehat{\psi}(f_i,t)\right] = \frac{b[\widehat{\psi}(f_i,t)]}{\psi(f_i,t)} \approx \frac{T^2}{48} \frac{d^2[\psi^2(f_i,t)]/dt^2}{\psi^2(f_i,t)} \tag{5}$$

where

$$b\left[\widehat{\psi}(f_i,t)\right] = E\left[\widehat{\psi}(f_i,t)\right] - \psi(f_i,t) = \text{bias error of estimate } \widehat{\psi}(f_i,t)$$

$d^2[\psi^2(f_i,t)]/dt^2 = $ second derivative of $\psi^2(f_i,t)$ with respect to t

By comparing Equations (4) and (5), it is seen that the computation of the maximum 1/3 octave band levels for nonstationary data requires a careful selection of an appropriate averaging time T that will provide a suitable compromise between the random and bias errors in the results. This is illustrated in Figure 1, which shows step-wise averages of the SPL versus time measured inside the
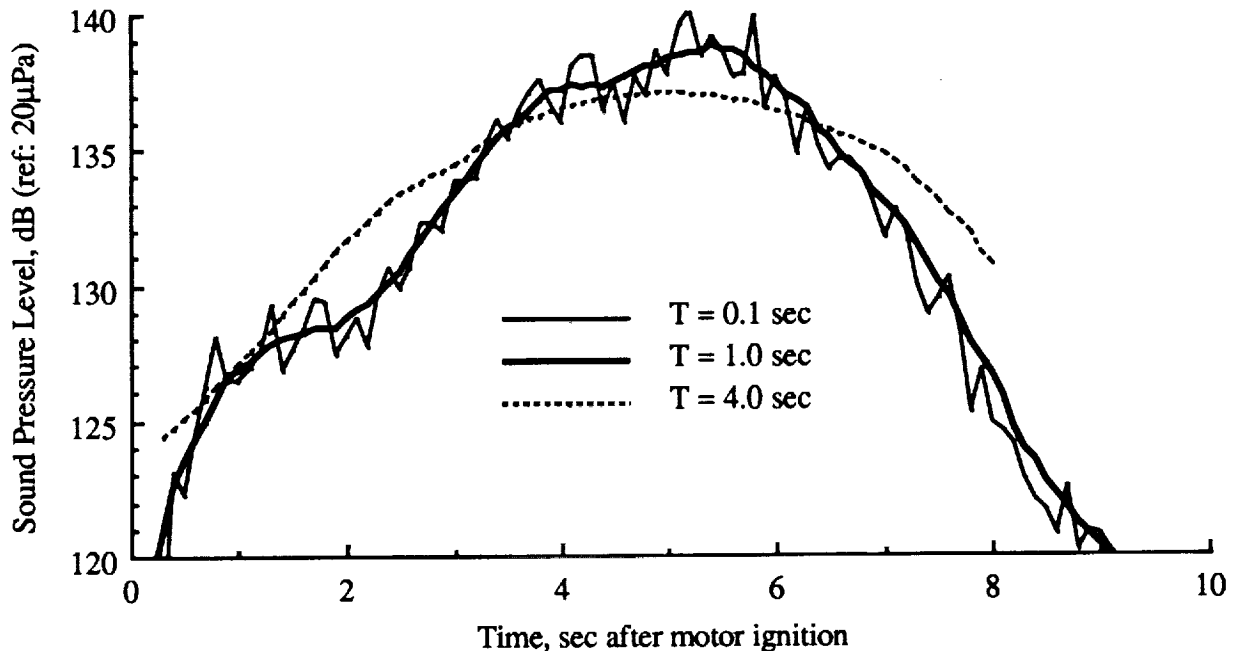
Figure 1. Running Averages of Overall Sound Pressure Level Inside Titan IV PLF During Lift-Off from VAFB.

Titan IV PLF during a typical launch from Vandenburg Air Force Base (VAFB). The step-wise rms computations were performed using Equation (2) with three different linear averaging times, namely, $T = 0.1$, 1.0, and 4.0 sec (in all three cases, the averaging operation was initiated every $k \Delta t = 0.1$ sec). It is clear from the results that the random fluctuations in the estimated SPL versus time decrease as the averaging time increases, as predicted by Equation (4). However, at the highest averaging time, it is also clear that the SPL versus time is being smoothed so as to underestimate the maximum SPL during the lift-off event, as predicted by Equation (5). The problem is to establish an averaging time that will provide an optimum compromise between these two sources of estimation error.

A common method in statistics for optimizing a compromise between random and bias errors in an estimate is to minimize the total mean square error given by

$$\varepsilon^2 = \varepsilon_r^2 + \varepsilon_b^2 \tag{6}$$

From Equations (4) and (5),

$$\varepsilon^2\left[\widehat{\psi}(f_i,t)\right] \approx \frac{1}{4B_iT} + \frac{T^4}{2304} \left[\frac{d^2[\psi^2(f_i,t)]/dt^2}{\psi^2(f_i,t)}\right]^2 \tag{7}$$

Taking the derivative of Equation (7) with respect to T, equating to zero, and solving for T yields

$$T_{oi} \approx \sqrt[3]{\frac{144}{B_i}\left[\frac{d^2[\psi^2(f_i,t)]/dt^2}{\psi^2(f_i,t)}\right]^{-2}} \tag{8}$$

where T_{oi} is the optimum averaging time that minimizes the total mean square error of the estimate.

5. EVALUATIONS OF TITAN IV LIFT-OFF DATA

The Titan IV is launched from two separate facilities, namely, Vandenburg Air Force Base (VAFB) in California and the Kennedy Space Center (KSC) in Florida. The available measurements of the acoustic levels inside the Titan IV PLF during lift-off from VAFB are broadly similar to each other in terms of the rate of change in their overall SPLs with time. The same is true of the overall SPL measured inside the PLF during lit-off from KSC. However, the overall SPLs versus time during lift-off from the two facilities are somewhat different, as illustrated in Figure 2.
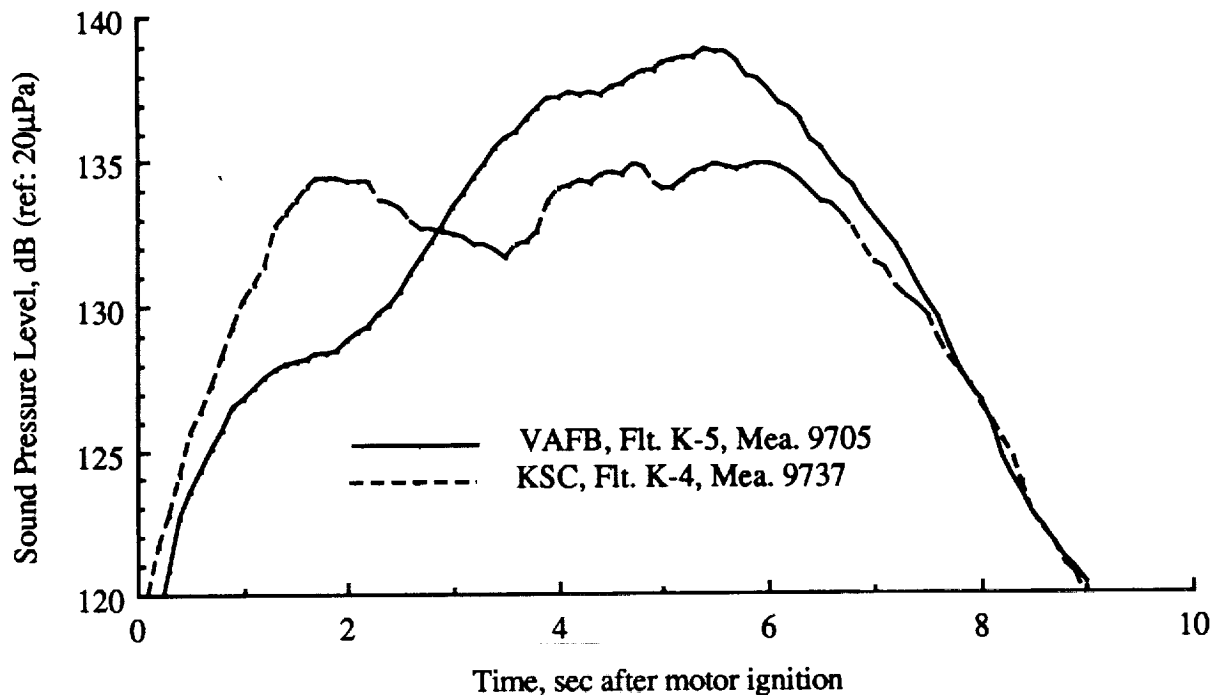
Figure 2. Running Averages of Overall Sound Pressure Levels Inside Titan IV PLF During Lift-Offs from VAFB and KSC, Averaging Time T = 1 sec.

It is seen in Figure 2 that the launches from KSC involve a more rapid increase in overall level and a longer duration near the maximum value than do the launches from VAFB. This is believed to be due to differences in the launch pads and motor exhaust deflectors at the two facilities. Beyond the launch facility effects, the spectra of the acoustic measurements vary somewhat at different locations within the PLF, and for launches with different payload configurations.

To account for the variations in the Titan IV lift-off acoustic levels, a total of eleven measurements made inside the PLF during two launches from KSC and one launch from VAFB were chosen for evaluation, as summarized in Table 2. These particular measurements had previously been evaluated by JPL as part of the derivation of design criteria for a specific JPL payload. The JPL evaluations established that the three acoustic measurements acquired during Flight K-5 from VAFB were generally of good quality, and provided an adequate signal-to-noise ratio in most of the 1/3 octave bands. However, the eight measurements obtained during Flights K-1 and K-4 from KSC were found to include intermittent telemetry noise spikes, particularly during the first 3.5 sec after motor ignition. Care was exercised to omit these identified noise spikes from the various evaluations of the measured acoustic signals from the KSC launches, but there is still a possibility that some of the results computed using the KSC data are influenced by data acquisition noise problems.

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Table 2. Titan IV Acoustic Measurements Selected for Evaluation.

Flight Number	Launch Facility	Water Injection*	Measurement Number	Station Number	Angle (degrees) to Flight Path
K-1	KSC	No	9700	155	90
K-1	KSC	No	9725	155	270
K-4	KSC	Yes	9737	370	350
K-4	KSC	Yes	9738	207	350
K-4	KSC	Yes	9739	207	180
K-4	KSC	Yes	9740	104	350
K-4	KSC	Yes	9741	104	90
K-4	KSC	Yes	9742	104	180
K-5	VAFB	Yes	9705	492	30
K-5	VAFB	Yes	9706	426	30
K-5	VAFB	Yes	9707	248	30

*Water injection for Titan IV is used to attenuate ignition transients, rather than lift-off noise.

All measurements in Table 2 were used to compute a space averaged autospectrum. Also, all measurements were reduced to step-wise linear averages in 1/3 octave bands computed with a T = 0.1 sec averaging time during lift-off, to assist the evaluations of SPL variations with time. However, only two measurements were used to perform the detailed quantitative evaluations of the SPL variations with time, namely (a) Measurement 9705 (Flight K-5) from VAFB, and (b) Measurement 9737 (Flight K-4) from KSC (the two measurements in Figure 2). These two measurements were selected for the quantitative studies because they represented typical launches with water injection at the two facilities, and provided reasonable signal-to-noise ratios.

Measurement 9705 on Flight K-5 from VAFB was made inside the PLF at Station 612, which is 60 inches above the cone-cylinder junction on the fairing. Measurement 9737 on Flight K-4 from KSC was made inside the PLF at Station 370, which is 62 inches below the cone-cylinder junction. These measurements were analyzed by both Martin Marietta Corporation and The Aerospace Corporation. However, only The Aerospace Corporation results were used for these studies because they were available on a compatible digital disk. The Aerospace data consisted of SPLs in 1/3 octave bands computed over T = 0.1 sec contiguous time segments during the lift-off event. The 1/3 octave band center frequencies ranged from f_1 = 20 Hz to f_{24} = 4000 Hz.

5.1 Overall Levels

In addition to the analysis averaging time, the random error in the overall SPL estimates from Equation (4) is a function of the equivalent statistical bandwidth of the overall signal, which in turn is a function of the autospectrum of the signal. The autospectra of the acoustic signals measured inside the PLF during lift-off vary with location inside the PLF, the PLF noise transmission characteristics, and the launch vehicle/facility geometry. Because of these variations, a random error expression is developed for an average autospectrum of the sound measured inside the PLF for launches from both facilities. On the other hand, the time resolution bias error in the overall SPL estimates is a function of the variations in the mean square value of the acoustic signal versus time. Since these variations appear to be different for launches from VAFB and KSC, the bias errors in the analysis of the launch data from the two facilities are evaluated separately.

