UNIFIED VISCOPLASTIC BEHAVIOR OF METAL MATRIX COMPOSITES

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ABSTRACT

The need for unified constitutive models was recognized more than a decade ago in the results of phenomenological tests on monolithic metals that exhibited strong creep–plasticity interaction. Recently, metallic alloys have been combined to form high–temperature ductile/ductile composite materials, raising the natural question of whether these metallic composites exhibit the same phenomenological features as do their monolithic constituents. Here, this question is addressed in the context of a limited, yet definitive (to illustrate creep/plasticity interaction) set of experimental data on the model MMC system W/Kanthal. Furthermore, it is demonstrated that a unified viscoplastic representation, extended for unidirectional composites and correlated to W/Kanthal, can accurately predict the observed longitudinal composite creep/plasticity interaction response and strain rate dependency. Finally, the predicted influence of fiber orientation on the creep response of W/Kanthal is illustrated.

NOMENCLATURE

Stresses

σ_{ij}	the Cauchy stress tensor
S_{ij}	the deviatoric stress tensor
a_{ij}	the deviatoric internal (or back) stress tensor
Σ_{ij}	the effective deviatoric stress tensor
σ_0	the reference uniaxial creep stress
Y	tensile threshold stress
K	shear threshold stress

Invariants

J_2	second invariant of the deviatoric stress (expressed for shear)
I_1	invariant representing transverse shear stress
I_2	invariant representing longitudinal shear stress
I_3	invariant representing maximum normal stress in the fiber direction
F	Bingham–Prager threshold function

Subscripts

$(\ )_L$ longitudinal properties

$(\ )_T$ transverse properties

i,j tensor indices, take on values of 1,2 or 3.

Material Parameters

μ represents the viscosity of material
H measure of the hardening
R measure of thermal recovery
n,m,β power law exponents
E elastic modulus
ν Poisson ratio
G shear modulus

Miscellaneous

d_i unit vector denoting local fiber direction

D_{ij} second order direction tensor; $d_i d_j$

δ_{ij} Kronecker delta function

$\dot{\epsilon}_{ij}$ inelastic strain rate tensor

ω ratio of longitudinal to transverse tensile stress
η ratio of longitudinal to transverse shear stress
$\mathcal{G}$ smoothing function for the internal stress during cyclic loading
τ time at which load history is constant
t time
< > Macauley bracket operator
Hv[] Heaviside unit function

INTRODUCTION

Structural alloys enter the *creep range* when the service temperature reaches approximately 0.3 to 0.4 of their melting temperature, T_m . In the terminology of Ashby deformation maps [1], this temperature delimits the boundary of the dislocation creep region. At temperatures within the creep range, as at lower temperatures, inelastic behavior results from dislocation motion, generation, and interaction but now with time-dependent manifestations of diffusion coming into play (e.g., dislocation climb, thermal recovery, etc.).

Although this represents the understanding of the material scientist regarding the creep of metals, it only recently has found its way into the mathematical representations (constitutive equations) that structural analysts and design engineers use to describe material behavior. The recognition that creep and plasticity have a common microstructural origin in this temperature range has led to the development of *unified* constitutive models [2,3].

The need for unified representations was recognized more than a decade ago in the results of phenomenological tests that exhibited strong *creep–plasticity interaction* [2]. Creep response was observed to be intimately affected by prior history of plasticity and vice versa. In terms of the mathematical forms appropriate for describing hereditary behavior, these observations prompted the introduction of internal state variables such as the *internal* (*or back*) *stress*. In this context the internal stress represents a macroscopic measure of the stress field associated with the dislocation microstructure. Incorporating (*Bailey–Orowan*) evolutionary laws that reflect competition between hardening and recovery mechanisms, unified constitutive models are found to accurately predict creep, rate sensitive plasticity and their mutual interactions.

When metallic alloys are combined to form a high–temperature ductile/ductile composite material, a natural question arises: *Do these metallic composites exhibit the* *same phenomenological features (e.g., creep–plasticity interaction) that led to unified constitutive relationships for their monolithic constituents ?* The objective of this study is to answer this question in the context of a limited, yet definitive (to illustrate creep/plasticity interaction) set of experimental data on a model metal matrix composite (MMC) system. We shall also demonstrate that a unified viscoplastic representation, extended for unidirectional composites, can accurately predict the observed composite response.

