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# UNIVERSAL, COMPUTER FACILITATED, STEADY STATE OSCILLATOR, CLOSED LOOP ANALYSIS THEORY AND SOME APPLICATIONS TO PRECISION OSCILLATORS 

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#### Abstract

The theory of oscillator analysis in the immittance domain was presented in Ref 1 which should be read in conjunction with the additional theory in this paper. The combined theory enables the computer simulation of the steady state oscillator. The simulation makes practical the calculation of the oscillator total steady state performance, including noise at all oscillator locations. Some specific precision oscilla- tors are analyzed.


## PART 1 THEORY

## 1. INTRODUCTION

The theory consists of all the material in Ref 1 plus the material of Sections 3.12 through 3.14 presented in this paper.

## 3. THE REAL OSCILLATOR

3.12 The circuit noise transformation function, ( $T R m(f)$, in the ZN configuration of Fig 2 of Reference 1

Eq 20 , repeated here for convenience,

$$
\left.\begin{gather*}
<\operatorname{term} 0>  \tag{20}\\
\mathcal{L}_{n n}(f)
\end{gathered}=\begin{gathered}
\langle\operatorname{term} 1> \\
\mathcal{L}_{V n}(f)
\end{gather*} \quad \bullet \right\rvert\, R T /\left(\left.Z t(f)\right|^{2}\right.
$$

may be considered a special case of general Eq 25

$$
\begin{gather*}
\langle\operatorname{term} 0\rangle  \tag{25}\\
\mathcal{L}_{m}(f)
\end{gathered}=\begin{gathered}
\langle\operatorname{term} 1\rangle \\
\mathcal{L}_{R}(f)
\end{gather*} \quad\langle\operatorname{term} 2\rangle
$$

In Eq 25, term 0 is the oscillator phase noise at location $m$, where, in Eq $20, m=I x$. Term 1 is called the residual phase noise [6] of the active or other device. Term 2 is called the circuit transformation of residual noise function at location $m$.
The importance of the residual noise lies in the fact that it can be measured independently of the oscillator. Then, one needs only to compute the applicable $C T R m(f)$ and then, using Eq 25, determine the oscillator noise, at any and every location.
Since it is stipulated that the noise is due to $V n$, then

$$
\begin{align*}
\mathcal{L} & =P S-V_{n}(f) /\left[V_{i n}(0)\right]^{2}  \tag{26}\\
& =\mathcal{L}_{V_{n}}(f) \bullet\left[V_{s}(0) / V_{i n}(0)\right]^{2} \tag{27}
\end{align*}
$$

where $V_{\text {in }}$ is the carrier input voltage at which the residual phase noise is measured and $V_{s}$ is defined in Fig 2 (in Ref 1) and used in Eq 21.

### 3.12.1 The computation of $C T R \mathrm{~m}(\mathbf{f})$

Fromeq 23;

$$
\begin{equation*}
\mathcal{L}_{m}=P S_{m}(f) / P S_{m}(0) \tag{28}
\end{equation*}
$$

From Eq 12a;

$$
\begin{align*}
P S_{m}(f) & =C F_{m V_{n}}(f) \bullet P S_{V_{n}}(f)  \tag{29}\\
& =C F_{m V_{n}}(f) \bullet \mathcal{L}_{V_{n}}(f) \bullet\left[V_{s}(0)\right]^{2} \tag{30}
\end{align*}
$$

From Eq 21

$$
\begin{equation*}
=C F_{m V_{n}}(f) \bullet \mathcal{L}_{R}(f) \bullet\left(V_{i n}\right)^{2} \tag{31}
\end{equation*}
$$

From Eq 27

Let

$$
=P S_{m}(0) /\left(V_{i n}\right)^{2}
$$

$$
\begin{equation*}
R O_{m}=P S(0) / P S_{V_{i} n}(0) \tag{322}
\end{equation*}
$$

Then

$$
\begin{equation*}
P S_{m}(0)=\left(V_{i n}\right)^{2} \bullet R O_{m} \tag{32b}
\end{equation*}
$$

$$
\begin{aligned}
& \stackrel{\mathcal{L}}{m}^{\text {we obtain; }}(f)=\mathcal{L}_{R}(f) \bullet C F_{m V n}(f) / R 0_{m}
\end{aligned}
$$

$$
\begin{array}{r}
\text { Combining Eqs } 28,31 \text { and } 32 \mathrm{~b}, \text { we obtain; } \mathcal{L}_{m}(f)=\mathcal{L}_{F} \tag{33}
\end{array}
$$

Comparing Eq 33 with Eq 25, we see that:

$$
\begin{equation*}
\left.C T R_{m}(f)=C F_{m V n}(f) / R\right)_{m} \tag{34}
\end{equation*}
$$

$C F_{m V n}$ is calculated as in Sect. 3.9.

