INVESTIGATION OF NONLINEAR E.M. PHENOMENA IN THE TETHERED MAGNETOSPHERIC CLOUD

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Abstract

Nonlinear effects of *parametric* and of *heating type*, produced in a plasma under the action of an electric field $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$, are considered in this work in connection with the so called Tethered Magnetospheric Cloud (TMC) accompanying the Tether Satellite System (TSS). The theoretical results presented below, particularly by some numerical calculations, show that these phenomena should appear in the ionosphere at high altitudes $Z \geq (150 - 200)\ km$, particularly, at $Z \simeq 300\ km$ of the TSS system orbit. Therefore, it is of a special interest to search these phenomena by such a *unique experiment* as the forthcoming first TSS-I and by the future, perhaps modified TSS missions. Because of the *parametric decay instability*, new branches of waves may be excited both around the electron and ion Lengmuir frequencies $\omega_0 = 2\pi f_0$ and $\Omega_0 = 2\pi F_0$ under the influence of high frequency (HF), ($f \leq 10^6$ to few $10^6\ Hz$), strong ($|\mathbf{E}| \sim V/m$ to tens V/m) electric waves. The *heating* of all the kinds of particles is growing up very quickly in the ionosphere with altitude in the extra low and very low (ELF and VLF) frequency ranges $F \simeq (1\ to\ 10^4)\ Hz$ discussed below. The temperatures (energies), for example, of the electrons accelerated by the electric field become larger than the ionization potential in this frequency range already at altitudes $Z \geq (150 - 200)\ km$ when the amplitude of the electric field is $|E_0| \sim (1-2)\ mV/m$. The sources of these electric field may be in the TSS-I mission the so called Phantom Loop (PL) - the Tethered Electrodynamic Tail (TET), and different kind of e.m. oscillations produced by different kind of instabilities in the TMC plasma. The growth rates of these instabilities will become very high in the TSS surrounding magnetoplasma. However, in *the future TSS missions* special artificial sources (generators) of electric fields should be used for these investigations.

Section I. Introduction

The nonlinear behavior of a magnetoplasma, let us say, of the ionosphere, under the action of an outer source of an electric field $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$ was studied by many authors. This problem has a long history. It began by the papers [1-4] (see Bonch Bruevich (1932), Tellegen (1933), Bailey and Martyn (1934), Főrsterling (1935)) and by many others in (1933-38) - after the discovery of the *crossmodulation of radiowaves* of different frequencies, reflected from the ionosphere, due to the nonlinear interaction of these waves. Shortly after, it was clear that the mechanism of this effect is created by the increasing of the velocities of the electrons v_e and of the other constituents of the plasma under the influence of the electric field. Namely, the velocities of all the constituent particles of the ionosphere and the collision frequencies ν between them, and their temperatures $T_{e,i,n}$ i.e. the conductivity of the plasma become functions of the amplitude E_0 and of the angular frequency ω of the electric field $\mathbf{E}$. I.e. $\nu = \nu(E_0, \omega)$, $v = v_{e,i,n}(E_0, \omega, \nu(E_0, \omega))$ and $T = T_{e,i,n}(E_0, \nu(E_0, \omega))$, where the indexes e, i, n denote, respectively, electrons, ions, and neutral particles. Thus, all the equations which determine these values and describe their behavior in the magnetoplasma *become nonlinear*. To learn theoretically different aspects of this problem, the selfconsistent solution of the adequate systems of equations must be solved.

It is selfevident that these *heating type phenomena*, occurring in a collisional plasma are acting in large plasma regions. The influence of the electric field covers regions with linear scales much larger than the mean free paths of the particles. These effects were described in dozens of papers and in some monographs. Many references of these works can be found in the comprehensive monograph in this field by Gurevich (1978, [5]). Earlier references are cited in the Russian book by Alpert (1947, [6]).

Another kind of nonlinear phenomena are the so called *parametric wave decay effects*. They were discovered about three and more decades after the discovery of the heating effects (see Silin 1965 [7], Du Bois and Goldman 1967 [8]). These phenomena are *local in space* and should not play a large role in the heating effects. The general theory of these phenomena is described in the comprehensive monograph by Silin 1973 [9]. These phenomena were studied just a little in connection with the ionosphere. The known theoretical results are used below, by considering some of these effects in the Tethered Magnetospheric Cloud [10].

However, the theory of the heating effects, known from the literature (see, for example, [5]), is working in a limited frequency band, namely when $\omega^2 \gg \omega_L^2 + \nu_e \nu_{in}$ ($\omega_L = 2\pi F_L$, F_L is the lower hybrid frequency), i.e. in the ionosphere at frequencies $F \geq (4 - 5)10^4\ Hz$. Additionally in these studies, the influence of the velocities of the ions and of all neutral particles, and also the influence of the collision frequencies ν_{in} between the ions and neutral particles, etc. were not taken into account in detail.

In this work, results of theoretical study of the heating of a magnetoplasma in all the frequency range are given, taking into account all kinds of collision frequencies and also their temperature dependencies, i.e. their dependence on the electric field $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$. Some results of numerical calculations of the temperatures T_e, T_i and T_n of the electrons, ions and neutral particles are presented here for the ionosphere in altitude, frequency and angle Θ dependencies in the regions $Z = (100 - 1000)km$, $F = (1 - 10^4)Hz$ (Θ is the angle between the wave's electric field $\mathbf{E_0}$ and the geomagnetic field $\mathbf{H}_0$). Some of these calculations are based on the *selfconsistent solution* of two systems of equations in the *hydrodynamic (microscopic)* approximation. One of these systems (see below (30)) consists of three vector (nine scalar) Lorentz equations of the derivatives of the velocities $\mathbf{V}_e$, $\mathbf{V}_i$, $\mathbf{V}_n$ of the particles. It is supposed that the magnetoplasma consists of one kind of neutral particles. The second system of three equations (see (31)) is of the derivatives of the temperatures $\frac{dT_e}{dt}$, $\frac{dT_i}{dt}$, $\frac{dT_n}{dt}$.

The general formulas of the *selfconsistent solution* are very complicated and bulky: they are absolutely immense. Only numerical results should be used to learn the $T_{e,i,n}$ and $\mathbf{V}_{e,i,n}$ dependencies in this case. However, in the ν = *const approximation*, i.e. when $\nu_{ei} = \nu_{ei,0}$, $\nu_{en} = \nu_{en,0}$, $\nu_{in} = \nu_{in,0}$, and the *electric field dependence of* ν is not taken into account, the analytical formulas obtained are sufficiently visible. These formulas were also used for some calculations.[1] It was important to evaluate the applicability of the formulas recommended in the literature, obtained for $\nu = const$. For this

[1] I am very grateful to Dr. Bob Estes who did all the calculations by computer.

purpose and also for the full study of this problem, the velocities and temperatures are calculated, in some cases, both, with the full systems of equations (3 and 3) of (30) and (31), and with the shortened systems of equations (3 and 2). In the last case, the neutral particles temperature $T_n = T_{n0} = const.$

The dependencies of the temperatures on time, namely, the process of their transition to the stationary state is also illustrated by some examples. The role of the neutral particles is important in this process. The system of equations of the temperatures $\left(\frac{dT_n}{dt}\right)_{e,i,n}$ has not a stationary solution. Mathematically it is clear: the determinant of this system $\Delta = 0$. Therefore T_n is growing up in time. It means that the temperature of the neutral particles *becomes even a source of the heating* of the electrons and ions. From the very beginning of the action of the electric field, the source of the heating of the neutral particles is the energy transferred to them by the collisions with the electrons and ions. The values $(T_e - T_n)$ and $(T_i - T_n)$ are positive. However, growing up in time, T_n becomes equal to T_i. From that moment, the degree of the growth of the temperatures becomes larger and larger, and the temperature of the electrons becomes larger than it would be without the influence of the neutral particles. The process as a whole *has not a stationary solution.* To stop the growth of the temperatures, in addition to collisions, other sources of loss of the energy accompanying the heating of the magnetoplasma must be taken into account. (see below Section III.2).

Section II. Nonlinear parametric phenomena

The mechanism, called *parametric decay instability in the strong pump waves*, $\mathbf{E}_p \cos \omega_p t$ can excite in a magnetoplasma resonance branches of waves and oscillations in the *extra low* (ELF, $0 < \omega \leq \Omega_H$), *very low* (VLF, $\Omega_H < \omega \leq \omega_L$), *low* (LF, $\omega_L < \omega \leq \omega_H$), and *high* (HF, $\omega > \omega_H$) frequencies. These effects become active when the energy density of the electric field is much larger than the pressure of the electrons, namely:

$$N_e \kappa T_e \ll \frac{E_p^2}{4\pi}. \tag{1}$$

It means that in the ionosphere, at the altitude $Z \simeq 300\ km$, where the electron density and temperature are equal to $N_e \simeq 1.8 \cdot 10^6\ cm^3$, $T_e \simeq$

$1.5 \cdot 10^3 K^\circ$, the electric field should be *rather strong*, namely $E_p > (60 - 70)\ V/m$. However, it is shown below that in our case, close to the resonance region $\omega_p \sim \omega_0$, namely when

$$0 < \omega_p - \sqrt{\omega_0^2 + \Omega_0^2} < \frac{3}{2} k^2 D^2 \sqrt{\omega_0^2 + \Omega_0^2} \,, \tag{2}$$

the amplitude E_0 of the electric field becomes much smaller: E_p is a few $Volts/m$ (see Section II.3). In (1 - 2) $k = \frac{2\pi}{\lambda}$ is the wave number, λ is the wavelength of the parametric excited oscillations, $D = \left(\frac{\kappa T_e}{4\pi N e^2}\right)^{1/2}$ is the Debye length, e and m are the charge and mass of the electrons, Ω_0 is the ion Langmuir frequency, and the Boltzman's constant $\kappa = 1.38 \cdot 10^{-16}\ erg \cdot deg^{-1}$.

Let us note here that when we are looking for the parametric effects in the ionosphere or in the magnetosphere under the action of artificial sources $E_p \cos \omega_p t$, located on rockets or satellites, or excited in the surrounding region of the magnetoplasma, the power of these sources should not be very large. Let us note that the growth rates γ_{eo} and γ_{eH} of excitation by beams of electrons of natural oscillations in the HF frequency band by the Cherenkov and cyclotron resonances

$$\begin{aligned} \gamma_{e0} &= \frac{N_b}{N_e} \left(\frac{V_b}{v_e}\right)^2 \omega_0 \cos^2 \Theta \,, \\ \gamma_{eH} &= \frac{\sqrt{\pi}}{4} \frac{N_b}{N_e} \frac{V_b}{v_e} \frac{\omega_0^2 \sin \Theta}{\omega_H} \end{aligned} \tag{3}$$

are very large. By discussing this problem for the TSS mission, the values of the growth rates become even comparable with ω_0 (Alpert 1989, 1990 [10]).

In the theory of the parametric nonlinear effects, a *characteristic parameter* appears

$$\rho_p = \frac{eE_p}{m\omega_p^2} \cdot \frac{2\pi}{\lambda} \tag{4}$$

(see Silin 1973 [9]). In the ionosphere at $Z \simeq 300\ km$

$$\rho_p = \frac{91}{s^2} \left(\frac{2\pi}{\lambda}\right) E_p, \quad \omega_p = s\omega_0, \quad s > 0 \tag{5}$$

To give a general presentation of the parametric effects by considering them at the altitude $Z = 300\ km$, some formulas for an isotropic plasma can be used for simplicity, because $\omega_0 \gg \omega_H$.

II.1 Dispersion equations, strong electric field $\mathbf{E} = \mathbf{E_p} \cos \omega_p t, \quad \omega_p \gg \omega_0, \omega_H, \quad \mathbf{Z} = 300\ \mathbf{km}.$

The dispersion equation of longitudinal oscillations of a magnetoplasma, excited under the action of a strong electric field of the angular frequency ω_p, which is much larger than the electronic angular Langmuir ω_0 and gyro ω_H frequencies, not taking into account the thermal velocities of the particles of the plasma, is the following:

$$\begin{aligned} D(\omega) &= \omega^8 \left[1 - \frac{(\omega_0^2 + \Omega_0^2)\cos^2\Theta}{\omega^2}\right]\left[1 - \left(\frac{\omega_H^2}{\omega^2}\right)\left(1 - \frac{\Omega_H^2}{\omega^2}\right)\right] + \\ &- \omega^6 \sin^2\Theta(\omega_0^2 + \Omega_0^2)\left(1 - \frac{\Omega_H \omega_H}{\omega^2}\right) + \qquad (6) \\ &+ \omega^4 \Omega_0^2 \omega_0^2 \left[1 - J_0^2(\rho_p)\right]\left(1 - \frac{\omega_H^2 \cos^2\Theta}{\omega^2}\right)\left(\omega^2 - \frac{\Omega_H^2 \cos^2\Theta}{\omega^2}\right) = 0 \end{aligned}$$

In (6), in addition to the notations given earlier, J_0 is the Bessel function, Ω_H is the angular ion gyrofrequency, and ρ_p - see (4), (5).

