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Solar Radiation on a Catenary Collector

M. Crutchik
Tel-Aviv University
Tel-Aviv, Israel

and

J. Appelbaum
Lewis Research Center
Cleveland, Ohio

SOLAR RADIATION ON A CATENARY COLLECTOR

M. Crutchik
Faculty of Engineering
Tel-Aviv University
Tel-Aviv 69978, Israel

J. Appelbaum†
National Aeronautics and
Space Administration
Lewis Research Center,
Cleveland, OH 44135

Abstract

A tent-shaped structure with a flexible photovoltaic blanket acting as a catenary collector is presented. The shadow cast by one side of the collector on the other side producing a self shading effect is analysed. The direct beam, the diffuse and the albedo radiation on the collector are determined. An example is given for the insolation on the collector operating on Mars surface for the location of Viking Lander 1 (VL1).

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†Current address: Tel-Aviv University, Faculty of Engineering, Tel-Aviv 69978, Israel.

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1. INTRODUCTION

Mission to Martian surface will require electric power. A power supply that requires little installation time, being light weight and stowed in a small volume can be accomplished by a photovoltaic (PV) array. A tent-shaped structure with a flexible PV blanket for solar power generation is proposed in [1], Fig. 1. The array is designed with a self-deploying mechanism using a pressurized gas expansion. The structural design for the array uses a combination of cables, beams and columns to support and deploy the PV blanket. The array is stowed with the blanket either folded or rolled. The main contribution to the stress in the structure is due to the tension in the cable which supports the PV blanket. The shape of the PV blanket is determined by an optimization between reduction in the cable tension and increase in blanket area. Under the force of gravity a cable carrying a uniformly distributed load will take the shape of a catenary curve, $f_c(y)$, with respect to the Y-Z plane, Fig. 2.

According to [2] and with some manipulations, $f_c(y)$ is given by:

$$f_c(y) = k\left[\cosh\left(\frac{D - y}{k}\right) - 1\right] \qquad (1)$$

The catenary constant k can be determined using the condition $f_c(0) = H$ and solving iteratively. However, when the blanket is fairly taut, the load may be assumed uniformly distributed along the y-axis and the catenary curve may be approximated by a parabola [2], i.e.,:

$$f_c(y) = \frac{H}{D^2} (D - y)^2 \qquad (2)$$

which simplifies the calculation.

Because of the shape of a catenary-tent-collector, a self-shading effect occurs on one of its sides (side B in Fig. 2). In this article we analyze the shadow shape and area. Based on these results, the beam insolation on the collector is calculated. We also determine the diffuse and the albedo insolation on the collector. An example for the planet Mars is given for the location of Viking Lander 1. Results for the parabolic approximation is given in Appendix B.

2. SHADOW CALCULATION

The catenary-tent-collector is self shading. The size of the shadow and the side which is shaded depend on the sun position. In general, both sides of the collector will be alternately shaded in a given day if at sunrise

$$\gamma_{sr} < \gamma_c - 90^o \qquad (3)$$

where

γ_c - the collector azimuth

γ_{sr} - the sun azimuth at sunrise.

The azimuth angles are measured from true south positively in a clockwise direction. In days when eq. (3) is not satisfied, only one side of the collector is shaded all the times. In this paper we analyze the shape and size of the shadow cast on a catenary-tent-collector facing the south-north direction. The results can be generalized for an arbitrary oriented tent by replacing the sun azimuth angle γ_s by the difference between the solar and the collector

azimuth angles, i.e.,

$$\gamma = \gamma_s - \gamma_c \tag{4}$$

With reference to Fig. 3, the shadow cast by line $\overline{ON}$ can be divided into three distinct cases:

$$\begin{aligned} &\text{(i)} \quad P_x < L \ , \ P_y < D \\ &\text{(ii)} \quad P_x < L \ , \ P_y \geq D \\ &\text{(iii)} \quad P_x \geq L \ , \ P_y < D \ . \end{aligned} \tag{5}$$

It will be shown later that for the case $P_x \geq L$, $P_y \geq D$ the shadow takes the shape either of case (ii) or case (iii).

The components P_x and P_y of the line $\overline{OP}$ was derived in [3,4] and are:

$$P_x = H \cos\delta \sin\omega/\sin\alpha \tag{6}$$

and

$$P_y = H(\sin\phi \cos\delta \cos\omega - \cos\phi \sin\delta)/\sin\alpha \tag{7}$$

where

$$\sin\alpha = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega \tag{8}$$

and

δ - solar declination angle

ϕ - local latitude

α - sun elevation angle

ω - solar hour angle

Case (i): $P_x < L$, $P_y < D$

In this case the solar rays penetrate the collector surface and the shadow on the collector takes the shape $NTEE_xM$ as shown in Fig. 4. Point E is a point where the ray penetrates the collector, and point T is a point where the ray is tangent to the collector. In order to calculate the shaded area, the components of the points E and T must be determined. Since the catenary function $f_c(y)$ is hyperbolic, the solution for the component y_E is obtained numerically from the solution of the following equation (see Appendix A):

$$\frac{k}{H}\cosh\left(\frac{D - y_E}{k}\right) + \frac{y_E}{P_y} = 1 + \frac{k}{H} \tag{9}$$

The component x_E is then given by (see Fig. 4)

$$x_E = \frac{P_x}{P_y} y_E \tag{10}$$

Point T is in y-z plane, i.e., $x_T = 0$ and y_T is obtained from the solution of (see Appendix A):

$$y_T = D - k \sinh^{-1}(H/P_y) \tag{11}$$

It is easier to obtain the shaded area A_{sh} from the "streched out" collector, Fig. 4(b), i.e.,

$$A_{sh} = \overset{\frown}{L\,N}\; E_y - 0.5\, x_E\, \overset{\frown}{T\,E_y} \tag{12}$$

The unshaded area A_{ush} is given by:

$$A_{ush} = A - A_{sh} = L(\overset{\frown}{N\,N_n} - \overset{\frown}{N\,E_y}) + 0.5\, x_E\, \overset{\frown}{T\,E_y} \tag{13}$$

where A is the area of one side of the collector, i.e.,

$$A = L\ \overset{\frown}{N\ N_n} \tag{14}$$

Defining a relative shaded area by ξ, we obtain

$$\xi = (2\ L\ \overset{\frown}{N\ E_y} - x_E\ \overset{\frown}{T\ E_y})/2L\ \overset{\frown}{N\ N_n} \tag{15}$$