### 3.12.2 The calculation of $R 0_{m}$ with the BPT program.

(Note that $R 0_{m}$ is independent of $V n$ and $d R$ )
a. Enter the applicable $Z N$ configuration of the oscillator.
b. Set $F=f_{o}$ and $d R=1 E-9 \mathrm{ohm}$.
c. Make $V n$ a $V W$ component (white noise voltage source) of convenient magnitude.
d. Execute Option C and note the magnitudes $V m$, at locations $m$ and $V_{i} n$.

Then

$$
\begin{equation*}
R O_{M}=\left(V_{m} / V_{i n}\right)^{2} \tag{35}
\end{equation*}
$$

or in $d B$ using the $D B$ option,

$$
\begin{equation*}
R)_{m}(d B)=V_{m} \text { referred to } V_{i n} \tag{35a}
\end{equation*}
$$

### 3.12.3 Notes for Sect $\mathbf{3 . 1 2}$

a. Validity of this section - The reader is reminded that, for flicker noise, this section is valid only for Fourier frequencies, $f$, at which $X_{t}(f) \gg R_{t}(f)$ as stipulated in Sect 3.11
b. Figure of merit - $C T R_{m}(f)$ is a very useful figure of merit of the transformation of device residual noise into oscillator noise at all locations.
c. If it is desired to ascertain the magnitudes of the voltages and currents at all locations at $f=0$, then

1. Make dR a value such as that of Step 3.12.2.b.
2. Set the Magnitude of $V n$ in step 3.12.2.c so that $V_{i n}(o s c)$ becomes equal to $V_{i} n$ (residual measurement input $V$ ) by means of Option E.
3. Execute Option C and record the data.

### 3.13 The oscillator $Q_{o p m}(f)$

$Q_{o p}$, the oscillator operating $Q$, is generally defined by

$$
\begin{equation*}
Q_{o p}=(d x / d f)_{f \rightarrow 0} \bullet f_{0} / 2 R T \tag{36}
\end{equation*}
$$

It is seen that $Q_{o p}$ applies only to low $f$.
It is proposed that Eq 36 be extended to be
(which includes $Q_{o p}$ )

$$
\begin{equation*}
Q_{o p m}(f)=(d x / d f) \bullet f_{0} / 2 R T \tag{37}
\end{equation*}
$$

as it will yield more information.
It can be shown that

$$
\begin{equation*}
Q_{o p n}(f) \approx f_{0} \bullet\left(V_{i n} / V_{x}\right) /\left\{2 \bullet S Q R\left[C T R_{i n}(f) \bullet f\right\}\right. \tag{38}
\end{equation*}
$$

Both $Q_{\text {opm }}$ and $C T R_{m}$ will become more important with the expanded use of oscillators, with more complicated resonators, for which Leeson's noise model [1],[5] may not apply.

### 3.14 Oscillator circuit configs.

Thus far the $Z$ and $N$ configurations have been described. There are additional useful configurations which are considered in detail in Ref 9 . Some of these are
3.14.1 N config - This is the raw complete oscillator circuit It is assigned node numbers and then entered into the computer.
3.14.1.1 $Z N$ config This is the $N$ config set up by means of the $Z$ config. The $N$ config of Fig 2 of Ref 1 is a $Z N$ config.
3.14.2 $Y$ config -This is the $Y$ dual of the $Z$ config and has $d G, G V$, and $B V$ instead of $d R, R V$, and $X V$.
3.14.3 YN config - This is the $N$ config. set up by means of the $Y$ config. In addition, it has a jumper to enable $Z$ measurements.
3.14.4 $Z Y N$ config - This is the $Z N$ configuration which has also been provided with $B V, G V$, and $d G$.
$Y$ and $Y N$ configurations have the important advantage of having fewer nodes.
The $Z$ configuration is preferred over the $Y$ configuration because of its much greater frequency capture range.
There are also many more possible $Y, Y N$, and $Z Y N$ configurations since tuning elements can be comected between any 2 nodes. nodes. Choosing the optimum node pair is difficult.