5.1.1 Statistical Sampling Error

To evaluate the statistical sampling (random) error in the overall SPL estimates, it is necessary to determine an appropriate value for the bandwidth of the measured signal x(t); i.e., a value of bandwidth that makes Equation (4) correct for the overall signal. This desired value of bandwidth is given by the so called "statistical bandwidth", defined as [5]

$$B_s = \frac{\left[\int_0^\infty G_{xx}(f)\,df\right]^2}{\int_0^\infty G_{xx}^2(f)\,df} \tag{9}$$

where $G_{xx}(f)$ is the autospectral density function of x(t). To arrive at the autospectrum, the average of the maximum 1/3 octave band rms acoustic pressures (in Pa) for eleven measurements inside the PLF during lift-off (see Table 2) was converted to an autospectral density function (in Pa^2/Hz). The resulting average autospectrum is shown in Figure 3. The statistical bandwidth of the autospectrum in Figure 3 was then computed using Equation (9) to obtain

$$B_s = \frac{(11010)^2}{378245} \approx 320\,\text{Hz} \tag{10}$$

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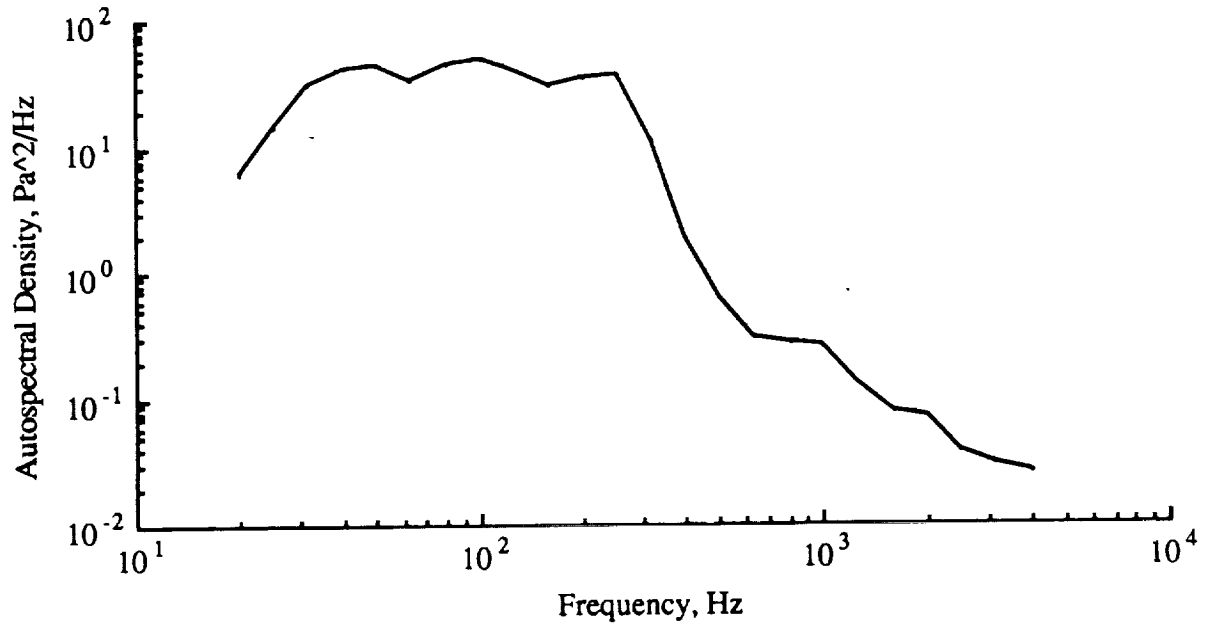
Figure 3. Average Autospectrum of Acoustic Pressures Measured Inside Titan IV PLF During Lift-Off.

The random error in the estimate of the overall rms value as a function of the averaging time T is then given by substituting the statistical bandwidth from Equation (10) into Equation (4) to obtain

$$\varepsilon_r[\hat{\psi}(t)] \approx \frac{0.028}{\sqrt{T}} \qquad (11)$$

For example, if an averaging time of T = 1 sec were used, the standard deviation of an rms value estimate would be about 2.8% of the true rms value of the signal, or about 0.25 dB.

5.1.2 Time Resolution Bias Error for VAFB Data

To evaluate the time resolution bias error in the lift-off rms pressure estimates, it is necessary to determine in Equation (5) the value of $d^2[\psi^2(t)]/dt^2$, which in turn requires a functional representation for the time-varying mean square value $\psi^2(t)$ during the lift-off event. This can be accomplished by curve fitting the mean square pressures computed using a T = 0.1 sec linear average time with a fourth order polynomial function. The results of the curve fit on Measurement 9705 from Flight K-5 over the time interval from 2 to 8 sec after motor ignition are shown in Figure 4. The squared correlation coefficient for the curve fit is $r^2 = 0.83$ (or r = 0.91), which constitutes a relatively good fit [4].

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$$\psi^2(t) = 139{,}850 - 155{,}780\,t + 59{,}912\,t^2 - 8{,}852.4\,t^3 + 441.08\,t^4$$

Figure 4. Curve Fit to Mean Square Estimates of Overall Acoustic Pressure During Titan IV Lift-Off from VAFB (Flight K-5, Measurement 9705).

The second derivative of the polynomial function in Figure 4 is computed to be

$$d^2[\psi^2(t)]/dt^2 = 119{,}800 - 53{,}110\,t + 5{,}293\,t^2 \tag{12}$$

Since it is an estimate of the maximum overall SPL during lift-off that is ultimately desired, the primary interest is in the time resolution bias error when the mean square value $\psi^2(t)$ is a maximum. Taking the derivative of the polynomial function in Figure 4 and equating to zero, it is found that $\psi^2(t)_{max}$ occurs at $t = 5.00$ sec. Not surprisingly, the second derivative function in Equation (12) reaches a maximum at essentially the same time, namely $t = 5.02$ sec. When the mean square value is a maximum,

$$\psi^2(t)_{max} = 27{,}875 \text{ Pa}^2 \text{ [or 138.4 dB (ref: 20}\mu\text{Pa)]}; \quad d^2[\psi^2(t)]/dt^2_{max} = -13{,}420 \text{ Pa}^2/\text{sec}^2 \tag{13}$$

Substituting the values from Equation (13) into Equation (5), the normalized time resolution bias error in the estimate of the overall SPL at its maximum is determined to be

$$\varepsilon_b[\widehat{\psi}(t)] \approx -0.010\,T^2 \tag{14}$$

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For example, if the averaging time were T = 1 sec, the time resolution bias of the maximum rms value estimate would be about -1.0% (or - 0.09 dB) below the true maximum rms value of the signal (the minus sign means that the maximum rms value is underestimated).

5.1.3 Time Resolution Bias Error for KSC Data

It is seen from Figure 2 that the overall level inside the PLF during lift-off from KSC appears to pass through two maxima of nearly equal magnitude, the first at about 1.5 to 2 sec and the second at about 5 to 6 sec after motor ignition. However, a review of all the acoustic measurements made inside the PLF during lift-off from KSC (see Table 2) indicate the overall level maximum at about 5 to 6 sec after motor ignition tends to be dominant. Hence, attention is restricted to the overall SPL around this second maximum.

As in Section 5.1.2, to determine the maximum value of $d^2[\psi^2(t)]/dt^2$ for the overall rms pressures inside the PLF during lift-off from KSC, the step-wise mean square values computed using a T = 0.1 sec averaging time are curve fitted with a fourth order polynomial. The results are shown in Figure 5. Note that the curve fitting operation is limited to the mean square values measure between 3.5 and 8 sec after motor ignition to eliminate the influence of the first maximum in the overall level discussed above.

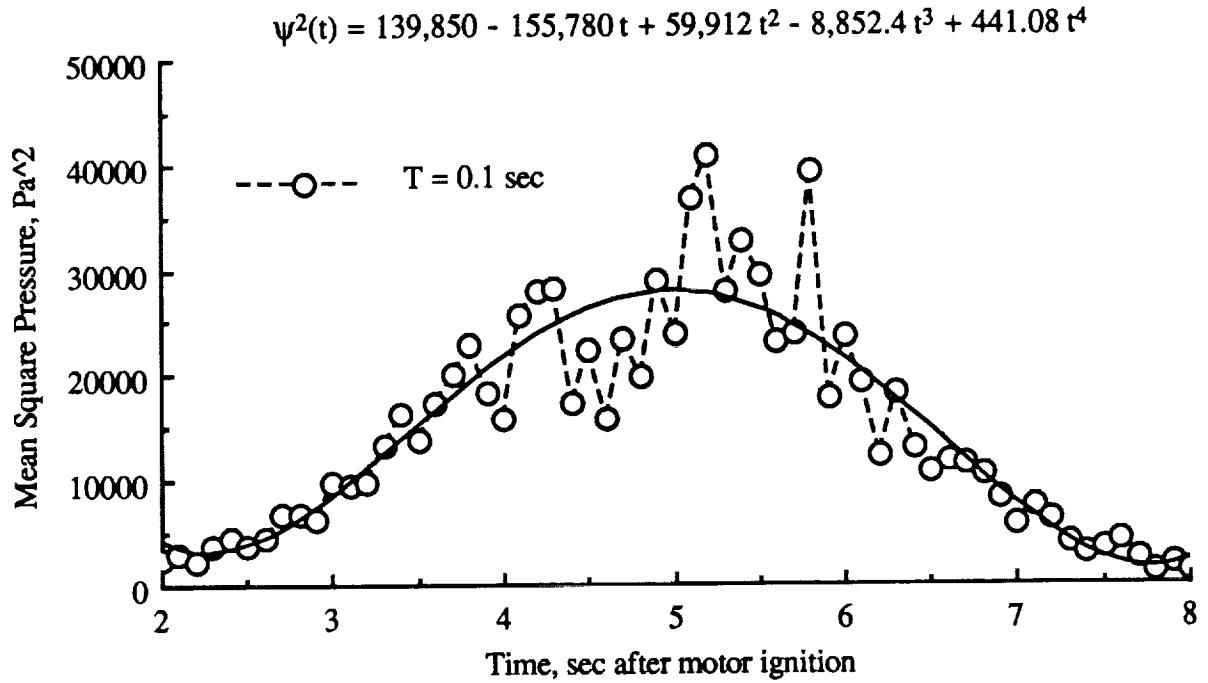$$\psi^2(t) = 121,100 - 108,070\,t + 35,044\,t^2 - 4,615.3\,t^3 + 211.26\,t^4$$
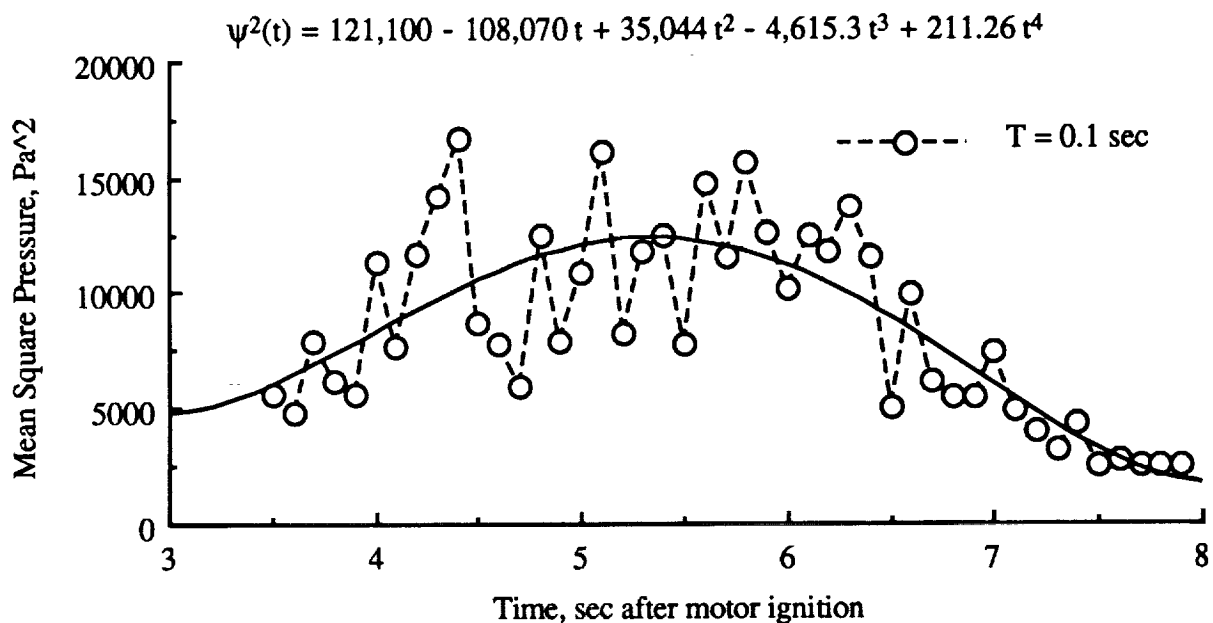
Figure 5.   Curve Fit to Mean Square Estimates of Overall Acoustic Pressure During Titan IV Lift-Off From KSC (Flight K-4, Measurement 9737).

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The squared correlation coefficient for the curve fit to the KSC lift-off data in Figure 5 is $r^2 = 0.63$ (or $r = 0.79$), which is somewhat less than the squared correlation coefficient of $r^2 = 0.83$ provided by the curve fit to the VAFB lift-off data in Figure 4, but still acceptably strong.

It should be mentioned that the KSC data were edited prior to the curve fitting operation to remove an unusually large mean square pressure value at 4.3 sec after motor ignition, which was believed to represent a noise spike. The suspicious value at 4.3 sec after motor ignition was replaced by a linear interpolation between the values computed 0.1 sec before and after 4.3 sec, but there is a possibility that one or both of these adjacent values were also contaminated by noise. To evaluate this potential problem, curve fitting operations were performed on various data configurations, included the unedited data, and very little difference was observed in the resulting time resolution bias error value.

