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OPTIMUM DESIGN OF NINETY DEGREE BENDS

Vijay Modi
Department of Mechanical Engineering
Columbia University

Abstract

An algorithm for the optimum design of an internal flow component to obtain the maximum pressure rise is presented. Maximum pressure rise in a duct with simultaneous turning and diffusion is shown to be related the control of flow separation on the passage walls. Such a flow is usually associated with downstream conditions that are desirable in turbomachinery and propulsion applications to ensure low loss and stable performance. The algorithm requires the solution of an "adjoint" problem in addition to the "direct" equations governing the flow in a body, which in the present analysis are assumed to be the laminar Navier-Stokes equations. Earlier studies have usually addressed such problems for the case of inviscid and/or irrotational flow. These assumptions may not be valid in flows that undergo sharp turning resulting in strong secondary flows and possibly separating and recirculating regions. The theoretical framework and computational algorithms presented in this study are for the steady Navier-Stokes equations.

A novel procedure is developed for the numerical solution of the adjoint equations. This procedure is coupled with a direct solver in a design iteration loop, that provides a new shape with a higher pressure rise. This procedure is first validated for the design of optimum plane diffusers in two-dimensional flow. The direct Navier-Stokes and the "adjoint" equations are solved using a finite volume formulation for spatial discretization in an artificial compressibility framework. The discretized equations are integrated using explicit Runge-Kutta time steps to obtain steady-state solutions. It is found that the computational work required to solve the "adjoint" problem is of the same order as that required to solve the direct problem. It is also found that the procedure converges within about ten iterations, and in addition, the number of design iterations are not sensitive to the grid used for the calculations. This is a significant computational advantage over heuristic design procedures based on point by point sensitivity analysis where the work increases with the refinement of the grid.

A simplified version of the above approach is then utilized to design ninety degree diffusing bends. The bend inlet is square with intermediate and exit cross-sections constrained to be rectangular. The location of bend walls is then determined in order to obtain the maximum pressure rise through the bend. Calculations were carried out for a mean radius ratio at inlet of 2.5 and Reynolds numbers varying from 100 to 500. While at this stage laminar flow is assumed it is shown that a similar approach can be conceived for turbulent flows.

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Vijay Modi, Assistant Professor

Hayri Cabuk, Post Doctoral Research Associate

Jian-Chun Huan, Graduate Student

Richard Quadracci, Graduate Student

Department of Mechanical Engineering,

Columbia University,

New York, New York 10027

MOTIVATION

How to shape internal flow passages

Combined turning and diffusing flow

Maximize pressure rise

Can not assume inviscid or 2-D flow

OBJECTIVES

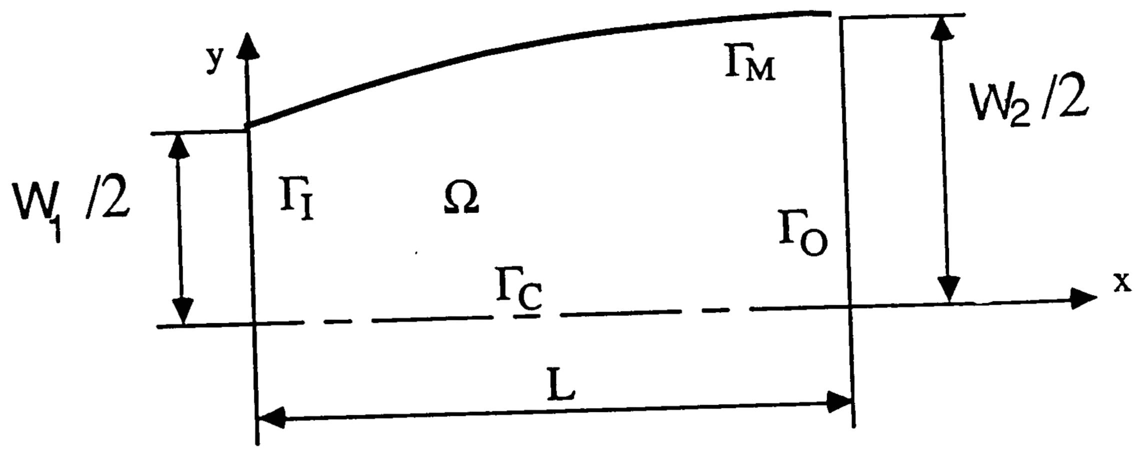
Develop theoretical framework – laminar 3-D

Develop Navier-Stokes and Adjoint solvers

Validate Navier-Stokes solver
(Laminar 90 degree bend, Taylor et al. 1982)

Validate Optimization Approach on
2-D straight diffusers

Apply to the design of ninety degree bends



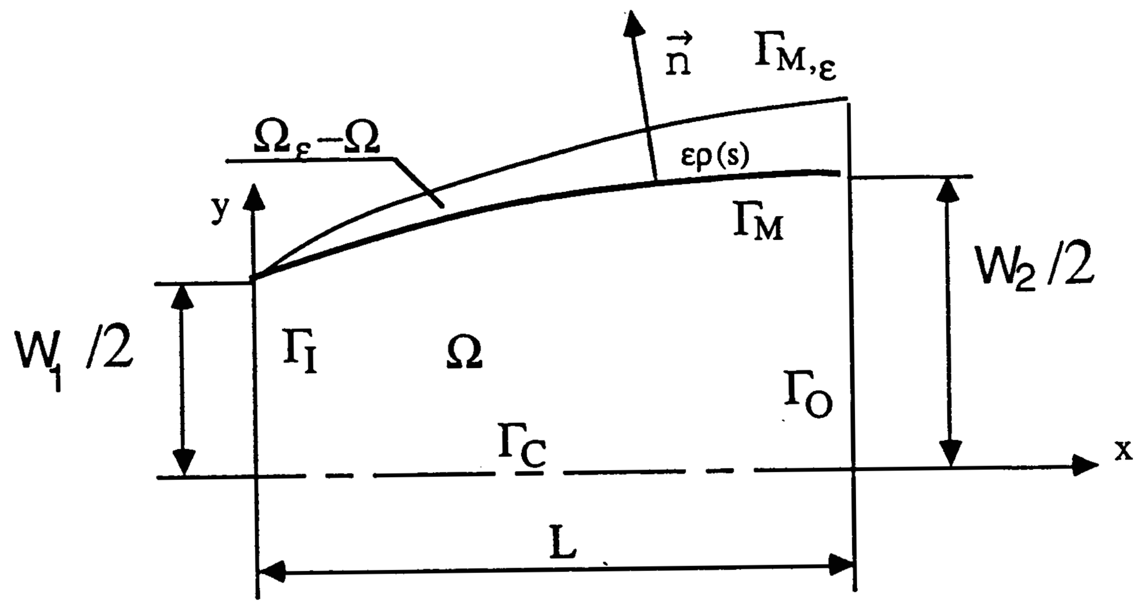
$$u_{i,i} = 0$$

$$u_j u_{i,j} = -p^*_{,i} + \nu u_{i,jj} \quad , \quad p^* = p/\rho$$

\# **No slip BC on** Γ_M

\# **Dirichlet BC for** u_i **at** Γ_I **and** Γ_O

$$J(\Gamma_M) = \int_{\Gamma_I} p^* u_i n_i ds + \int_{\Gamma_O} p^* u_i n_i ds$$



$$[u_i^\varepsilon, p^\varepsilon] \equiv \text{ Solution to NS in } \Omega_\varepsilon$$