The solution of (6), estimates, in the absence of the electric field, when $E_0 = 0$ and $J_0^2(\rho) = 1$, the wellknown *three resonance branches* of the plasma waves

$$\begin{aligned} \Theta = 0 \ : \ \omega_1^2 &= \Omega_H, \ \omega_2^2 = \omega_H^2, \ \omega_3^2 = \omega_0^2 + \Omega_0^2 \\ \Theta = \frac{\pi}{2} \ : \ \omega_1^2 &= 0, \ \omega_2^2 = \omega_L^2 = (\omega_H \Omega_H)\frac{\left(1 + \frac{\omega_H \Omega_H}{\omega_0^2}\right)}{\left(1 + \frac{\omega_H^2}{\omega_0^2}\right)}, \\ \omega_3^2 &= \omega_U^2 = \left(\omega_0^2 + \Omega_0^2 + \omega_H^2 + \Omega_H^2\right) . \qquad (7) \end{aligned}$$

Taking into account that $\frac{\Omega_H}{\omega_H}, \ \frac{\Omega_0^2}{\omega_0^2} = \frac{m}{M} \ll 1$ the lower and upper hybrid frequencies are

$$\omega_L^2 = \frac{\Omega_H \omega_H}{\left(1 + \frac{\omega_H^2}{\omega_0^2}\right)}, \ \omega_U^2 \simeq \left(\omega_0^2 + \omega_H^2\right) . \qquad (8)$$

In the presence of the electric field, the solution of (8) gives *four resonance branches*

$$\begin{aligned}\Theta = 0\ :\ \omega_1^2 &= \Omega_H^2,\ \omega_{2,p}^2 = \Omega_p^2 = \frac{\omega_0^2\Omega_0^2[1 - J_0^2(\rho_p)]}{\omega_0^2 + \Omega_0^2} \simeq \Omega_0^2\left[1 - J_0^2(\rho_p)\right] \\ \omega_3^2 &= \omega_H^2,\ \omega_U^2 = \omega_0^2 + \Omega_0^2 \simeq \omega_0^2 \qquad (9)\end{aligned}$$

and

$$\begin{aligned}\Theta = \frac{\pi}{2}\ :\omega_1^2 &= 0,\ \omega_2^2 = 0, \\ \omega_3^2 &= \omega_{Lp}^2 = \omega_L^2 + \frac{\Omega_0^2[1 - J_0^2]}{1 + \frac{\omega_H^2}{\omega_0^2}}, \qquad (10) \\ \omega_4^2 &= \omega_{Up}^2 = \omega_0^2 + \Omega_0^2 + \omega_H^2 + \Omega_H^2 + \omega_{Lp}^2 \simeq \omega_0^2 + \omega_H^2 + \omega_{Lp}^2\ .\end{aligned}$$

In (6) - (10) Ω_H, ω_L, ω_H are the ion gyro, low hybrid and electron gyro frequencies, Ω_0 and ω_0 are the ion and electron Lengmuir frequencies, ω_U is the hybrid frequency and in the formula of ω_{Lp}^2, is used the inequality $\Omega_H^2 \ll \Omega_0^2$.

Approximate theoretical dependencies of these branches of wave on the angle Θ_p between the wave vector $\mathbf{k}$ and the geomagnetic field $\mathbf{H}_0$ at the altitude $Z = 300\ km$ are given for illustration on Fig. 1 both for $E_0 = 0$ and $E_0 \neq 0$. The characteristic values of the frequencies used in these calculations are

$$\omega_0 = 7.6 \cdot 10^7,\ \ \Omega_0 = 7.9 \cdot 10^5,\ \ \omega_H = 7.7 \cdot 10^6,\ \ \Omega_H = 4.14 \cdot 10^2$$

(see the Table in Section IV), and $\rho_p = 1,\ 0.1$ and $5 \cdot 10^4$. For excitation of the new parametric branch of oscillations $\omega_{2,p} = (\Omega_p \to 0)$, the amplitude E_0 of the electric field should be about a few hundreds $Volts/m$ when $\omega_p^2 \gg \omega_0^2$.

By increasing ρ_p, the frequency of Ω_p is also increasing. For example, when $\rho_p = 1$, $\Omega_p \simeq 0.51\Omega_0$ and approaches quickly the ion Langmuir frequency Ω_0. By decreasing ρ_p, namely when $\rho_p \ll 1$, $J_0^2(\rho_p) \simeq \left(1 - \frac{p^2}{2}\right)$ and

$$\Omega_p = \Omega_0\left[1 - J_0^2(\rho_p)\right]^{1/2} = \frac{\rho_p}{\sqrt{2}} = \frac{64}{s^2}\left(\frac{2\pi}{\lambda}\right)E_p \qquad (11)$$

(see (5)). The frequency Ω_p becomes small and can reach ELF frequencies $\omega <, \ll \Omega_H$ in the bound of the Alfven waves (see (5)). The amplitude of the electric field needed for the excitation of these very low frequency waves remains large when $\omega_p \gg \omega_0$. However, by approaching the resonance region $\omega_p \sim \omega_0$ (see below II.3) the electric field becomes much smaller, namely, $E_0 \sim$ a few $Volts/m$.

The new branch of oscillations $\omega_{2,p} = (\Omega_p \rightarrow 0)$ is of special interest. Taking into account the thermal velocities of the charged particles v_e and v_i, i.e. in the *kinetic approximation*, the equation of this resonance branch is the following:

$$\omega_{2p}^2 = \cos^2\Theta\left[\Omega_H^2 \cdot K(...) + 3k^2 v_i^2\right] , \qquad (12)$$

where

$$\begin{aligned} K(...) &= \frac{\Omega_0^2\left[1 - J_0^2(\rho_p)\right]\left(1 + k^2 D_e^2\right)^{-1}}{\Omega_H^2 + \Omega_0^2 \sin^2\Theta\left[1 - J_0^2(\rho_p)\right]\left(1 + k^2 D_e^2\right)^{-1}}, \\ \cos\Theta &\gg \left(\frac{2\pi}{\lambda}\right) \cdot \frac{v_i}{\omega} = k\frac{v_i}{\omega}, \quad \sin\Theta \gg \left(\frac{2\pi}{\lambda}\right) \cdot \frac{v_i}{\Omega_H} . \end{aligned} \qquad (13)$$

At $Z = 300\ km$, the Debye length $D_e = V_e/2\omega_0 \simeq 0.2\ cm$ and because of the large wave length of the excited oscillations $k^2 D_e^2 \ll 1$ the second limitation of (13) is fulfilled when

$$\sin\Theta \gg \frac{v_i}{\Omega_H}\left(\frac{\omega n}{c}\right) = \frac{\omega}{\Omega_H} \cdot \frac{v_i}{c} \cdot n \simeq 10^{-2}\frac{\omega}{\Omega_H}, \qquad (14)$$

and equation (12) becomes simpler and similar to the adequate equation in an isotropic plasma ($\Theta \sim 0$). Namely,

$$\omega_p^2 = \Omega_0^2\left[1 - J_0^2(\rho_p)\right] + 3k^2 v_i^2 \qquad (15)$$

when

$$\frac{\Omega_0^2}{\Omega_H^2}\sin^2\Theta \ll 1 , \quad \sin\Theta \ll 5.2 \cdot 10^{-4}. \qquad (16)$$

In (14) and (15) the coefficient of refraction $n = \frac{\Omega_0}{\Omega_H}$, $v_i = 1.5 \cdot 10^5\ cm/s$ and $c = 3 \cdot 10^{10}\ cm/s$. From (15) it follows that, at the altitude $Z = 300\ km$, the value of $\frac{\omega_{2,p}}{\Omega_H} \leq 5.2 \cdot 10^{-2}$ is very small.

Such ELF and VLF parametric oscillations were called by Aliev and Silin (see 1965 [7]) *anisotropic sound waves.* In our case in the ionosphere, they are ELF *longitudinal waves* propagating along the direction of the electric field $\mathbf{E}_0$. Indeed, when $\omega_{2,p} \ll \Omega_H \ll \Omega_0$ equation (15) becomes

$$\omega_p^2 = \frac{1}{2}\left(\Omega_0^2\rho_p^2\right) + 3k^2v_i^2 = \frac{1}{2}\left(\frac{e}{m}\frac{\Omega_0}{\omega_p^2}\right)^2 (\mathbf{k}\mathbf{E}_0)^2 + 3k^2v_i^2 = 3k^2(V_p^2 + v_i^2), \quad (17)$$

where (see (4) and (5))

$$\mathbf{V}_p = \frac{1}{\sqrt{2}}\frac{e\Omega_0}{m\omega_p^2}\mathbf{E}_0 = \frac{5\cdot 10^7}{s^2}\cdot \mathbf{E}_0 \quad (18)$$

is the velocity of this new parametric wave.

The discovering of such waves in the ionosphere will be a very nice and an important contribution to the plasma physics. This can also be one of the methods of diagnostic of the parameters of the plasma.The wave vector of these waves $k = 2\pi/\lambda$ may be estimated by these experiments.

II.2 Resonance oscillations in the frequency band $s\omega_p = \omega_0$, $0.25 < s < 2$, $s = 1$

Let us discuss here the behavior of the low frequency branch $\omega_{2,p} = (\Omega_p \to 0)$, when the frequency of the electric field is not large and crosses the first resonance region $\omega_p = s\omega_0$ $(s = 1)$. The formulas of the isotropic plasma are used for illustration of this dependence, particularly, because $\omega_0 \gg \omega_H$. In this case, under the influence of the electric field the infinite spectra of oscillation generated in the plasma are

$$\omega = \omega_0\left\{1 + \frac{1}{2}\sum_{s=-\infty}^{s=\infty} J_s^2(\rho_p)\frac{\Omega_0^2}{(s\omega_p + \omega_0)^2}\right\} \quad (19)$$

and

$$\omega^2 = \Omega_0^2\left\{\sum_{s=-\infty}^{s=\infty} J_s^2(\rho_p)\frac{(\omega + s\omega_p)^2}{(\omega + s\omega_p)^2 - \omega^2}\right\}, \quad (20)$$

where in (19), (20) $s = \pm 1,\ 2,\ \ldots$. Taking into account that in (20) $\omega \ll s\omega_p$, its denominator may be changed by $|s\omega_p|^2 - \omega_0^2$ and far enough from the resonance regions where $|s\omega_p|^2 \simeq \omega_0^2$, the LF branch of waves (20) is estimated by the approximate equation

$$\omega^2 \simeq \Omega_0^2 [1 - \Phi_r(\rho_p)]\,, \quad \Phi_s = \frac{\pi r}{\sin \pi r} J_r(\rho_p) J_{-r}(\rho_p), \tag{21}$$

where $r = \frac{\omega_0}{\omega_p}$. In the resonance regions

$$\omega^2 = \frac{\omega_0^2}{8} \Delta_s^2 \left\{ 1 \pm \sqrt{1 + \left(\frac{\Delta_p}{\Delta_s}\right)^3} \right\} \tag{22}$$

where

$$\Delta_s = \left[\left(\frac{\omega_0}{s\omega_p}\right)^2 - 1 \right], \quad \Delta_p = \left[32 J_s^2(\rho_p) \frac{\Omega_0^2}{\omega_0^2} \right]^{1/3}, \quad \Delta_s \ll 1 \tag{23}$$

(Silin 1973 [9]).

It is selfevident from equations (20 - 23) that by crossing the resonance regions $\omega_0^2 = \omega_{sp}^2$ the frequency ω^2 becomes negative. I.e. that $\omega = i\gamma$ and becomes the growth rate of the increasing disturbances of the plasma. Instabilities appear in the plasma when $\Delta_s < 0$, $0 < \Phi_{\frac{\omega_0}{\omega_p}}$ and > 1 (see (21)). In the resonance region $\Delta_s < 1$ and from (23) it follows that

$$\frac{\omega}{\omega_0} = \pm \frac{\Delta s}{4} \left\{ \left[\left(\frac{\Delta p}{\Delta s}\right)^{3/2} + 1 \right]^{1/2} \pm i \left[\left(\frac{\Delta p}{\Delta s}\right)^{3/2} - 1 \right]^{1/2} \right\} \tag{24}$$

By using equations (21), the dependencies of $\frac{\omega}{\Omega_0}$ and $\frac{\gamma}{\omega_0}$ on $r = \frac{\omega_0}{\omega_p}$ sufficiently far from the resonance region, were calculated for $Z = 300\ km$ and $\rho_p = 1$. The formulas (22) - (24) were used to calculate these dependencies around the resonance region $\omega_p = \omega_0$, $s = 1$. They are given on Fig. 2. Let us note here the following nice and important properties of this parametric branch of oscillations.

When the frequency ω_p of the electric field $\mathbf{E}_{p0}$ is comparable with the electron Lengmuir frequency, then in the resonance region, close to $\frac{\omega}{\omega_p} \sim 1$:

1. The frequency Ω_p of the nonlinear parametric oscillations is no longer comparable with the ion Lengmuir frequency Ω_0 (see Fig. 1). It reaches in our case the value $\Omega_{p,\,max} = 91\Omega_0$ by $\omega_0 = 0.927\omega_p$. Really it will be smaller in the collisional plasma (see II.3).
2. A narrow region of an instability appears with a maximum of the growth rate $\gamma_{max} = 44\Omega_0$ by $\omega_p = 0.972\omega_0$. $\gamma = 0$ when $\frac{\omega}{\omega_p} = 1$.
3. A broad region of an instability appears with a narrow maximum $\gamma_{max} = 64\Omega_0$ by $\omega_p = 1.035\omega_0$. Really, both the growth rates are smaller when the collision between the electrons and ions are taken into account (see next Section).

To search and to find these wave branches and instabilities will be an important contribution to the TSS mission.