Following the procedure in Section 5.1.2, the second derivative of the polynomial function in Figure 5 is computed to be

$$d^2[\psi^2(t)]/dt^2 = 70,090 - 27,691\,t + 2,535.1\,t^2 \tag{15}$$

Taking the derivative of the polynomial function in Figure 5 and equating to zero, it is found that $\psi^2(t)_{max}$ occurs at $t = 5.35$ sec. However, the second derivative function in Equation (15) reaches a maximum at about the same time, namely $t = 5.46$ sec, probably because of the distortion of the polynomial curve fit caused by the unexplained peak in the running average data at 4.3 sec after motor ignition. Assuming that the maximum value of the second derivative in Equation (15) actually occurs at the same time as the maximum SPL,

$$\psi^2(t)_{max} = 12,297 \text{ Pa}^2 \text{ [or } 134.9 \text{ dB (ref: } 20\mu\text{Pa})]; \quad d^2[\psi^2(t)]/dt^2_{max} = -5,524 \text{ Pa}^2/\text{sec}^2 \tag{16}$$

and the maximum time resolution bias error in the estimation of the overall SPL during lift-off from KSC is approximated from Equation (5) to be

$$\varepsilon_b[\hat{\psi}(t)] = -0.0094\,T^2 \tag{17}$$

The estimated error in Equation (17) is close enough to the value of $-0.01\,T^2$ computed for the VAFB lift-off data in Section 5.1.2 to assume that Equation (14) applies to Titan IV launches from either KSC or VAFB.

5.1.4 Averaging Time for Minimum Mean Square Error

From Equations (7), (11), and (14), the mean square error for estimates of the maximum SPL inside the PLF during a Titan IV lift-off from either VAFB or KSC is

$$\varepsilon^2[\hat{\psi}(t)] = \varepsilon_r^2[\hat{\psi}(t)] + \varepsilon_b^2[\hat{\psi}(t)] \approx \frac{7.8 \times 10^{-4}}{T} + 1.0 \times 10^{-4} T^4 \tag{18}$$

From Equation (8), the optimum averaging time to minimize the mean square error in Equation (18) is $T_0 = 1.14$ sec, giving a minimum rms error (the positive square root of the mean square error) for estimates of the maximum overall SPL of $\varepsilon[\hat{\psi}(t)]_{min} \approx 0.029$. Plots of the normalized random error, bias error, and rms error for various values of the averaging time T are shown in Figure 6. It is seen in Figure 6 that the rms error indeed reaches a minimum value of $\varepsilon = 0.029$ (or about 0.25 dB) at $T_0 = 1.14$ sec, as predicted by Equation (18), but is less than 0.036 (about 0.31 dB) for all averaging times between $T = 0.6$ and $T = 1.7$ sec. Hence, any averaging time selected within about $\pm 50\%$ of $T_0 = 1.14$ sec would provide an SPL estimate with an rms error within 25% of the minimum. However, the rms error increases rapidly as the averaging time moves below or above this range.
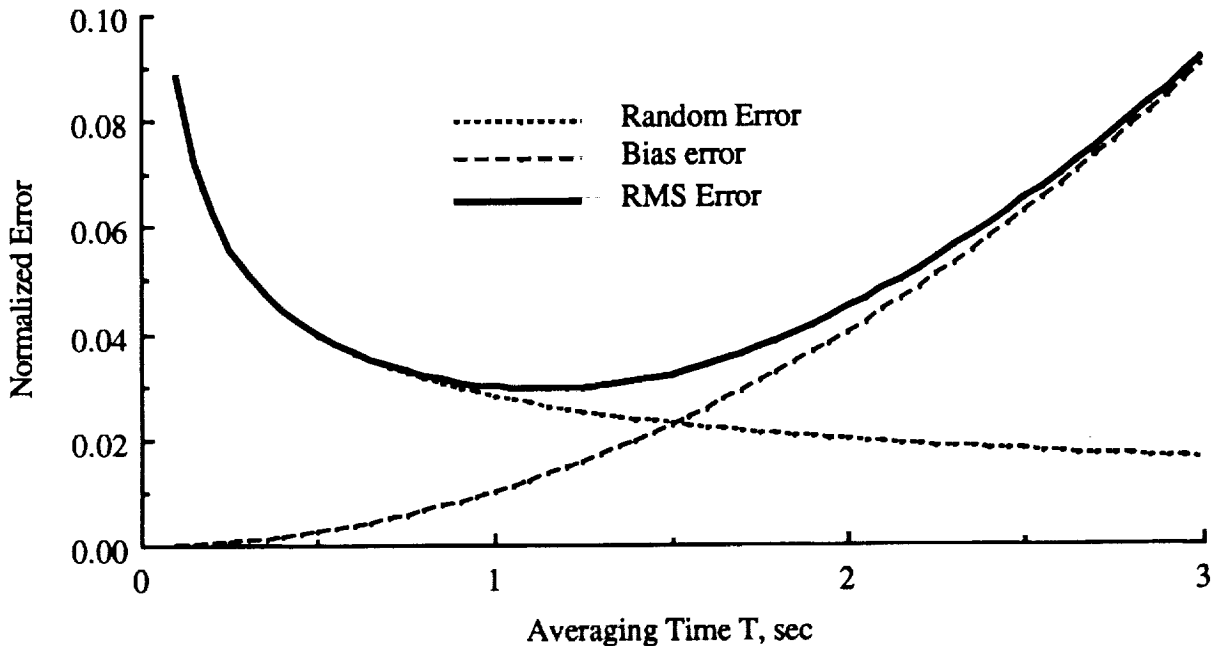

Figure 6. Normalized Errors Versus Averaging Time for Estimates of Maximum Overall Sound Pressure Level Inside Titan IV PLF During Lift-Off.

5.1.5 Smoothness of Overall Sound Pressure Level Variations with Time

Referring back to Figure 1, it is seen that the overall SPL inside the PLF during lift-off from VAFB, when computed with a linear averaging time of T = 0.1 sec, displays rapid variations with time, particular in the region between 4 and 6 sec after motor ignition where the level is passing through a maximum. It is also seen in this figure that the variations are smoothed out by an analysis with an averaging time near the optimum T = 1.14 sec determined in Section 5.1.4. The issue is whether the time variations seen in the SPLs computed with the T = 0.1 sec averaging time are physically meaningful, or the result of normal statistical sampling errors in the level estimates. To evaluate this matter, Measurement 9705 from Flight K-5 launched from VAFB is used because there is a high confidence that the data from Flight K-5 do not include intermittent noise spikes that would invalidate the analysis.

To proceed, let it be hypothesized that the variation in the true SPL versus time during lift-off is smooth, as represented by a fourth order polynomial curve fit to the mean square pressures computed with the T = 0.1 sec averaging time. If the time variations in the SPL estimates with the T = 0.1 sec averaging time are simply random estimation errors, then the vast majority of the estimates should fall within a 99% probability interval about the polynomial curve fit. To establish a 99% probability interval for the estimates, rather than use the approximate error expression in Equation (4), the more exact chi-square distribution for variances (equal to mean square values since the mean value is zero) is used. Specifically, from [4],

$$\frac{\psi^2 \chi^2_{n;0.005}}{n} < \hat{\psi}^2 \leq \frac{\psi^2 \chi^2_{n;0.995}}{n} \tag{19}$$

where

ψ^2 = hypothesized variance (polynomial curve fit)

$\hat{\psi}^2$ = estimated variance (T = 0.1 sec averaging time)

$\chi^2_{n;0.005}$ = 0.005 percentile of chi-square with n degrees-of-freedom

$\chi^2_{n;0.995}$ = 0.995 percentile of chi-square with n degrees-of-freedom

$n \approx 2B_s T$

From Equation (10), B_s = 320 Hz. Hence, for T = 0.1 sec, it follows that n ≈ 64. From any table of chi-square distribution values (e.g., [4]), $\chi^2_{64;0.005} \approx 39$ and $\chi^2_{64;0.995} \approx 97$. Substituting these values into Equation (19), the 99% probability interval for the variance of the pressures computed with an averaging time of T = 0.1 sec is as plotted in Figure 7.

15
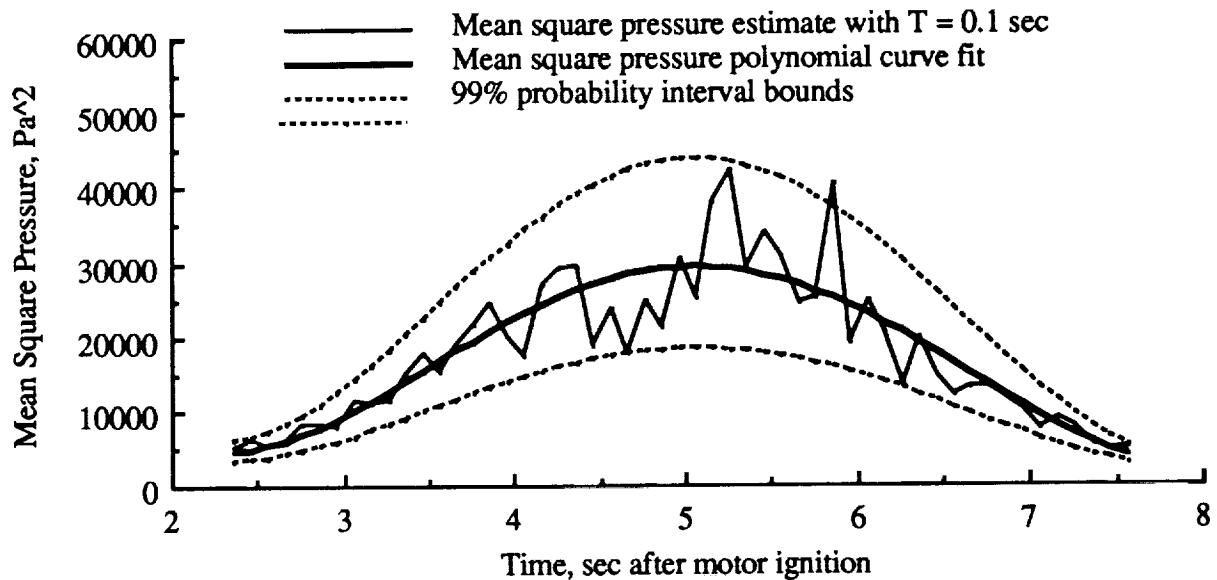
Figure 7. Error Bounds on Mean Square Estimates of Overall Acoustic Pressure During Titan IV Lift-Off from VAFB (Flight K-5, Measurement 9705).

Note in Figure 7 that the mean square pressure estimates at all times (with only one exception) fall within the 99% probability interval, meaning there is no reason to question the variation of the actual mean square pressure versus time is smooth; i.e, it can be assumed that the deviations from the polynomial curve fit by the levels estimated with an averaging time of T = 0.1 sec are due to statistical sampling errors. It follows that there is no statistically significant reason to believe that the estimates computed with an averaging time of T ≈ 1.14 sec will smooth through physically significant variations in the lift-off data.

A plot of the overall SPL in dB (ref: 20μPa) during lift-off computed with the averaging time of T = 1.1 sec (the closest averaging time to T_0 = 1.14 sec that could be achieved) is shown in comparison to the polynomial curve fit in Figure 8. Note that the maximum estimated overall SPL occurs slightly later than the maximum of the curve fit, but the values at the maxima agree to within 0.2 dB (a discrepancy of less than 2%).

It should be mentioned that the results in Figure 8 suggest that an optimum analysis of Titan IV lift-off acoustic data could be accomplished by a polynomial curve fit to the squared pressure values, rather than by a step-wise (or running) average of the squared pressures with the derived optimum averaging time. However, most data analysis facilities are better equipped to compute step-wise averages rather than polynomial curve fits.

16
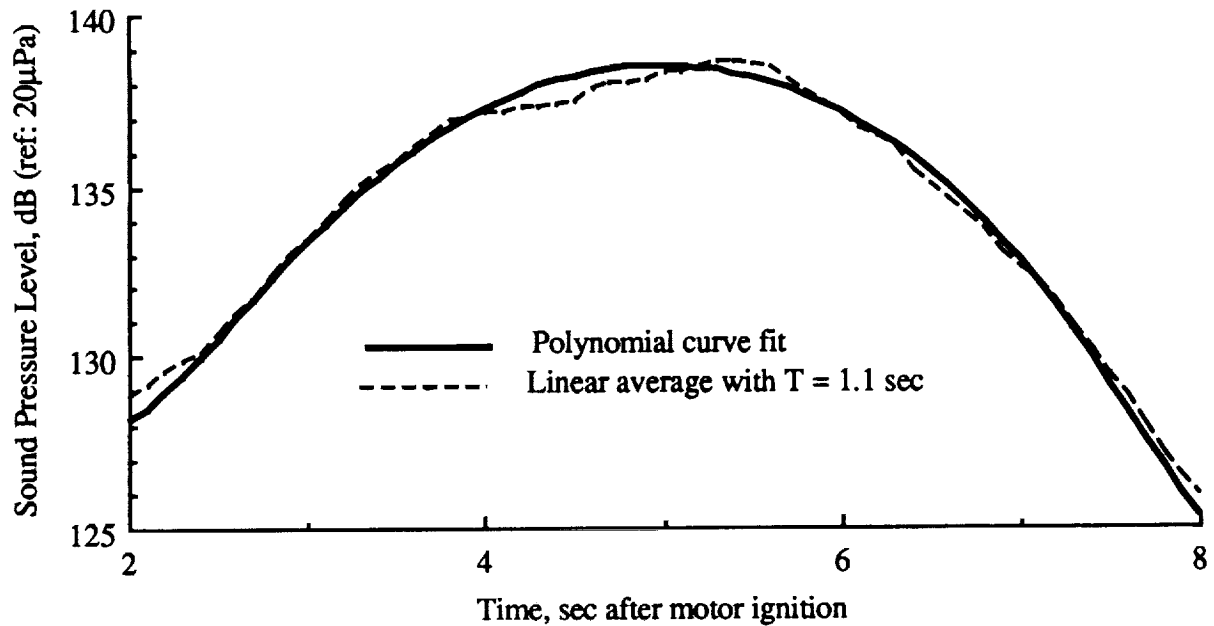
Figure 8. Overall Sound Pressure Level Estimates During Titan IV Lift-Off from VAFB (Flight K-5, Measurement 9705).