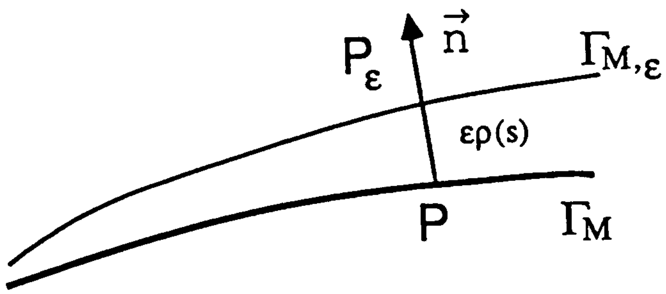
$$u_i^\varepsilon = u_i + \epsilon\phi_i$$

$$p^\varepsilon = p^* + \epsilon\pi$$

$$\phi_{i,i} = 0$$

$$u_j\phi_{i,j} + \phi_j u_{i,j} = -\pi_{,i} + \nu\phi_{i,jj}$$

$$\phi_i = 0 \qquad \text{on } (\Gamma - \Gamma_M)$$



$$
\begin{aligned}
u_i^\varepsilon]_{P_\varepsilon} &= u_i^\varepsilon]_P + \epsilon\rho\left(\frac{\partial u_i^\varepsilon}{\partial n}\right)_P + O(\epsilon^2) \\
&= u_i]_P + \epsilon\phi_i]_P + \epsilon\rho\left(\frac{\partial u_i}{\partial n}\right)_P + O(\epsilon^2)
\end{aligned}
$$

$$
\phi_i = -\rho\left(\frac{\partial u_i}{\partial n}\right) \qquad \text{on } \Gamma_M .
$$

$$J(\Gamma_{M_\epsilon}) - J(\Gamma_M) = \epsilon\delta J + O(\epsilon^2)$$

$$\delta J = \int_{\Gamma_I} \pi u_i n_i ds + \int_{\Gamma_O} \pi u_i n_i ds$$

$$\begin{aligned} z_{i,i} &= 0 && \text{in } \Omega \\ \nu z_{i,jj} + u_j(z_{i,j} + z_{j,i}) - r_{,i} &= 0 && \text{in } \Omega \\ z_i &= u_i && \text{on } \Gamma \end{aligned}$$

$$\delta J = \nu \int_{\Gamma_M} \rho(s) \left(\frac{\partial u_i}{\partial n}\right) \left(\frac{\partial z_i}{\partial n}\right) ds$$

$$\rho(s) = \omega(s) \left(\frac{\partial u_i}{\partial n}\right) \left(\frac{\partial z_i}{\partial n}\right)$$

 # Our “Adjoint” equation:

$$\nu z_{i,jj} + u_j(z_{i,j} + z_{j,i}) - r_{,i} = 0 \qquad \text{in } \Omega$$

$$w_i = \frac{1}{2}(z_i - u_i)$$

$$q = \frac{1}{2}\left(r - p^* + (1/2)u_j^2 - 2u_j w_j\right)$$

 # Pironneau’s “Adjoint” equation:

$$w_{i,i} = 0 \qquad \text{in } \Omega$$

$$\nu w_{i,jj} + u_j w_{i,j} + \underline{\underline{w_j u_{j,i}}} - q_{,i} = \underline{\underline{-u_j u_{i,j}}} \qquad \text{in } \Omega$$

$$w_i = 0 \qquad \text{on } \Gamma$$

$$\delta J = \nu \int_{\Gamma_M} \rho(s) \left(\frac{\partial u_i}{\partial n}\right)\left(\frac{\partial u_i}{\partial n} + 2\frac{\partial w_i}{\partial n}\right) ds$$

$$\frac{dw}{dt} = w(\text{const}) + \cdots$$

NAVIER-STOKES SOLVER

$$p_t = -\beta^2 u_{i,i}$$

$$u_{i,t} + u_j u_{i,j} = -p_{,i} + \nu u_{i,jj}$$

a) **Artificial Compressibility**

b) **Runge-Kutta Time Integration**

c) **Finite Volume Discretization**

d) **Artificial Dissipation**

e) **Local Time Stepping**

e) **Implicit Residual Smoothing**

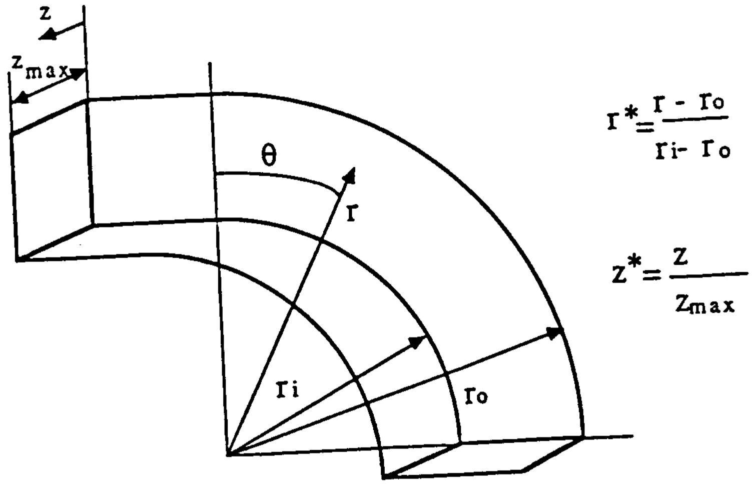


Figure 4: Geometry of a circular bend with square cross section.