The length of an arc between two points y_1 and y_2 is given by:

$$\overset{\frown}{y_1\ y_2} = \int_{y_1}^{y_2} \sqrt{1 + [f_c'(y)]^2}\ dy \tag{16}$$

where $f_c'(y)$ is the derivative of the function $f_c(y)$. The length of the arcs in eq. (15) thus become:

$$\overset{\frown}{N\ N_n} = \int_0^D \sqrt{1 + [f_c'(y)]}\ dy = k\ \sinh(D/k) \tag{17}$$

$$\overset{\frown}{N\ E_y} = \int_0^{y_E} \sqrt{1 + [f_c'(y)]^2}\ dy = k\left[\sinh(D/k) + \sinh\left(\frac{y_E - D}{k}\right)\right] \tag{18}$$

$$\overset{\frown}{T\ E_y} = \int_{y_T}^{y_E} \sqrt{1 + f_c'(y)^2}\ dy = k\left[\sinh\left(\frac{y_E - D}{k}\right) + \sinh\left(\frac{D - y_T}{k}\right)\right] \tag{19}$$

Case (ii): $P_x < L$, $P_y \geq D$

This case is shown in Fig. 5. The array touches the collector at the foot, point E, and at point U. The coordinates of point E are:

$$y_E = D \tag{20}$$

As derived in Appendix A,

$$x_E = \frac{P_x}{P_y} (D - y_U) \tag{21}$$

where y_U is determined from the solution of (see Appendix A):

$$k\left[\cosh\left(\frac{D - y_U}{k}\right) - 1\right] + \frac{H}{P_y} y_U = \frac{DH}{P_y} \tag{22}$$

The coordinates of point T are the same as for case (i), i.e., $x_T = 0$ and y_T is given by eq. (11). The shaded and unshaded areas, Fig. 5(b), are, respectively:

$$A_{sh} = A - 0.5\, x_E\, \overset{\frown}{T\,N_n} \tag{23}$$

$$A_{ush} = 0.5\, x_E\, \overset{\frown}{T\,N_n} \tag{24}$$

The relative shaded area is obtained as:

$$\xi = 1 - \frac{x_E\, \overset{\frown}{T\,N_n}}{2\, L\, \overset{\frown}{N\,N_n}} \tag{25}$$

where the arc length $\overset{\frown}{T\,N_n}$ is given by:

$$\overset{\frown}{T\,N_n} = \int_{y_T}^{D} \sqrt{1 + [f_c'(y)]^2}\, dy = k \sinh\left(\frac{D - y_T}{k}\right) \tag{26}$$

and $\overset{\frown}{N\,N_n}$ is given in eq. (17).

Case (iii): $P_x \geq L$, $P_y < D$

The solar ray may either intersect the collector or may fall outside the collector. The case of intersection was already analysed in case (i). To determine whether the solar ray intersects the collector, eqs. (9) and (10) are solved.

If $x_E < L$ than there is an intersection. If $x_E > L$ than there is no intersection, see Fig. 6. The coordinates of point E are:

$$x_E = L \tag{27}$$

and y_E is determined from the solution of (see Appendix A):

$$k\left[\cosh\left(\frac{D - y_E + L\,\frac{P_x}{P_y}}{k}\right) - \cosh\left(\frac{D - y_E}{k}\right)\right] = \frac{L\,H}{P_x} \tag{28}$$

The coordinates of point T are $x_T = 0$ and y_T is given by eq. (11). The shaded and unshaded areas, Fig. 6, are respectively:

$$A_{sh} = L\,\overset{\frown}{N\,E_y} - 0.5\,L\,\overset{\frown}{T\,E_y} \tag{29}$$

$$A_{ush} = L(\overset{\frown}{N\,N_n} - \overset{\frown}{N\,E_y} + 0.5\,\overset{\frown}{T\,E_y}) \tag{30}$$

The relative shaded area is:

$$\xi = (2\,\overset{\frown}{N\,E_y} - \overset{\frown}{T\,E_y})/2\,\overset{\frown}{N\,N_n} \tag{31}$$

where $\overset{\frown}{N\,N_n}$, $\overset{\frown}{N\,E_y}$ and $\overset{\frown}{T\,E_y}$ are given in eqs. (17)-(19).

The case $P_x \geq L$, $P_y \geq D$ corresponds either to case (ii) or to case (iii) depending on the ratio P_x/P_y of the ray $\overline{OP}$ (see Fig. 7).

For $P_x/P_y < L/D$ the solution is as for case (ii) and

for $P_x/P_y \geq L/D$ the solution is as for case (iii)

For a south-north oriented collector, the shadow as a function of time is symmetrical with the solar noon. For the after noon, the shadow calculations are with respect to line O'M (see Fig. 3).

3. SOLAR RADIATION CALCULATION

With the results developed in the previous section, the beam irradiance in W or W/m^2 and the beam insolation in Whr/day or Whr/m^2-day on the catenary-tent-collector can be determined. The diffuse and albedo components will be added to the beam to get the global irradiance and insolation. A north-south facing catenary-tent-collector will be considered. A generalization to any arbitrary oriented tent may be obtained using eq. (4).

Beam Irradiance

The beam irradiance on both sides of the tent depends on the self-shading condition of eq. (3). In calculating the irradiance on an unshaded side we resort to Fig. 8. The beam irradiance P, in watts (W), on an unshaded side is given by:

$$dP_b = G_b \cos\theta \, dA \tag{32}$$

where G_b is the beam irradiance in W/m^2 normal to the solar rays, and θ is the angle between the solar rays and the normal to the surface. $\cos\theta$ and dA are given by

$$\cos\theta = \cos\beta(y) \sin\alpha + \sin\beta(y) \cos\alpha \cos\gamma_s \tag{33}$$

and

$$dA = L \, dS = L \sqrt{1 + [f_c'(y)]^2} \, dy \tag{34}$$

where dS is the unit length of the collector (Fig. 8) and

$$\sin\beta(y) = |f_c'(y)|/\sqrt{1 + [f_c'(y)]^2} \tag{35}$$

$$\cos\beta(y) = 1/\sqrt{1 + [f_c'(y)]^2} \tag{36}$$

Therefore, eq. (32) reduces to:

$$dP_b = G_b\, L(\sin\alpha + |f_c'(y)|\cos\alpha\cos\gamma_s)dy \qquad (37)$$

Integrating eq. (37) in the interval [0,D], we obtain the beam irradiance, in W, on the unshaded side of the collector, i.e.,

$$P_b = G_b\, L(D\sin\alpha + H\cos\alpha\cos\gamma_s) \qquad 38)$$

Using the angle ϵ as defined in Fig. 8, we obtain:

$$P_b = G_b\, L(H^2 + D^2)^{1/2}(\cos\epsilon\sin\alpha + \sin\epsilon\cos\alpha\cos\gamma_s) \qquad 39)$$

i.e., the beam irradiance on a catenary collector is equivalent to that of a flat plate collector MNM_sN_s. This conclusion is also valid for any non-flat shape collector.