EXPERIMENTS

A general experimental approach to investigating history–dependent behavior is to subject *identical* specimens (over a time period $t = 0$ to $t = \tau$) to loading histories that differ in some respect. Then for $t > \tau$, each specimen is subjected to the *same* loading conditions, during which time their respective responses are compared. Only if the material has no memory of its earlier loading history during $t < \tau$ will the responses be the same for $t > \tau$. Well defined tests of this kind are not only useful in studying hereditary behavioral features but also in identifying appropriate state variables. Here, we focus on a pair of tests of this kind [4] that are designed specifically to identify creep/plasticity interaction effects. In these tests the different initial histories ($t < \tau$) of the two *identical* specimens correspond to different amounts of plasticity on load–up. The common loading ($t > \tau$) is creep at the same constant stress (σ_0).

The test material is a unidirectional W/Kanthal composite of 35 percent fiber volume ratio. Plate (coupon) specimens were tested at 600°C under a uniaxial stress along the fiber (0°) direction. This temperature corresponds to approximately 0.2 T_m for the tungsten fibers and 0.45 T_m for the Kanthal matrix. The load–up and creep responses for both of the test specimens are shown in Fig.1; both tests are performed

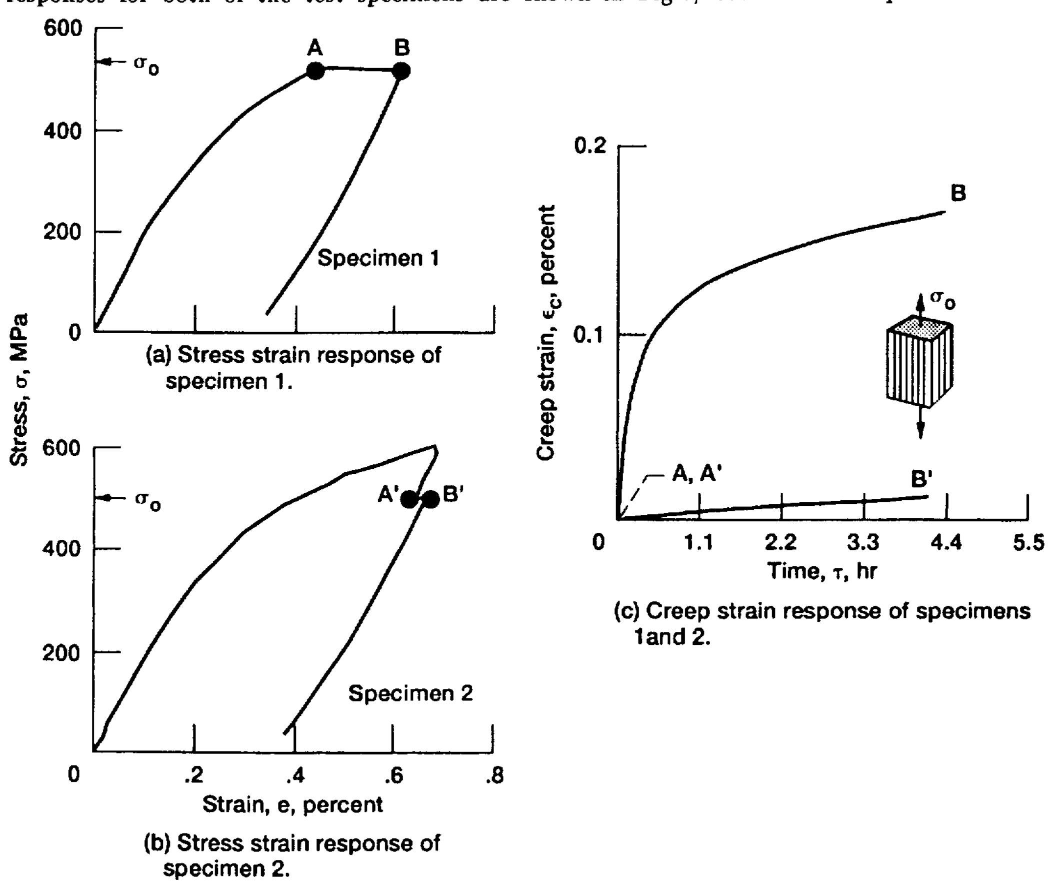


Figure 1.—Experimental observation of creep-plasticity interaction in W/Kanthal at 600 °C with a fiber volume ratio of 35%.