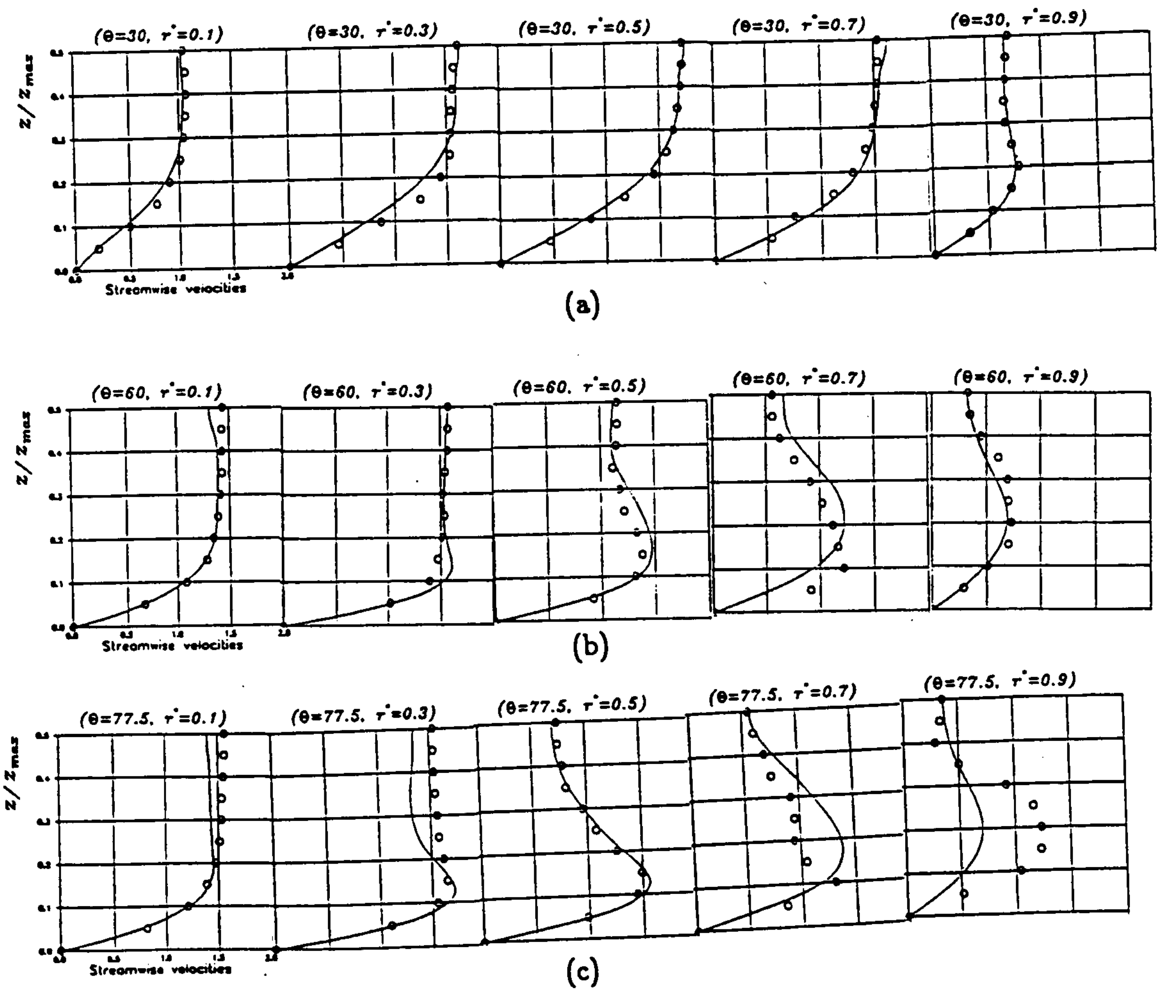


Figure 5: Streamwise velocities in a bend: a) $\Theta = 30$ degrees, b) $\Theta = 60$ degrees, c) $\Theta = 77.5$ degrees.

ADJOINT EQUATION SOLVER

$$r_t^* = -\beta^2 z_{i,i}$$

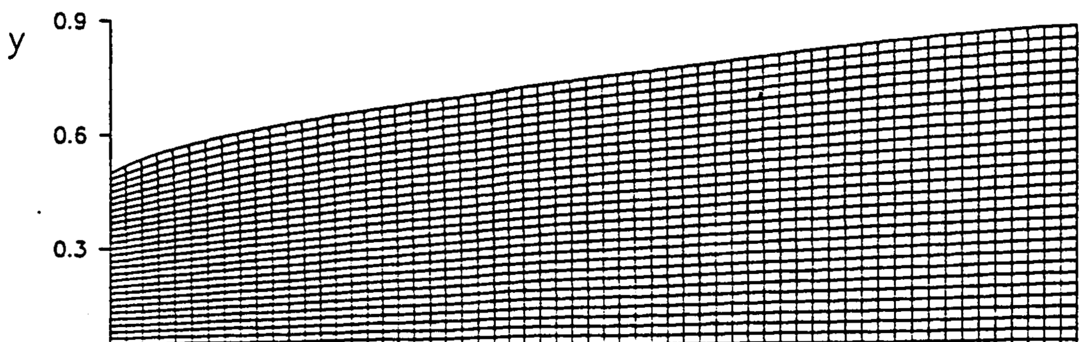
$$z_{i,t} = \nu z_{i,jj} + u_j(z_{i,j} + z_{j,i}) - \frac{1}{2}\left(z_k z_k\right)_{,i} - r^*_{,i}$$

MESH GENERATION

Thompson et al.

$$x_{\xi\xi} + x_{\eta\eta} = 0$$

$$y_{\xi\xi} + y_{\eta\eta} = 0$$



Boundary Conditions

No-slip bc on the walls

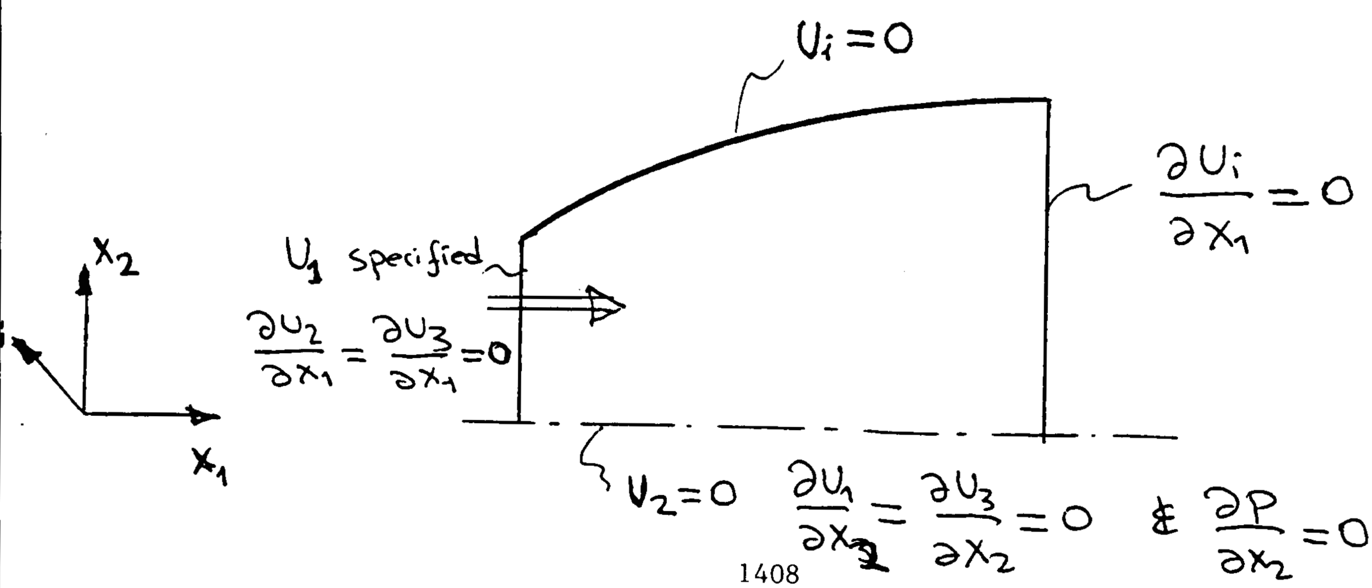
Zero normal derivatives at exit

Streamwise velocity component is specified at entrance

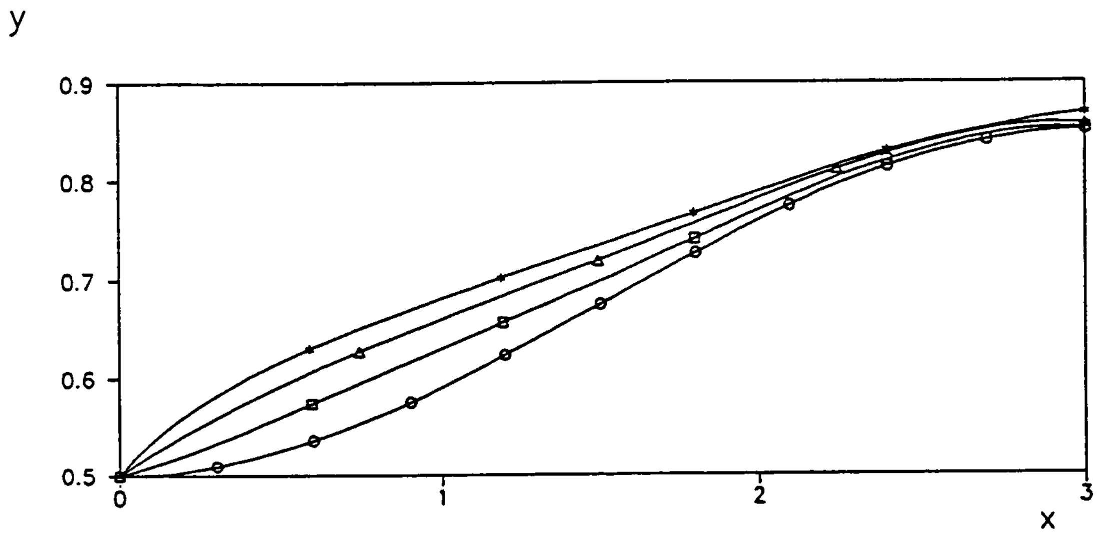
Zero normal derivatives for remaining velocities at entrance