5.2 1/3 Octave Band Levels

The SPLs in the various 1/3 octave bands measured during a Titan IV lift-off have different statistical bandwidths from the overall data. Hence, the random sampling errors for the 1/3 octave band level estimates will be different from those shown in Figure 6. The largest random error in the 1/3 octave band data would be expected in the lowest frequency band (centered at $f_1 = 20$ Hz for the Titan IV data), because it has the smallest bandwidth (B = 4.5 Hz from Table 1). On the other hand, the smallest random error should be in the highest frequency band ($f_{24} = 4000$ Hz for the Titan IV data), which has the widest bandwidth (B = 920 Hz).

Concerning the time resolution bias error, it might be anticipated that the variations of the SPLs with time in the various 1/3 octave bands are similar to the time variations of the overall level. If so, the time resolution bias error for the overall level estimates in Figure 6 would apply to the 1/3 octave band level estimates as well. A qualitative review of the step-wise average SPLs computed with T = 0.1 sec in the various 1/3 octave bands for various measurements on flights from both VAFB and KSC indicates the time variations of the 1/3 octave band levels are consistent with those determined for the overall in Sections 5.1.2 and 5.1.3. Nevertheless, this qualitative conclusion is quantitatively verified using selected 1/3 octave band data from Measurement 9705 on Flight K-5 launched from VAFB, and Measurement 9737 on Flight K-4 launched from KSC.

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5.2.1 Statistical Sampling Error

As a first order of approximation, assume the autospectrum of the sound pressure in each 1/3 octave band is a constant (i.e., white noise). From Equation (9), the statistical bandwidth of the signal in each 1/3 octave band is then equal the bandwidth of the 1/3 octave band filter. Consider three 1/3 octave bands, namely, the lowest frequency band centered at 20 Hz, an intermediate frequency band centered at 250 Hz, and the highest frequency band centered at 4000 Hz. From Table 1, $B_1 \approx 4.5$ Hz for the band centered at $f_1 = 20$ Hz, $B_{12} \approx 57$ Hz for the band centered at $f_{12} = 250$ Hz, and $B_{24} \approx 920$ Hz for the band centered at $f_{24} = 4000$ Hz.. It follows from Equation (4) that the normalized random errors for estimates in the 20, 250, and 4000 Hz bands are given by

$$\varepsilon_r[\hat{\psi}(20,t)] \approx \frac{0.236}{\sqrt{T}} \quad ; \quad \varepsilon_r[\hat{\psi}(250,t)] \approx \frac{0.0662}{\sqrt{T}} \quad ; \quad \varepsilon_r[\hat{\psi}(4000,t)] \approx \frac{0.0165}{\sqrt{T}} \tag{20}$$

Comparing the result for the 4000 Hz band to the result for the overall in Equation (11), it is seen that random error in the 4000 Hz band is predicted to be less than the random error in the overall. This is due to the fact that the autospectrum of the overall varies dramatically with frequency, while the autospectrum in the 4000 Hz band is assumed to be constant. Of course, the actual autospectra of the acoustic pressures within the various 1/3 octave band are probably not constant, particularly at the higher center frequencies, meaning the statistical bandwidths of the acoustic pressures are undoubtedly less than the half-power point bandwidths of the 1/3 octave band filters. However, because the optimum averaging time in Equation (8) is so insensitive to the value of the signal bandwidth (it is inversely proportional to the one-fifth power of bandwidth), the assumption of a uniform autospectrum within each 1/3 octave band is considered an acceptable approximation.

5.2.2 Time Resolution Bias Error for VAFB Data

Following the procedure in Section 5.1.2, a fourth order polynomial is fit to the mean square values computed in the 1/3 octave bands centered at 20, 250, and 4000 Hz using a step-wise linear average with $T = 0.1$ sec. The resulting curves fits over the time interval from 2 to 8 sec after motor ignition are shown in Figure 9. It is seen in Figure 9 that the squared correlation coefficient for the curve fit in the 20 Hz band is a weak $r^2 = 0.19$ (or $r = 0.44$), reflecting the high random error for the estimates in this band. On the other hand, in the 4000 Hz band, the squared correlation coefficient is a stronger $r^2 = 0.74$ (or $r = 0.86$), and would be even larger except for the curious peak in this band at 5.1 sec.

$$\psi^2(20,t) = 273.09 - 314.90\,t + 129.68\,t^2 - 20.571\,t^3 + 1.1050\,t^4$$

(a) 20 Hz 1/3 Octave Band
- - -O- - · T = 0.1 sec

$$\psi^2(250,t) = -3,086.4 - 1,015.6\,t + 1,874.3\,t^2 - 402.85\,t^3 + 23.916\,t^4$$

(b) 250 Hz 1/3 Octave Band
- - -O- - · T = 0.1 sec

$$\psi^2(4000,t) = 101.07 - 89.455\,t + 34.889\,t^2 - 5.1871\,t^3 + 0.25892\,t^4$$

(c) 4000 Hz 1/3 Octave Band
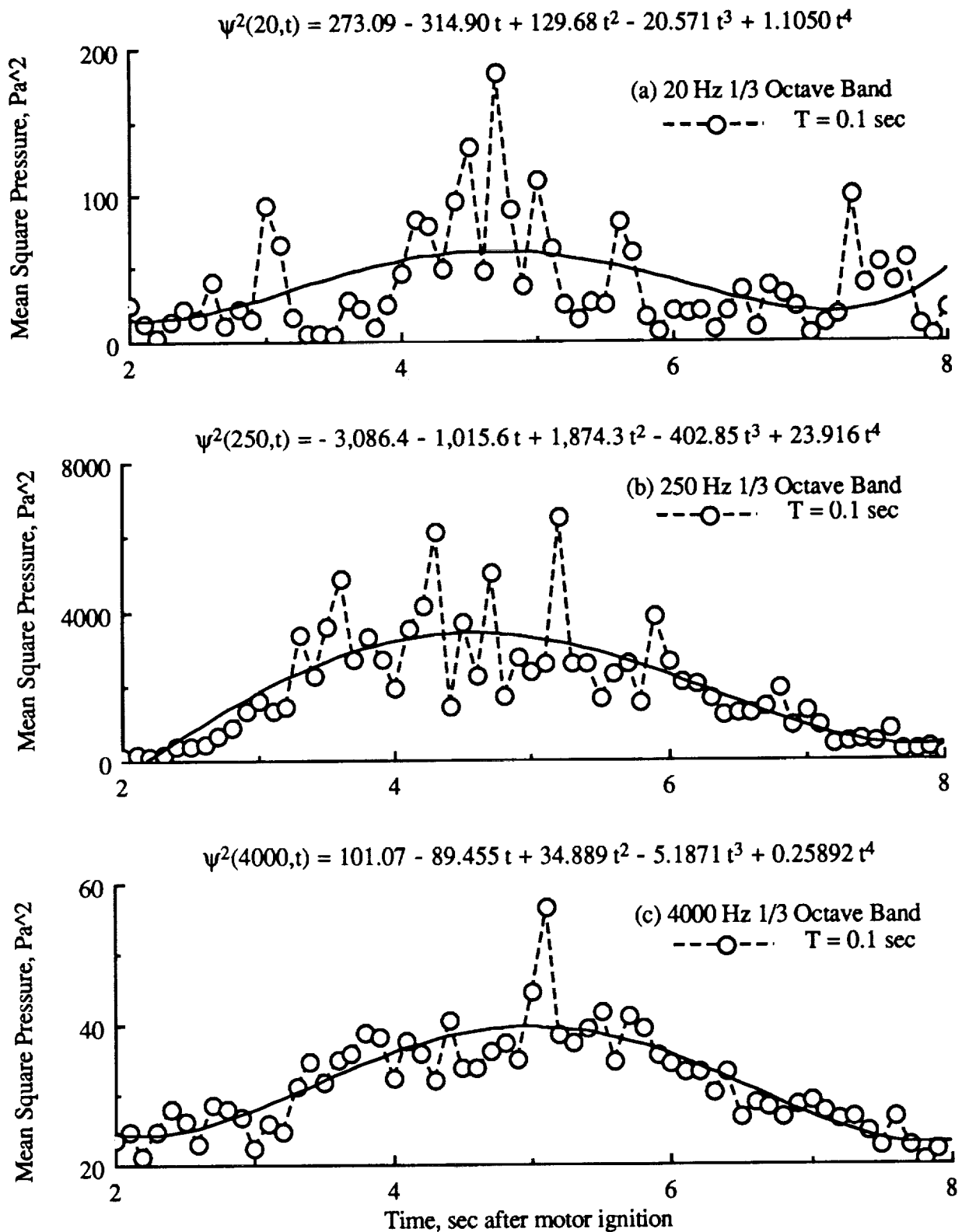- - -O- - · T = 0.1 sec

Time, sec after motor ignition

Figure 9. Curve Fits to Mean Square Estimates of Acoustic Pressures in Selected 1/3 Octave Bands During Titan IV Lift-Off from VAFB (Flight K-5, Measurement 9705).

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20 Hz 1/3 Octave Band

For the 20 Hz band data, the second derivative of the polynomial function in Figure 9(a) is

$$d^2[\psi^2(20,t)]/dt^2 = 259.4 - 123.4\,t + 13.26\,t^2 \ Pa^2/sec^2 \tag{21}$$

The maximum value of this second derivative occurs at $t = 4.65$ sec after motor ignition, while the maximum value of the polynomial function occurs at $t = 4.70$ sec. The values of the polynomial function and its second derivative at $t = 4.7$ sec are

$$\psi^2(20,t)_{max} = 61.15 \ Pa^2 \ [\text{or } 111.8 \ dB \ (\text{ref: } 20 \ \mu Pa)]; \quad d^2[\psi^2(20,t)]/dt^2{}_{max} = -27.67 \ Pa^2/sec^2 \tag{22}$$

Substituting the values from Equation (22) into Equation (5) yields the time resolution bias error for estimates of the maximum SPL in the 1/3 octave band centered at 20 Hz as

$$\varepsilon_b[\hat{\psi}(20,t)] \approx -0.0094\,T^2 \tag{23}$$

250 Hz 1/3 Octave Band

For the 250 Hz band data, the second derivative of the polynomial function in Figure 9(b) is

$$d^2[\psi^2(250,t)]/dt^2 = 3,749 - 2,417\,t + 287.0\,t^2 \ Pa^2/sec^2 \tag{24}$$

The maximum value of this second derivative occurs at $t = 4.21$ sec after motor ignition, while the maximum value of the polynomial function occurs at $t = 4.57$ sec. The values of the polynomial function and its second derivative at $t = 4.5$ sec are

$$\psi^2(250,t)_{max} = 3,399 \ Pa^2 \ [\text{or } 129.3 \ dB \ (\text{ref: } 20 \ \mu Pa)]; \quad d^2[\psi^2(250,t)]/dt^2{}_{max} = -1,303 \ Pa^2/sec^2 \tag{25}$$

Substituting the values from Equation (25) into Equation (5) yields the time resolution bias error for estimates of the maximum SPL in the 1/3 octave band centered at 250 Hz as

$$\varepsilon_b[\hat{\psi}(250,t)] \approx -0.0080\,T^2 \tag{26}$$

4000 Hz 1/3 Octave Band

For the 4000 Hz band data, the second derivative of the polynomial function in Figure 9(c) is

$$d^2[\psi^2(4000,t)]/dt^2 = 69.78 - 31.12\,t + 3.107\,t^2 \; Pa^2/sec^2 \tag{27}$$

The maximum value of this second derivative occurs at t = 5.01 sec after motor ignition, while the maximum value of the polynomial function occurs at t = 4.98 sec. The values of the polynomial function and its second derivative at t = 5.0 sec are

$$\psi^2(4000,t)_{max} = 39.46 \; Pa^2 \; [\text{or } 109.9 \; dB \; (\text{ref: } 20 \; \mu Pa)]; \; d^2[\psi^2(4000,t)]/dt^2_{max} = -8.14 \; Pa^2/sec^2 \tag{28}$$

Substituting the values from Equation (28) into Equation (5) yields the time resolution bias error for estimates of the maximum SPL in the 1/3 octave band centered at 4000 Hz as

$$\epsilon_b[\hat{\psi}(4000,t)] \approx -0.0043\,T^2 \tag{29}$$

Comparing the results in Equations (23), (26), and (29) with the computed time resolution bias error for the overall value in Equation (14), it is seen that the errors for the signals in the 20 and 250 Hz bands are similar to the error for the overall. On the other hand, the error for the signal in the 4000 Hz band is less than half the error for the overall. This reduction in the indicated error at 4000 Hz is believed to be due to the poor signal-to-noise ratio in this band (the maximum SPL is only about 3 db above the instrumentation noise floor), which smooths the indicated variations of the SPL with time. Hence, it will be assumed that the time resolution bias error for the overall value given by Equation (14) and shown in Figure 6 applies to all the 1/3 octave band SPL measurements made inside the Titan IV PLF during lift-off from VAFB.

5.2.3 Time Resolution Bias Error for KSC Data

It is well known that inside payload enclosures during lift-off, 1/3 octave band acoustic and vibration levels at different center frequencies commonly reach maxima at different times [6]. However, for the Titan IV launches from KSC (Flights K-1 and K-4), this appears to occur in an extreme manner; i.e., some measurements (including 9737) show the 1/3 octave band SPLs at center frequencies below 100 Hz reaching maxima as early as 1.5 sec after motor ignition, while the higher

frequency levels reach maxima as late as 6 sec after motor ignition. This wide variation in the times that the 1/3 octave band levels appear to reach their maxima during lift-off from KSC is not fully understood at this time, but is probably related to the launch pad and motor exhaust defector configuration at this facility. To evaluate a typical time resolution bias error for 1/3 octave band estimates during lift-off from KSC, the step-wise average (T = 0.1 sec) SPL levels computed in the 1/3 octave band centered at 250 Hz are curve fitted. This 1/3 octave band produces the highest levels during lift-off. As for the evaluation of the overall levels in Section 5.1.3, only the levels computed during the time interval between 3.5 and 8.0 sec after motor ignition are used for the curve fit. The results are shown in Figure 10.