The beam irradiance on a collector that is partially shaded can not be obtained by multiplying the incoming beam irradiance with the factor $(1-\xi)$ as for a flat plate collector since the beam irradiance on the collector is not uniform along the y axis of the collector, i.e., the angle θ varies with y. Using eqs. (33),(35) and (36), the variation of the beam irradiance, in W/m^2, along the collector is:

$$G_b(y) = \frac{G_b}{\sqrt{1 + [f_c'(y)]^2}}\,[\sin\alpha + \cos\alpha\cos\gamma_s|f_c'(y)|]$$

and with eq. (1) we obtain:

$$G_b(y) = \frac{G_b}{\cosh\left(\frac{D-y}{k}\right)}\left[\sin\alpha + \cos\alpha\cos\gamma_s\left|\sinh\left(\frac{D-y}{k}\right)\right|\right] \qquad (40)$$

With reference to Fig. 9(a), the beam irradiance on the partially shaded side of the

collector for case (i) is given by:

$$P_b^{sh} = \int_{y_T}^{y_E} G_b(y)\, dA_1 + \int_{y_E}^{D} G_b(y)\, dA_2$$

where $G_b(y)$ is given by eq. (40), and

$$dA_1 = x\, dS = x \cosh\left(\frac{D-y}{k}\right) dy \tag{41}$$

$$dA_2 = L \cosh\left(\frac{D-y}{k}\right) dy \tag{42}$$

where x and y are related by (Fig. 9(a))

$$x = \left(\frac{x_E}{y_E - y_T}\right)(y - y_T) \tag{43}$$

Substituting these results, we obtain the beam irradiance on the partially shaded side of the collector:

$$P_b^{sh} = G_b \left\{ \int_{y_T}^{y_E} \left[\sin\alpha + \cos\alpha \cos\gamma_s \left| \sinh\left(\frac{D-y}{k}\right) \right| \right] \left(\frac{x_E}{y_E - y_T}\right) (y - y_T)\, dy \right.$$

$$\left. + L \int_{y_E}^{D} \left[\sin\alpha + \cos\alpha \cos\gamma_s \left| \sinh\left(\frac{D-y}{k}\right) \right| \right] dy \right\} \tag{45}$$

Similarly, the beam irradiance for case (ii) (Fig. 9b) is:

$$P_b^{sh} = G_b \int_{y_T}^{D} \left[\sin\alpha + \cos\alpha \cos\gamma_s \left| \sinh\left(\frac{D-y}{k}\right) \right| \right] \left(\frac{x_E}{D - y_T}\right) (y - y_T)\, dy \tag{46}$$

and the beam irradiance for case (iii) (Fig. 9c) is:

$$P_b^{sh} = G_b \left\{ \int_{y_T}^{y_E} \left[\sin\alpha + \cos\alpha \cos\gamma_s \left| \sinh\left(\frac{D-y}{k}\right) \right| \right] \left(\frac{L}{y_E - y_T}\right) (y - y_T)\, dy \right.$$

$$\left. + L \int_{y_E}^{D} \left[\sin\alpha + \cos\alpha \cos\gamma_s \left| \sinh\left(\frac{D-y}{k}\right) \right| \right] dy \right\} \qquad (47)$$

In winter, side A (Fig. 2) remains always unshaded, therefore the beam irradiance P_{bA} on this side is given by eq. (38) or eq. (39). Side B may be partially shaded and the beam irradiance P_{bB}^{sh} is given by eqs. (45), (46) or (47) where the collector azimuth is 180°, therefore, γ_s is replaced by $(\gamma_s - 180^o)$. The total beam irradiance on the catenary-tent-collector in winter is:

$$P_b = P_{bA} + P_{bB}^{sh} \qquad (48)$$

In summer, early in the morning and late in the afternoon, side A is partially shaded, therefore, the beam irradiance P_{bA} is given by eqs. (45), (46) or (47). Side B is unshaded, and the beam irradiance P_{bB} is given by eq. (38) with the azimuth $\gamma_s - 180^o$, i.e.,

$$P_{bB} = G_b\, L[D \sin\alpha + H \cos\alpha \cos(\gamma_s - 180^o)] \qquad (49)$$

thus the beam irradiance on both sides of the tent collector for early morning and late afternoon hours is:

$$P_b = P_{bA}^{sh} + P_{bB} \qquad (50)$$

For other hours of the day, side B will be partially shaded and eq. (48) applies.

Diffuse Irradiance

The diffuse irradiance on an incremental area dA is given by

$$dP_d = G_{dh}\, dA\, F_{dA-S} \tag{51}$$

where

G_{dh} - diffuse irradiance on a horizontal surface

F_{dA-S} - view factor of dA with respect to the sky

For sufficiently small dS, the view factor for dA is given by:

$$F_{dA-S} = 0.5[1 + \cos\beta(y)] \tag{52}$$

Using eqs. (34), (36) and (52) for eq. (51) we obtain:

$$dP_d = 0.5\, G_{dh}\, L(\sqrt{1 + [f_c'(y)]^2} + 1)dy$$

Integrating for y in the interval [0,D] we get the diffuse irradiance on one side of the tent:

$$P_d = 0.5\, G_{dh}\, L(\overset{\frown}{N\, N_n} + D) = 0.5\, G_{dh}\, L[k \sinh(D/k) + D] \tag{53}$$

Since the diffuse irradiance is independent of the orientation of the collector (for isotropic skies), eq. (53) applies also to the other side of the tent.

The view factor of a catenary collectors can now be calculated by defining:

$$P_d = G_{dh}\, A\, F_{A-S} \tag{54}$$

using eqs. (17) and (53) one obtains:

$$F_{A-S} = 0.5\left[1 + \frac{D}{k \sinh(D/k)}\right] \tag{55}$$

Albedo

The albedo can be determined by using the expression:

$$P_{al} = al\ G_h\ A\ F_{A-G} \tag{56}$$

where al - the albedo factor

G_h - global irradiance on a horizontal surface

F_{A-G} - view factor of area A with respect to ground

The view factor of area A with respect to the sky, F_{A-S}, and with respect to the ground, F_{A-g}, are related by [5]:

$$F_{A-S} + F_{A-G} = 1 \tag{57}$$

obtaining with eq. (55):

$$F_{A-G} = 0.5 \left[1 - \frac{D}{k\ \sinh(D/k)}\right] \tag{58}$$

Using eq. (14), (17), (56) and (58) we obtain the albedo irradiance on one side of the tent as:

$$P_{al} = 0.5\ al\ G_h\ L[k\ \sinh(D/k) - D] \tag{59}$$

Since the albedo irradiance is independent on the orientation of the collector eq. (59) applies also to other side of the tent.