Typical bc's at symettry planes

Zero second derivatives for pressure (a computational bc)



Diffuser Profile History



Re=200, 61 by 31 grid

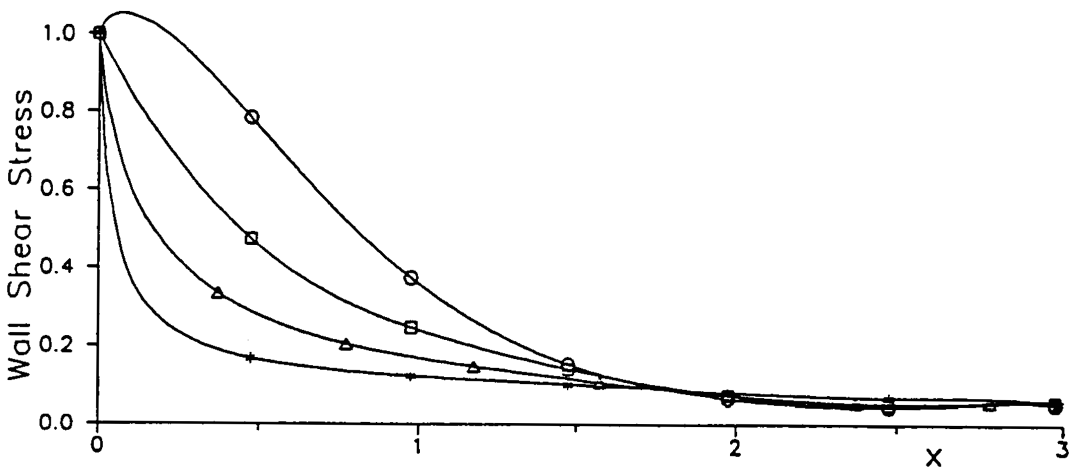
○ : N=1

⊓ : N=2

△ : N=5

∗ : N=10

Skin Friction History



Re=200, 61 by 31 grid

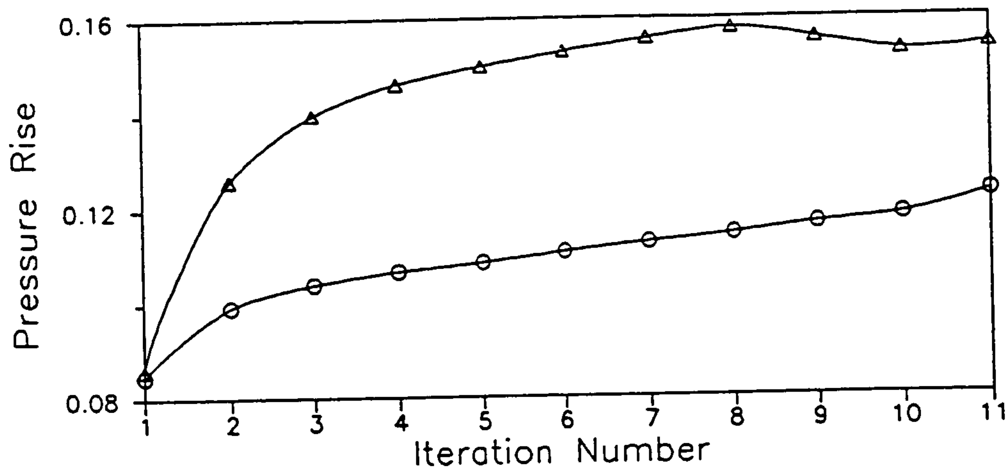
○ : N=1

□ : N=2

△ : N=5

*** : N=10**

Pressure Rise History

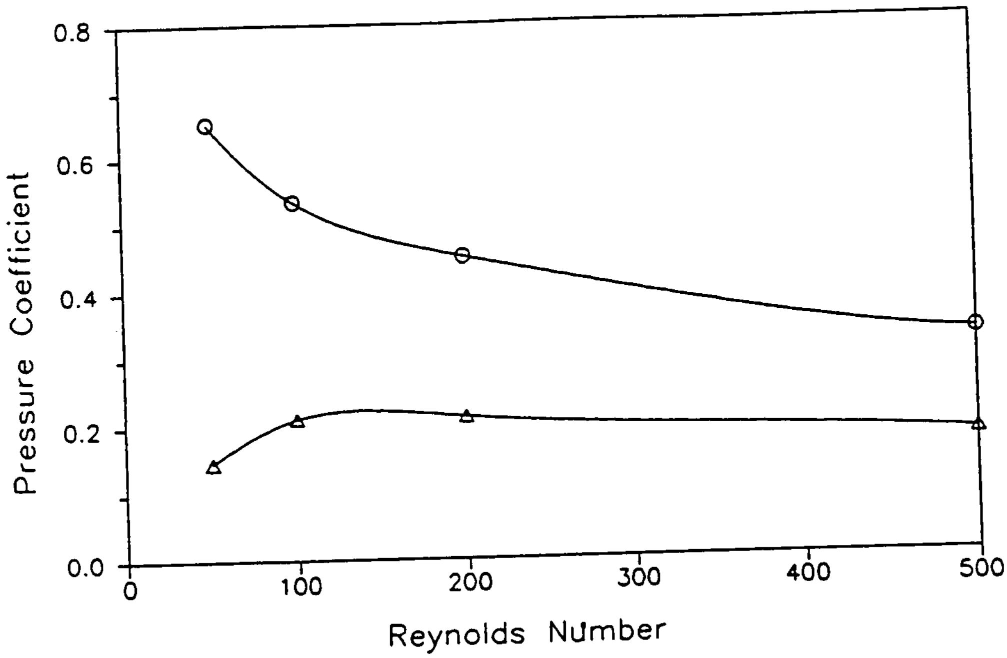


Re=200, 61 by 31 grid

○ : Area-averaged pressure rise

△ : Flow-averaged pressure rise

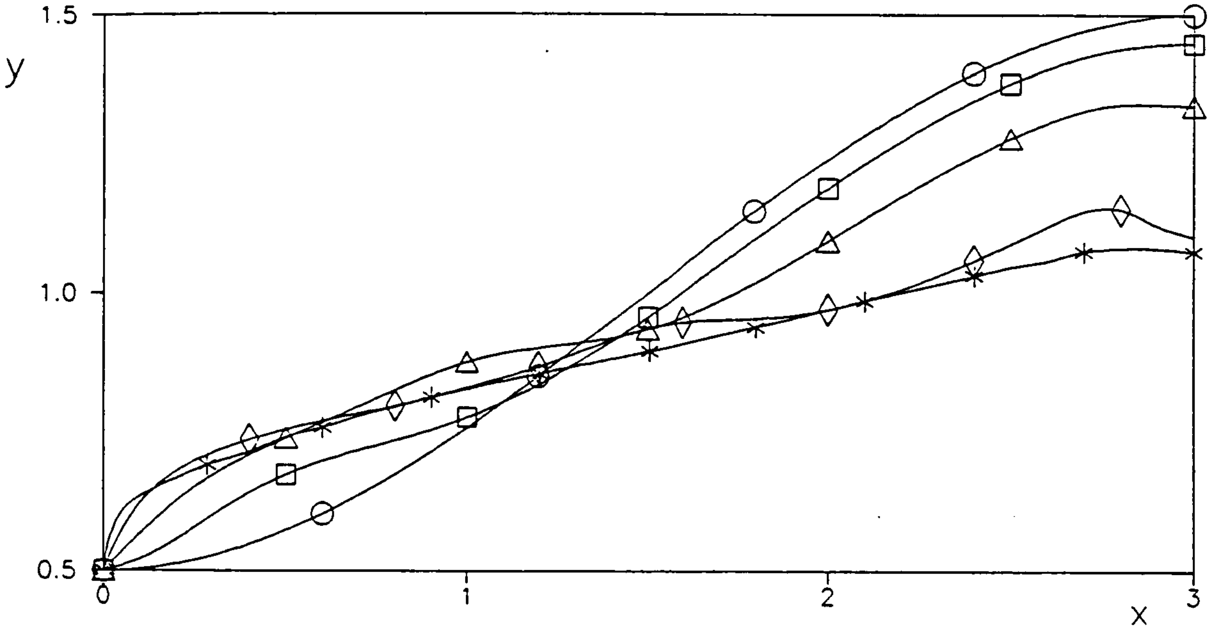
$$C_p \equiv \frac{\text{Actual Pressure Rise}}{\text{Ideal Pressure Rise}}$$



