NASA-CR-190998

HQ GRANT
IN-43-CR
128410
p. 25

FINAL REPORT

Title: Experimental Measurement and Theoretical Modeling Of Microwave Scattering and the Structure of the Sea Surface Influencing Radar Observations from Space

Sponsor by: National Aeronautics and Space Administration

Contract number: Grant NAGW-1272

Research Organization: Center for Electromagnetic Theory and Applications
Research Laboratory of Electronics
Massachusetts Institute of Technology

OSP number: 70391

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Period covered: March 1, 1988 – September 30, 1992

(NASA-CR-190998) EXPERIMENTAL MEASUREMENT AND THEORETICAL MODELING OF MICROWAVE SCATTERING AND THE STRUCTURE OF THE SEA SURFACE INFLUENCING RADAR OBSERVATIONS FROM SPACE Final Report, 1 Mar. 1988 - 30 Sep. 1992 (MIT) 25 p

N93-12006

Unclas

G3/43 0128410

EXPERIMENTAL MEASUREMENT AND THEORETICAL MODELING OF MICROWAVE SCATTERING AND THE STRUCTURE OF THE SEA SURFACE INFLUENCING RADAR OBSERVATIONS FROM SPACE

Electromagnetic Bias Theory

The electromagnetic (EM) bias ε is an error present in radar altimetry of the ocean due to the non-uniform reflection from wave troughs and crests. The electromagnetic bias is defined as the difference between the mean reflecting surface and the mean sea surface. A knowledge of the electromagnetic bias is necessary to permit error reduction in mean sea level measurements by satellite radar altimeters. Direct measurements of the EM bias were made from a Shell Offshore oil production platform in the Gulf of Mexico for a six month period during 1989 and 1990. Measurements of the EM bias were made at 5 GHz and 14 GHz.

During the EM bias experiments by *Melville et al.* [1990,1991] a wire wave gauge was used to obtain the modulation of the high frequency waves by the low frequency waves. It became apparent that the EM bias was primarily caused by the modulation of the short waves. This was reported by *Arnold et al.* [1989,1990,1991]. The EM bias is explained using physical optics scattering and an empirical model for the short wave modulation. Measurements of the short wave modulation using a wire wave gauge demonstrated a linear dependence of the normalized bias on the short wave modulation strength M. The theory accurately predicts this dependence by the relation $\varepsilon = -\alpha M H_{1/3}$. The wind speed dependence of the normalized bias is explained by the dependence of the short wave modulation strength on the wind speed. While other effects such as long wave tilt and curvature will have an effect on the bias, the primary cause of the bias is shown to be due to the short wave modulation.

This report will present a theory using physical optics scattering and an empirical model of the short wave modulation to estimate the EM bias. The estimated EM bias will be compared to measurements at C and Ku bands.

1. EM Bias Dependence on Short Wave Modulation

The back scattered power from a small patch on the ocean surface depends on the displacement of the patch under observation from mean sea level. It has been observed that more power is reflected from the troughs of waves than from their crests. A typical measurement of the relative back scatter coefficient as a function of wave displacement is shown in figure 1.

The EM bias ε can be defined mathematically as the ratio of the first two moments of the back scatter coefficient profile σ_η^0 give by

$$\epsilon = \frac{E[\eta\sigma_\eta^0\,(\eta)]}{E[\sigma_\eta^0\,(\eta)]} \tag{1}$$

where η is the surface displacement, and E[ ] denotes an ensemble average. The back scatter coefficient σ^0 is related to the back scatter coefficient profile as

$$\sigma^0 = \int_{-\infty}^{\infty} d\eta\ \sigma_\eta^0(\eta)\ p_\eta(\eta) \tag{2}$$

where $p(\eta)$ is the surface displacement probability density function. The task is to develop a theory to predict the back scatter coefficient profile from which the EM bias can be calculated.