Following the analysis procedure in Section 5.2.2, the second derivative of the polynomial function in Figure 10 is

$$d^2[\psi^2(250,t)]/dt^2 = 11,685 - 4,847\,t + 453.7\,t^2 \ Pa^2/sec^2 \tag{30}$$

$$\psi^2(250,t) = 16,079 - 16,395\,t + 5,842.7\,t^2 - 807.78\,t^3 + 37.812\,t^4$$
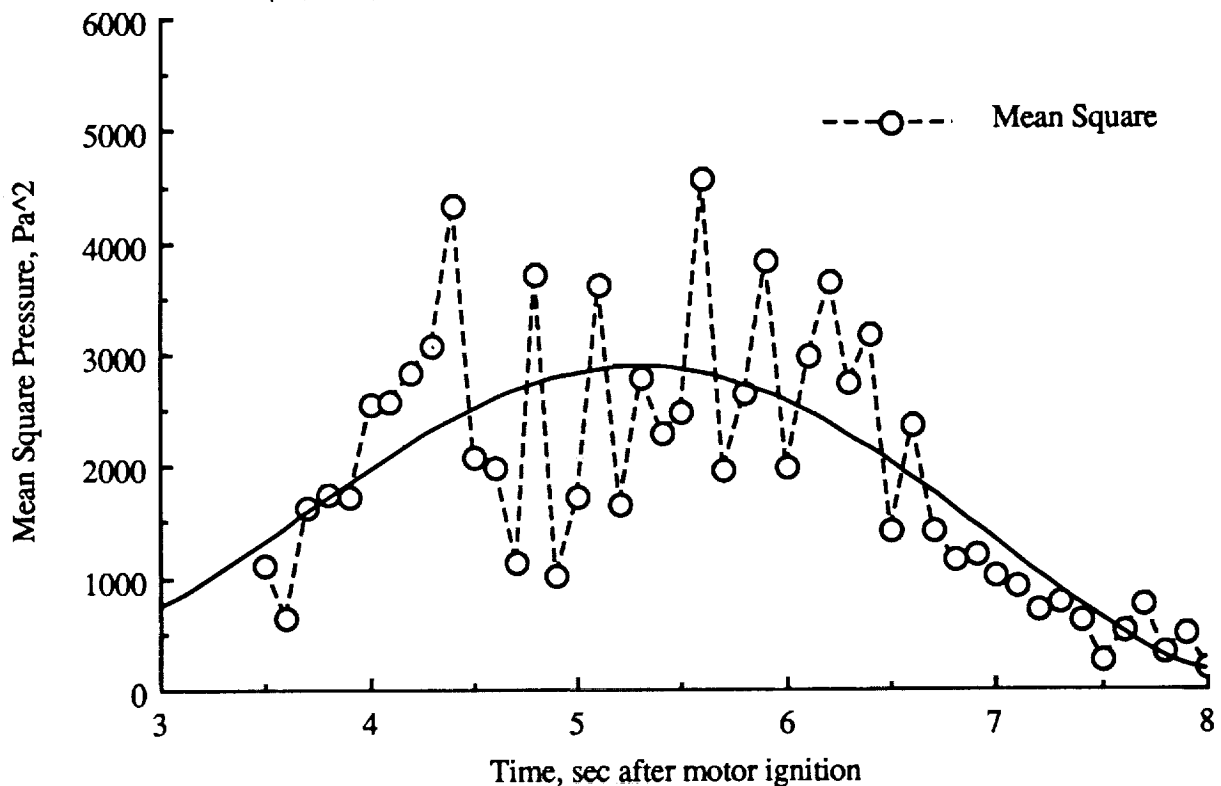

Figure 10. Curve Fit to Mean Square Estimates of Acoustic Pressure in 1/3 Octave Band Centered at 250 Hz During Titan IV Lift-Off from KSC (Flight K-4, Measurement 9737).

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The maximum value of this second derivative occurs at t = 5.34 sec after motor ignition, while the maximum value of the polynomial function occurs at t = 5.29 sec. The values of the polynomial function and its second derivative at t = 5.3 sec are

$$\psi^2(250,t)_{max} = 2883 \ Pa^2 \ [\text{or } 128.6 \ dB \ (\text{ref: } 20 \ \mu Pa)]; \ d^2[\psi^2(250,t)]/dt^2_{max} = -1256 \ Pa^2/sec^2$$

(31)

Substituting the values from Equation (31) into Equation (5) yields the time resolution bias error for estimates of the maximum SPL in the 1/3 octave band centered at 250 Hz as

$$\varepsilon_b[\hat{\psi}(250,t)] \approx -0.0091 \ T^2$$

(32)

Based upon the result in Equation (32), it is considered reasonable to assume that Equation (14) provides an adequate approximation to the time resolution bias error in the estimation of 1/3 octave band SPLs inside the Titan IV PLF during lift-off from KSC.

5.2.4 Averaging Time for Minimum Mean Square Error

Using the random error expressions in Equation (20) and the bias error in Equation (14), the normalized rms errors versus averaging time for the estimation of the maximum SPLs in the 1/3 octave bands centered at 20, 250, and 4000 Hz during Titan IV lift-offs from either VAFB or KSC are computed using the procedures detailed in Section 5.1.4. The results are plotted in Figure 11.
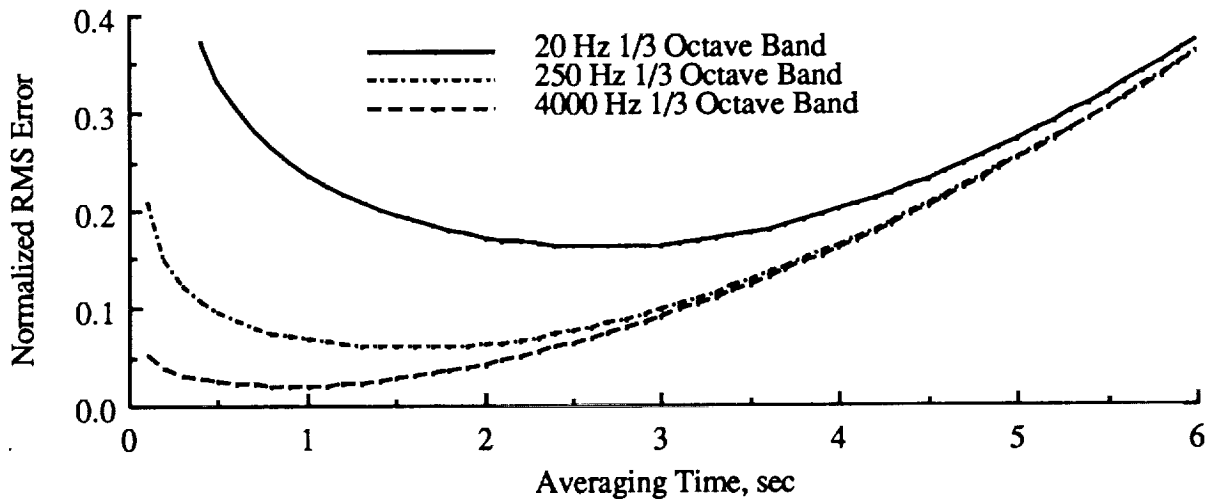

Figure 11. Normalized RMS Errors for Maximum Sound Pressure Level Estimates in 20, 250, and 4000 Hz 1/3 Octave Bands During Titan IV Lift-Off.

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The optimum averaging times for the computation of the maximum SPLs in the 1/3 octave bands centered at 20, 250, and 4000 Hz are computed using Equation (8) to be T_{oi} = 2.68, 1.61, and 0.92 sec, respectively. Using Equation (8) with the bandwidths in Table 1, the optimum averaging times with ± 50% bounds for the estimation of the maximum SPLs in all 1/3 octave bands are plotted in Figure 12. The optimum averaging times and minimum rms errors for the 1/3 octave band estimates are listed in Table 3.

Note from Figure 12 that the optimum averaging time versus 1/3 octave band center frequency plots as a straight line on log-log paper. Hence, it can be described in equation form by

$$T_{oi} = 4.88 \, f_i^{-0.2} \tag{33}$$

where f_i is the center frequency of the ith 1/3 octave band. In all cases, however, a relatively wide range of averaging times will yield an rms error near the minimum value. Specifically, as for the rms error curve for the overall level estimates in Figure 6, the rms errors for the 1/3 octave band estimates fall within 25% of the minimum value for any averaging time within ± 50% of T_{oi}. It follows that the data in all 1/3 octave bands above 200 Hz could be analyzed using the averaging time appropriate for the overall estimate (T ≈ 1.1 sec) with acceptable results. However, in the bands with lower center frequencies where the minimum rms errors are already high, this may not be acceptable.
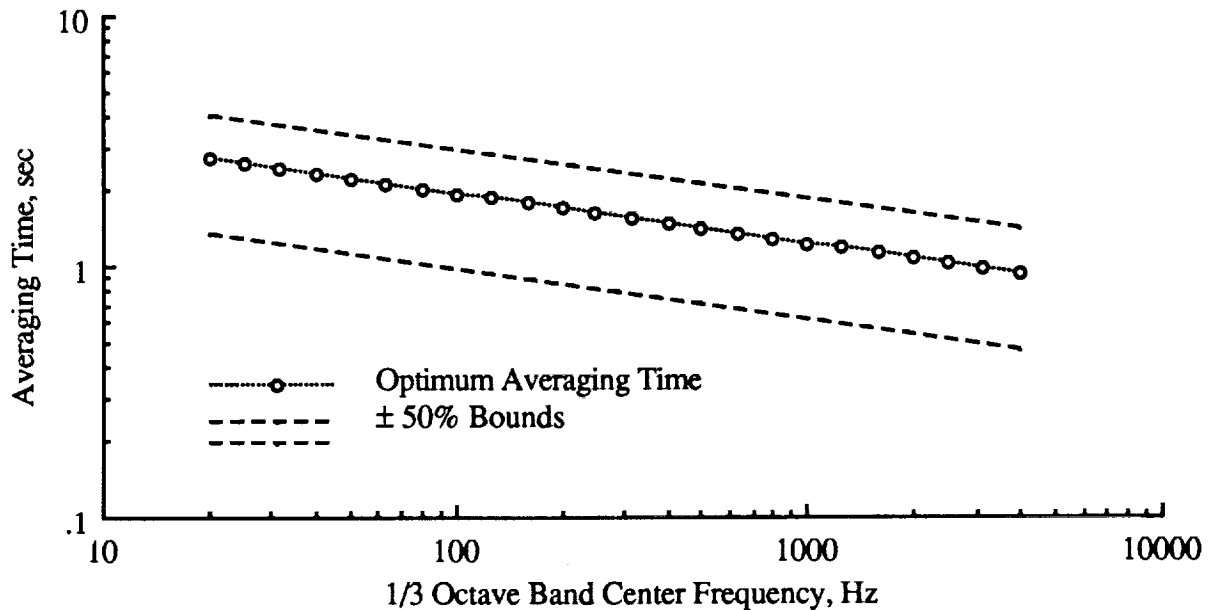

Figure 12. Optimum Averaging Times for Analysis of 1/3 Octave Band Sound Pressure Levels Inside Titan IV PLF During Lift-Off.

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Table 3. Optimum Averaging Times and Minimum Normalized RMS Errors for Analysis of 1/3 Octave Band Sound Pressure Levels Inside Titan IV PLF During Lift-Off.

Center Freq. (Hz)	Band-width (Hz)	Optimum Averaging Time (sec)	Minimum Normalized RMS Error	Center Freq. (Hz)	Band-width (Hz)	Optimum Averaging Time (sec)	Minimum Normalized RMS Error
20	4.5	2.68	0.16	315	75	1.53	0.052
25	5.7	2.56	0.15	400	92	1.47	0.048
31.5	7.5	2.42	0.13	500	113	1.41	0.044
40	9.2	2.32	0.12	630	150	1.33	0.040
50	11.3	2.23	0.11	800	185	1.28	0.037
63	15.0	2.11	0.10	1000	225	1.23	0.034
80	18.5	2.02	0.091	1250	300	1.16	0.030
100	22.5	1.94	0.084	1600	360	1.12	0.028
125	30	1.84	0.075	2000	450	1.07	0.026
160	36	1.77	0.070	2500	570	1.02	0.023
200	45	1.69	0.064	3150	750	0.96	0.021
250	57	1.61	0.058	4000	920	0.92	0.019

To illustrate the error problem in the low frequency bands, the 20 Hz 1/3 octave band signal from Measurement 9705 (Flight K-5) during the Titan IV lift-off from VAFB is analyzed using the optimum averaging time of T = 2.7 sec (the closest averaging time to 2.68 sec that could be achieved), as well as an averaging time of T = 1.1 sec, with the results shown in Figure 13. Also shown in Figure 13 are the basic data computed with the T = 0.1 sec averaging time, and the fourth order polynomial fit to these data.