II.3 Excitation of oscillations close to the resonance region

For excitation of the parametric oscillation, in particular, of the interesting for us here branches $\frac{\omega_{2,p}}{\Omega_0} \sim \left(\frac{\Omega_p}{\Omega_0} \to 0\right)$, given on Figs. 1 and 2, the amplitude of the electric field is much smaller than when $\omega_p \gg \omega_0, \omega_H$ close to resonance region. By the kinetic approximation it is determined by the condition

$$0 < \left[\Delta\omega = \omega_p - \sqrt{(\omega_0^2 + \Omega_0^2)}\right] < \frac{3}{2}k^2 D_e^2 \cdot \sqrt{(\omega^2 + \Omega_0^2)}. \qquad (25)$$

Let us illustrate shortly this effect by the following.

In the *frequency band* (25), the characteristic parameters of this process are described by the formulas (see [9])

$$r_E^2 = \frac{8\nu_{ei}}{\omega_0} D_e^2 \,, \quad k^2 = \frac{\nu_{ei} + 2\Delta\omega}{3D_e^2\Delta\omega} \,, \quad E_0^2 = \frac{2\pi r_E^2}{D_e^2} N_e \kappa T_e \qquad (26)$$

where r_E, cm is the amplitude of the oscillations of the electrons under the influence of electric field $\mathbf{E}_0 e^{i\omega t}$, ν_{ei} is the collision frequency between the electrons and ions and it is taken that $N_e\kappa T_e = N_i\kappa T_i$, $D_e = D_i$, and $k^2 D_e^2 \ll 1$.

When $\Delta\omega < 0$, i.e. $\left[\omega_p - \sqrt{\omega_0^2 + \Omega_0^2}\right] < 0$ and the *threshold values* of $\Delta\omega$ and E_0 are determined by (26), the plasma becomes unstable and the process of exciting the parametric oscillations of the plasma begins.

In the ionosphere, at the altitude $Z \simeq 300\ km$ (see the Table in Section III)

$$\nu_{ei} = 3 \cdot 10^3\ , \quad N\kappa T \simeq 3.7 \cdot 10^{-7}\ erg\ , \quad D_\epsilon^2 \simeq 0.2\ cm^2 \tag{27}$$

and from (26) and (27) it follows that the threshold values of the characteristics of this process are

$$\begin{aligned} \Delta\omega &= -\frac{\nu_{ei}}{2}\frac{1}{(1 - 3k^2 D_e^2)} \simeq -\frac{\nu_{ei}}{2} = -1.5 \cdot 10^3\ s^{-1}\ , \\ r_E^2 &= 6 \cdot 10^{-5}\ cm^2\ , \quad E_0 = 3.83 \cdot 10^{-5}\ CGSE = 1.1\ V/m \end{aligned} \tag{28}$$

and the maximum growth rate of these oscillation is estimated by

$$\gamma_{max} \simeq -\frac{\omega_0}{16}\frac{r_E^2}{D_0^2} \simeq \frac{\gamma_{ei}}{2} \tag{29}$$

Section III. Nonlinear heating of a magnetoplasma

In the following parts of this section, we give the results of the theoretical study of the heating of a magnetoplasma under the action of the electric field $\mathbf{E}_0 e^{i\omega t}$ in the microscopic (hydrodynamic) approximation. They are used for some numerical calculations, mainly of the temperature dependencies of the electrons at different altitudes Z of the ionosphere and different angular frequencies ω, in particular, in connection with the orbit ($Z \simeq 300\ km$) of the TSS-1 mission (Section IV.1). However, by the general statement of this problem, the microscopic theory has difficulties from the very beginning. The appropriate system of equations has not a stationary solution.This point becomes crucial at high altitudes of the ionosphere ($Z >\ ,\ \gg 300\ km$) and, especially, in the magnetosphere and in a full ionized plasma, where the collision frequency between the ions and neutral particles $\nu_{en} \sim\ ,\ = 0$. In this case, the theory must be improved by introducing thermal losses in the adequate equations. Some of this effects are discussing shortly below (see III.4).

III.1 Statement of the problem. Microscopic theory

Let us consider a magnetoplasma under the action of an alternative electric field $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$, characterized at the starting point of this process $t = 0$, $E_0 = 0$ by the following.

- it is an isothermal plasma, $T_{e0} = T_{i0} = T_{n0}$ - the temperatures of all the particles are equal,
- it is a quasineutral plasma, $N_{e0} = N_{i0}$ - the densities of the electrons and ions are equal, and
- it has one kind of neutral particles N_n. The effective mass of these neutral particles and of the ions is equal to $M_n = M_i = \frac{\Sigma N_s M_s}{N_n}$, where $N_n = \Sigma N_s$ and N_s, M_s are the densities and masses of neutral particles of different kind.

Then the temperatures and velocities of the electrons, ions, and neutral particles under the influence of $\mathbf{E}_0 e^{i\omega t}$ can be calculated by the following systems of equations:

$$\begin{aligned}
\frac{d\mathbf{V}_e}{dt} &= -\frac{e\mathbf{E}}{m} - \frac{e}{mc}(\mathbf{V}_e \times \mathbf{B}) - \nu_{ei}(\mathbf{V}_e - \mathbf{V}_i) - \nu_{en}(\mathbf{V}_e - \mathbf{V}_n), \\
\frac{d\mathbf{V}_i}{dt} &= \frac{e\mathbf{E}}{M} + \frac{e}{Mc}(\mathbf{V}_i \times \mathbf{B}) + \frac{m}{M}\nu_{ei}(\mathbf{V}_e - \mathbf{V}_i) - \nu_{in}(\mathbf{V}_i - \mathbf{V}_n), \\
MN_n\frac{d\mathbf{V}_n}{dt} &= mN_e\nu_{en}(\mathbf{V}_e - \mathbf{V}_n) + MN_i\nu_{in}(\mathbf{V}_i - \mathbf{V}_n), \qquad (30)
\end{aligned}$$

and

$$\begin{aligned}
\frac{dT_e}{dt} &= -\frac{2}{3}e\mathbf{V}_e \cdot \mathbf{E} - \delta_{ei}\nu_{ei}(T_e - T_i) - \delta_{en}\nu_{en}(T_e - T_{no}) \\
\frac{dT_i}{dt} &= \frac{2}{3}e\mathbf{V}_i \cdot \mathbf{E} + \delta_{ei}\nu_{ei}(T_e - T_i) - \delta_{in}\nu_{in}(T_i - T_{no}) \\
N_n\frac{dT_n}{dt} &= N_e\delta_{en}\nu_{en}(T_e - T_{no}) + N_i\nu_{in}\delta_{in}(T_i - T_{no}) \qquad (31)
\end{aligned}$$

where δ_{en}, δ_{ei}, δ_{in} are, respectively, the energy lost by the electrons due to their chaotic collisions with the neutral particles and ions and energy lost by the ions due to their collisions with the neutral particles. In the calculation the values $\delta_{en} \simeq \delta_{ei} \simeq 2 \cdot 10^{-3}$ and $\delta_{in} \simeq 1$ were used (see Gurevich [5]).

In addition to equations (30) and (31), the following formulas of the collision frequencies must be used

$$
\begin{aligned}
\nu_{ei}(E_0, \omega) &= \nu_{ei,0}\left(\frac{T_{e0}}{T_e(\mathbf{E})}\right)^{3/2} \cdot \frac{\ln\left[0.37\frac{T_e(\mathbf{E})}{e^2 N_e^{1/3}}\right]}{\ln\left[0.37\frac{T_{e,0}}{e^2 N_e^{1/3}}\right]} \\
\nu_{en}(E_0, \omega) &= \nu_{en,0}\left(\frac{T_e(\mathbf{E})}{T_{e,0}}\right)^{1/2} \\
\nu_{in}(E_0, \omega) &= \nu_{in,0}\left(\frac{T_i(\mathbf{E})}{T_{i,0}}\right)^{1/2} \qquad (32)
\end{aligned}
$$

where $\nu_{ei,0}$, $\nu_{en,0}$ and $\nu_{in,0}$ are the initial values of the collision frequencies $E_0 = 0$, and the members ln[...] are the, so called, Coulomb logarithms.

Thus, the systems of equations are interconnected and a selfconsistent solution of (30 - 32) should be used to study in sufficient completeness the problem considered here. Namely this general statement is the crucial point, both for a full and correct understanding of the process of heating of a magnetoplasma in the frames of the microscopic (hydrodynamic) approximation and also for a correct quantitative estimation of the temperatures.

III.2 Velocities $\mathbf{V}_{e,i,n}$. Approximation $\nu = \mathrm{const}$

By solution of the system of equations (34), the formulas of the velocities of the particles are visible when the dependence of the collision frequencies on the electric field is not taken into account - it is the *approximation* $\nu = Const$. The components of the electron velocities are the following:

$$
V_{ex} = \frac{-e\tilde{\omega}_H}{m\omega} E_y \cdot \left\{ \frac{\left[1 - \tilde{\nu}_{in}^2 - \tilde{\Omega}_H^2 + \mu\tilde{\nu}_{ei}(\tilde{\nu}_{ei} - \tilde{\nu}_{en})\right] - i(\mu\tilde{\nu}_{ei} + \tilde{\nu}_{in})}{\left[(1 - \tilde{\nu}_e\tilde{\nu}_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e\right]^2 - \tilde{\omega}_B^2(1 + i\tilde{\nu}_{in})^2} \right\},
$$

$$V_{ey} = \frac{-e}{m\omega} E_y \cdot \left\{ \frac{i\left\{(1+i\tilde{\nu}_{in})\left[(1-\tilde{\nu}_e\tilde{\nu}_{in})+i\tilde{\nu}_e\right] - \tilde{\Omega}_H^2\left[1+\left(\tilde{\nu}_{en} - n\tilde{\nu}_{in}\frac{\nu_{in}}{\omega+in\nu_{in}}\right)\right]\right\}}{\left[(1-\tilde{\nu}_e\tilde{\nu}_{in}-\tilde{\omega}_L^2)+i\tilde{\nu}_e\right]^2 - \tilde{\omega}_H^2(1+i\tilde{\nu}_{in})^2} \right\},$$

$$V_{ez} = \frac{-e}{m\omega} E_z \cdot \left\{ \frac{i(1+i\tilde{\nu}_{in})}{(1-\tilde{\nu}_e\tilde{\nu}_{in})+i\tilde{\nu}_e} \right\}. \qquad (33)$$

The components of the ion velocity are

$$V_{ix} = \frac{-e\tilde{\Omega}_H}{m\omega} E_y \cdot \left\{ \frac{\mu\left[1+i(\mu\tilde{\nu}_{ei}+\tilde{\nu}_{in})\right]^2 - \tilde{\omega}_L^2 + \left(\tilde{\nu}_{ei} + in\tilde{\nu}_{en}\frac{\nu_{en}}{\omega+in\nu_{in}}\right)\left(2+i\tilde{\nu}_{in}\right)}{\left[(1-\tilde{\nu}_e\tilde{\nu}_{in}-\tilde{\omega}_L^2)+i\tilde{\nu}_e\right]^2 - \tilde{\omega}_H^2(1+i\tilde{\nu}_{in})^2} \right\},$$

$$V_{iy} = \frac{e\mu}{m\omega} E_y \cdot \left\{ \frac{i\left[1+i\left(\tilde{\nu}_{en} - n\tilde{\nu}_{en}\frac{\nu_{en}}{\omega+in\nu_{in}}\right)\right]\left[\left(1-\tilde{\nu}_e\tilde{\nu}_{in}\right)+i\tilde{\nu}_e\right]}{\left[(1-\tilde{\nu}_e\tilde{\nu}_{in}-\tilde{\omega}_L^2)+i\tilde{\nu}_e\right]^2 - \tilde{\omega}_H^2(1+i\tilde{\nu}_{in})^2} - \Phi_i(\ldots) \right\}$$

$$\Phi_i(\ldots) = \left\{ \frac{i\tilde{\omega}_H^2\left[1+i\left(\tilde{\nu}_{in} - n\tilde{\nu}_{in}\frac{\nu_{in}}{\omega+in\nu_{in}}\right)\right]}{\left[(1-\tilde{\nu}_e\tilde{\nu}_{in}-\tilde{\omega}_L^2)+i\tilde{\nu}_e\right]^2 - \tilde{\omega}_H^2(1+i\tilde{\nu}_{in})^2} \right\},$$

$$V_{iz} = \frac{e\mu}{m\omega} E_z \cdot \left\{ \frac{i\left[1+i\left(\tilde{\nu}_{en} - n\tilde{\nu}_{en}\frac{\nu_{en}}{\omega+in\nu_{in}}\right)\right]}{(1-\tilde{\nu}_e\tilde{\nu}_{in})+i\tilde{\nu}_e} \right\}, \qquad (34)$$

and the velocity of the neutral particles $\mathbf{V}_n$, estimated by the velocities of the electrons and ions $\mathbf{V}_e$, $\mathbf{V}_i$, is equal to

$$\mathbf{V}_n = \frac{(\mathbf{V}_e + \frac{\tilde{\nu}_{in}}{\mu\tilde{\nu}_{en}}\mathbf{V}_i)}{(1+\frac{\tilde{\nu}_{in}}{\mu\tilde{\nu}_{en}} - \frac{i}{n\mu\tilde{\nu}_{en}})} \qquad (35)$$

The equations (33 - 35) are calculated in the coordinate system (x, y, z) where the magnetic field $\mathbf{H}_0 = \mathbf{H}_0(0, 0, H_z)$, i.e. it is parallel to the axis z, and the electric field $\mathbf{E}_0$ lies in the plane (yz) i.e. $\mathbf{E}_0 = \mathbf{E}_0(0, E_y, E_z)$, $(0, E_0 \sin\Theta, E_0\cos\Theta)$. Additionally, in (4) - (6) dimensionless collision frequencies

$$\tilde{\nu}_{ei} = \frac{\nu_{ei}}{\omega}, \tilde{\nu}_{en} = \frac{\nu_{en}}{\omega}, \tilde{\nu}_{in} = \frac{\nu_{in}}{\omega}, \tilde{\nu}_e = \frac{\nu_{ei} + \nu_{en}}{\omega}, \tilde{\nu}_e\tilde{\nu}_{in} = \frac{\nu_e\nu_{in}}{\omega^2},$$
$$\tilde{\Omega}_H = \frac{\Omega_H}{\omega}, \quad \tilde{\omega}_H = \frac{\omega_H}{\omega}, \quad \tilde{\omega}_L = \frac{\omega_L}{\omega} \tag{36}$$

are used where Ω_H, $\omega_L \simeq (\Omega_H\omega_H)^{1/2}$ and ω_H are, respectively, the angular ion, lower-hybrid and electron gyrofrequencies, $\mu = \frac{m}{M}$ and $n = \frac{N_e}{N_n} = \frac{N_i}{N_n}$.