The primary cause of the EM bias at C and Ku bands will be assumed to be the modulation of the short wave amplitude by the long waves. The short wave amplitude modulation will found empirically by measuring the energy in the short waves as a function of long wave displacement. Physical optics scattering will be used to predict the back scatter coefficient profile from the short wave modulation profile.

A model relating the EM bias to the important parameters describing the ocean surface will be developed. It will be shown that the bias can be described by a relationship between wave height and a short wave modulation strength parameter.

1.1 Short Wave Modulation Model

The ocean wave field is separated into long and short waves at a separation wave length L with corresponding wave number k_s (see figure 2). The separation wave length is chosen to be much larger than the electromagnetic wave length λ_{EM} of the microwave scatterometers and much smaller than the dominant wave length λ_0 of the ocean waves. The short waves have a variance σ_s^2 and the long waves have a variance σ_l^2. The scatterometers used to make the measurements had illuminated spot sizes on the order of one meter; which was chosen as the separation wave length satisfying the relationship:

$$[\lambda_{EM} = O(10^{-2}m)] \ll [L = O(1m)] \ll [\lambda_0 \approx O(10m)] \tag{3}$$

The long waves can be modeled as a surface tilt and curvature. The tilt and curvature of the surface will be considered to be of secondary important to the short wave modulation and thus will be neglected. The possible effects of neglecting the tilt and curvature of the surface will be discussed later.

The wave number modulation of the short waves will be neglected allowing the short wave spectrum to be described by a constant spectral shape. The wavefield at scales less than L will be modelled as unidirectional with a k^{-p} spectral shape. The short wave modulation can then be described by a short wave height variance which varies with long wave displacement. The short wave model spectrum for p greater than one is given by

$$S_p(k,\eta) = \begin{cases} (p-1)\sigma_s^2(\eta) k_s^{p-1} k^{-p} & k \geq k_s = \frac{2\pi}{L} \\ 0 & k < k_s \end{cases} \tag{4}$$

so that the variance of the short waves is given by

$$\sigma_s^2(\eta) = \int_{k_s}^{\infty} S_p(k,\eta) \tag{5}$$

The corresponding autocorrelation function and correlation coefficient are

$$R_p(x,\eta) = \sigma_s^2(\eta)\ C_p(x) \tag{6}$$

$$C_p(x) = \int_{k_s}^{\infty} dk(p-1)\ k_s^{p-1}\ k^{-p} \cos kx \tag{7}$$

The correlation coefficient can be rewritten as

$$C_p(k_s x) = (p-1)(k_s x)^{p-1} \int_{k_s x}^{\infty} du\ u^{-p} \cos u \tag{8}$$

This transform can be performed resulting in a power series representation for small argument and an asymptotic series representation for large argument (see Appendix A for details). Values of $p = 2.5$ and 3.0 will be used later in comparing with measured results. A value of $p = 3.0$ for a unidirectional surface corresponds to a k^{-4} tow-dimensional spectrum and a value of $p = 2.5$ corresponds to $k^{-3.5}$ tow-dimensional spectrum. These values of p were chosen to agree with the measurement of *Banner et al.* [1988], *Shemdin et al.* [1988] and *Jahne and Riemer* [1990]. The exponent is decreased by one when integrating over the direction of no variation of the unidirectional surface.

1.2 Physical Optics Scattering Theory

Physical optics, or the Kirchhoff approximation, is a well known scattering theory, having been used for rough surface scattering by *Beckman and Spizzichino* [1963], *Hagfors* [1966], *Fung and Moore* [1966], *Holliday et al.* [1986] and many others. The physical optics integral at normal incidence for a unidirectional surface is given by (for derivation, see Appendix B)

$$\sigma^0 = \left(\frac{k_{EM}^2 A_0}{\pi}\right) \int_{-1}^{1} du(1-|u|)\ e^{-4\sigma_s^2\ K_{EM}^2[1-C_p(k_s L u)]} \tag{9}$$

where σ^0 is the backscatter coefficient, k_{EM} is the electromagnetic wave number, A_0 is the illuminated area, σ_s is the RMS wave height of the short wave, k_s is the separation wave number of the surface spectrum, L is the illuminated spot diameter and C_p is the surface correlation coefficient given by equation (7). Two assumptions were made in the development of equation (9); namely, a tangent plane approximation for the electric surface current and a Gaussian probability density distribution for the surface displacement.