4. Example

The example refers to a south-north facing catenary-tent-collector deployed on the Mars surface [6] at the locations of Viking Lander VLI (Latitude - 22.3°N, Longitude -

47.9°W) and in an autumn day L_s = 200° [6]. For this time of the year eq. (3) does not apply and, therefore, only side B will be partially shaded during the day. The dimensions of the catenary collector are: D = 3m, H = 2m and L = 1.5m. Since $f_c(0)$ = H, this results in k = 2.53 and the catenary equation is given by:

$$f_c(y) = 2.53[\cosh(1.186 - 0.395y) - 1] \tag{60}$$

Figure 10(a) shows the shadow shapes on side B of the catenary collector for ϕ = 22.3°N for different hours of the day. The shadow calculations are based on section 2. For comparison, the shadow shapes for ϕ=32° are shown in Fig. 10(b). As expected, the self-shading effect is more pronounced for higher latitudes. It is interesting to note that for ϕ=22.3°N, the shadow effect is quite small during the noon hours. In summer, the shadow will be even less.

The insolation of the catenary-tent-collector for ϕ=22.3°N, L_s=200° and albedo al=0.22 is shown in Table 1 based on radiation data at VLI [6]. The table shows the beam, diffuse and the albedo insolations on sides A and B in kWhr-day and kWhr/m^2-day. As expected, the beam insolation on side B is lower by 59.6% than on side A. It is interesting to note that the diffuse insolation comprises 46.6% of the global insolation. This characteristic is typical for Mars, a place where the atmosphere consists mainly of dust particles.

Table 1. - Insolation on a catenary-tent collector, ϕ=22.3°N, L_s=200°, al=0.22

	BEAM COMPONENT			DIFFUSE	ALBEDO	GLOBAL
	side A	side B	Total	COMPONENT	COMPONENT	INSOLATION
Q(kWhr-day)	12.65	5.11	17.76	16.27	0.89	34.92
q(kWhr/m^2-day)	2.246	0.907	1.577	1.445	0.079	3.101

5. CONCLUSIONS

The article analyses the performance of a catenary-tent-collector [1] (a flexible blanket that falls freely on both sides of a central support). This kind of collectors has characteristics (portability and simplicity) that are desirable for solar power plants on outer space planets. Because of its shape, there is a self-shading effect that must be taken into account in the solar radiation calculation. Therefore, the shape and area of the shadow on the collector is calculated and used in the determination of the beam radiation. The diffuse and albedo radiation were also calculated to determine the global radiation on the collector. The numerical example is based on solar radiation data on Mars.

6. NOMENCLATURE

A	Collector area, [m^2]
A_{sh}	shaded area of a collector, [m^2]
A_{ush}	unshaded area of a collector, [m]
D	length of the collector, [m]
F_{A-G}	view factor of area A with respect to ground
F_{A-S}	view factor of area A with respect to sky
F_{dA-S}	view factor of incremental area dA with respect to sky
G_b	direct beam irradiance, [W/m^2]
G_{dh}	diffuse irradiance on a horizontal surface, [W/m^2]
G_h	global irradiance on a horizontal surface, [W/m^2]
H	height of collector, [m]
k	catenary constant

L	collector width, [m]
L_s	areocentric longitude of the sun (for Mars)
P_{al}	albedo irradiance, [W]
P_b	direct beam irradiance on an unshaded side of a collector, [W]
P_b^{sh}	direct beam irradiance on a partially shaded side of a collector, [W]
P_{bA}, P_{bB}	direct beam irradiance on an unshaded side A and B of a collector, respectively, [W]
P_{bA}^{sh}, P_{bB}^{sh}	direct beam irradiance on a partially shaded side A and B of a collector, respectively, [W]
P_x, P_y	x and y components of a shadow length, [m]
Q	insolation, [kWhr-day]
q	insolation, [kWhr/m^2-day]
α	sun altitude
γ_c	collector azimuth
γ_s	sun azimuth
γ_{sr}	sun azimuth at sunrise
δ	solar declination angle
ϵ	characteristic angle of the collector
θ	angle between solar ray and the normal to the collector
ξ	relative shaded area
ϕ	local latitude
ω	solar hour angle

7. REFERENCES

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APPENDIX A

Calculation of points E and T

Calculation of Point E and T for case (i)

Point E

Point E may be calculated from the projection of the solar ray $\overline{NP}$ on the z-y plane, Fig. A1 (a). The projection line equation is:

$$z = H(1 - y/P_y) \tag{A.1}$$

The intersection of the projection line, eq. (A1) with the catenary equation $f_c(y)$ yields the coordinate y_E, i.e., the solution of:

$$\frac{k}{H}\cosh\left(\frac{D - y_E}{k}\right) + \frac{y_E}{P_y} = 1 + \frac{k}{H} \tag{A.2}$$

The coordinate x_E is on line $\overline{OP}$, i.e.,

$$x_E = \frac{P_x}{P_y} y_E \tag{A.3}$$

and finally the coordinate z_E is calculated from eq. (A1).

Point T

Point T is determined by a sun ray that its projection on the z-y plane is tangent to the arc $\overset{\frown}{N\ N_n}$, Fig. A1 (b), i.e., at this point, the derivative of $f_c(y)$ is equal to the slope of line $\overline{AB}$. From eq. (1)

$$f_c'(y) = -\sinh\left(\frac{D - y}{k}\right) \tag{A.4}$$

Since $\overline{AB}$ and $\overline{N\ P_y}$ are parallel lines, the slope of line $\overline{AB}$ is $-H/P_y$, thus obtaining:

$$y_T = D - k \sinh^{-1}(H/P_y) \tag{A.5}$$

The coordinate $x_T = 0$ and the coordinate z_T is calculated from the $f_c(y)$ equation.

Calculation of y_U and x_E for case (ii)

A sun ray that touches the foot of the tent at point E passes through point U on the blanket, Figs. 5 and A.1.(a). It follows that

$$(D - y_U)/x_E = P_y/P_x \tag{A.6}$$

and

$$\frac{z_U}{D - y_U} = \frac{f_c(y_U)}{D - y_U} = \frac{H}{P_y} \tag{A.7}$$

from which y_U is determined by solving:

$$f_c(y_U) + \frac{H}{P_y} y_U = \frac{DH}{P_y} \tag{A.8}$$

where $f_c(y)$ is given in eq. (1). The coordinate x_E is then calculated from eq. (A.6).

Calculation of y_E for case (iii)

Referring to Figs. 6 and A.1 (b), we may write

$$(y_E - y_U)/L = P_y/P_x \tag{A.9}$$

and

$$\frac{z_U - z_E}{y_E - y_U} = \frac{f_c(y_U) - f_c(y_E)}{y_E - y_U} = \frac{H}{P_y} \tag{A.10}$$

From eqs. (A.9) and (A.10), y_E is determined by:

$$f_c\left(y_E - L\,\frac{P_y}{P_x}\right) - f_c(y_E) = \frac{L\,H}{P_x} \qquad (A.11)$$

where $f_c(y)$ is given in eq. (1).