III.3 Temperatures $T_e(E)$ and $T_i(E)$ of the electrons and ions

By calculating the temperatures, it is convenient and physically more acceptable to use the dimensionless values ω/ν instead of ν/ω because $\nu_e = \nu_{ei} + \nu_{en}$ is never equal to zero. Then, for example, the components of the velocities $V_{e,z}$ and $V_{i,z}$ (they are the same as in an isotropic plasma) are equal to

$$V_e = V_{ez} = -\frac{eE_z}{m\nu_e} \cdot \frac{\left(1 - i\frac{\omega}{\nu_{in}}\right)\left(1 - \frac{\omega^2}{\nu_e\nu_{in}} + i\frac{\omega}{\nu_{in}}\right)}{\left(1 - \frac{\omega^2}{\nu_e\nu_{in}}\right)^2 + \frac{\omega^2}{\nu_{in}^2}},$$
$$V_i = V_{iz} = -\frac{eE_z}{M\nu_e} \cdot \frac{\nu_{en}}{\nu_{in}} \cdot \frac{\left(1 - \frac{n\nu_{en}}{\omega + in\nu_{in}} - i\frac{\omega}{\nu_{en}}\right)\left(1 - \frac{\omega^2}{\nu_e\nu_{in}} + i\frac{\omega}{\nu_{in}}\right)}{\left(1 - \frac{\omega^2}{\nu_e\nu_{in}}\right)^2 + \frac{\omega^2}{\nu_{in}^2}}, \tag{37}$$

and their real parts are

$$R_e(V_e) = \frac{-eE_0}{m} \cdot \frac{\nu_e\omega^2 - \nu_{in}(\omega^2 - \nu_e\nu_{in})}{(\omega^2 - \nu_e\nu_{in})^2 + \nu_e^2\omega^2},$$
$$R_e(V_i) = \frac{-eE_0}{M} \cdot \frac{\nu_e\omega^2 - \nu_{en}(\omega^2 - \nu_e\nu_{in})}{(\omega^2 - \nu_e\nu_{in})^2 + \nu_e^2\omega^2}. \tag{38}$$

Let us note here that by the approximation used in the earlier studies (see [5]), the following equations of the velocity $\mathbf{V}_e$ of electrons

$$\mathbf{V}_e = -\frac{e}{m}\frac{e^{i\omega t}}{\omega_B^2 + (\nu_e - i\omega)^2}\left\{\mathbf{E}_0(\nu_e - i\omega) + \frac{\omega_H^2(\mathbf{E}_0\mathbf{H}_0)}{H_0^2(\nu_e - i\omega)}\mathbf{H} - \frac{\omega_H(\mathbf{E}_0\mathbf{H}_0}{H_0}\right\} \tag{39}$$

and of $R_e(\mathbf{V}_e)$, when $\mathbf{H}_0 = 0$,

$$R_e(V_e) = \frac{-eE_0}{m} \cdot \frac{\nu_e}{\omega^2 + \nu_e^2} \ . \tag{40}$$

were used. By comparison of (39) and (40) with (33) and (38), it follows that the equations of $\mathbf{V}_e$ and certainly also of the temperatures, used in these studies, are effective only when

$$\omega \gg \nu_{in} \ , \ \ \omega^2 \gg \nu_e \nu_{in} \ , \ \ and \ \ \omega^2 \gg \omega_L^2 + \nu_e \nu_{in} \ . \tag{41}$$

Visible, but rather complicated formulas of the temperatures of the electrons and ions $T_e(E_0, \omega)$ and $T_i(E_0, \omega)$ can be obtained only by the *approximation* $\nu = Const$ from the selfconsistent solution of the system of equations (30) and the shortened system of equation (31). Namely, when in the third equation of (35) $\frac{dT_n}{dt} = 0$. These formulas are as the following

$$\frac{T_e}{T_{n0}} = 1 + \frac{e^2 E_0^2}{3m\delta_e \nu_e^2 T_{n0}} \cdot \Phi(...) \ , \tag{42}$$

where

$$\begin{aligned} \Phi(...) \ &= \ \left\{A_1(\nu)\left[\cos^2\Theta R_e(V_{ez}) + \sin^2\Theta R_e(V_{ey})\right]\right\} + \\ &+ \ \left\{A_2(\nu)\left[\cos 2\omega t F_{e,1}(\nu,\omega) + \sin 2\omega t F_{e,2}(\nu,\omega)\right]\right\} \ . \end{aligned} \tag{43}$$

In formula (43), the members proportional to $\frac{m}{M} \cdot (R_e\{V_{ey}, z\})$, and $\frac{m}{M} \cdot I_m\{V_{ez}\}$ are omitted and the following notations are used:

$$\begin{aligned} A_1(\nu) \ &= \ \frac{\nu_e + \frac{\nu_{in}}{\delta_e}\left(1 + \frac{\nu_{en}}{\nu_{ei}}\right)}{\nu_{en} + \frac{\nu_{in}}{\delta_e}\left(1 + \frac{\nu_{en}}{\nu_{ei}}\right)} \ , \\ A_2(\nu) \ &= \ \left[\left(\nu_{en} + \frac{\nu_{in}}{\delta_e}\right)^2 + 4\nu_{ei}^2\right]^{-1/2} \ , \end{aligned} \tag{44}$$

$$\begin{aligned} F_{e1}(\nu,\omega) \ &= \ \frac{A_{3,1}B_1 - 2\omega B_2}{A_{3,1}^2 + 4\omega^2} - \frac{A_{3,2}B_1 - 2\omega B_2}{A_{3,2}^2 + 4\omega^2} \ , \\ F_{e2}(\nu,\omega) \ &= \ \frac{A_{3,1}B_2 + 2\omega B_1}{A_{3,1}^2 + 4\omega^2} + \frac{A_{3,2}B_2 + 2\omega B_1}{A_{3,2}^2 + 4\omega^2} \ , \end{aligned} \tag{45}$$

$$
\begin{aligned}
B_1(\nu) &= \delta_e \nu_{ei} \left(1 + \frac{\nu_{in}}{\delta_e \nu_{ei}}\right) \left[\cos^2 \Theta R_e\{V_{ez}\} + \sin^2 \Theta R_e\{V_{ey}\}\right] + \\
&+ 2\omega \left[\cos^2 \Theta I_m\{V_{ez}\} + \sin^2 \Theta I_m\{V_{ey}\}\right] , \\
B_2(\nu) &= \delta_e \nu_{ei} \left(1 + \frac{\nu_{in}}{\delta_e \nu_{ei}}\right) \left[\cos^2 \Theta I_m\{V_{ez}\} + \sin^2 \Theta I_m\{V_{ey}\}\right] - \\
&- 2\omega \left[\cos^2 \Theta R_e\{V_{ez}\} + \sin^2 \Theta R_e\{V_{ey}\}\right] , \qquad (46)
\end{aligned}
$$

In addition, the values $R_e\{V_{ey}\}$, $R_e\{V_{ez}\}$, $I_m\{V_{ey}\}$ and $I_m\{V_{ez}\}$ are the real and imaginary parts of the terms in braces of the appropriate components of the velocity $\mathbf{V}_e$ (see (33)).

It is seen from equations (42, 43) that the temperature T_e has two members. One of them estimates the stationary ($t \to \infty$) temperature, not depending on time. The establishment of the stationary temperature is illustrated below by some results of calculations. The second member of (43) describes the harmonic periodical variation of the temperature; it is changing in time by the double frequency $2\pi f$ of the electric field $\mathbf{E} \sim e^{i2\pi ft}$. In the following, we discuss only the stationary temperature.

It is useful to introduce a characteristic field E_h (see [5]), which can be, in its own way, a measure of the intensity E_h^2 of the field needed for a remarkable heating of the magnetoplasma. When $\Theta = 0$, i.e. in an isotropic plasma, from equations (42) and (43) it follows that

$$
E_h^2 = \frac{3m\delta_e \nu_e^2 T_{n0}}{e^2} \cdot \frac{A(\nu, \omega)}{A_1(\nu)} , \qquad (47)
$$

and the stationary temperature of the electrons is determined by

$$
\left(\frac{T_e(E_0, \omega)}{T_{n0}}\right)_{stat} = 1 + \frac{E_0^2}{E_h^2} . \qquad (48)
$$

The stationary ($t \to \infty$) temperature of the ions is determined by the following equation

$$
\frac{T_i(E_0, \omega)}{T_{n0}} = 1 + \frac{E_0^2}{E_h^2} \left[1 + \frac{1}{\delta_e} \frac{\nu_{in}}{\nu_e} \left(1 + \frac{\nu_{en}}{\nu_{ei}}\right)\right]^{-1} , \qquad (49)
$$

where $A_1(\nu)$ is given by (44) and

$$A(\nu,\omega) = \frac{\left(1 - \frac{\omega^2}{\nu_e \nu_{in}}\right)^2 + \frac{\omega^2}{\nu_{in}^2}}{\left(1 - \frac{\omega^2}{\nu_e \nu_{in}}\right) + \frac{\omega^2}{\nu_{in}^2}} = [\nu_e R_e\{V_{ez}\}]^{-1} \ . \qquad (50)$$

Let us note here that by comparison of (42) and (49) we can see that the temperature $T_i(E_0,\omega)$ of the ions is smaller than the temperature $T_e(E_0,\omega)$ of the electrons. However, really in the ionosphere this is remarkable only at the altitudes $Z \leq (100 \rightarrow 200)$ km. At $Z \geq 200$ km, $T_i(E_0,\omega) \simeq T_e(E_0,\omega)$.

The time-dependent part of the ion temperature is determined by the same formulas as for the electrons (see (42, 43)), by replacing in the functions $F(...)$ an $B(...)$ (see (45) and (46)) the values V_{ey} , V_{ez} of the velocity components of electrons by the values V_{iy} and V_{iz} of the ions. The formulas (33) - (35) given above were used by some numerical calculations of the temperatures with the $\nu = const$ *approximation* (see Section IV). Because of the diverse character of the problem studied here, let us briefly emphasize again some of its aspects.

III.4 Some peculiarities of the problem.

It is obvious that in the frame of the microscopic (hydrodynamic) approximation used in this study, the magnetoplasma should be a collisional media. However, if it would be, for example, a full ionized magnetoplasma, i.e. if it is characterized only by the constituent particles $N_e = N_i$ ($N_n = 0$) - by one kind of collisions ν_{ei} between the electrons and ions - then the used equations of the temperature would not have a stationary solution. The temperatures of the electrons would grow up continuously all the time of action of the electric field. It is because the loss of the energy of the electrons accelerated by the electric field is equal to the energy transmitted by them to the ions. Mathematically it means that the determinant Δ_2 of the first two equations of the system (2) becomes equal to zero. But this namely should happen, for example, when we consider these processes in the magnetoplasma at far distances from the Earth where $\nu_{en} \rightarrow 0$.

Within the limits of this study, the problem of the absence of a stationary solution of the three equations of (31) exists from the very beginning if the equation of the derivative of the temperature $\frac{dT_n}{dt}$ of the neutral particles are taken into account. Mathematically it is because the determinant of the system (35) $\Delta_3 = 0$. The temperature T_n of the neutral particles is growing up continuously under the action of the electric field, and can become even a source of heating of the electrons and ions. The crucial region of the ionosphere, where this process becomes highly noticeable is at the altitudes $Z > 300\ km$ (see below). The values T_e and T_i become larger in this region than they would be by the solution of the shortened system of equations (35).

Thus it was important to learn in this study the applicability of the microscopic approximation used in many studies and here by considering the behavior of all the temperatures T_e, T_i, and T_n by the two approximations: $\nu = const$ and $\nu(\mathbf{E})$. Especially it was important to search the role of the neutral particles. Let us note shortly here how the continuous growth of the temperatures of the plasma can be stopped.

The heating of the magnetoplasma is accompanied and regulated, in addition to the collisions ν_{ei}, ν_{en} and ν_{in}, by many other processes. Their role should be of different degree. However, for a complete solution of this problem, they should be taken into account. Then the general solution of this task will become stationary. One of these most important processes is the ionization of the neutral particles by the accelerated electrons under the action of the electric field $\mathbf{E}_0 e^{i\omega t}$.