Barrick [1970] argued that the only valid use of physical optics was in the limit as electromagnetic frequency becomes infinite, resulting in geometric optics or specular point theory. Specular point theory has been discussed by *Kodis* [1966], *Barrick* [1968], and *Barrick and Bahar* [1981], where it was shown that specular point theory depends only on the slope statistics of the surface, not the shape of the correlation coefficient. *Fung and Chang* [1971], *Fung and Eom* [1981] and *Chan and Fung* [1988] showed, using method of moment calculations, that high frequency restriction of *Barrick* [1970] is too restrictive. The validity of the Kirchhoff approximation requires only that the average radius of curvature of the surface be large compared to the electromagnetic wavelength. This insures that the surface will be smooth enough for the tangent plane surface current approximation to be applicable. Stated explicitly, the validity of equation (9) requires

$$\frac{\lambda_{EM}}{\rho_C} \ll 1 \tag{10}$$

where ρ_C is the average radius of curvature of the surface.

The high frequency portion of the ocean wave spectrum causes the average radius of curvature to be small. This apparently renders invalid the use of either specular point or physical optics scattering theory when observations of the ocean surface are made at microwave frequencies. However, *Tyler* [1976] showed that the high frequency features of a surface should be smoothed prior to application of specular point theory. The common practice is to include only the portion of the ocean surface with wave lengths longer than the electromagnetic wave length [*Valenzuela,* 1978].

The physical optics integral of equation (9) does not require a filter function as proposed by *Tyler* [1976] to be explicitly applied to the ocean surface. A filter function is inherent in the physical optics integral because the high frequency waves are weighted less because of their small heights. These statements will be verified by comparing the results of the physical optics integral to results obtained using exact method of moment calculation for the employed short wave model.

Axline and Fung [1978] used the method of moments and a Monte Carlo simulation to find the exact scattering coefficients of a random surface with specified spectrum and correlation coefficient. The physical optics integral was evaluated numerically for a surface with a spectrum given by equation (4) with $p = 3$. The results are shown by the solid line in figure 3. The physical optics scattering coefficients computed using a Monte Carlo simulation are shown by the squares. The method of moments technique was used to obtain the exact scattering coefficients for $k_{\mathrm{EM}} = 104.7$, and the results are shown by the triangles. It can be seen that the physical optics theory provides excellent agreement with exact method of moment results. This establishes the validity of using physical optics scattering for the employed short wave spectrum model.

In order to study the implicit filter function in the physical optics integral, the following spectrum with a variable high wave number cut off will be used:

$$S_p(k) = \begin{cases} 0 & k > k_h \\ S_0 k^{-p} & K_s \leq k \leq k_h \\ 0 & k > k_s = \frac{2\pi}{L} \end{cases} \tag{11}$$

A corresponding correlation coefficient can easily be found similar to equation (7).

For a separation length $L = 2m$, a typical short wave height $\sigma_s = 2cm$, $k_{\mathrm{EM}} = 105$ and 293 corresponding to C and Ku bands, and with $p = 3$, figure 4 shows the back scatter coefficient as a function of k_h normalized by the scatter coefficient for an effective infinite k_h. Physical optics integral results are shown by the solid line. Physical optics scattering coefficients computed using a Monte Carlo simulation are shown by the squares, and the exact method of moments results are shown by the triangles. It is easily recognized that the back scatter coefficient is less influenced by shorter waves. The same dependence of exact method of moment and physical optics results on high wave number cutoff, demonstrates that physical optics applies the correct filter function implicitly.