APPENDIX B

Using the parabolic approximation for $f_c(y)$ greatly simplifies the mathematical calculation. The error is small and the results are very similar to the catenary case. The procedure for calculating the points for the shadows are the same as in Appendix A but now $f_c(y)$ is the parabolic approximation (eq. (2)), i.e.,

$$f_c(y) = H\left(1 - \frac{y}{D}\right)^2 \quad , \quad f_c'(y) = \frac{-2H}{D}\left(1 - \frac{y}{D}\right) \qquad (B.1)$$

Case (i)

The coordinates of points E and T are:

$$x_E = D \cdot \frac{P_x}{P_y}\left(2 - \frac{D}{P_y}\right) \quad , \quad y_E = D\left(2 - \frac{D}{P_y}\right) \quad , \quad z_E = H\left(\frac{D}{P_y} - 1\right)^2 \qquad (B.2)$$

$$x_T = 0 \qquad , \quad y_T = D\left(1 - \frac{D}{2P_y}\right) \quad , \quad z_T = H\left(\frac{D}{2P_y}\right)^2 \qquad (B.3)$$

With these results, the arcs $\overset{\frown}{N\,N_n}$, $\overset{\frown}{N\,E_y}$ and $\overset{\frown}{T\,E_y}$ can now be determined. The length of an arc is given by eq. (16) and for the parabolic approximation we have:

$$g(y) = \int \sqrt{1 + [f_c'(y)]^2}\,dy = \int \sqrt{1 + \left[\frac{2H}{D^2}(D - y)\right]^2}\,dy$$

the solution of the integral is:

$$g(y) = \frac{D^2}{4H} \sinh^{-1}\left[\frac{2H}{D^2}(y - D)\right] + \frac{1}{2}(y - D)\sqrt{1 + \left[\frac{2H}{D^2}(y - D)\right]^2} \qquad (B.4)$$

The arc lengths are therefore:

$$\overset{\frown}{N\,N_n} = g(D) - g(0) \qquad (B.5)$$

$$\overset{\frown}{N\,E_y} = g(y_E) - g(0) \qquad (B.6)$$

$$\overset{\frown}{T\,E_y} = g(y_E) - g(y_T) \qquad (B.7)$$

Case (ii)

Solving eq. (A.8) with eq. (2) for y_U (Fig. 5) results in:

$$y_U = D\left(1 - \frac{D}{P_y}\right) \qquad (B.8)$$

and using eq. (A.6) we obtain the coordinates of x_E. The coordinates of points E and T are:

$$x_E = P_x\left(\frac{D}{P_y}\right)^2 \quad , \quad y_E = D \quad , \quad z_E = 0 \qquad (B.9)$$

$$x_T = 0 \quad , \quad y_T = D\left(1 - \frac{D}{2P_y}\right) \quad , \quad z_T = H\left(\frac{D}{2P_y}\right)^2 \qquad (B.10)$$

The expression for $\overset{\frown}{T\,N_n}$ is given by:

$$\overset{\frown}{T\,N_n} = g(y_T) - g(D) \qquad (B.11)$$

where g(y) is given in eq. (4).

Case (iii)

Solving eq. (A.11) with eq. (2) for y_E (Fig. 6) results in:

$$y_E = D\left(1 - \frac{D}{2P_y}\right) + L\,\frac{P_y}{2P_x} \qquad \text{(B.12)}$$

The coordinates of points E and T are:

$$x_E = L \quad , \quad y_E = D\left(1 - \frac{D}{2P_y}\right) + L\,\frac{P_y}{2P_x} \quad , \quad z_E = H\left(\frac{D}{2P_y} - \frac{L\,P_y}{2DP_x}\right)^2$$

$$x_T = 0 \quad , \quad y_T = D\left(1 - \frac{D}{2P_y}\right) \quad , \quad z_T = H\left(\frac{D}{2P_y}\right)^2$$

Table 1. - Insolation on a catenary-tent collector, ϕ=22.3°N, L_s=200°, al=0.22

	BEAM COMPONENT			DIFFUSE	ALBEDO	GLOBAL
	side A	side B	Total	COMPONENT	COMPONENT	INSOLATION
Q(kWhr-day)	12.65	5.11	17.76	16.27	0.89	34.92
q(kWhr/m^2-day)	2.246	0.907	1.577	1.445	0.079	3.101

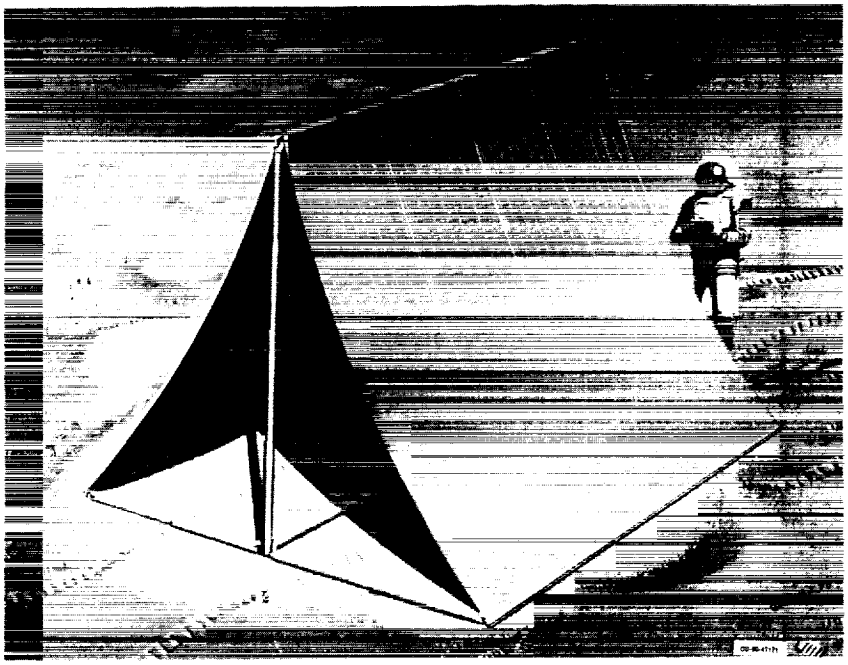

Figure 1.—Artist's view of a catenary-tent-collector.