under load control, with an engineering stress rate of 0.193 MPa/s. In each case, the resulting elastic strain rate on load-up is approximately 1 $\mu\epsilon$/sec. Specimen 1 is loaded directly to the creep stress $\sigma_0 =$ 520 MPa; specimen 2 is first overloaded to 610 MPa and then unloaded to $\sigma_0 = 520$ MPa. The subsequent creep responses during the 4.1 hour hold period, AB for specimen 1 and A′B′ for specimen 2, are also shown in Fig.1. *Evidently, creep varies markedly with load-up history.* The initial creep rate of specimen 2, whose loading history involves the overload, is at least two orders of magnitude less than that of specimen 1; see Fig. 1c. A significant permanent strain is also shown in each specimen upon unloading. The W/Kanthal composite viewed phenomenologically as a material in its own right, evidently exhibits plasticity, creep and their mutual interaction, even as tested here along the strong fiber (0°) direction.

UNIFIED CONSTITUTIVE MODEL

We now state an anisotropic deformation theory to be used in modeling the W/Kanthal behavior of Fig.1. The continuum based theory [5,6] considers the composite as pseudo homogeneous and locally transversely isotropic with its own properties that can be measured for the composite as a whole. A complete experimental procedure for characterizing a particular composite in terms of this theory is specified in [5]. An isothermal, multiaxial statement of the model is as follows:

Flow law

$$2\mu\, \dot{\epsilon}_{ij} = \langle F \rangle^n\, \Gamma_{ij} \tag{1}$$

Evolutionary law

$$\dot{a}_{ij} = \frac{H}{\mathcal{G}^{\beta}}\, \dot{\epsilon}_{ij} - R\, \mathcal{G}^{m-\beta}\pi_{ij} \tag{2}$$

in which,

$$F = \frac{1}{K_T^2}\left[I_1 + \frac{1}{\eta^2}I_2 + \frac{9}{4(4\omega^2-1)}I_3\right] - 1 \tag{3}$$

$$\hat{G} = \frac{1}{K_T^2}\left[\hat{I}_1 + \frac{1}{\eta^2}\hat{I}_2 + \frac{9}{4(4\omega^2-1)}\hat{I}_3\right] \tag{4}$$

$$\begin{aligned}\Gamma_{ij} &= \Sigma_{ij} - \xi(D_{ki}\Sigma_{jk} + D_{jk}\Sigma_{ki} - 2I_0D_{ij}) \\ &\quad - \tfrac{1}{2}\zeta I_0(3D_{ij} - \delta_{ij})\end{aligned} \tag{5}$$

$$\begin{aligned}\pi_{ij} &= a_{ij} - \xi(D_{ki}a_{jk} + D_{jk}a_{ki} - 2\hat{I}_0D_{ij}) \\ &\quad - \tfrac{1}{2}\zeta\hat{I}_0(3D_{ij} - \delta_{ij})\end{aligned} \tag{6}$$

$$\mathcal{G} = \langle \hat{G} - \hat{G}_0 \rangle\ \mathrm{Hv}[S_{ij}\pi_{ij}] + \hat{G}_0 \tag{7}$$

$$\xi = \frac{\eta^2-1}{\eta^2} \qquad 0 \le \xi \le 1 \tag{8}$$

$$\zeta = \frac{4(\omega^2-1)}{4\omega^2-1} \qquad 0 \le \zeta \le 1 \tag{9}$$

where $\omega = Y_L/Y_T$ and $\eta = K_L/K_T$ represent the ratio of longitudinal to transverse tensile and longitudinal to transverse shear threshold stresses, respectively. These equations incorporate the following invariants, whose physical meanings are discussed in [6]:

$$I_1 = J_2 - I + \frac{1}{4} I_3 \tag{10}$$

$$I_2 = I - I_3 \tag{11}$$

$$I_3 = I_0^2 \tag{12}$$

with

$$J_2 = \frac{1}{2}\Sigma_{ij}\Sigma_{ji} \tag{13}$$

$$I = D_{ij}\Sigma_{jk}\Sigma_{ki} \tag{14}$$

$$I_0 = D_{ij}\Sigma_{ji} \tag{15}$$

and

$$\Sigma_{ij} = S_{ij} - a_{ij} \tag{16}$$

$$D_{ij} = d_i d_j \tag{17}$$

where ϵ_{ij} is the inelastic strain, S_{ij} is the applied deviatoric stress, a_{ij} is the deviatoric internal stress and d_i is a unit vector designating the local fiber direction [6]. The invariants $\hat{I}_1$, $\hat{I}_2$ and $\hat{I}_3$ in Eqs. (1) to (6) are found from Eqs. (10) to (15) with Σ_{ij} replaced by a_{ij}.