○ : **Optimal diffusers**

△ : **Straight diverging diffusers**

Separating Initial Profile



Re=100, 31 by 11 grid

○ : N=1

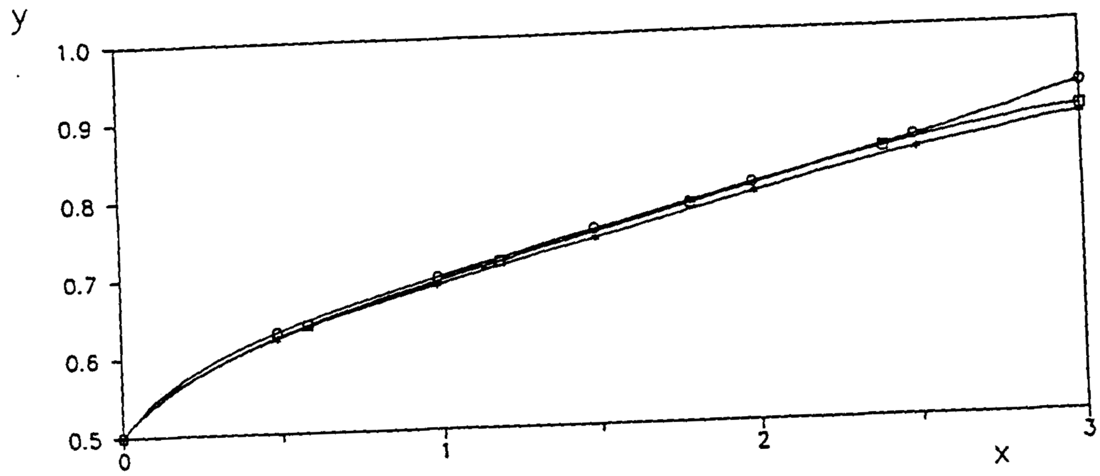
□ : N=2

△ : N=3

◇ : N=4

∗ : N=11

Grid Study

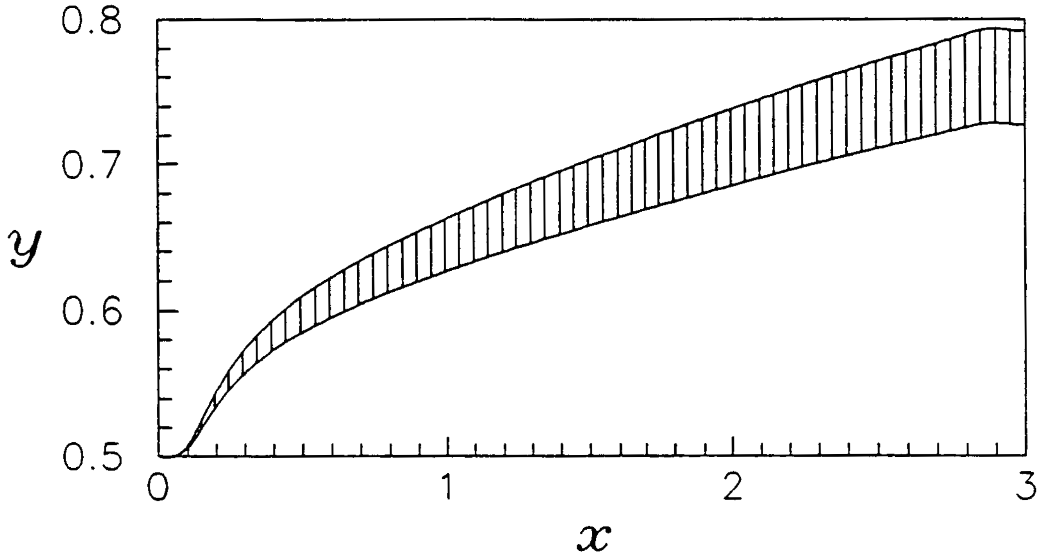


Re=200

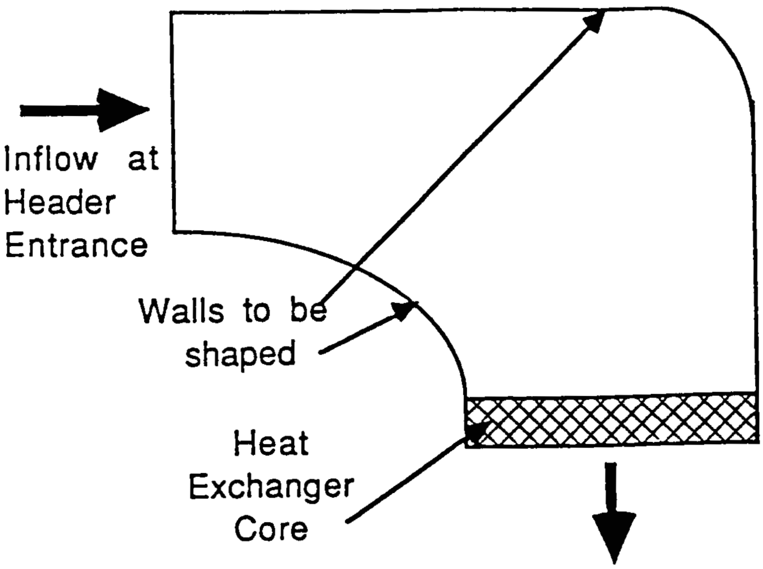
○ : 31 by 16

□ : 61 by 31

*** : 121 by 61**



The error in the location of the optimal diffuser profile corresponding to a 2 percent error in the total pressure rise. The optimum shape lies between the high and low y–values shown in the graph (Re=500, grid size is 61 by 21, and $L/W_1=3$).