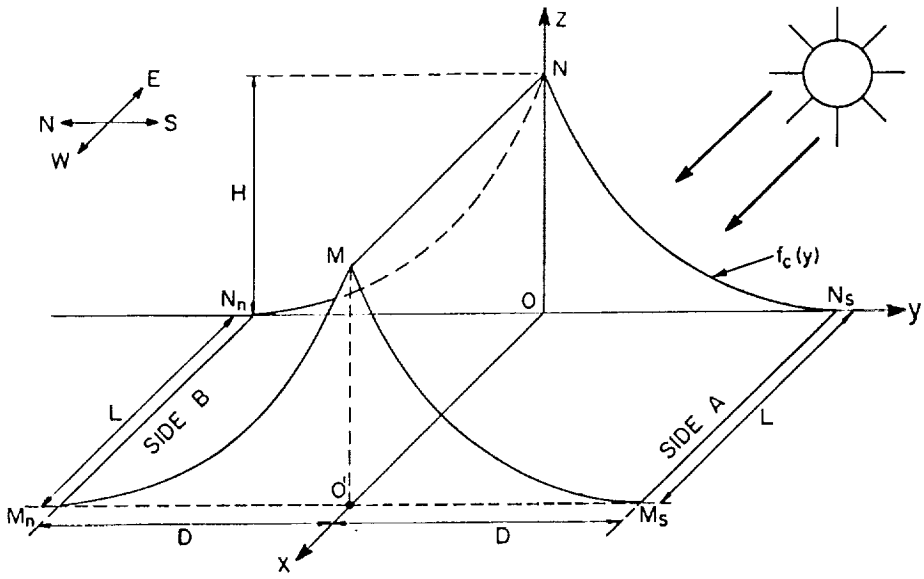

Figure 2.—Catenary-tent-collector: definitions and orientation.

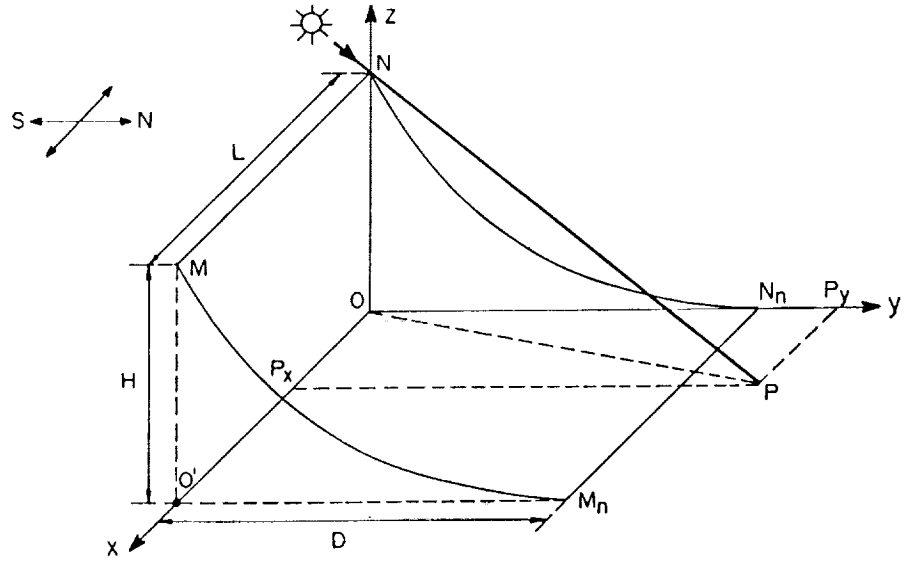

Figure 3.—Catenary collector: basic shadow parameters.

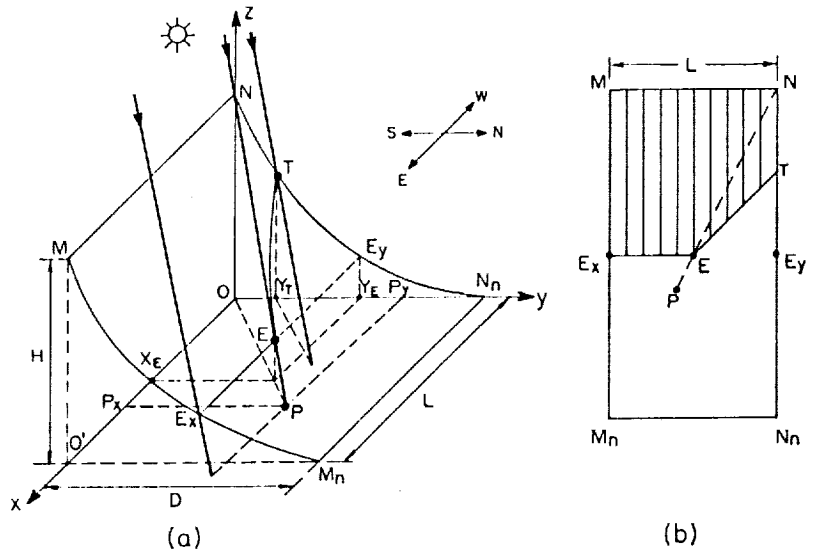

Figure 4.—Catenary collector - case (i) (a) shadow parameters, (b) stretched-out shadow shape.

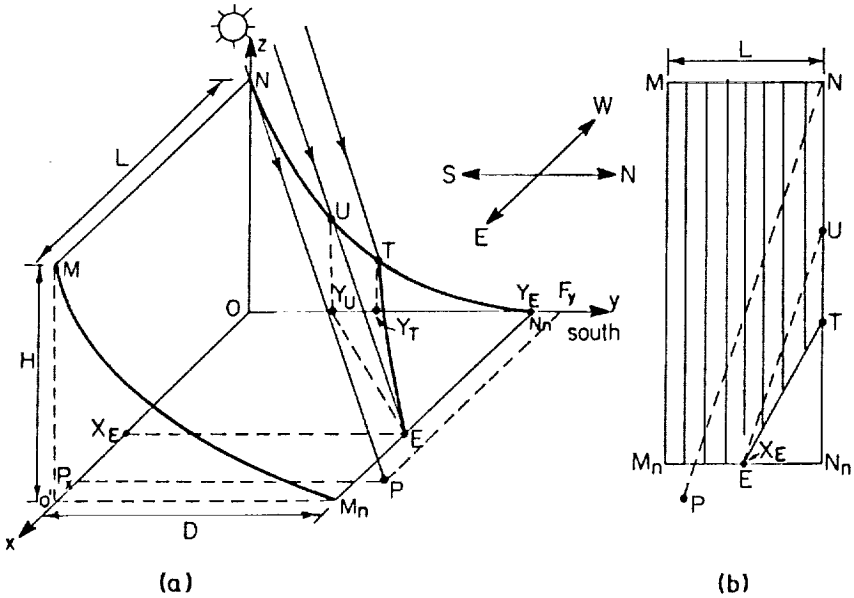

Figure 5.—Catenary collector - case (ii) (a) shadow parameters, (b) stretched-out shadow shape.