## PART 2 SOME APPLICATIONS TO PRECISION OSCILLATORS

## 7. INTRODUCTION

This part describes some applications to the precision oscillators likely to be found in PTTI systems.
The data was obtained with program BPT as directed by the user guided by the above theory. The circuit is entered into the computer as a NETLIST via a file or the keyboard. The computer translates the netlist into a PARTS LIST which is readily understood by any user. The user then interactively directs the computer to generate the desired data.
The program is basically an elaborate laboratory simulator with extensive stockroom, fabrication, instrument room, measurement, housekeeping, and recordkeeping facilities, unmatchable in any real laboratory. At present, the program is available for the IBM PC, AT etc. and compatible computers. The user proceeds, controls and operates the program as if he or she were constructing, testing, and then modifying the "simulated breadboard circuit", as directed by the user and the program, in exactly the same manner as in a real laboratory, but much more expeditiously, accurately, thoroughly. and with much greater understanding. The important difference is that the simulated breadboard includes only the information as directed by the user but the real breadboard also includes intrinsic information, unknown to the user, such as parasitic components and frequencies. This difference signifies that only about $90 \%$ of the real laboratory testing can be eliminated by the computer simulation.

From this program description, is seen that the general analysis and design modification procedures consists of the following 5 steps all performed within the program environment.

1. Construction of the oscillator.
2. Trimming this oscillator to the desired operating frequency.
3. Analyzing and evaluating the oscillator performance with the aid of the extensive measurement facilities within the program.
4. Modifying the oscillator to improve the performance.
5. Repeating steps 3 and 4 until the desired performance is obtained.
6. The analysis is then confirmed by constructing and testing the real oscillator to check the correct entrance of the parts and layout data into the computer and to be alerted of important omissions in the data.

It will be noted that above steps 1 to 5 are exactly those followed in the real laboratory but slightly modified for use with the above described theory. The effort and time required to perform these steps will be a small fraction of those for step 6 .
The main difference between the this method of analysis and the customary present methods are

1. The circuit of the device being analyzed is that of the full real oscillator and not a possibly poor approximation incapable of producing all the important and correct data.
2. The type of data obtained closely resembles that for the real oscillator and, in addition, types, practically, unobtainable in the real laboratory.

The difference is primarily due to the closed loop analysis and the noise source, amplifier and filter oscillator model, made possible by the computer and the above theory, as contrasted with the customary open loop analysis. It should be remembered that the real oscillator operates closed, and not open, loop.

The applications are:

1. A 10 MHz 1 resonator oscillator.
2. A 10 MHz 2 resonator oscillator.

Application 1 has been and is being manufactured in very large quantities and it is difficult to appreciate the value of a detailed analysis at this stage in its design history. However, the analysis is still useful, at this time, in the following respects:

1. It provides a greater understanding of the oscillator operation.
2. It clearly demonstrates the validity of the complete design basis including the optimum noise performance.
3. It serves as a production control tool for quickly determining the effect of changes in part characteristics upon the total oscillator performance and thus providing information as to the permissibility of substituting parts with these changes. Such changes are very often found necessary during production.

Application 2 is an example of the use of the theory and program as a research and development tool. This oscillator has never been built and it is advisable to conduct a preliminary computer study to explore its behavior and desirability prior to more intensive computer studies and expensive experimental efforts.
The data for these applications are presented in the form of simplified schematics, a typical netlist, a typical parts list, and plots of the more important, and infrequently or not previously published, operating characteristics. Comments on the data are also included.
The oscillator plots are for 2 quantities versus the Fourier frequency, $f$.
The circuit transformation of residual noise at location $m, C T R_{m}(f)$. The magnitude of the closed loop impedance, $Z_{i n}(f)$, at the input terminals of the active device.

The $C T R_{m}$ function is described in Sect 3.12
If the noise performance of the oscillator, $\mathcal{L}_{m}(f)$, has been experimentally determined and $C T R_{m}(f)$ has been calculated, then the residual noise can then be calculated from Eq 25.
The $Z_{i n}$ quantity determines the contribution of the active device input noise current, $I_{n}$, to the oscillator noise as it produces a noise voltage, $E_{n}=I_{n} \bullet Z_{i n}$, across the active device input terminals. It is therefore very important, when measuring the device residual noise, that the device be terminated to simulate the impedances present in the closed loop oscillator.

In this connection, the noise currents may be determined by measuring the residual noises at the calculated terminations and then calculating the corresponding noise currents (see Sect 3.12.3c).