To interpret the results in Figure 13, it is necessary to make an important assumption, namely, the actual variation in the 20 Hz band SPL with time is relatively smooth, as indicated by the polynomial curve fit; i.e., the fluctuations in the step-wise linear average computed with the T = 0.1 sec averaging time are due solely to random estimation errors, as substantiated for the overall value estimates in Figure 7 (a statistical test similar to that outlined in Section 5.1.5 will easily accept this hypothesis for the 20 Hz band data as well). Under this assumption, the results in Figure 13 indicate the running average with the optimum averaging time of T = 2.7 sec indeed produces a more accurate estimate of the maximum SPL in the 20 Hz band than the T = 1.1 sec averaging time. To be specific, the maximum SPL estimated from the step-wise average with T = 2.7 sec is 111.3 dB,
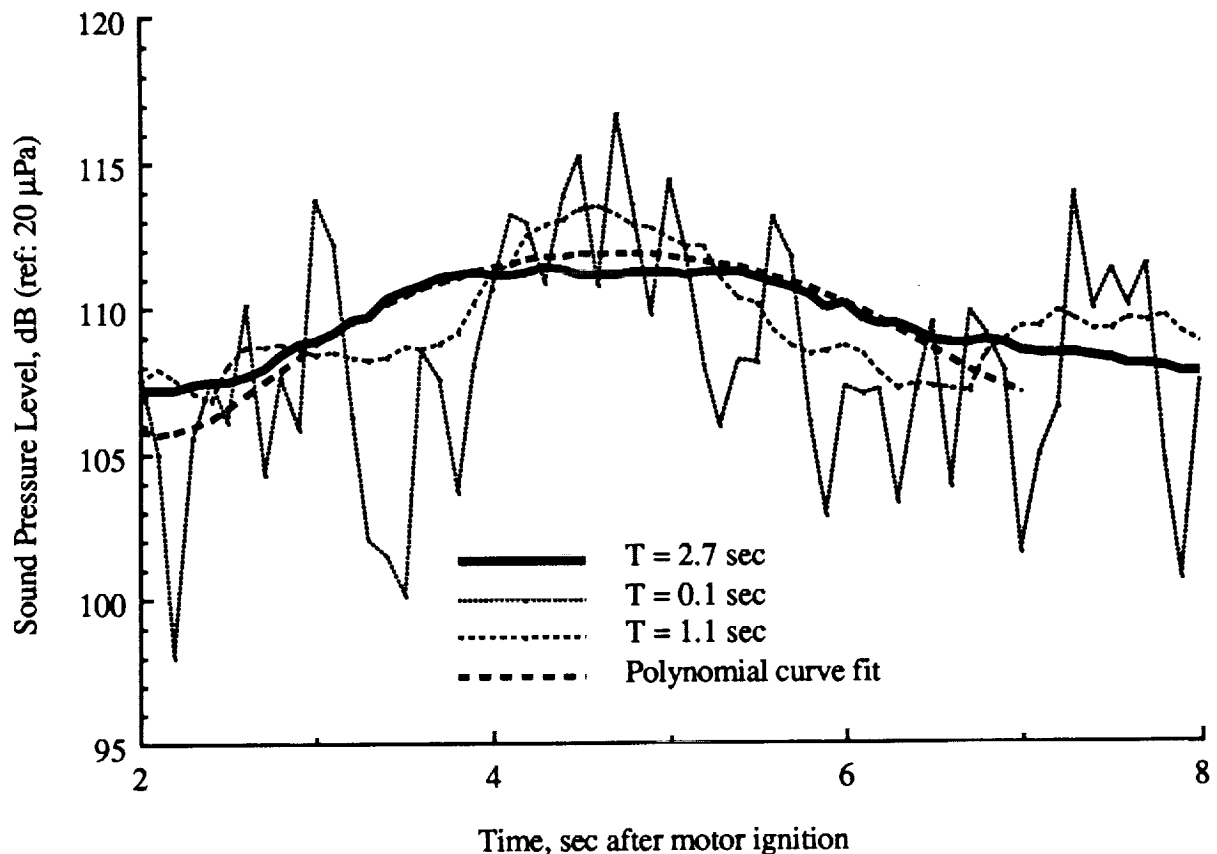
Figure 13.   Running Averages of Sound Pressure Level in 1/3 Octave Band Centered at 20 Hz During Titan IV Lift-Off from VAFB (Flight K-5, Measurement 9705).

which is only 0.5 dB below the maximum of 111.8 dB from the polynomial curve fit. On the other hand, the maximum SPL estimated from the step-wise average with T = 1.1 sec is 113.4 dB, which is 1.6 dB above the maximum value of the curve fit and 2.1 dB above the maximum of the estimate with the optimum averaging time. These results agree with expectations, as follows:

(1)   With the optimum averaging time of T = 2.7 sec, the time resolution bias error (which always causes an underestimate) is being weighted equally with the random error (which usually causes an overestimate due to upward random error fluctuations in the step-wise average), meaning the estimate will commonly be close to the true maximum for the time-varying SPL.

(2)   With the averaging time of T = 1.1 sec, the random errors are dominant and, thus, an over-estimate of the true maximum for the time-varying rms value is very likely due to the upward random error fluctuations in the step-wise average.

6. EVALUATIONS OF SPACE SHUTTLE LIFT-OFF DATA

The Space Shuttle is currently launched from only one facility, namely, the Kennedy Space Center (KSC) in Florida, so there is no problem with variations in the lift-off SPLs due to differences in launch facilities. However, the acoustic measurements inside the Space Shuttle orbiter payload bay (PLB) do vary somewhat with location. To account for these spatial variations, the results from 60 acoustic measurements made inside the PLB during six launches (Flights STS-1 through 5 and 9) [7] were used to arrive at the average acoustic spectrum needed to define the statistical sampling (random) error in Equation (4). Similar to Titan IV launches from a given facility, a qualitative evaluation indicates the variations in the Space Shuttle PLB SPLs with time are similar from one launch to the next. Hence, one measurement was selected for a detailed evaluation of the lift-off SPL versus time, namely, Flight STS-1, Measurement V08Y9219A [8], which was made at orbiter locations X_o863, Y_o-100, and Z_o381. This measurement was selected because it provided a good signal-to-noise ratio, and because Flight STS-1 was carried out with a light payload, meaning the acceleration of the vehicle during lift-off was near a maximum for typical launches. This should produce a near maximum (conservative) value for the time resolution bias error in Equation (5). The basic analysis of this measurement was perform by the NASA Goddard Space Flight Center, and consisted of SPL computations in 1/3 octave bands during the lift-off event using a continuous exponentially-weighted average, as defined in Equation (3), with an RC averaging time constant of $K = 0.1$ sec. An exponentially-weighted average with a time constant of $K = 0.1$ sec corresponds statistically to a linear average in Equation (4) with an averaging time of $T = 2K = 0.2$ sec [9].

6.1 Overall Levels

Following the analysis approach used for the Titan IV lift-off data in Section 5.1, the random and time resolution bias errors in the estimation of the overall SPL during lift-off of the Space Shuttle from KSC are computed as follows:

6.1.1 Statistical Sampling Error

To compute the random error in the overall SPL estimates during lift-off, it is necessary to determine a representative "statistical bandwidth" for the PLB acoustic measurements, as defined in Equation (9). To this end, the average of the autospectra for the 60 lift-off acoustic measurements inside the PLB detailed in [7] was computed with the results shown in Figure 14. Using this average autospectrum, the statistical bandwidth for the lift-off acoustic data is computed to be
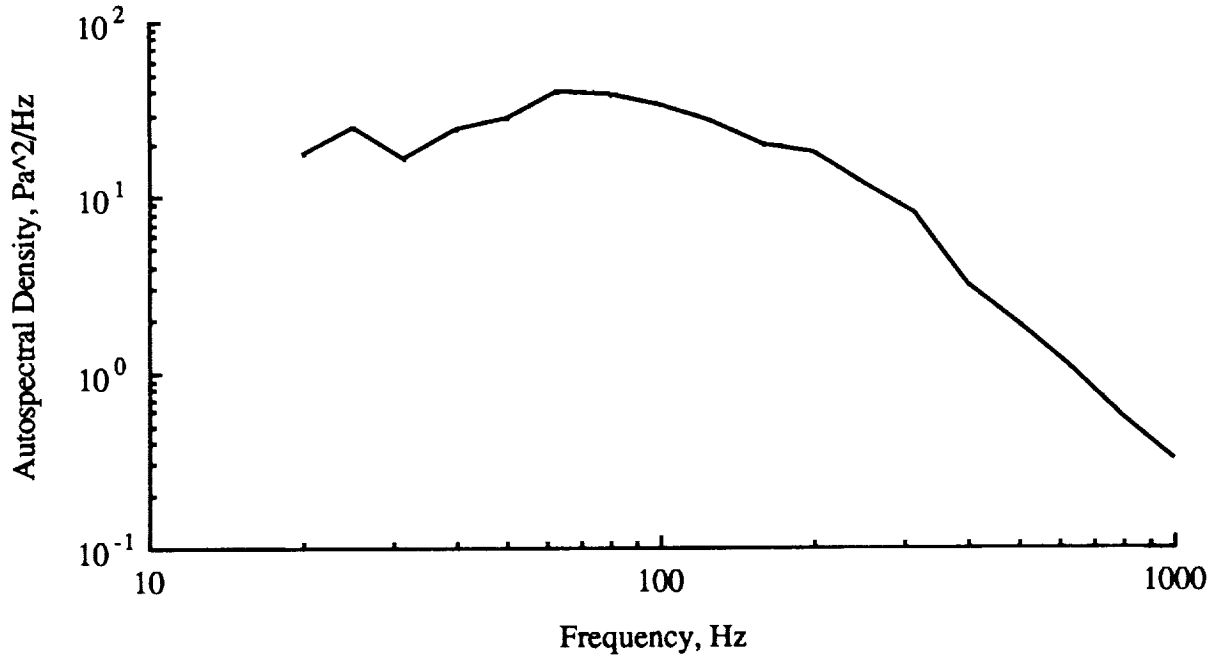
Figure 14. Average Autospectrum of Acoustic Pressures Measured Inside Space Shuttle PLB During Lift-Off.

$$B_s = 340 \, Hz \qquad (34)$$

which interestingly is very close to the statistical bandwidth of $B_s = 320$ Hz computed for the Titan IV lift-off data in Equation (10).

From Equation (4), the normalized random error in the estimate of the overall rms value of the lift-off acoustic pressures as a function of the averaging time T is then given by

$$\varepsilon_r[\widehat{\psi}(t)] \approx \frac{0.027}{\sqrt{T}} \qquad (35)$$

6.1.2 Time Resolution Bias Error

To compute the time resolution bias error in the estimates of the overall level in the Space Shuttle PLB during lift-off, the exponentially weighted running average of STS-1 Measurement V08Y9219A computed by the Goddard Space Flight Center [8] was converted to discrete values for the average SPL every 0.2 sec. These data were then curve fitted using a fourth order polynomial with the results shown in Figure 15. Note that the squared correlation coefficient for the curve fit is a strong $r^2 = 0.94$ (or $r = 0.97$).

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$$\psi^2(t) = 4{,}148.0 - 2{,}988.0\,t + 2{,}690.7\,t^2 - 463.96\,t^3 + 21.239\,t^4$$
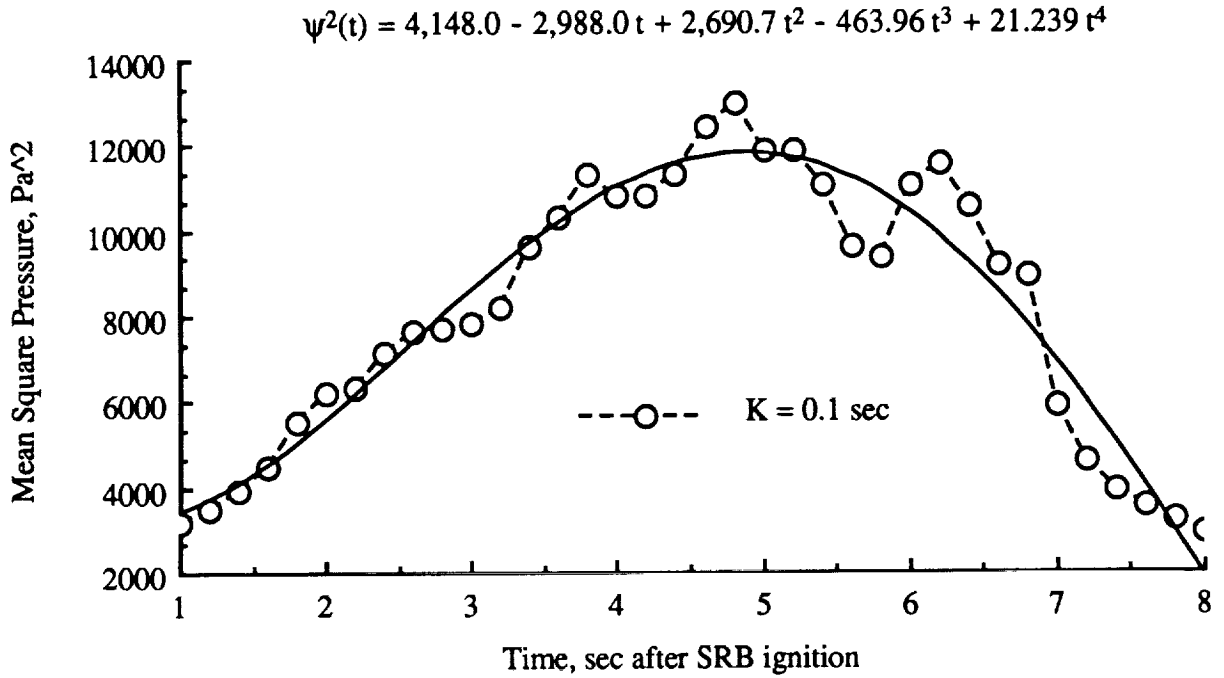
Figure 15.   Curve Fit to Mean Square Estimates of Overall Acoustic Pressure During Space Shuttle Lift-Off from KSC (Flight STS-1, Measurement V08Y9219A).