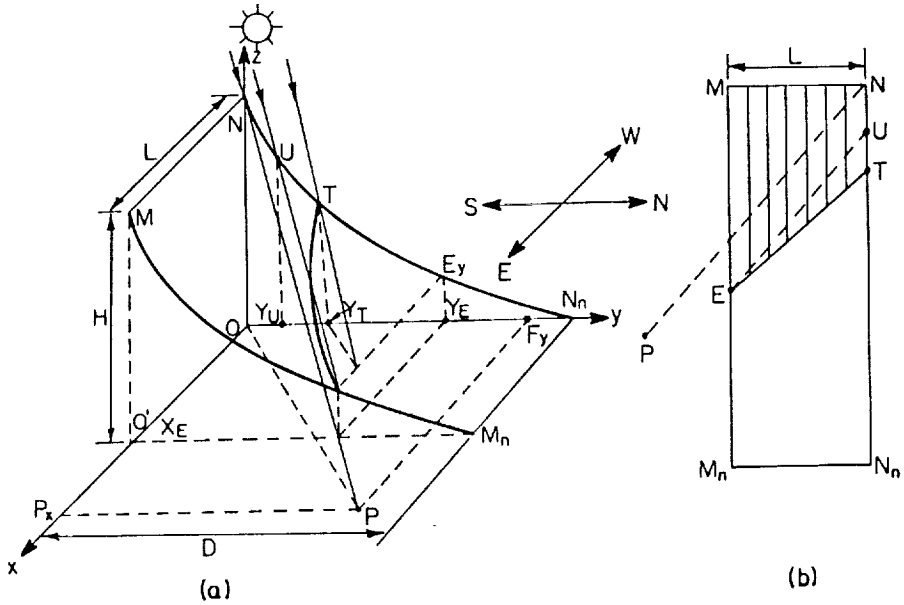

Figure 6.—Catenary collector - case (iii) (a) shadow parameters, (b) stretched-out shadow shape.

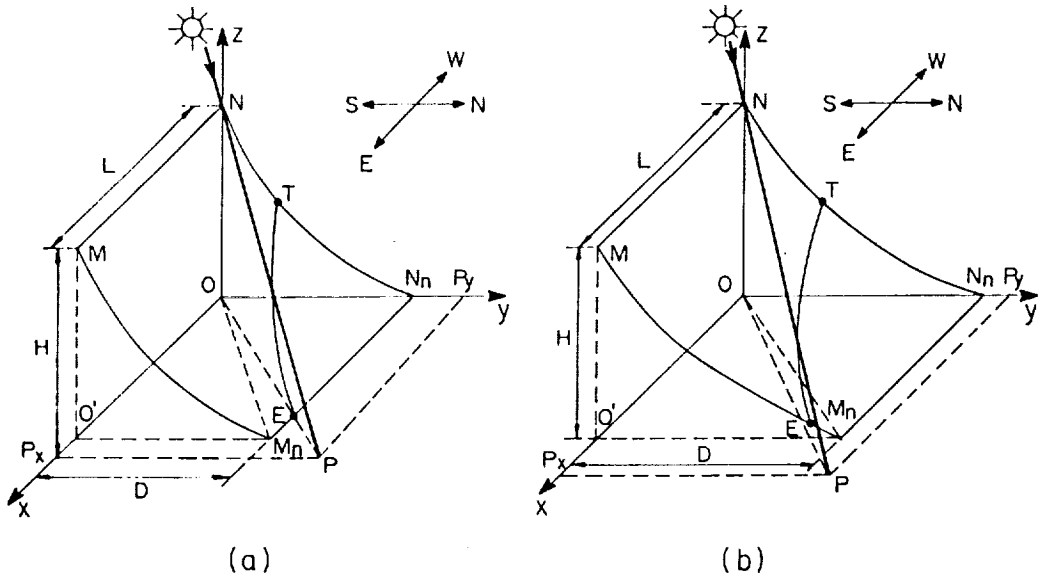

Figure 7.—Catenary collector - shadow shape for $P_x \geq L$, $P_y \geq D$ (a) $P_x/P_y \geq L/D$, (b) $P_x/P_y < L/D$.

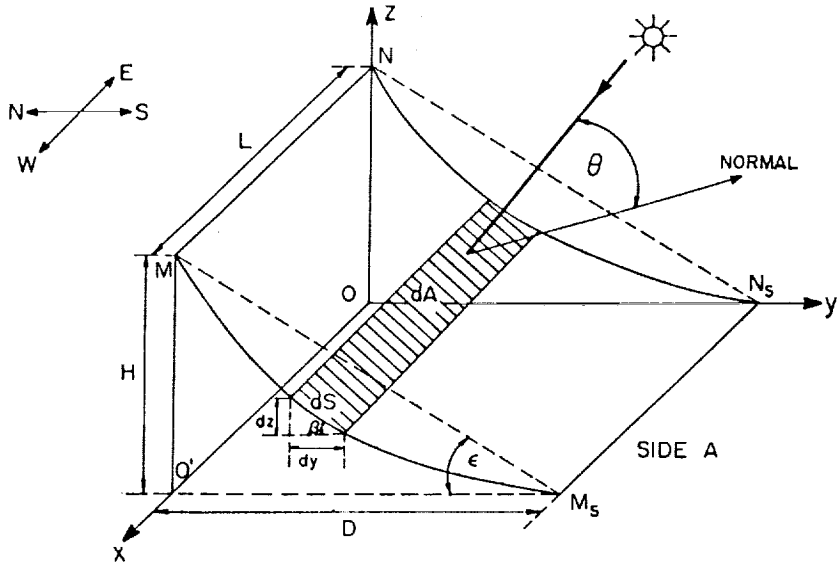

Figure 8.—Catenary collector - solar radiation calculation on an unshaded side.