## 8. 10 MHz 1 RESONATOR OSCILLATOR

Fig 4 is the schematic diagram of this oscillator, called OSC1.
It is the familiar Colpitts type with an SC cut 3rd overtone crystal resonator, $X L$, having $\mathrm{R} 1=70$ Ohms, $\mathrm{C} 1=2.1 \mathrm{E}-16$ Farad and $\mathrm{Qx}=1.083 \mathrm{E} 6$.
There are 5 additional components which are critical and therefore must be carefully controlled; CA, LA, CN, LN, and C'L. CA and LA make up the resonator mode selector network, X1 (see Ref 3). CN and LN make up the resonator overtone selector network, X2. It is possible to combine the overtone and mode selector functions into 1 three element network, either in X1 or X2. However, in production, the control of the elements becomes very difficult.
C'L is the tuning element of the resonator. It may be a capacitance, inductance, or a network including a tuning diode.
The use of elements consuming RF power has been minimized so that the calculated oscillator $Q_{o p}$, 1.080 E 6 , is very close to $Q_{x}$. This is true only when RL is 1 Megohm. For RL $=10$ Kilohms, $Q_{o p}$ becomes 9.605 E 5 and for $\mathrm{RL}=1$ Kilohm, $Q_{o p}=4.885 \mathrm{E} 5$ and quickly decreases with further reductions in RL.

Fig 4 shows both $V_{i n}$ and $V_{s}$ (see Sect 3.12) defined as if RE were an integral part of the transistor

Q1. This is done to ensure that the measurement of the residual noise, $\mathcal{L R}(f)$, in Q 1 includes the, well known, marked reduction in flicker noise due to RE. . Fig 5 shows $C T R_{m}$ and $Z_{i n}$ plotted versus $f$.
CTR data is presented for 2 locations, RL and $I_{x}$. RL is the normal output location. However, the curves indicate that the $I_{x}$ noise performance is superior past $f=10 \mathrm{~Hz}$ and much superior at high values of $f$. Therefore consideration should be given to extracting the output from $I_{x}$. One method of doing this without significant deterioration in $Q_{o p}$ and the low frequency noise performance is described in Ref 8.
The curves include data for both the upper and lower sidebands, $+f$ and $-f$, of the spectrum since they may not be symmetrical. symmetrical. The asymmetry is caused by the fact that the signal, at the location being observed, is the sum of at least 2 signals arriving via different paths. If there is only 1 major llator noise source then the signals are correlated and must be combined as phasors.
The relative phase varies with the frequency $f$, and at $f=f_{a}$ the signals will be in phase in one sideband and out of phase in the other sideband. The out of phase signals causes dips in the CTR function in the region of $f_{a}$. The value of $f_{a}$ has a strong dependence upon $Q_{o p}$, being closer to $f_{o}$ the greater the $Q_{o p}$, because the phase shifts more rapidly.
The magnitude of the dip is a function of the equality of the magnitudes of the 2 signals. Curve B of Fig 5 shows a dip of about 20 db at about 20 Hz below the carrier. There is no conspicuous dip in the resonator current, $I_{x}$, noise because of the resonator filtering action. This effect may be of great importance in systems which require an usually low noise signal in a relatively narrow $f$ region close to the carrier.
A strong dip also exists in curve G , the curve for $Z_{\text {in }}-f$, at a somewhat higher magnitude $f_{a}$.
The $Z_{\text {in }}$ plots show an increase of over X 100 as $f$ varies from 100 to .1 Hz .

## 9. 10 MHz 2 RESONATOR OSCILLATOR

The following reports on the result of a preliminary computer study to determine whether it merits additional computer and experimental studies.
Fig 6 is the working but unoptimized schematic diagram of the ac circuits of this oscillator, called OSC2.
It is a modified Pierce type with 2 resonators, XL1 and XL2, identical to XL1 of Fig 4, capacitively coupled by Cc.
The oscillator parts list in shown in Fig 7 and the netlist is that in Fig 8.
For simplicity, the mode selector and overtone selector networks are not included but they can be similar to those of OSC1. It is interesting to observe that their omission is tolerable in the computer oscillator but may be disastrous in the real oscillator.
C'L1 and C'L2 are the tuning adjustments for their respective resonators. These adjustments also serve to set the oscillator frequency, $f_{o}$, and to shape the oscillator phase noise curve at low Fourier frequencies.
A +1 Hz shift in the effective $f_{s}$ of XL1 corresponds to +.43 Hz shift in the oscillator $f_{o}$.
A +1 Hz shift in the effective $f_{s}$ of XL2 corresponds to +.55 Hz shift in the oscillator $f_{o}$. This data
shows that the resonators are almost equally important in determining the oscillator long term frequency stability.
$Q_{o p}=1.47 \mathrm{E} 6$ which is about 40 -because of the 2 resonators.
At $f<8 \mathrm{~Hz}$ the noise is identical at all locations and equal to those of OSC1 except for the 3 dB improvement due to the higher $Q_{o p}$.