The applicability of physical optics becomes obvious by considering figure 4, which also shows equation (10) computed as a function of k_h. It is seen that the ratio of electromagnetic wave length to the average radius of curvature is less than one for the portion of the spectrum that contributes to the scattering, thus satisfying the validity criteria of equation (10).

The physical optics integral of equation (9) is difficult to solve using standard asymptotic techniques. The exponential argument in equation (9) contains the correlation coefficient given by equation (7). The correlation coefficient is not well represented by the first few terms of a Taylor series as used in standard asymptotic techniques. An alternate method is to use a series with a fractional power term to represent the exponential argument of equation (9) as

$$4\sigma_s^2 k_{EM}^2 \left[1 - C_p(k_s L u)\right] = \left(\frac{k_s L u}{Z_0}\right)^{2/\alpha} + \cdots \tag{12}$$

Assuming the parameter $\sigma_s K_{\text{EM}}$ is large, only the first term of equation (12) is needed. Substituting equation (12) into equation (9), and extending the limits to infinity gives

$$\sigma^{\circ} = 2\left(\frac{k_{EM}^2 A_0}{\pi}\right) \int_0^{\infty} du\ e^{-\left(\frac{k_s L u}{Z_0}\right)^{2/\alpha}} \tag{13}$$

Making a variable substitution and using the definition of the gamma function

$$\Gamma(p) = \int_0^{\infty} dx\ x^{p-1}\ e^{-x} \tag{14}$$

gives equation (9) as

$$\sigma^0 = \left(\frac{k_{EM}^2 A_0}{\pi}\right) \alpha \Gamma\left(\frac{\alpha}{2}\right) \frac{Z_0}{k_s L} \tag{15}$$

The exponential argument of equation (9) is expanded in equation (12) about $k_s L u = z_0$ where z_0 is chosen according to the relation

$$4\sigma_s^2 k_{EM}^2 [1 - C_p(Z_0)] = 1 \tag{16}$$

so as to provide a good fit at the e^{-1} point of the exponential of equation (9). Keeping the first term of equation (12), differentiating both sides with respect to $k_s L u$, and solving for α about z_0 gives

$$\alpha = \frac{1}{2\sigma_s^2 k_{EM}^2 [-C_p'(z_0)] z_0} \tag{17}$$

Figure 5 compares the back scatter coefficient, calculated using the asymptotic result of equations (15),(16) and (17) with the numerical integration of equation (9) for $k_s L = 2\pi$. As seen in figure 5, the asymptotic back scatter coefficient is a good approximation when $\sigma_s k_{\text{EM}} > 1$.

1.3 EM Bias - Linear Short Wave Modulation Model

A model relating the EM bias directly to the short wave modulation will be described. It will be shown that the bias can be described by a simple relationship with the wave height and a short wave modulation strength parameter.

The short wave modulation profile is approximated linearly by

$$\sigma_s(\eta) = \sigma_m(1 + m\frac{\eta}{\sqrt{n^2}}) \tag{18}$$

where σ_s is the local RMS short wave height, σ_m is the global RMS short wave height, and m is a measure of the modulation strength. The back scatter coefficient profile can be computed by substituting equation (18) into equation (9). For the modulation strength much less than one, the back scatter coefficient profile becomes

$$\sigma^0(\eta) = \frac{k^2 A_0}{\pi} \int_{-1}^{1} du(1-|u|)\; e^{-4(\sigma_m k_E M)^2 (1+2m\eta/\sqrt{\eta^2})\;[1-C_p(k_s Lu)]} \tag{19}$$

Representing the back scatter coefficient profile by its Taylor series expansion about mean sea level gives

$$\sigma^0(\eta) = \sigma^0(0) + \eta\frac{\partial\sigma^0(0)}{\partial\eta} + \cdots \tag{20}$$

where the bias can be determined using equation (1) giving

$$\epsilon = \frac{\overline{\eta}^2\frac{\partial\sigma^0(0)}{\partial\eta} + \cdots}{\sigma^0(0)} \tag{21}$$