The velocity of the electrons becomes in the ionosphere, depending on the frequency ω and altitude Z, larger than the ionization potential E_i of the neutral particles, namely, the thermal, chaotic velocity $v_e(\mathbf{E})$ plays the prime role by this process. It is considerably larger than the directed velocity $V_e(\mathbf{E})$. For instance, in an isotropic plasma

$$v_e(\mathbf{E} = \left(\frac{2\kappa T_e(E_0,\omega)}{m}\right)^{1/2} \sim \frac{R_e\{V_e(\mathbf{E})\}}{\sqrt{\delta}} \sim 20 R_e\{V_e(\mathbf{E}) \qquad (51)$$

(see (34) and (38)). The ionization potential of the atomic hydrogen H_1 particles (they are the main constituent at the high altitudes of the ionosphere) is equal to $E_i = 13.54\ eV$. Therefore, the process of the ionization begins when $v_E(\mathbf{E}) = v_{e(i)} \geq 2.2 \cdot 10^8\ cm/s$. It becomes sufficiently remarkable, i.e. that the *additional density* of the electrons $N_e(\mathbf{E})$ can become even considerably larger than the regular density of the plasma N_e when the amplitude of the electric field is rather small, even when $E_0 = (1-2)\ mV/m$. This is seen from the following.

The dependence of the cross-section $\sigma_e(E_i)$ of ionization of the atomic hydrogen on the energy of the electrons is given in Fig. 3 (Massy at al. 1969 [12]). The value of σ_e increases rapidly with E_i up to a sharp maximum $\sigma_{e,max} \simeq (7-8)10^{-17}\ cm^2$ and then it is smoothly decreasing up to $\sigma_{e,stat} \simeq 2 \cdot 10^{-17}\ cm^2$. I.e. that at the altitude $Z = 300\ km$, where the density of the atomic hydrogen $N_n \simeq 3 \cdot 10^9$, the production of the electrons

$$I_e = \sigma_e \cdot N_n \cdot v_e(\mathbf{E}) \geq (10\ to\ 50)\ s^{-1} \tag{52}$$

and the additional ionization is described by the recombination equation

$$\frac{dN}{dt} = I_e N - \alpha_e N^2. \tag{53}$$

It follows from (53), that growing up in time the electron concentration reaches a maximum by $\frac{dN}{dt} = 0$ equal to $N = 10^9$, even if the coefficient of recombination $\alpha_e \sim 10^{-8},\ cm^3 \cdot s^{-1}$. However, in this region of the ionosphere, $\alpha_e <, \ll 10^{-9}$. Thus, under the action of the electric field, the surrounding plasma can become *full ionized* i.e. $N_e \simeq 10^9\ cm^{-3}$. As a result, the collision frequency between the electrons and ions $\nu_{ei}(\mathbf{E}) \propto N_e(\mathbf{e})/T_e(\mathbf{E})^{3/2}$ becomes very large (see (3), and the $T_e(\mathbf{E})$ dependencies given in IV.1). The growth of T_e is stopped very quickly because the electron concentration is very quickly increasing. Indeed, in the point of inflection of the time dependence of the $N(\mathbf{E}, t)$, where $\frac{d^2N}{dt^2} = 0$, $N = \frac{I_e}{2\alpha}$. Thus, at that point $\frac{dN}{dt} = \frac{I_e^2}{4\alpha_e}$ is very large and the tangential of $N(\mathbf{E}, t)$ is almost vertical. Other losses of energy which can stop the growth of the temperature of the magnetoplasma are, for example, the following.

The *thermal emission* of the constituent particles of the plasma i.e. the volume Stefan- Boltzman emission of the plasma can play a remarkable role.

The density of this emission is approximately equal to

$$I_p(\omega) = \frac{\kappa T \omega^2}{4\pi^3 c^3} \cdot n_{gr}(\omega) n^2(\omega), \ erg \cdot s \cdot cm^{-3} \tag{54}$$

In (54) $n(\omega)$ is a coefficient of refraction of the plasma and $n_{gr} = n + \omega\frac{dn}{d\omega}$ is the, so called, group velocity coefficient of refraction.

The *heating kinetic losses* of the plasma should be considered and, in some cases, they can also be remarkable. I.e. the *electron-electron* and *ion-ion* collisions ν_{ee} and ν_{ii} and possibly other kind of kinetic thermal losses which, in a sense, are similar to the Landau damping of e.m. waves in a plasma, should also be taken into account.

To a certain extent, as it is seen below, we can omit in this work the discussion in detail of the noted above points and some other processes, accompanying the heating of the magnetoplasma by an electric field. They are beyond the frame of this study. They are also not of the decisive importance at the altitudes of the ionosphere of the TSS Mission.

Section IV. Numerical results. Conclusions $\mathbf{Z = (10^2 \text{ to } 10^3)}$ km, $\mathbf{F = (1 \text{ to } 10^4)}$ Hz

In the following section, we give results of numerical calculations mainly of the temperature of the electrons and, for comparison, a few examples of the temperatures of the ions and neutral particles. The following model of the ionosphere (see Table) and the amplitude of the electric field $E_0 = 1\ mV/m$ are used in this calculations. The altitude dependencies of the electron and ion densities and of the Lengmuir frequencies $f_0 = \omega_0/2\pi$ and $F_0 = \Omega_0/2\pi$, given in the Table, correspond to the day-time conditions of the ionosphere. The altitude, frequency and angle Θ dependencies of the temperatures are presented in plots (Figs. 4 to 11). The frequencies $F = \omega/2\pi = 1\ and\ 10^3\ Hz$ and the angle between the magnetic and electric field $\Theta = 0$ and the altitude $Z = 300\ km$ are selected by the calculations, because the orbit of the coming in the future TSS Satellite should pass at this altitude.

It is seen below, that the growth of the temperature with altitude in the frequency range $F \simeq (1\ to\ 100)\ Hz$ is very large. It becomes about $10^2\ to\ 10^3$ and more times larger than the initial temperature of the plasma already at $Z \sim (200-300)\ km$. Therefore, the frequency $F = 1\ Hz$ is used to illustrate more dramatically the region where it is necessary to stop the temperature growth by including in the theory additional losses of energy (see above III.4). The zone of ionization of the plasma by the accelerated electrons is therefore marked on the plots by a thin line and vertical arrows. The frequency band $F = (1\ to\ 10^4)\ Hz$, that we have used, is much smaller, but also becomes somewhat larger than the gyrofrequency $F_H = \frac{\Omega_H}{2\pi}$ of the ions in the altitude range discussed. It is also smaller and much smaller than the low hybrid frequency $F_L = \frac{\omega_L}{2\pi}$. In addition, the value $\nu_e \nu_{in} = (\nu_{ei} + \nu_{in})\nu_{in}$ is much larger than this frequencies at low altitudes and becomes comparable and also much larger than at high altitudes. Therefore, the formulas used in the literature are ineligible for these calculations (see page 18 and (41)). The used angle $\Theta = 0$ corresponds to the isotropic plasma and characterize the behavior of the maximum values of the temperature. However, the evaluation of these dependencies on different values of Θ are also given and the influence of the magnetic field $\mathbf{H}_0$ is shown by these data.

The results of the numerical calculations mainly correspond to the approximation $\nu = const$, i.e. that the shorted system of (31) is used and it is assumed that $\frac{dT_n}{dt} = 0$, $T_n = T_{n0}$. However, results of the selfconsistent solution of the full systems of (30) to (32) are also given by some examples and the role of the neutral particles is discussed.

Let us now describe in more detail the numerical results by the plots, presented here .

Table 1. Parameters of the ionosphere.

Z, km	100	120	150	200	300	400	500	800	1000
N_n, cm^{-3}	$3.7 \cdot 10^{13}$	$3 \cdot 10^{12}$	$6 \cdot 10^{11}$	$5 \cdot 10^{10}$	$3 \cdot 10^{9}$	$5 \cdot 10^{8}$	$5 \cdot 10^{7}$	$2 \cdot 10^{6}$	$4 \cdot 10^{5}$
$N_e = N_i$	10^5	$1.2 \cdot 10^5$	$1.5 \cdot 10^5$	$5 \cdot 10^5$	$1.8 \cdot 10^6$	$1.5 \cdot 10^6$	10^6	$4 \cdot 10^5$	$2.3 \cdot 10^5$
$n = N_e/N_n$	$\sim 3 \cdot 10^{-9}$	$4 \cdot 10^{-8}$	$2.5 \cdot 10^{-7}$	10^{-5}	$6 \cdot 10^{-4}$	$3 \cdot 10^{-3}$	$2 \cdot 10^{-2}$	$2 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$
M^+/M^+_{H1}	28	25	20	12	5	3	2	1.2	1
$(\mu = m/M)10^4$	$1.94 \cdot 10^{-1}$	$2.36 \cdot 10^{-1}$	$2.72 \cdot 10^{-1}$	$3.4 \cdot 10^{-1}$	$5.4 \cdot 10^{-1}$	1.81	2.72	3.62	5.44
$\sqrt{\mu} \cdot 10^3$	4.40	4.85	5.21	5.83	7.35	13.4	16.5	19.0	23.3
T, K°	220	400	600	1000	1500	~ 1800	~ 1800	~ 1800	2000
$v_e \cdot 10^{-7}$, cm/s	0.3	0.46	0.63	1.1	2.0	2.8	3.5	4.5	5.2
ν_{en}, s^{-1}	$2 \cdot 10^5$	$2 \cdot 10^4$	$3 \cdot 10^3$	$3 \cdot 10^2$	$3 \cdot 10^1$	6.0	0.7	10^{-1}	$8 \cdot 10^{-3}$
ν_{in}, s^{-1}	$8.8 \cdot 10^2$	$9.7 \cdot 10^1$	15.6	1.75	0.22	$8 \cdot 10^{-2}$	$1.15 \cdot 10^{-2}$	$1.9 \cdot 10^{-3}$	$1.8 \cdot 10^{-4}$
ν_{ei}, s^{-1}	10^3	$9 \cdot 10^2$	$6 \cdot 10^2$	$2 \cdot 10^3$	$3 \cdot 10^3$	$1.5 \cdot 10^3$	$3 \cdot 10^2$	$1.5 \cdot 10^2$	70
$(\nu_e \cdot \nu_{in})$	$1.76 \cdot 10^8$	$1.1 \cdot 10^6$	$5.6 \cdot 10^4$	$4.02 \cdot 10^3$	$6.67 \cdot 10^2$	$1.2 \cdot 10^2$	3.45	$2.8 \cdot 10^{-1}$	$1.3 \cdot 10^{-2}$
$(\omega_0 \cdot 10^{-6})$, Hz	17.84	19.54	21.85	39.89	75.69	69.09	56.4	35.6	27.1
$(\Omega_0 \cdot 10^{-3})$ Hz	78.70	91.20	114	268	790	931	930	758	637
$(\omega_B \cdot 10^{-6})$, Hz	8.40	8.29	8.20	8.02	7.66	7.31	7.02	6.22	5.68
$(\Omega_B \cdot 10^{-3})$, Hz	0.163	0.180	0.223	0.272	0.414	1.33	1.91	2.25	3.09
$(\omega_L \cdot 10^{-4})$ Hz	3.70	3.86	4.28	5.40	7.93	9.86	11.58	13.24	13.25
n_A	490	500	517	859	1326	698	486	289	153
$V_A \cdot 10^{-7}$, cm/s	6.1	6.0	5.8	3.5	2.3	4.3	6.2	10.3	19.6

IV.1 Altitude dependencies of temperatures. a.$\frac{dT_n}{dt} = 0$, b.$\frac{dT_n}{dt} \neq 0$.

a.$\frac{dT_n}{dt} = 0$. First of all let us consider the altitude dependencies of the ratio of the temperatures $T_e(E_0, \omega)$ and $T_i(E_0, \omega)$ to the initial temperature of the plasma T_{n0}, shown on Fig. 4 for $F = (1$ *and* 10^3 Hz, calculated by the shorted system of equations (31) and by the two approximations: $\nu = const$ and $\nu = \nu(T) = \nu(E_0, \omega)$. How the collision frequencies $\nu(\mathbf{E}, t)$ reaches their stationary values at the altitude $Z = 10^3$ km and $F = 1$ Hz, is also shown for sake of illustration, on Fig. 5.

The following is seen by examination of this figure.

1. The temperatures of the electrons and ions are growing up quickly on both frequencies. They become equal in the altitude region $Z \sim (200 - 250)$ km.

2. The temperatures, when $F = 10^3$ Hz, have a maximum at $Z \sim 200$ km by the approximation $\nu = \nu(E_0, \omega)$. The same happens by approximation $\nu = const$ when $F = 1$ Hz.

3. At $Z \geq 300$ km the temperatures *are smaller* by the approximation $\nu = \nu(E_0, \omega)$ than by the approximation $\nu = const$.

4. The process of ionization by the accelerated electrons is acting almost in all the altitude region when $F = 1$ Hz and *does not play a role*, when $F = 10^3$ Hz, especially at $Z \geq 300$ km. Thus, the process of the ionization should be stopped when $F = 1$ Hz already at the altitude $Z = (120 - 150)$ km.