The second derivative of the polynomial function in Figure 15 is computed to be

$$d^2[\psi^2(t)]/dt^2 = 5{,}381.4 - 2{,}783.8\,t + 254.87\,t^2 \tag{36}$$

Taking the derivative of the polynomial function in Figure 15 and equating to zero, it is found that $\psi^2(t)_{max}$ occurs at $t = 4.88$ sec, while the second derivative function in Equation (36) reaches a maximum about one-half sec later, namely, at $t = 5.46$ sec. To be conservative, assume the maximum value of the second derivative in Equation (36) occurs at the same time as the maximum SPL. The needed quantities are then

$$\psi^2(t)_{max} = 11{,}770 \ Pa^2 \ [\text{or } 134.7 \ dB \ (\text{ref: } 20\mu Pa)]; \quad d^2[\psi^2(t)]/dt^2_{max} = -2{,}220 \ Pa^2/sec^2 \tag{37}$$

and the time resolution bias error in the estimation of the maximum overall SPL during lift-off from KSC is approximated from Equation (5) to be

$$\epsilon_b[\hat{\psi}(t)] \approx -0.0039\,T^2 \tag{38}$$

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The estimated error in Equation (38) is only about 40% of the value of $-0.010\,T^2$ computed for the Titan IV lift-off data in Section 5, meaning for a fixed averaging time T, the estimation of lift-off SPLs for the Space Shuttle involves a smaller time resolution bias error than for the Titan IV.

6.1.3 Averaging Time for Minimum Mean Square Error

From Equations (7), (35), and (38), the mean square error for estimates of the maximum overall SPL inside the Space Shuttle PLB during lift-off from KSC is

$$\varepsilon^2[\widehat{\psi(t)}] = \varepsilon_r^2[\widehat{\psi(t)}] + \varepsilon_b^2[\widehat{\psi(t)}] \approx \frac{7.3 \times 10^{-4}}{T} + 1.5 \times 10^{-5}\,T^4 \tag{39}$$

From Equation (8), the optimum averaging time to minimize the mean square error in Equation (39) is $T_0 = 1.65$ sec, giving a minimum rms error (the positive square root of the minimum mean square error) for estimates of the maximum overall SPL of $\varepsilon\big[\widehat{\psi(t)}\big]_{min} \approx 0.024$. Plots of the normalized random error, bias error, and rms error versus the averaging time T are shown in Figure 16. It is seen in Figure 16 that the rms error reaches a minimum value of $\varepsilon = 0.024$ (about 0.2 dB) at $T_0 = 1.65$ sec, as predicted by Equation (39), but is less than 0.03 (about 0.25 dB) for all averaging times between $T = 0.8$ and $T = 2.5$ sec. Hence, any averaging time selected within about $\pm$ 50% of $T_0 = 1.65$ sec would provide an rms error within 25% of the minimum. However, the rms error increases rapidly as the averaging time moves below or above this range.

6.1.4 Smoothness of Overall Sound Pressure Level Variations with Time

Using the procedures detailed in Section 5.1.5, but without presenting the detailed computations, it has been confirmed that the short time averaged estimates for the overall mean square pressure in the Space Shuttle PLB, as shown in Figure 15, fall well within a 99% probability interval about the fourth order polynomial curve fit. Hence, like the Titan lift-off SPLs, there is no reason to question that the variations of the actual SPL versus time during lift-off are smooth; i.e, it can be assumed that the deviations from the polynomial curve fit by the mean square pressure levels estimated with an exponentially weighted averaging time constant of $K = 0.1$ sec (equivalent to a linear averaging time of $T = 0.2$ sec [9]) are due to random sampling errors. It follows that there is no reason to believe that the estimates computed with an averaging time of $T = 1.65$ sec will smooth through physically significant variations in the lift-off data.
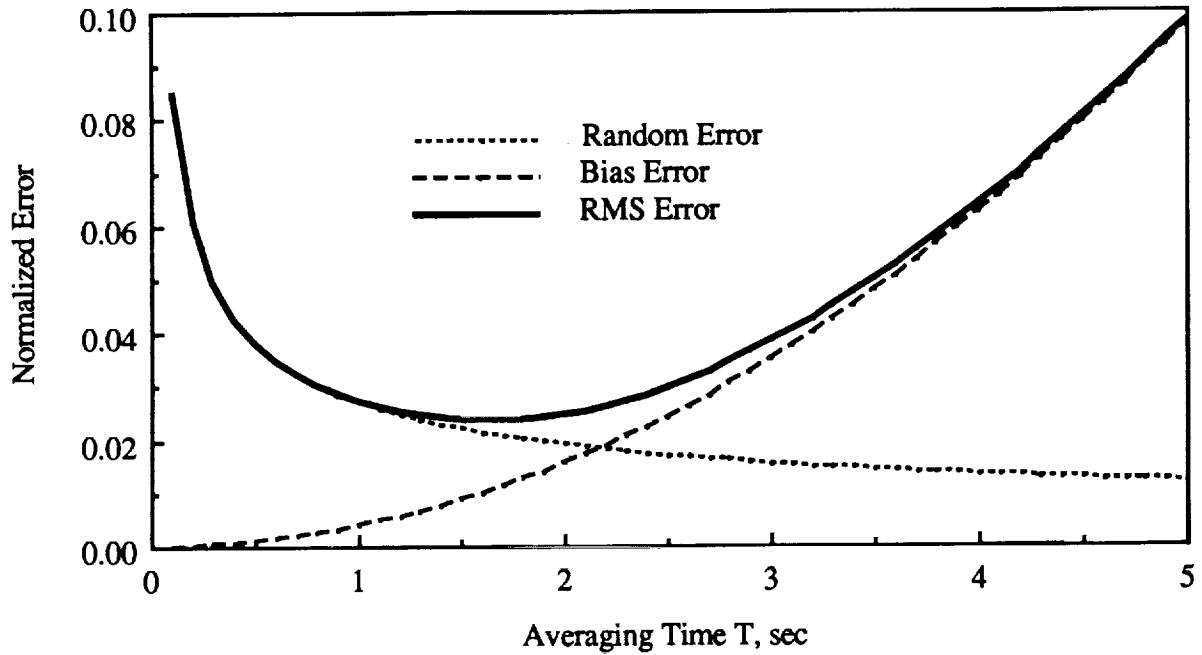
Figure 16.   Normalized Errors Versus Averaging Time for Estimates of Maximum Overall Sound
Pressure Level Inside Space Shuttle PLB During Lift-off.

A plot of the overall SPL in dB (ref: 20 μPa) during lift-off computed with an averaging time of T
= 1.6 sec (the closest averaging time to T_0 = 1.65 sec that could be achieved) is shown in compar-
ison to the polynomial curve fit in Figure 17.   Note that the maximum overall SPLs estimated from
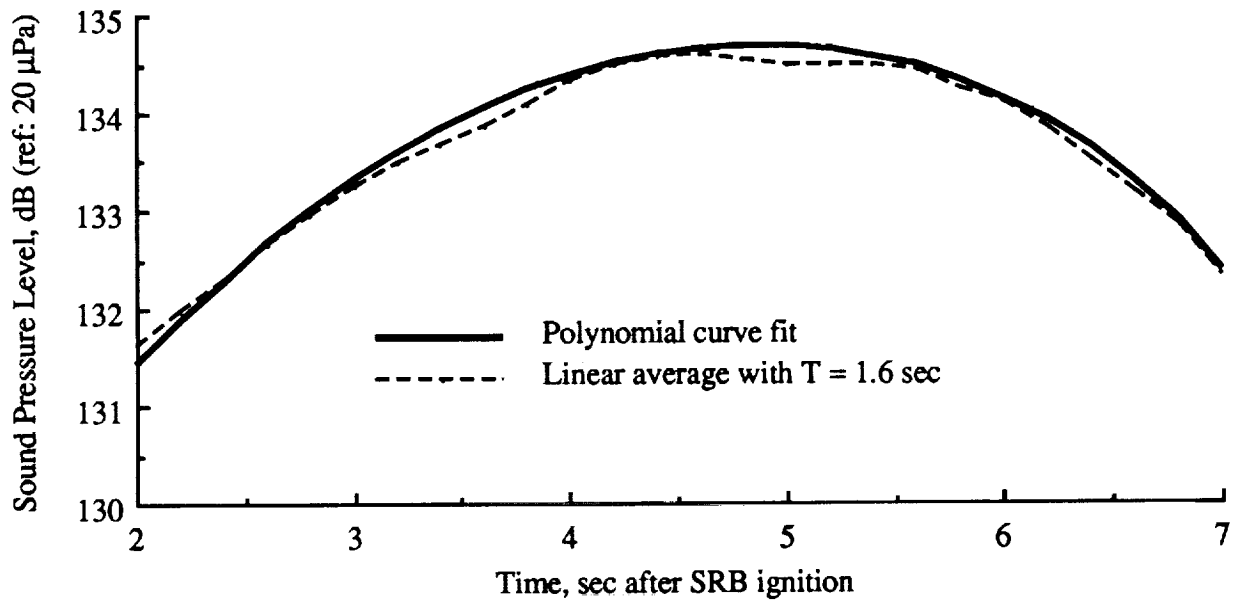
Figure 17.   Overall Sound Pressure Level Estimates During Space Shuttle Lift-Off from KSC
(Flight STS-1, Measurement V08Y9219A).

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the linear average occurs at a slightly earlier time than the maximum of the curve fit, but the values of the maxima agree within 0.2 dB (a discrepancy of less than 2%), exactly as occurred for the equivalent Titan IV estimates in Figure 8.

6.2 1/3 Octave Band Levels

Following the procedures detailed in Section 5.2 for the Titan IV lift-off data, the averaging times that produce 1/3 octave band estimates of the sound levels in the Space Shuttle PLB during lift-off with a minimum mean square error are formulated, as follows:

6.2.1 Statistical Sampling Error

From Equation (4), as a first order of approximation, the random errors in the 1/3 octave band SPL estimates during lift-off are assumed to be a function only of the 1/3 octave bandwidth B_i; i = 1, 2, ..., and the averaging time T. Hence, the random errors for the 1/3 octave band levels estimated in the Space Shuttle PLB during lift-off are the same as determined for the 1/3 octave band levels estimated in the Titan IV PLF in Section 5.2.1.

6.2.2 Time Resolution Bias Error

A review of the exponentially weighted 1/3 octave band SPLs versus time in the Space Shuttle PLB during lift-off in [7] indicates the assumption verified for the Titan IV lift-off data applies to Space Shuttle as well, namely, the 1/3 octave band SPLs reach their maxima at slightly different times, but otherwise their variations with time are broadly similar to those shown for the overall level in Figure 15. Hence, it is assumed that the time resolution bias error computed for the overall SPL estimates in Equation (38) applies to the 1/3 octave band levels as well.

6.2.3 Averaging Time for Minimum Mean Square Error

Using Equation (8) with the bandwidths in Table 1 and the values in Equation (37), the optimum averaging times for the estimation of maximum SPLs in 1/3 octave bands with a minimum mean square error are as plotted in Figure 18 and listed in Table 4. Since the optimum averaging time plots as a straight line on log-log paper, it can be described in equation form by

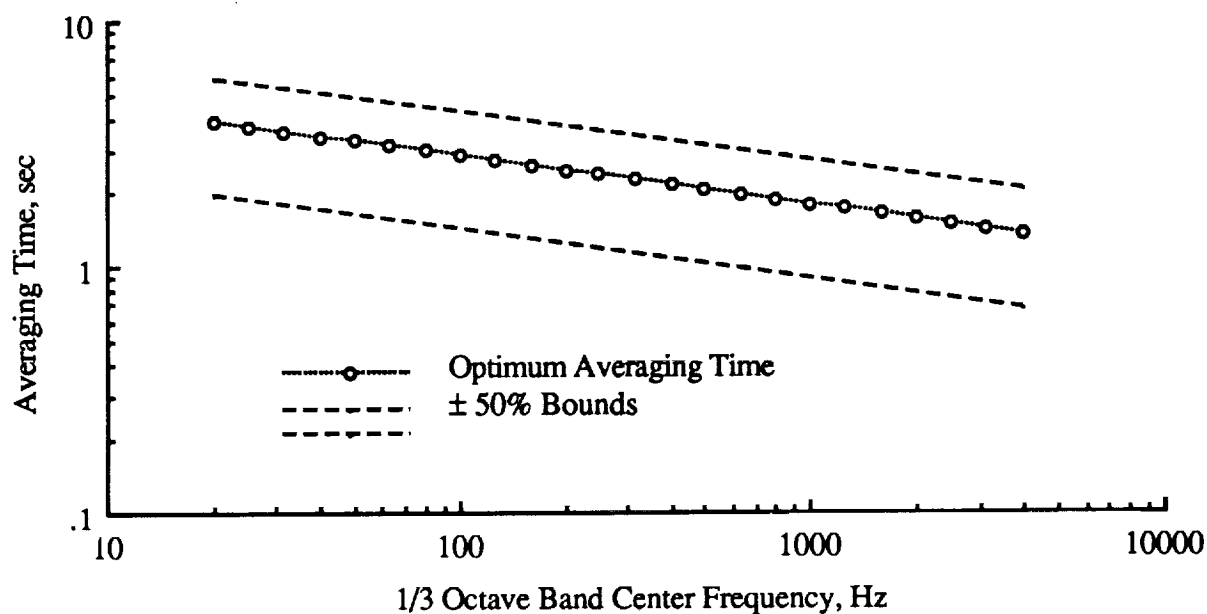$$T_{oi} = 7.10 \, f_i^{-0.2} \tag{40}$$

Figure 18. Optimum Averaging Times for Analysis of 1/3 Octave Band Sound Pressure Levels Inside Space Shuttle PLB During Lift-Off.