The material constants are:

$$\mu,\ n,\ H,\ \beta,\ R,\ m,\ K_T,\ G_0,\ \omega,\ \text{and}\ \eta$$

As stated earlier, they must be determined for a specific composite using the procedures outlined in [5].

A compatible anisotropic elasticity theory is necessary for the subsequent correlations and predictions. Here, we use the anisotropic elasticity model developed by Arnold [7]. The necessary elastic moduli and Poisson ratios are:

$$E_L,\ E_T,\ G_L,\ \nu_L\ \text{and}\ \nu_T$$

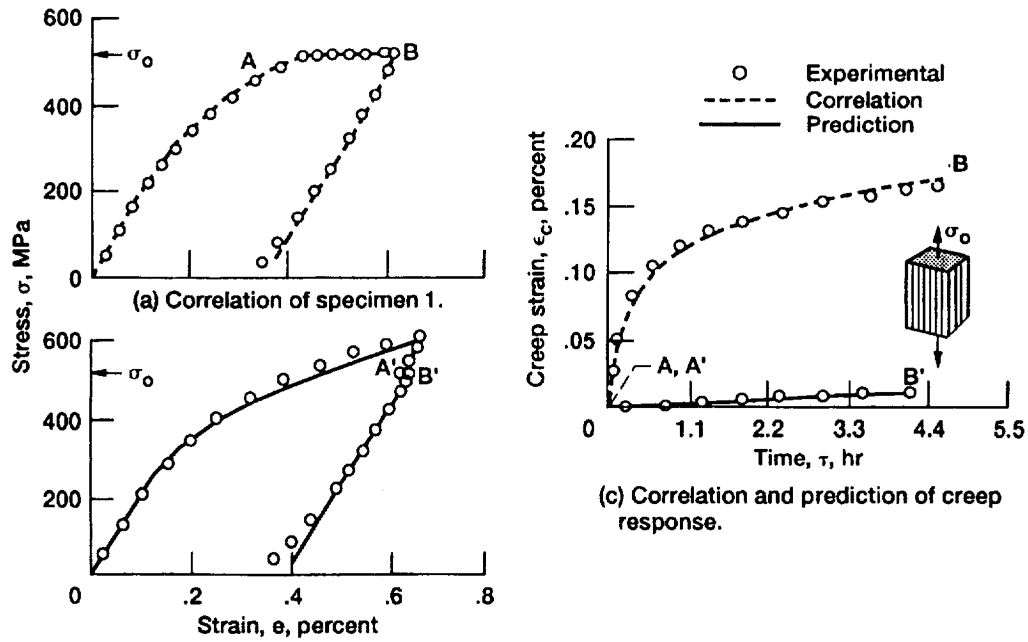
where the subscripts L and T denote longitudinal (along the fiber direction) and transverse (normal to the fiber direction), respectively.

MODEL CHARACTERIZATION – SPECIMEN 1

Given that the complete set of characterization tests specified in [5] are not available for W/Kanthal at 600°C, a limited data set consisting of the tensile and creep response of specimen 1 (Figs. 1a and 1c) is used. Characterization is thus accomplished by reducing the multiaxial equations of the previous section to the uniaxial (0°) form and then determining optimal values of the material constants to match key features of the experimental data, viz., the tensile load-up and the transient creep response of specimen 1. The procedure outlined in [5] was followed as closely as possible allowing for the very limited data set. The values of the relevant constants so obtained are:

<u>Inelastic</u>	<u>Elastic</u>
$\mu = 2.75 \times 10^{13}(3.0/(4\omega^2-1))$	
$n = 5.3$	$E_L = 206850.$
$H = 3.45 \times 10^5$	$E_T = 158047.$
$\beta = 0.55$	$\nu_L = 0.4$
$R = 8.87 \times 10^{-8}((4\omega^2-1)/3.0)$	$\nu_T = 0.25$
$m = 1.63$	$G_L = 68950.$
$K_T^2 = 4.548(3.0/(4\omega^2-1))$	
$\hat{G}_0 = 0.05$	

These values are consistent with the units of MPa for stress and hr for time. As the predictions in the following section are made for the same uniaxial direction (0°) along which the correlations are based, the material constants ω and η, designating the degree of anisotropy, need not be specified. Figures 2a and 2c show a comparison of the experimental (open circles) and predicted responses (dashed lines) of specimen 1 based on this limited data correlation. The accuracy of the correlation is very good.