The properties noted above of the nonlinear heating process of the ionosphere *are typical*. Namely, the behavior of T_e and T_i for $F = 1$ Hz is typical for the frequency band $F = (1$ *to* $10^2)$ Hz and at frequencies a little higher. The behavior for $F = 10^3$ Hz is typical for the frequency band $F > (10^2 - 10^3)$ Hz up to the value $F = 10^4$ Hz used here. This is seen from the following dependencies of $T_e(E_0, \omega)/T_{n0}$ on Z:

- **in Figure 6**, by the curves calculated by the approximation $\nu = const$ for $F = (1,\ 10^2,\ 10^3$ *and*$10^4)$ Hz,

- **in Figure 7**, by the curves for the angles $\Theta = (0,\ 30,\ 60,\ 75,\ 85,\ and\ 90)$ degrees also calculated by the approximation $\nu = const$,
- **in Figure 8**, by the curves, calculated for $F = 10^3\ Hz$ and the angles $\Theta = (0\ and\ 75)$ degrees by the $\nu = \nu(E_0, \omega)$ approximation.

The characteristics of these altitude dependencies of the temperature are in agreement with the noted above points 1 to 4.

b. $\mathbf{\frac{dT_n}{dt} \neq 0}$. Some results of the selfconsistent solution of the full systems of equations (30)- (32) are presented in Figure 9. Not to overload this figure, only the T_e and T_n dependencies are given in it. The main new important characteristics of these dependencies in addition to the given above in **a.** of this section are the following

1. The temperature T_n of the neutral particles is *growing up* very quickly at altitudes $Z > 200\ km$. It approaches the temperature T_e of the electrons at $Z > (400 - 500)\ km$ and *becomes close to* T_e similarly to the temperature of ions, T_i.

2. At the altitudes $Z > 400\ km$, the temperatures T_e *are larger* on both the frequencies by the approximation $\frac{dT_n}{dt} \neq 0$ than by the approximation $\frac{dT_n}{dt} = 0$.

3. At least at $Z \geq 300\ km$, the heating of the neutral particles becomes a source of the heating of the electrons and ions. The growth of T_n should be stopped, similarly to the case $\frac{dT_n}{dt} = 0$ at these altitudes (see III.4).

4. In general, the *heating of the neutral particles is an important part* in many cases of the process of heating of the plasma.

How the temperature $T_e(E_0, t)$ reaches the stationary temperature at the altitude $Z = 200\ km$ is shown on Fig. 10 for $F = 1\ Hz$ and $\Theta = 0$. It is seen how slow this process is going on when $\frac{dT_n}{dt} \neq 0$, in comparing with the case $\frac{dT_n}{dt} = 0$.

IV.2 Frequency dependencies of the temperature. Z = 300 km.

For the interested to us altitude $Z \simeq 300\ km$, the frequency dependencies of T_e are given on **Fig. 11**. They were calculated for the both approximations and $E_0 = 1\ mV/m$. In a sense, this is a *a transition region.* The influence of the heating of the neutral particles is still not too large in this region (see Fig. 9).

The behavior of the frequency dependencies of T_e is presented on Fig. 11, for different angles Θ, are in agreement with the given above conclusions in the sections **IV.1.a** and **IV.1.b**. The heating of all kind of particles is very high in this region. Even by amplitudes of the field $E_0 \sim (10^{-2} - 10^{-1})\ V/m$, the temperatures of the electrons and ions can reach rapidly, at least, values $T_e(E_0, \omega) \geq (10 - 10^2)T_{n0}$ in a broad range of angles Θ between the electric field $\mathbf{E}_0$ and the Earth's magnetic field $\mathbf{H}_0$. Temperatures $T_e(E_0, \omega) > 10^2 T_{n0}$ should, at least, be stopped by the ionization and recombination and other processes, accompanying the heating of the plasma by the electric field $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$. This does not mean that the temperatures of the electrons and ions *can not become larger than the critical temperature* $T_{e,(i)}$ of ionization, let us say, of the atomic hydrogenH_1, which is equal to

$$T_{e,(i)} = 13.5\ eV = 1.16 \cdot 10^4 \cdot 13.5 = 1.57 \cdot 10^5,\ K^{\circ}. \tag{55}$$

However, to answer this question, theoretical calculation must be done taking into account losses of energy, in particular, noted in Section III.4.

Summary

Some nonlinear parametric and heating effects in a magnetoplasma were considered in this work, particularly, in connection with the forthcoming Tethered Satellite Mission (TSM). For this aim, the theory of nonlinear heating of a collisional magnetoplasma, was extended, taking into account the heating of all the kinds of particles and all the kind of collisions between them and also the interconnection of these processes. It seems to the author, that at first the generating of parametric branches of oscillations due to the decay instabilities are discussed first in this work in connection with ionosphere. This was done on the basis of the known theories.

The numerical results describe in detail the altitude, frequency etc. temperature dependencies. Conclusions, as they follow by describing of different aspects of the discussed problems, are given.

These phenomena can play a great role in the plasma surrounding the TSS System. It will be an important contribution both to plasma physics field and for diagnostic of many properties of the plasma to search and to investigate them in this unique experiment. By the theoretical treatment of the related experimental data, the wave numbers and other local parameters of different branches of waves can be estimated. The processes of the growth of generation of different branches of e.m. oscillations and waves and of the artificial ionization of the plasma, and many other phenomena can be learned as they are developing in the course of this experiment. Despite of the fact, that the TS System will not have special artificial sources of alternating electric fields, the electromagnetic oscillations, generated in the TM Cloud by *other kinds* of instabilities can become such sources. Possibly, they will produce heating and parametric effects.

The great importance, for many reasons, to search for these effects in the ionosphere and magnetosphere, makes it advisable, that generators of alternating electric fields in the ELF, VLF and HF bands should be added to the TSS System in the future TSS Missions.

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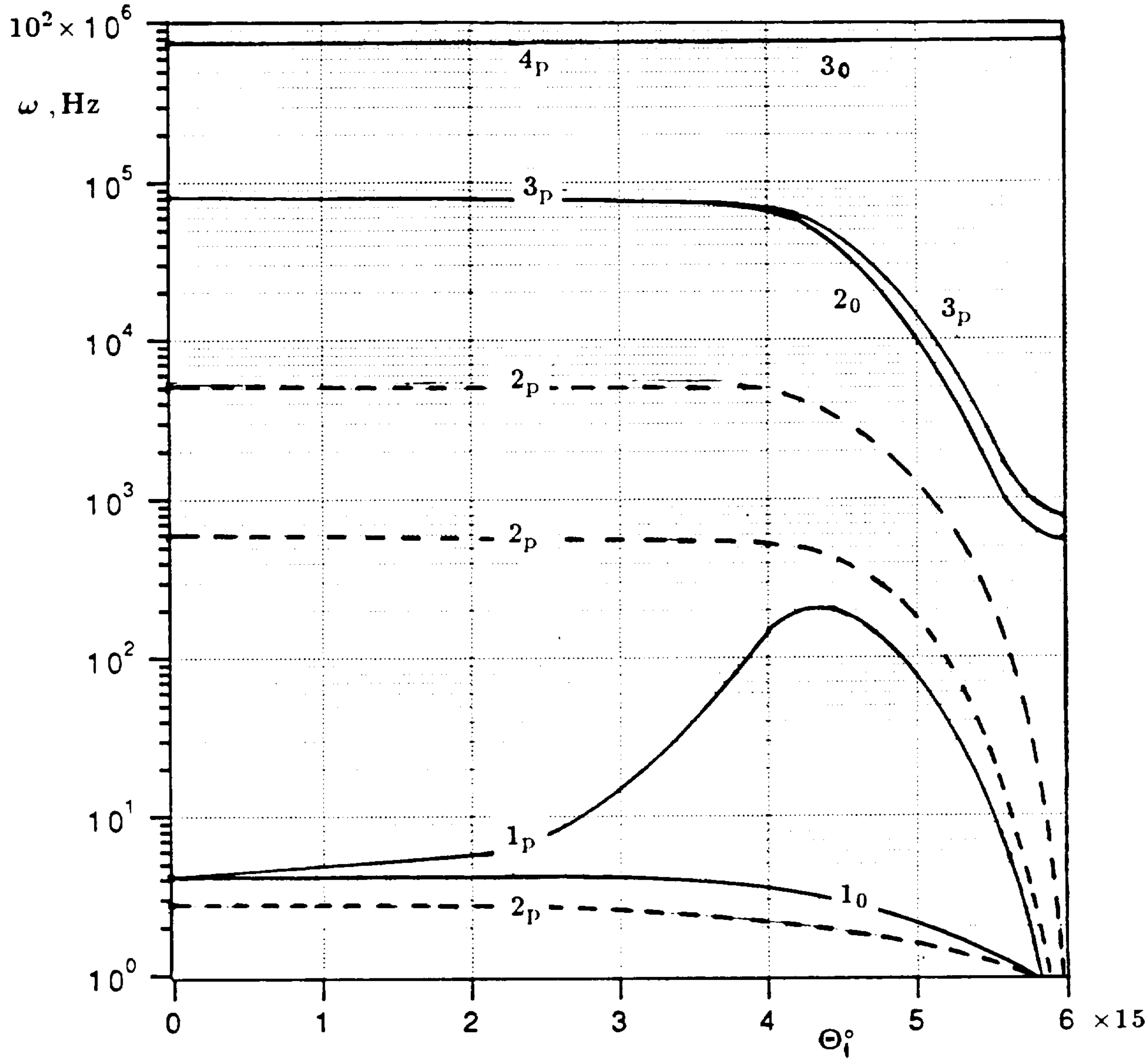

Figure 1. Resonance branches of a collisionless magnetoplasma
Electric field $\mathbf{E}_0 = 0$: $\omega_1 = (\Omega_H \rightarrow 0) \rightarrow \mathbf{1_0}$; $\omega_2 = (\omega_H \rightarrow \omega_L) \rightarrow \mathbf{2_0}$;
$\omega_3 = (\omega_0 \rightarrow \omega_U) \rightarrow \mathbf{3_0}$.
Electric field $\mathbf{E} = \mathbf{E}_0 \cos\omega t$: $\omega_1 = (\Omega_H \rightarrow 0) \rightarrow \mathbf{1_p}$;
$\omega_2 = (\omega_{2,p} = \Omega_p \rightarrow 0) \rightarrow \mathbf{2_p}$; $\Omega_p = 2.7 \cdot 10^2\ Hz\ (\rho_p = 5 \cdot 10^{-4})$.
$\Omega_p = 5.6 \cdot 10^4\ Hz\ (\rho_p = 0.1)$, $\Omega_p = 5.1 \cdot 10^5\ Hz\ (\rho_p = 1)$;
$\omega_3 = (\omega_H \rightarrow \omega_{Lp}) \rightarrow \mathbf{3_p}$, $\omega_{Lp} = 5.5 \cdot 10^4\ Hz\ (\rho_p = 0.1)$;
$\omega_4 = (\omega_H \rightarrow \omega_{U,p}) \rightarrow \mathbf{4_p}$.

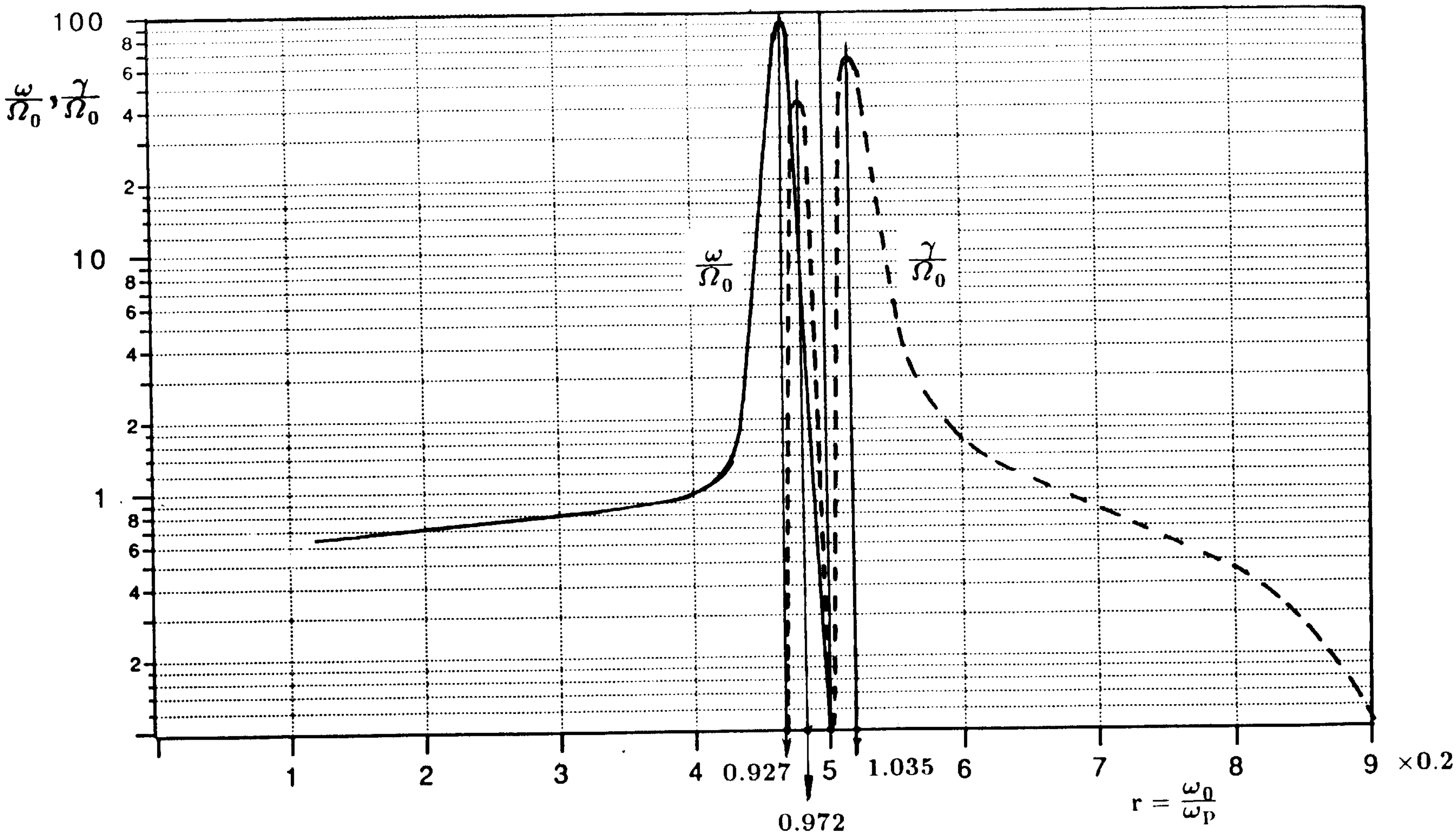

Figure 2. Dependencies of the ratios of $\frac{\omega}{\Omega_0}$ and of the growth rates $\frac{\gamma}{\Omega_0}$ of the parametric resonance branch $\omega_{2,p}$ on $r = \frac{\omega_0}{\omega_p}$, around the first resonance region $s = 1$.