Table 4. Optimum Averaging Times and Minimum Normalized RMS Errors for Analysis of 1/3 Octave Band Sound Pressure Levels Inside Space Shuttle PLB During Lift-Off.

Center Freq. (Hz)	Band-width (Hz)	Optimum Averaging Time (sec)	Minimum Normalized RMS Error	Center Freq. (Hz)	Band-width (Hz)	Optimum Averaging Time (sec)	Minimum Normalized RMS Error
20	4.5	3.90	0.061	315	75	2.22	0.026
25	5.7	3.72	0.055	400	92	2.13	0.025
31.5	7.5	3.52	0.050	500	113	2.05	0.025
40	9.2	3.38	0.047	630	150	1.93	0.024
50	11.3	3.24	0.043	800	185	1.85	0.024
63	15.0	3.06	0.039	1000	225	1.78	0.024
80	18.5	2.94	0.037	1250	300	1.68	0.024
100	22.5	2.82	0.035	1600	360	1.62	0.024
125	30	2.67	0.032	2000	450	1.55	0.024
160	36	2.57	0.031	2500	570	1.48	0.024
200	45	2.46	0.029	3150	750	1.40	0.024
250	57	2.34	0.028	4000	920	1.34	0.024

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As discussed in Section 5.2.4, the error problem is most severe in the low frequency 1/3 octave bands where the signal bandwidth is a minimum. To illustrate this problem for the Space Shuttle lift-off data, the 20 Hz 1/3 octave band signal from Measurement V08Y9219A (Flight STS-1) is analyzed using a near-optimum averaging time of T = 4 sec (the closest averaging time to 3.9 sec that could be achieved) with the results shown in Figure 19. Also shown in Figure 19 are the basic data computed with an RC averaging time constant of K = 0.5 sec (equivalent to a linear averaging time of T = 1 sec) and the fourth order polynomial curve fit to these basic data. Again, as in Section 5.2.4, if the polynomial curve fit is assumed to represent an accurate estimate of the SPL variations with time, the estimated maximum SPL in the 20 Hz 1/3 octave band during lift-off computed with the near-optimum linear averaging time of T = 4 sec is 115.2 dB, as compared to 115.7 dB from the polynomial curve fit and 117.3 dB from the equivalent linear averaging time of T = 1 sec. The analysis with the optimum averaging time results in a underestimate of the maximum SPL by 0.5 dB, but this is within the range of the expected rms error of $\varepsilon = 0.061$ (a standard deviation of 0.5 dB), and is substantially less than the discrepancy of 1.6 dB provided by the estimate with the T = 1 sec averaging time.
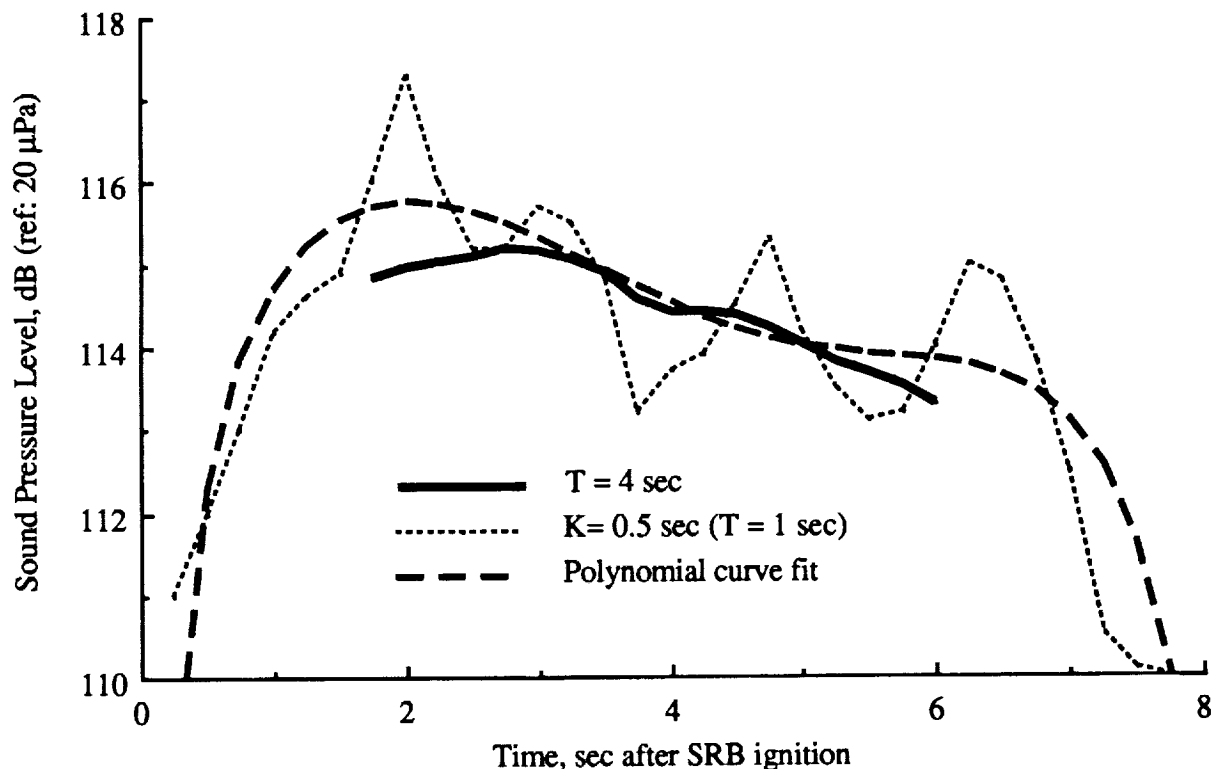

Figure 19.   Running Averages of Sound Pressure Level in 1/3 Octave Band Centered at 20 Hz During Space Shuttle Lift-Off from KSC (Flight STS-1, Measurement V08Y9219A).

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As a concluding point of interest, the original Space Shuttle launch acoustic data presented in the DATE reports (e.g., [7]) were analyzed using an exponentially-weighted average with a time constant of K = 0.5 sec, which is statistically equivalent to a linear average with an averaging time of T = 1 sec [9]. This T = 1 sec equivalent linear averaging time was established by trial-and-error procedures, where the overall acoustic measurements from the first flight [7] were analyzed with various averaging times. From Section 6.1.3, the empirically-determined value of T = 1 sec is well within the range of the analytically-determined optimum averaging time of T_0 = 1.64 sec ± 50% (about 0.8 to 2.4 sec) for estimates of the overall levels in the Space Shuttle PLB during lift-off. However, in the DATE reports, this same averaging time (K = 0.5 sec equivalent to T = 1 sec) is used to analyze all of the 1/3 octave band signals as well. The results derived herein (see Table 4) indicate that T = 1 sec is too short an averaging time for the accurate estimation of the maximum SPLs in the 1/3 octave bands below 2500 Hz.

7. CONCLUSIONS

The specific conclusions drawn from this study may be summarized as follows:

1. The available acoustic data measured inside the Titan IV payload fairing (PLF) and the Space Shuttle payload bay (PLB) support the conclusion that the variation in the sound pressure level (SPL) with time is relative smooth during the lift-off event, and that the rapid variations seen in the launch SPLs computed with short averaging times are due to random estimation errors.

2. From the first conclusion, the short time-averaged mean square values of the overall and 1/3 octave band SPLs can be fitted by fourth order polynomial functions with reasonable accuracy. These polynomial functions can be used directly to estimate the maximum SPLs during the lift-off event. However, they can also be used to derive time resolution bias errors for step-wise linear averaging operations, which in turn allow the derivation of optimum averaging times that will minimize the mean square errors in the maximum SPLs determined from step-wise averages.

3. The optimum averaging times for computing a step-wise average of the overall SPLs measured inside the Titan IV PLF and the Space Shuttle PLB during lift-off are

Titan IV: T_0 = 1.14 sec
Space Shuttle: T_0 = 1.64 sec

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The evaluations indicate that any averaging time within ± 50% of the above optimum values should provide acceptable results (an rms error within 25% of the minimum achievable error).

4. The optimum averaging times for computing a step-wise linear average of the 1/3 octave band SPLs measured inside the Titan IV PLF and the Space Shuttle PLB during lift-off are

$$\text{Titan IV: } T_{oi} = 4.88 \, f_i^{-0.2} \text{ sec}$$
$$\text{Space Shuttle: } T_{oi} = 7.10 \, f_i^{-0.2} \text{ sec}$$

where f_i is the center frequency of the ith 1/3 octave band. As for the overall, any averaging time within ± 50% of the above optimum values for each 1/3 octave band should provide acceptable results.

5. If an exponentially-weighted average (RC lowpass filter) is used to compute the SPLs, the RC averaging time constant K should be one-half the linear averaging time T stated in the third and fourth conclusions above (i.e., $K = T/2$).

8. RECOMMENDATIONS

Based upon the conclusions in Section 7, it is recommended that the analysis of acoustic measurements made inside the Titan IV PLF and the Space Shuttle PLB during lift-off be performed using the averaging times detailed in Table 5. Of course, the more precise averaging times given in Section 7 can be used if desired, but the values in Table 5 will simplify the analysis and provide results with an rms error within 25% of the minimum achievable error.

Table 5. Recommended Averaging Times for Titan IV and Space Shuttle Lift-Off Acoustic Data

Vehicle	Frequency Range	Averaging Time (sec)*
Titan IV	Overall and all 1/3 octave bands above 250 Hz	T = 1.0 or K = 0.5
	All 1/3 octave bands at 250 Hz and below	T = 2.0 or K = 1.0
Space Shuttle	Overall and all 1/3 octave bands above 250 Hz	T = 1.5 or K = 0.75
	All 1/3 octave bands at 250 Hz and below	T = 2.5 or K = 1.25

* T = linear averaging time; K = exponentially-weighted averaging time constant.

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APPENDIX
TIME RESOLUTION BIAS ERROR IN RMS VALUE ESTIMATES

Consider a nonstationary random signal x(t) with a mean square value at any instant given by

$$\psi^2(t) = E[x^2(t)] \tag{A1}$$

where E[ ] denotes "expected value" of [ ]. Assuming $T \gg 1/f_1$, where f_1 is the lowest frequency in the signal x(t), an estimate of the mean square value over a time interval T centered at the instant t is given by

$$\widehat{\psi}^2(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x^2(\tau)\,d\tau \tag{A2}$$

where the hat (^) over $\psi^2(t)$ denotes "estimate of", and τ is a dummy variable of integration. The expected value of the estimate is given by

$$E\left[\widehat{\psi}^2(t)\right] = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} E\left[x^2(\tau)\right]d\tau = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \psi^2(\tau)\,d\tau \tag{A3}$$

Expand $\psi^2(\tau)$ in Equation (A3) into a Taylor series about the point $\tau = t$. Assuming the second derivative of $\psi^2(\tau)$ with respect to τ does not vary substantially over the interval T, the first three terms of the Taylor series should provide an adequate approximation for $\psi^2(t)$, namely,

$$\psi^2(\tau) \approx \psi^2(t) + (\tau - t)\frac{d\left[\psi^2(t)\right]}{dt} + (\tau - t)^2\frac{d^2\left[\psi^2(t)\right]}{dt^2} \tag{A4}$$

Substituting Equation (A4) into Equation (A3) and noting that

$$\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} (\tau - t)\,d\tau = 0 \quad \text{and} \quad \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{(\tau - t)^2}{2}\,d\tau = \frac{T^3}{24} \tag{A5}$$

it follows that

$$E\left[\widehat{\psi}^2(t)\right] \approx \psi^2(t) + \frac{T^2}{24} d^2[\psi^2(t)]/dt^2 \approx \psi^2(t)\left[1 + \frac{T^2}{24}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)}\right] \qquad (A6)$$

Assume the random (statistical sampling) errors in the mean square value estimate are negligible so that

$$E\left[\widehat{\psi}^2(t)\right] \approx \widehat{\psi}^2(t) \qquad (A7)$$

The estimate of the time-varying root mean square (rms) value $\psi(t)$ of the nonstationary random signal is then

$$\widehat{\psi}(t) = \sqrt{\widehat{\psi}^2(t)} \approx \psi(t) \sqrt{1 + \frac{T^2}{24}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)}} \qquad (A8)$$

Further assume that the second term under the radical is less than, say 0.4, so that

$$\widehat{\phi} = \phi \sqrt{1 + \varepsilon} \approx \phi\left(1 + \frac{\varepsilon}{2}\right) \qquad (A9)$$

Then Equation (A8) can be further approximated by

$$\widehat{\psi}(t) \approx \psi(t)\left[1 + \frac{T^2}{48}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)}\right] \qquad (A10)$$

The time resolution bias error in the estimate is defined as

$$b\left[\widehat{\psi}(t)\right] = \widehat{\psi}(t) - \psi(t) \approx \psi(t)\left[1 + \frac{T^2}{48}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)}\right] - \psi(t) = \psi(t)\left[\frac{T^2}{48}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)}\right] \qquad (A11)$$

In terms of a normalized time resolution bias error, Equation (A11) can be written as

$$\varepsilon_b\left[\widehat{\psi}(t)\right] = \frac{b\left[\widehat{\psi}(t)\right]}{\psi(t)} \approx \frac{T^2}{48}\frac{d^2[\psi^2(t)]/dt^2}{\psi^2(t)} \qquad (A12)$$

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