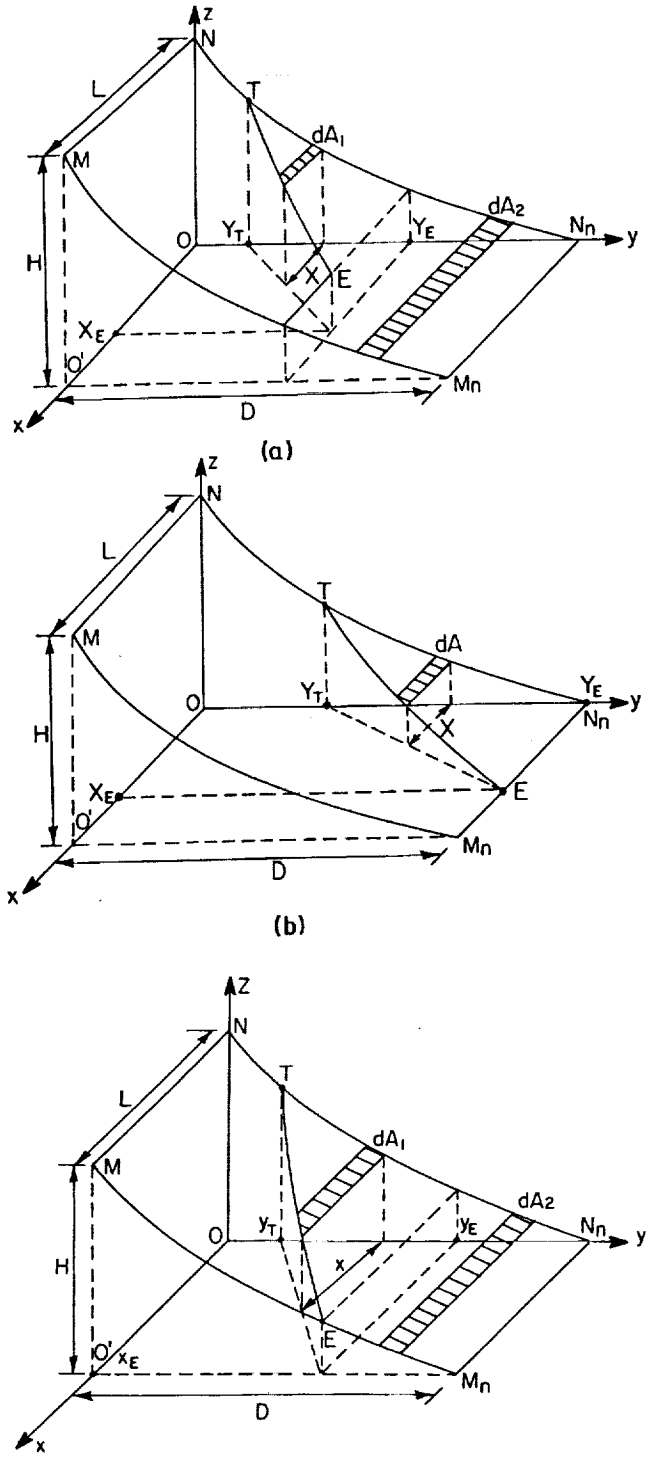

Figure 9.—Catenary collector - beam radiation on a partially shaded side (a) (i), (b) case (ii), (c) case (iii).

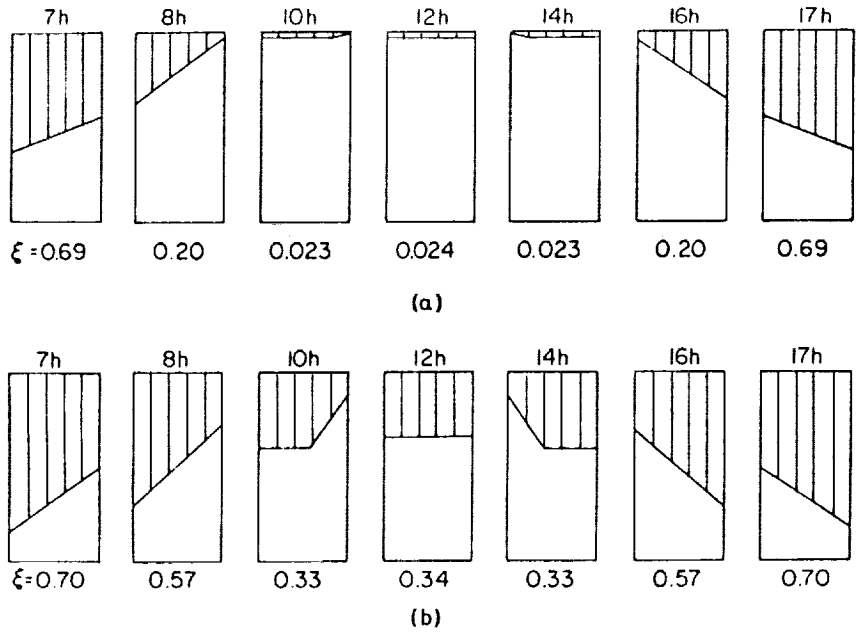

Figure 10.—Catenary collector - shadow shapes on side B at different hours of the day for (a) ϕ = 22.3.°N, (b) ϕ = 32°N.

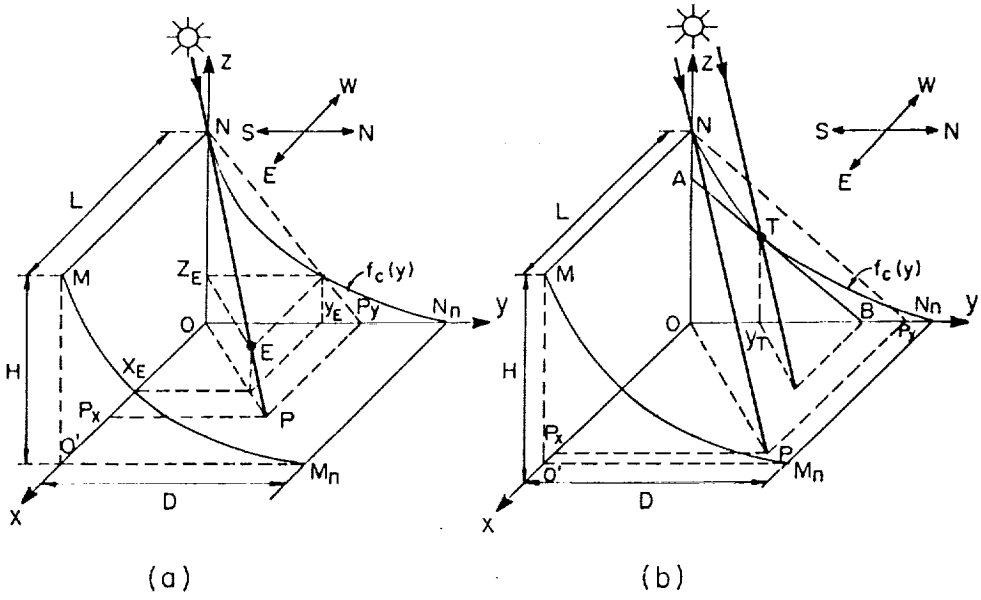

Figure A.1—Catenary collector - calculation of points (a) E, (b) T.

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M. Crutchik, Tel-Aviv University, Faculty of Engineering, Tel-Aviv 69978, Israel. J. Appelbaum, NASA Lewis Research Center, Cleveland, Ohio 44135.

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13. ABSTRACT (*Maximum 200 words*)

A tent-shaped structure with a flexible photovoltaic blanket acting as a catenary collector is presented. The shadow cast by one side of the collector on the other side producing a self shading effect is analyzed. The direct beam, the diffuse and the albedo radiation on the collector are determined. An example is given for the isolation on the collector operating on Mars surface for the location of Viking Lander 1 (VL1).

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