(b) Prediction of specimen 2.

Figure 2.—Comparison of experimental (symbols) and analytical predictions (lines) of creep-plasticity interaction in W/Kanthal at 600 °C with a fiber volume ratio of 35%.

PREDICTED RESPONSE – SPECIMEN 2

Having used the data of specimen 1 for characterization, we now employ the constitutive model with the specified constants to predict the response of specimen 2. This is done by numerically integrating the constitutive equations over the prescribed loading history. Figures 2b and 2c compare this prediction (solid lines) with the experimental response of specimen 2 (open circles). The agreement is remarkably good, showing definitively that the unified model [5,6] accurately represents the observed hereditary effects and the interaction of plasticity and creep in these experiments.

The experiments described here were conducted exclusively along the 0°–fiber direction of the unidirectional W/Kanthal composite. If inelastic/hereditary effects were to be suppressed, it is expected this would occur under testing, as here, along the strong (0°) fiber direction. *Evidently, this is not the case.* However, this raises the question of how hereditary features might vary as the anisotropic composite is loaded in other directions (e.g., normal to the fiber direction). Such predictions for other fiber orientations can be made for W/Kanthal at 600° using the results obtained here, provided the remaining constants ω and η are specified [5]. The determination of these constants, characterizing the degree of anisotropy, and the required experiments analogous to the pair described here but at, say 90°, are left for future research. Qualitative predictions of the influence of fiber orientation are made in a subsequent section based on assumed values of ω and η.

STRAIN RATE DEPENDENT PLASTICITY

Another feature of elevated temperature response of monolithic metals is strain rate dependence. This feature can also be represented by unified models [2,3]. As with hereditary effects, we might ask if strain rate dependence, such as observed in monolithic alloys, similarly carries over to MMCs.

To address this question, we include the load-up response of a third W/Kanthal specimen tested (under strain control) at 600°C and at a strain rate of $100\mu\epsilon$/sec, (two orders of magnitude greater than that for specimens 1 and 2, described earlier). Figure 3 compares the tensile response of this specimen with that of specimen 2. *Rate dependence is evident,* showing approximately a 15–20% increase in strength for the 100–fold increase in strain rate. This is approximately the same degree of rate dependence exhibited by many monolithic alloys at temperatures in this range.

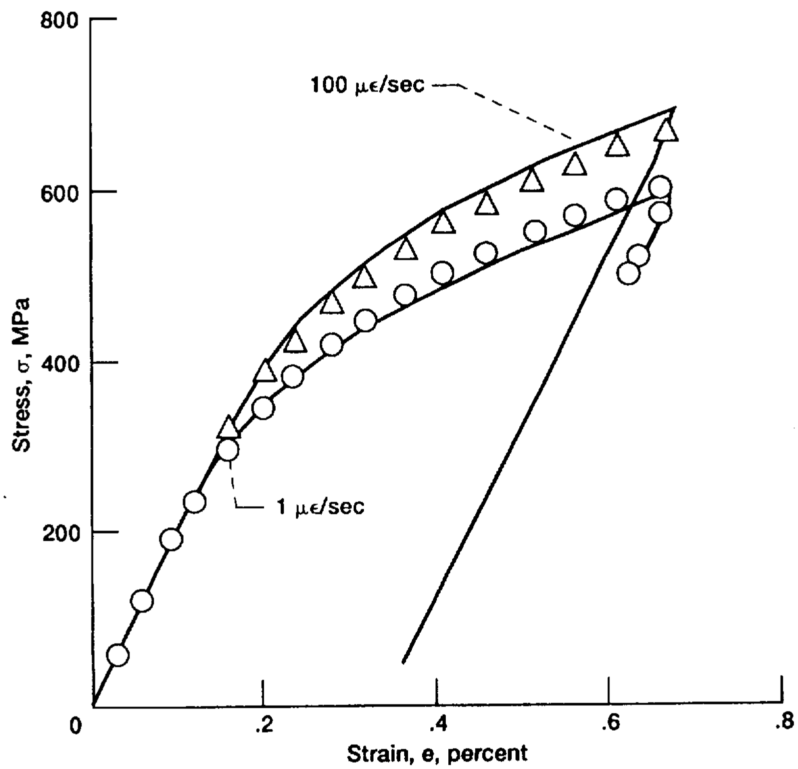


Figure 3.—Comparison of experimental (symbols) and analytical predictions (solid lines) of stress-strain response at different strain rates of 1 $\mu\epsilon$/sec and 100 $\mu\epsilon$/sec.