Sketch of the inlet header

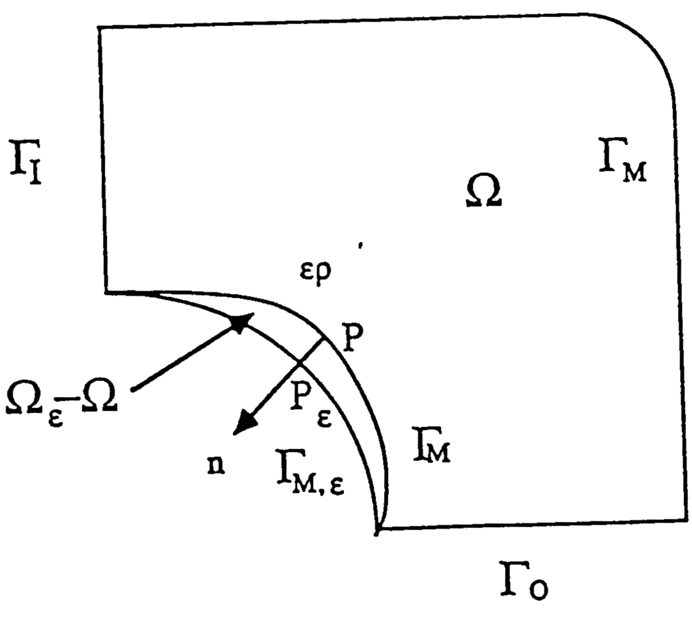


Figure 3: A representative section of a three-dimensional diffuser. Flow enters at upstream boundary Γ_I and exits at downstream boundary Γ_O. The walls to be shaped are Γ_M.

ISSUES

\# Geometry Constraints

In general: Move walls by $\epsilon\rho(s)$ along the normal direction, everywhere

New shape may not satisfy

- specified mean passage location
- specified cross-sectional shape
- overall system geometry

\# Present Work

- No correction on side walls ($z = 0$, $z = z_{\max}$)
- Apply mid-plane ($z = z_{\max}/2$) correction to all z locations
- Hence all cross-sections are rectangular

\# Laminar flow results ($Re < 500$)

Governing Equations:

$$u_{i,i} = 0$$

$$u_j u_{i,j} = -p_{,i} + \nu u_{i,jj}$$

Design Objective:

Maximize Static Pressure Rise

$$J = \int_{\Gamma_I} p \, ds + \int_{\Gamma_O} p \, ds$$

Cabuk and Modi, 1990

$$\left(\frac{\partial u}{\partial n}\right)_{\text{wall}} = 0$$

$$\left(\frac{\partial u}{\partial n}\right)_{\text{wall}} = \epsilon$$

DESIGN ALGORITHM

1: Choose an initial shape.

2: Generate the computational grid.

3: Solve the N-S equations.

4: Compute shear stress on the walls.

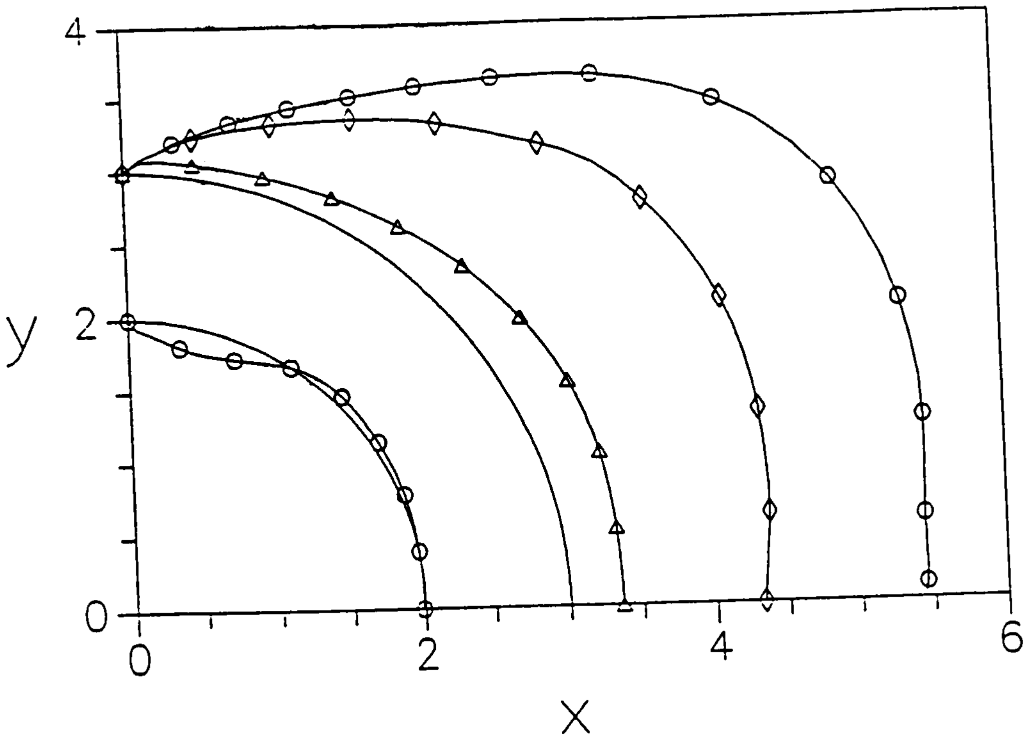
5: Compare wall shear stress to target distribution and determine the amount of boundary movement $\rho(s)$.

6: Update the shape.

7: Go to step (2)

$$\rho(s) = \omega(s) \left[\left(\frac{\partial u}{\partial n} \right)_{\text{wall}} - \left(\frac{\partial u}{\partial n} \right)_{\text{target}} \right]$$

Iteration History (Re=200)