At $f>20 \mathrm{~Hz}$ the noise performance may be much superior to that of OSC1. The best performance, that at location C2, is also plotted on Fig 5 to facilitate the comparison of the noise performance of the 2 oscillators. It will be seen, from that figure, that at 10 KHz , the OSC2 performance is potentially better by about 60 dB .

The $Z_{\text {in }}$ plots are appreciably better than those of OSC1.
The following carrier signal levels were calculated by BPT after setting $d R$ so that $I_{x}$ of XL1 $=1$ mA , corresponding to $d R=1.36 \mathrm{E}-5 \mathrm{ohm}$ :
$I_{x}$ of XL2 $=0.17 \mathrm{~mA}$
V of $\mathrm{C} 1=0.099 \mathrm{~V} ., \mathrm{V}$ of $\mathrm{Cc}=0.015 \mathrm{~V} ., \mathrm{V}$ of $\mathrm{C} 2=0.026 \mathrm{~V}$.
The calculated Ide of Q1, is, assuming ALC limiting, 0.75 mA .

## 10. ADDITIONAL NOISE SOURCES

A large part of the just reported very good noise performance of OSC1 and the even better performance of OSC2 may be nullified by the following important additional noise sources:

Resonator noise (See Section 3.3)- Resonator noise, which is mainly flicker frequency noise, produces $f^{-3}$ phase noise which, in good circuit designs, swamps the circuit flicker noise and thus effectively determines the total oscillator phase noise, at low $f$.

Additive noise- Noises, produced by passive component thermal and other noise sources and noises generated in active devices such as buffer and output amplifiers, set effective limits to the total oscillator noise floors.

Those readers, not used to the $C T R_{m}$ and residual noise concepts but are familiar with the customary $\mathcal{L}(f)$ noise data, are reminded that, since $\mathcal{L}(1 E 4)$ of a good active device is better than -140 dBc , the 10 KHz point on curve E of Fig 9 corresponds to a highly improbable $\mathcal{L}_{C 2}(1 E 4)$ of $(-140-125)=-265 \mathrm{dBc}$.

## 11. CONCLUSIONS for PART 2

In spite of its relatively complex circuit, requiring 2 high performance resonators, the following conclusion are reached:

In view of its potentially excellent noise performance, the 2 resonator oscillator merits further computer and experimental study including the possibility of also using the 2 resonators as part of a vibration noise cancellation system.

Much additional effort is desirable to decrease the effect of the noise sources described in Sect 10 .

## 12. REFERENCES

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FIG 4 1E MHZ $\begin{gathered}1 \times T A L \\ Z N \\ \text { CONFIG OSCILLATOR }\end{gathered}$


PARZEN


FIG 6 1日 MHZ $\begin{array}{r}\text { Z XTAL OSCILLATOR } \\ \text { ZN CONFIG }\end{array}$



## FIG 7 PARTS LIST 2 RESONATOR OSCILIATOR

10 mhz $2 x t a l$ osc $z$ config, 1
cl, $\mathrm{C}, 1,0,1 \mathrm{~L}-010,0,0,0$
$\{[2\}, I, 1,10,1,0,0,0$
$\{d R\}, R, 10,11,1.36 E-005,0,0,0$
\{RV\}, R, $11,12,-3.917113507084587 \mathrm{E}-002,0,1,0$
$\{X V\}, X, 12,13,-1.876242564109969 \mathrm{E}-007,0,0,0$
(R1)XL1,R, $13,2,70,0,9,0$
(C1), C, $2,3,2.1 \mathrm{E}-016,0,9,10000000$
(Co), C, 1, 3, 1E-050, $0,9,0$
C'LL,L, 3, $1,5.386228087475492 \mathrm{~B}-007,0,1,0$
Cc, C, 4, 0, 1E-009, 0, 0,0
(R1) XL2,R, 4, 5, 70, 0, 9, 0
$(C 1), C, 5,6,2 B-016,0,9,10000000$
(Co) $\mathrm{C}, 4,6,1 \mathrm{~B}-050,0,9,0$
C'L2,L, $6,7,1 E-006,0,1,0$
$C 2, C, 7,0,1 E-010,0,0,0$
mm, $N, 9,0,1,2.859460045734375 \mathrm{E}-002,100,1000$
(rbe), R, 0, $7,3497.163744224224,0,7,0$
(Cbed), C, 0, 7, 4.550972008524029E-012, 0, 7,0
$\mathrm{Vn}, \mathrm{Ww}, 7,9,0,0,0,0$
Vins, TP, $0,7,1 B+020,0,0,0$
FIG 8 NEILIST 2 RESONATOR OSCILLATOR