INFLUENCE OF FIBER ORIENTATION

Finally, the predicted influence of fiber orientation on the creep response of W/Kanthal is illustrated in Fig. 4 for fiber angles (relative to the loading direction) of 0, 30, 60 and 90°. These calculations are made with assumed values of $\omega = 5.0$ and $\eta = 2.5$ designating the degree of anisotropy and so are to be considered only qualitative. The creep stress in Fig. 4 is 200 MPa and the equivalent elastic strain rate is $1\mu\epsilon$/sec.

As expected, the inelastic response significantly increases, during load-up as the fiber angle is increased, see Fig. 4a, such that in the transverse case (90°) a total strain equal to approximately 0.4% is achieved at 200 MPa instead of 520 MPa, as in the longitudinal (0°) case shown in Fig. 2a. This difference in maximum achieved stress (by a factor of 2.5) indicates a significant decrease in transverse load carrying capacity relative to that in the longitudinal direction, as expected. In comparing the 60° case to that of the 90° case (see, Fig. 4b) it is interesting to note that the amount of creep strain incurred in the 60° case exceeds that incurred in the 90° case, whereas the 0°

and 30° cases are ordered as expected. Past experience has shown, however, that this change in ordering is dependent upon the relative magnitude of the material parameters chosen. Clearly, if ω and η (the degree of anisotropy) are found experimentally to differ greatly from the above assumed values, so too will the predicted amount of inelasticity.

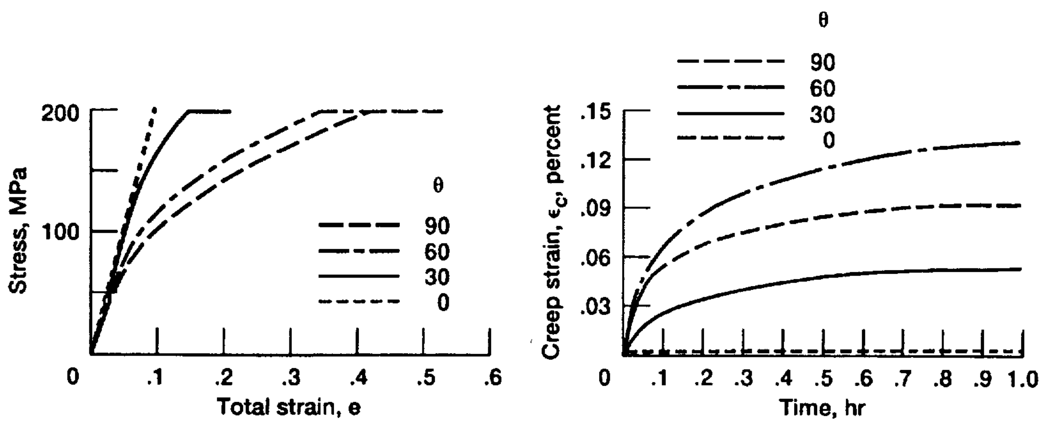


Figure 4.—Analytical predictions of the creep response of W/Kanthal at varying fiber angles. θ = 0, 30, 60, 90°, with a fiber volume ratio of 35%.

CONCLUSIONS

Generally, we conclude from this study that at 600°C W/Kanthal, viewed here as a model ductile/ductile MMC, shows qualitatively the same features of time–dependent, hereditary behavior observed in monolithic alloys at comparable temperatures. That is to say, W/Kanthal exhibits rate sensitive plasticity, creep and their intimate interaction. These effects are evident even as tested here uniaxially (along the direction of maximal constraint, i.e., 0°) in the fiber direction.

We further conclude that the unified anisotropic viscoplasticity theory [6] used to model the W/Kanthal response is capable of representing the key features of the inelastic behavior, for example creep–plasticity interaction, strain–rate sensitivity, and the effect of fiber orientation. Here, we have demonstrated, with a limited set of data, the accuracy of this model in the longitudinal direction and leave for future work the validation of the off–axis response (i.e., effect of fiber orientation).

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13. ABSTRACT ***(Maximum 200 words)***

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