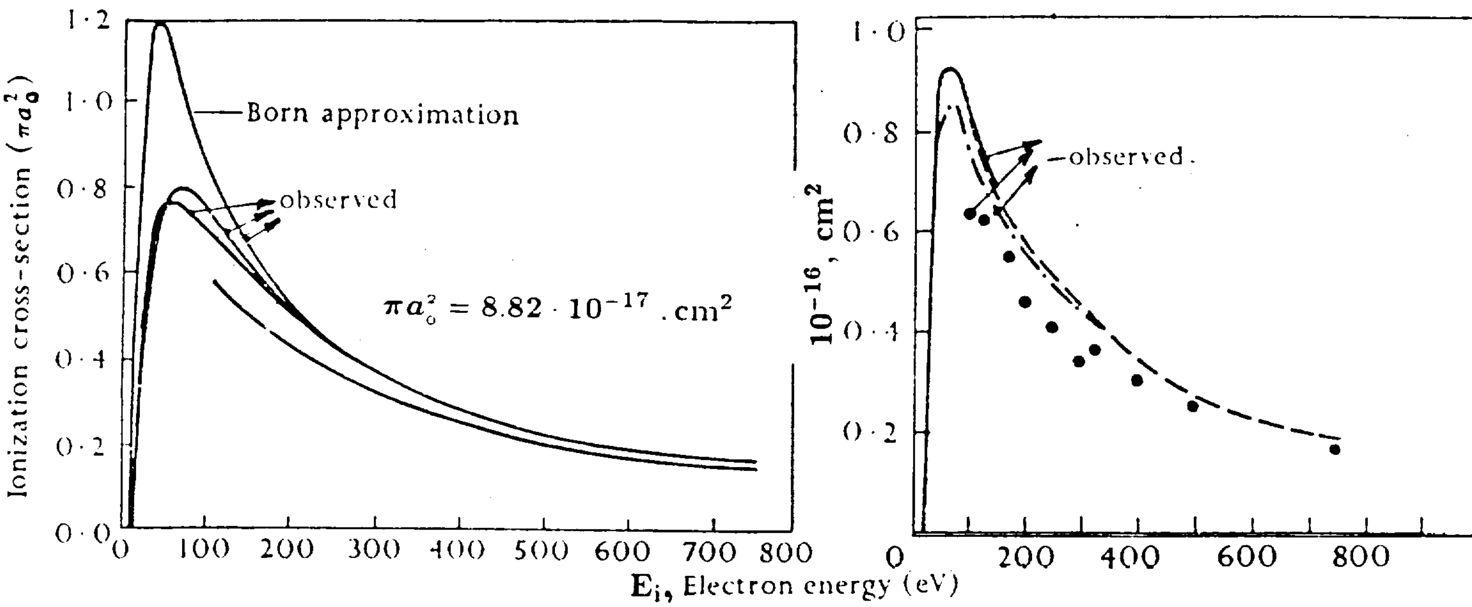

Figure 3. Experimental and theoretical dependencies of the cross-section σ_e on the potential E_i,eV of ionization of the atomic hydrogen H_1 by accelareted electrons (Massey et al. 1969 [12]).

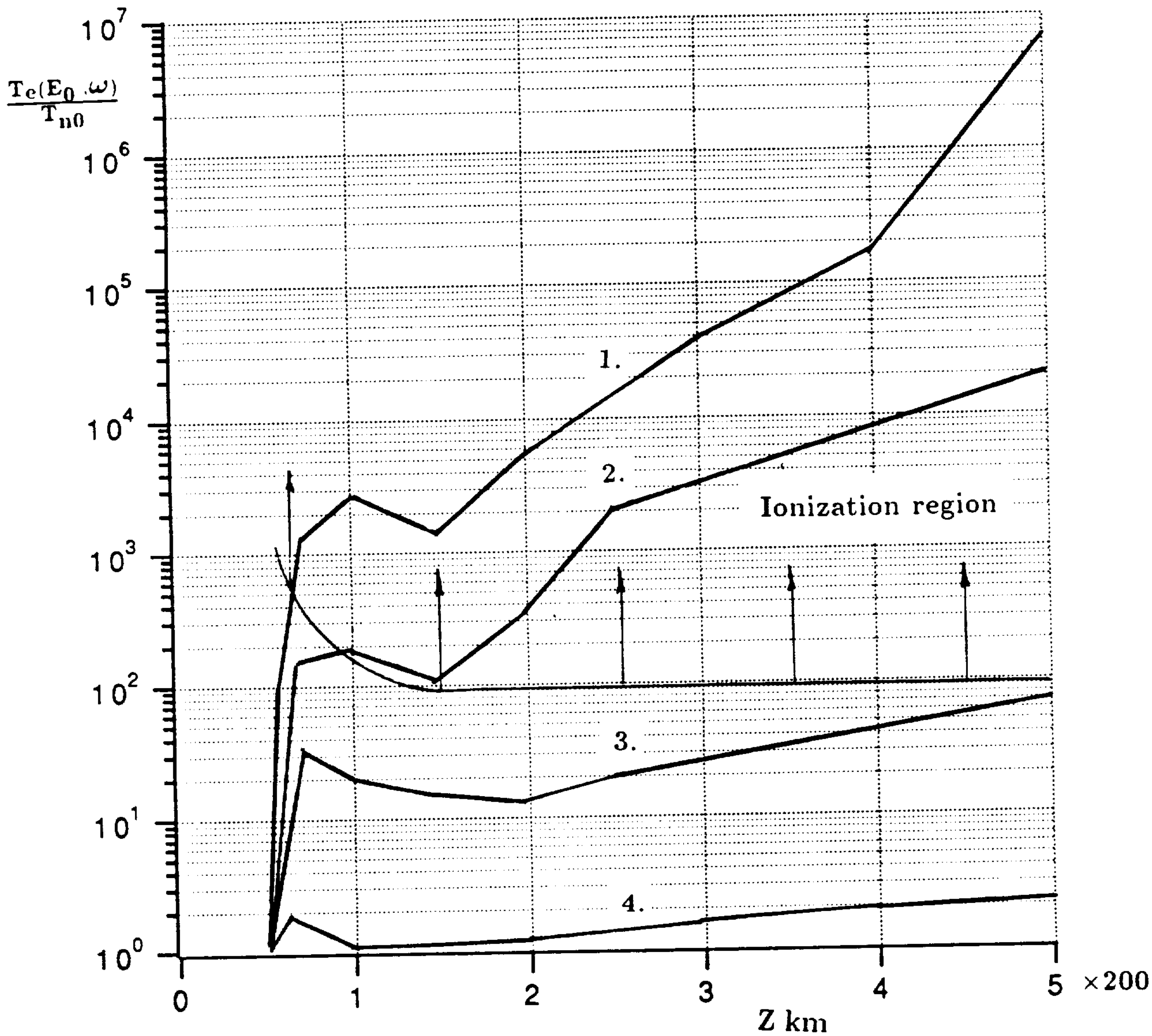

Figure 6. The same as on Fig. 4. Approximation $\nu = const$, $\Theta = 0$.
1. $\rightarrow F = 1\ Hz$, **2.** $\rightarrow F = 10^2\ Hz$, **3.** $\rightarrow F = 10^3\ Hz$,
4. $\rightarrow F = 10^4\ Hz$

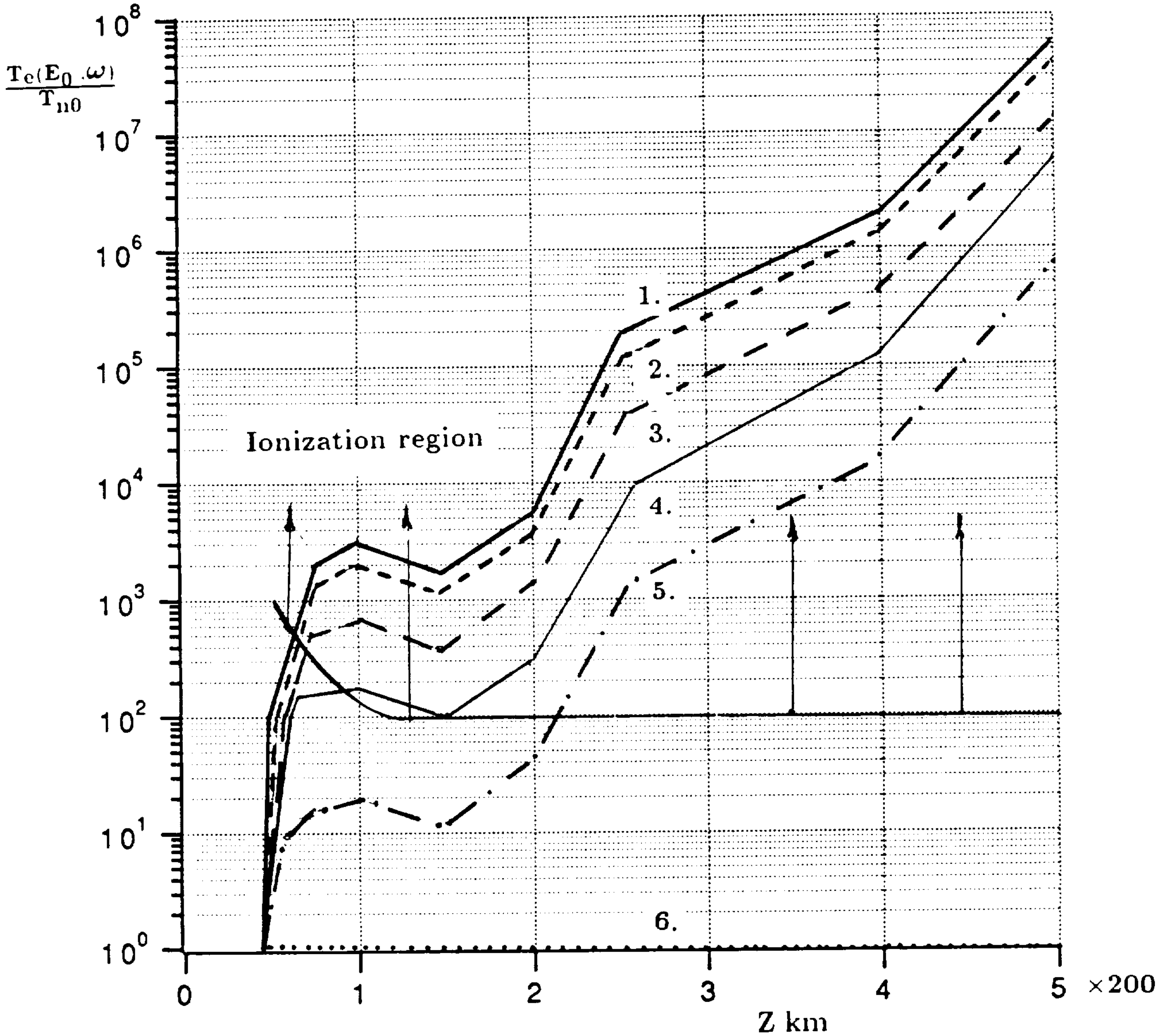

Figure 7. The same as on Fig. 4. Approximation $\nu = const$. $F = 1$ Hz.
1. , **2.** , **3.** , **4.** , **5.** , and **6.** → Θ = 0 , 30 , 60 . 75
85 . 90) degrees between the vectors $\mathbf{E}_p$ and $\mathbf{H}_0$.