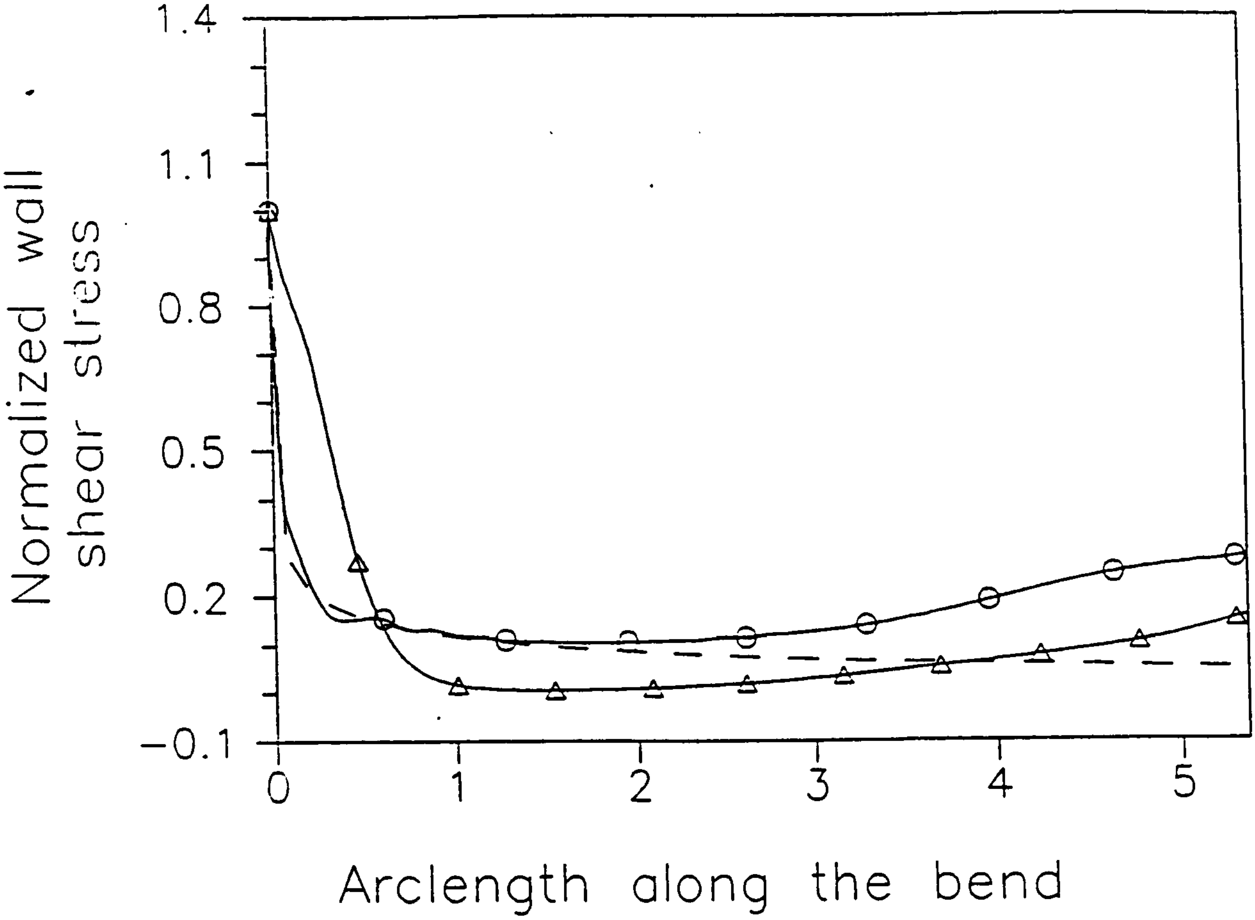


Dashed Curve : N=1

○ : **N=8**

△ : **N=2**

◇ : **N=5**

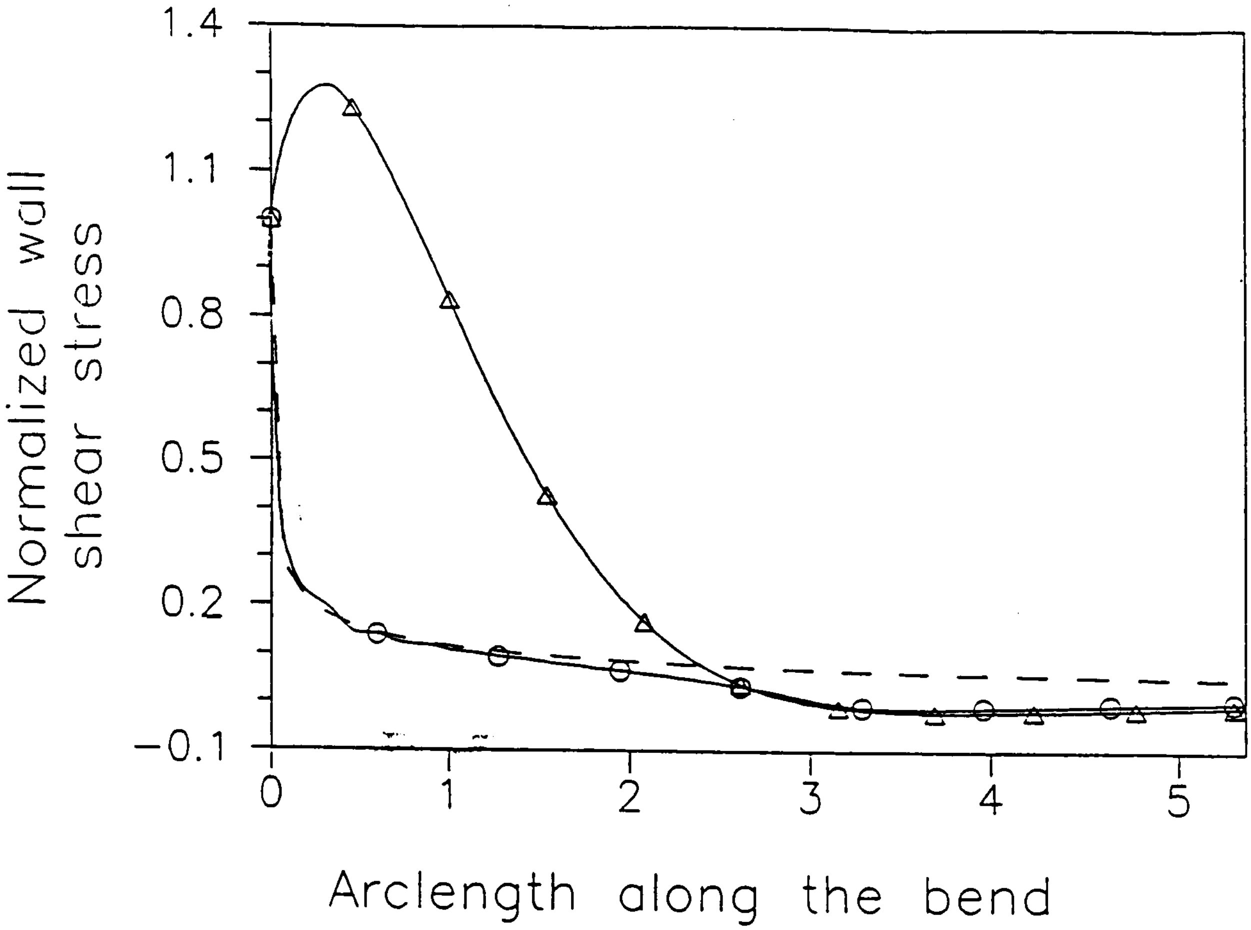


Wall shear stress along the outer wall, Re=100

○ : Optimum diffusing bend

△ : Elliptic diffusing bend

Dashed Curve : Target distribution



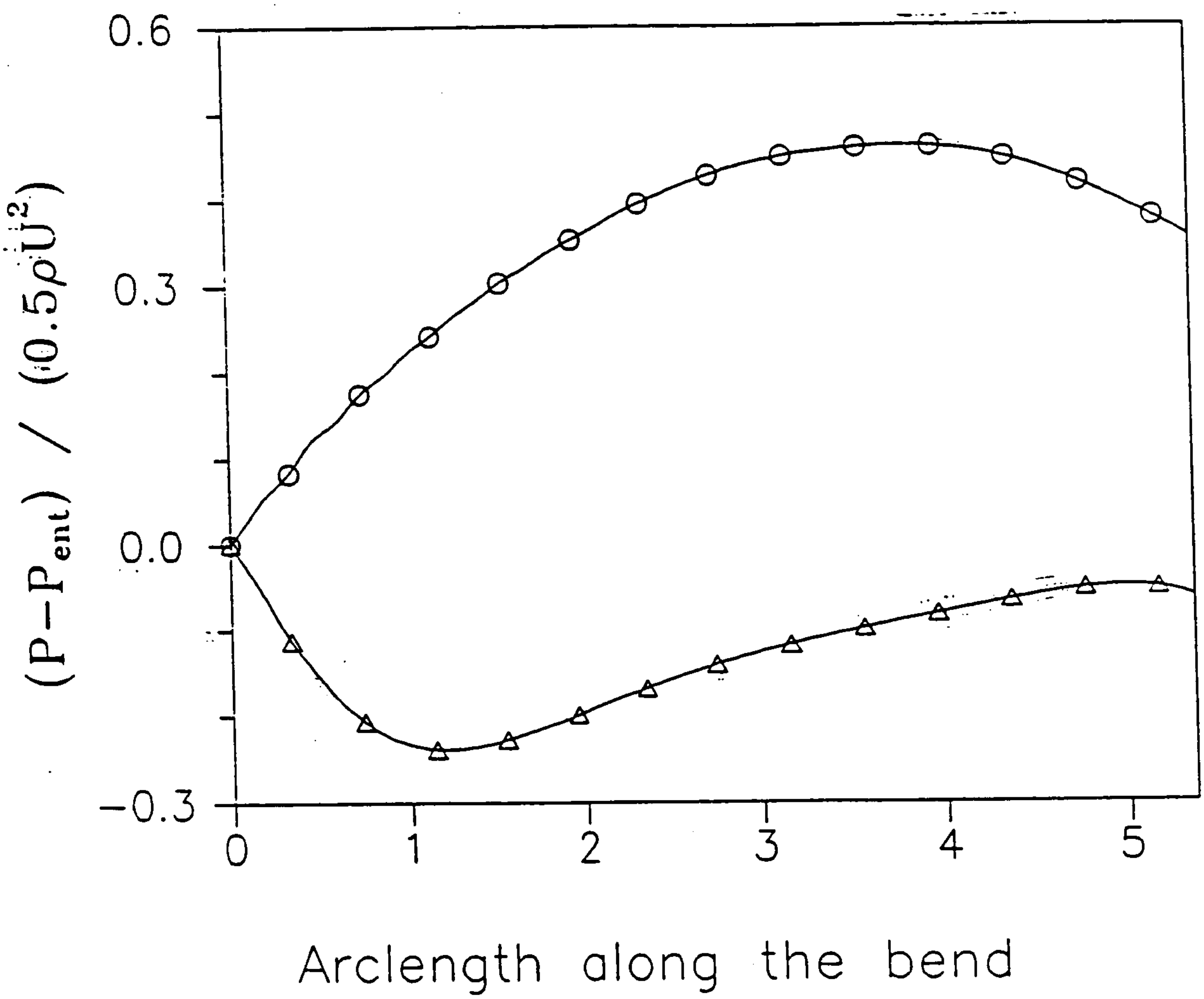
Wall shear stress along the inner wall, Re=100

○ : Optimum diffusing bend

△ : Elliptic diffusing bend

Dashed Curve : Target distribution

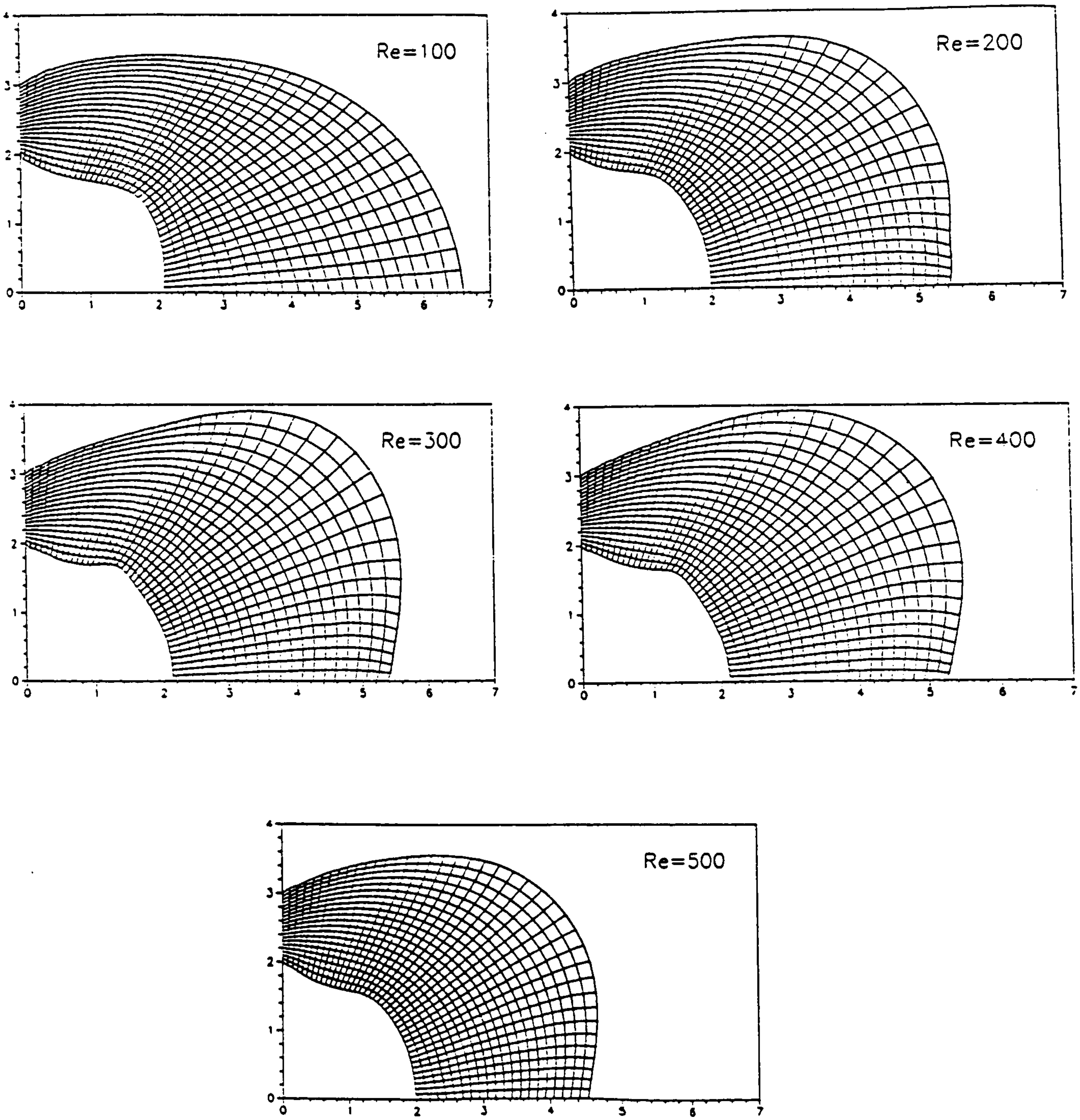
Pressure rise along the header (Re=100)



○ : **Optimum header**

△ : **Elliptic header**

Optimum Shapes



Performance = f(shape)

Possible Applications:

- 90 Degree Bend
- Turn Around Ducts
- Transition Ducts
- S-shaped Ducts
- Straight or Curved Diffusers
- Turbine Blades
- Engine Inlets
- Turning Vanes

CONCLUSIONS

Theory

\# Theoretical Framework for Design with Navier-Stokes equations

\# Determine $\rho(s)$ from Direct+Adjoint or from Direct alone

Computational

\# Direct and Adjoint Solvers Validated for Plane Diffusers

\# Design of 90 degree Bend with Specified Cross-section, Max. Δp

\# Number of Design Cycles < 10

\# "Flow" Interpretation of Adjoint Problem

Future Plan

\# Apply to 3-D turbulent flow

\# Specify mean line, vary cross-section

\# Other objectives : Min. Distortion