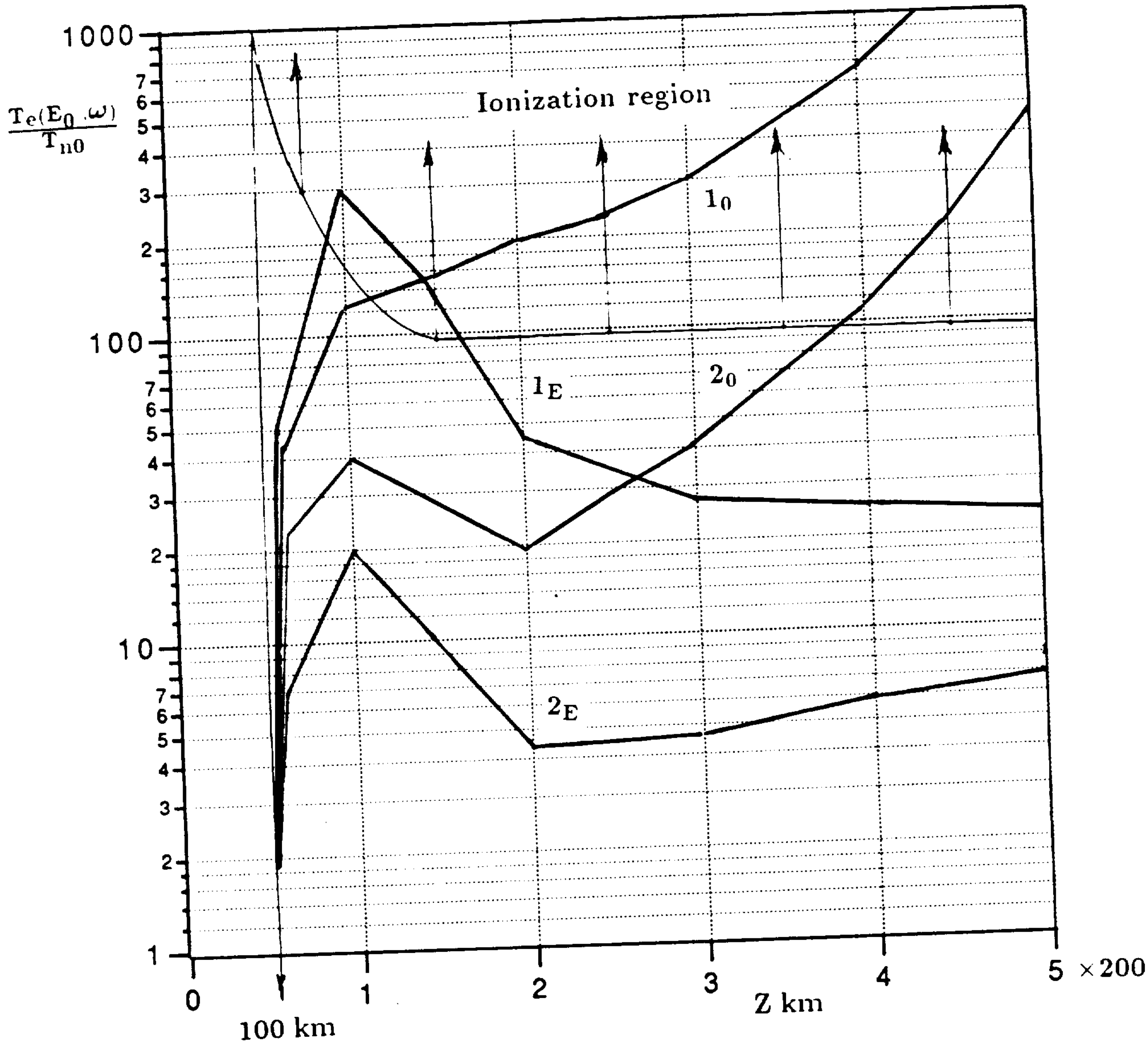

Figure 8. The same as on Fig. 4 for $F = 10^3\ Hz$.
Approximation $\nu = const$: $\mathbf{1_0} \rightarrow \Theta = 0$: $\mathbf{2_0} \rightarrow \Theta = 75°$
Approximation $\nu = \nu(E_0, \omega)$: $\mathbf{1_E} \rightarrow \Theta = 0$; $\mathbf{2_E} \rightarrow \Theta = 75°$

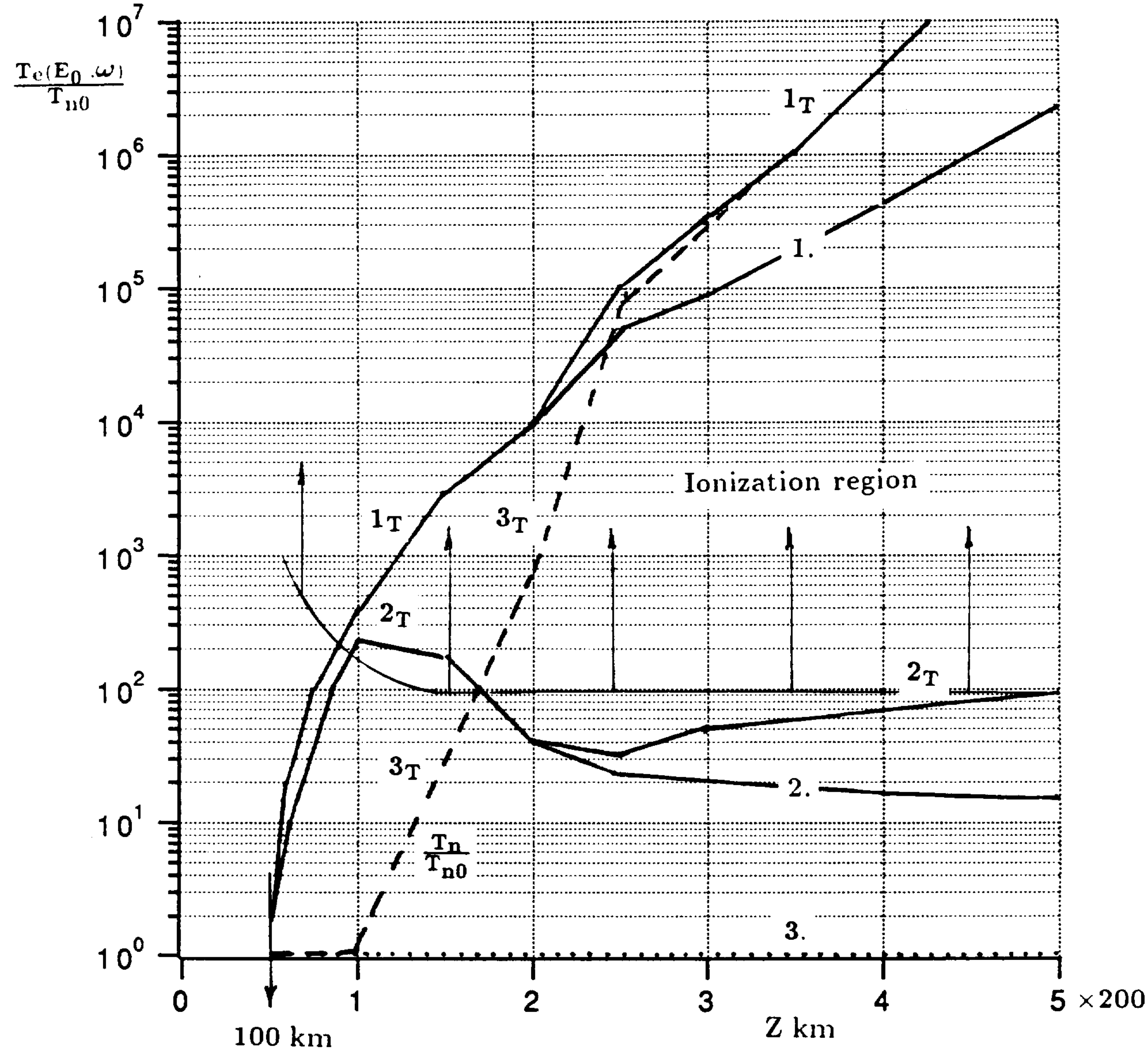

Figure 9. Altitude dependencies of the ratios of the temperatures, Approximation $\boldsymbol{\nu} = \boldsymbol{\nu}(E_0, \omega)$, $\Theta = 0$.

$\frac{T_e(E_0,\omega)}{T_{n0}}$: $F = 1\ Hz$, **1.** $\rightarrow \frac{dT_n}{dt} = 0$, $\mathbf{1_T} \rightarrow \frac{dT_n}{dt} \neq 0$

$F = 10^3\ Hz$, **2.** $\rightarrow \frac{dT_n}{dt} = 0$, $\mathbf{2_T} \rightarrow \frac{dT_n}{dt} \neq 0$

$\frac{T_n(E_0,\omega)}{T_{n0}}$: $F = 1\ Hz$, **3.** $\rightarrow \frac{dT_n}{dt} = 0$, $\mathbf{3_T} \rightarrow \frac{dT_n}{dt} \neq 0$

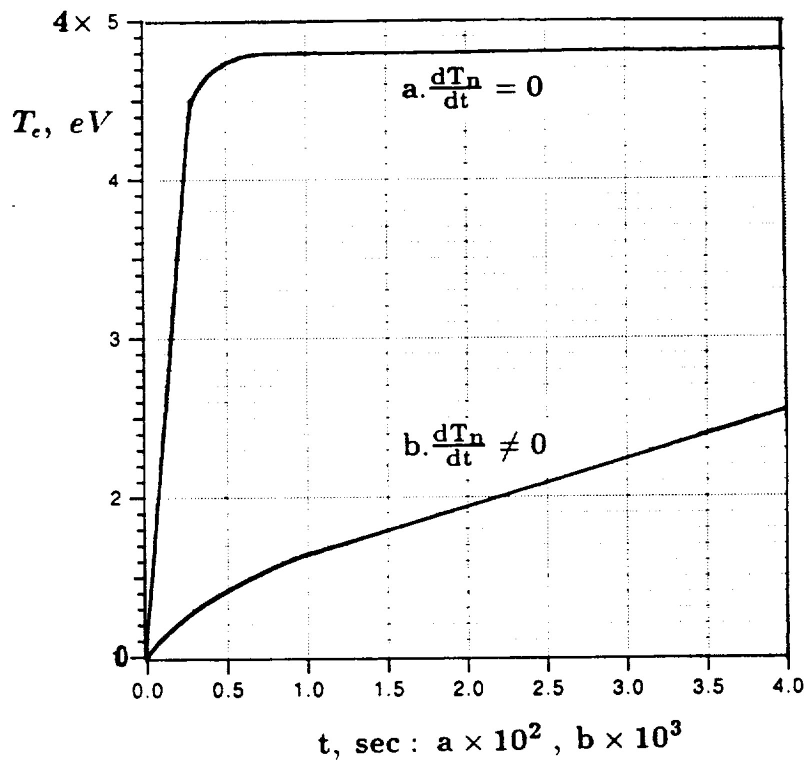

Figure 10. The establishment of the temperature $T_e(E_0, \omega)$ in time. $F = 1\ Hz$, $\Theta = 0$, $Z = 200\ km$. Approximation $\boldsymbol{\nu} = \boldsymbol{\nu}(E_0, \omega)$. a. $\rightarrow \frac{dT_n}{dt} = 0$, b. $\rightarrow \frac{dT_n}{dt} \neq 0$.

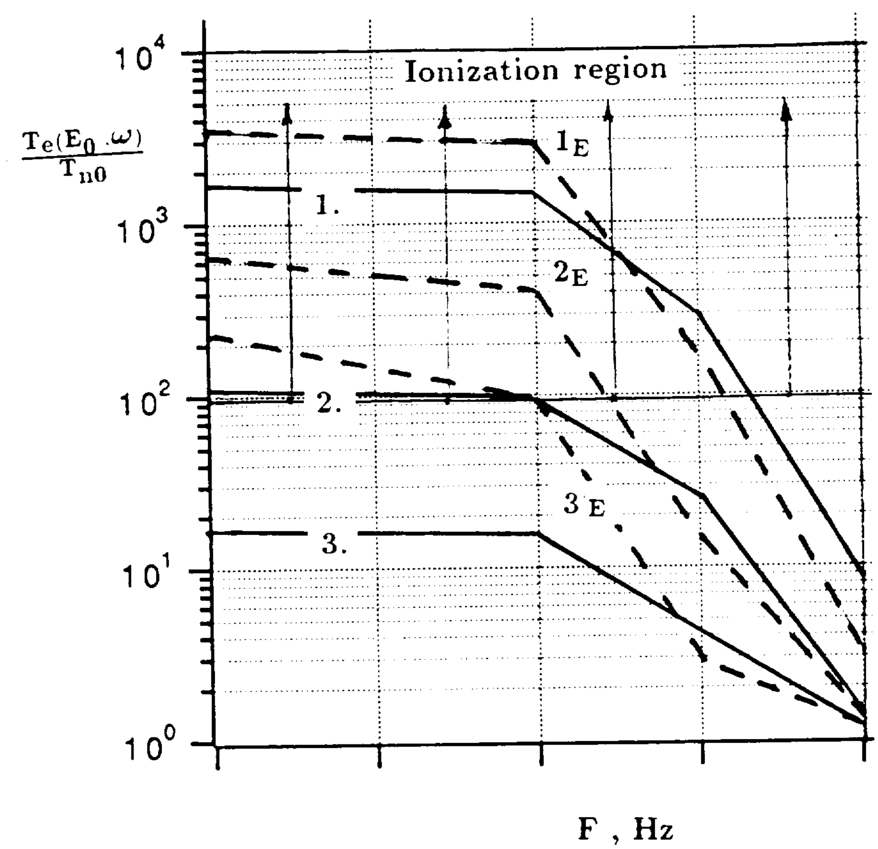

Figure 11. Frequency dependencies of the ratios $\frac{T_e(E_0,\omega)}{T_{n0}}$. $Z = 300\ km$.
Approximation $\nu = const$: **1.** $\rightarrow \Theta = 0$; **2.** $\rightarrow \Theta = 75°$. **3.** $\rightarrow \Theta = 85°$
Approximation $\nu = \nu(E_0, \omega)$: $\mathbf{1_E} \rightarrow \Theta = 0$; $\mathbf{2_E} \rightarrow \Theta = 75°$; $\mathbf{3_E} \rightarrow \Theta = 85°$