Substituting equation (19) into (21), the bias can be written in terms of a short wave modulation strength parameter m and the wave height as

$$\epsilon = -\alpha m\sqrt{\overline{\eta}^2} \tag{22}$$

where

$$\alpha = \frac{\int_0^1 du(1-u)8(\sigma_m k_{EM})^2[1-C_p(k_s Lu)]e^{-4(\sigma_m k_{EM})^2[1-C_p(k_s Lu)]}}{\int_0^1 du(1-u)\; e^{-4(\sigma_m k_{EM})^2}\;[1-C_p(k_s Lu)]} \tag{23}$$

The linear dependence of the EM bias on wave height has been well known from experimental observations for some time [*Walsh et al.*, [1989]. A correspondence between the short wave modulation profile and the EM bias was established by *Arnold et al.* [1990], but equation (22) specifies the correspondence by showing the bias to be proportional to the short wave modulation strength.

Equation (23) can be solved asymptotically by using the methods of the last section given by equations (12)-(17). Equation (23) is found to be well approximated by equation (17) for $\sigma_m k > 1$. Figure 6 compares the numerical integration of equation (23) with the asymptotic result with $p = 2.5$ and 3 and $k_s L = 2\pi$. For $\sigma_m k > 1$, the asymptotic results are good approximations.

2. Experiment Description

An experiment to measure the EM bias at C and Ku bands (the frequencies of the TOPEX/Poseidon altimeters) was conducted from December 1989 through May 1990 [*Melville et al*, 1990] from a Shell Offshore production complex (Brazos-19) in 40 meters of water off the coast of Texas in the Gulf of Mexico. Nadir looking coherent scatterometers at 5 and 14 GHz and a Thorn/EMI IR wave gauge were mounted 18 meters above sea level in the middle of a 60 meter bridge joining two platforms. For short periods of the experiment, a capacitance wire wave gauge was mounted adjacent to the footprints of the scatterometers. The wind speed and direction, air and sea temperature, humidity and rain fall were measured by an R. M. Young instrument package. The data contained in this report comes from a week of data taken during May 1990.

The EM bias was measured using the back scatter and doppler of the C and Ku band scatterometers. The wave displacement was obtained by integrating the Doppler centroid, which is proportional to the vertical wave velocity, over time to give the displacement. The simultaneous measurements of back scatter and wave displacement were then used to calculate the EM bias.

The capacitance wire wave gauge was used to measure the short wave modulation. The short wave RMS height was measured by calculating th energy in the high pass filtered wave gauge output. The wave gauge output was high pass filtered at 0.88 Hz corresponding to a two meter separation wave length (estimated using the linear deep-water dispersion relation of $\omega^2 = gk$.)

3. Results

The investigation of the effect of short wave modulation on the bias is begun by first examining the relationships between the measured RMS short wave height and the back scatter coefficients. Figure 7 shows hourly averages of the C and Ku band relative back scatter coefficients, RMS short wave height, and wind speed. A visual examination reveals the back scatter coefficients decrease as the RMS short wave height increases and vice versa. This is easily explained by noting that a rougher surface will scatter less energy in the back scatter direction. It also establishes a correlation between the back scatter coefficients at C and Ku bands and short waves with wavelengths of the order of one meter and less.

Figure 8 shows a direct comparison between the scatter coefficients and the RMS short wave height. The solid curve represents the back scatter coefficient as computed using the physical optics integral of equation (9). The circles and triangles are measured C and Ku band back scatter coefficients respectively. An absolute calibration of the scatterometers was not possible because of a slow drift in the RF electronics. Therefore, the measured back scatter coefficients have been adjusted by constant gains so as to fit the physical optics integral curve. The bias does not depend on a constant gain difference in the back scatter power. It depends only on the relative relationship between the back scattered power and the wave displacement, because, as seen in equation (1), the bias is normalized by the mean back scattered power. Figure 8 shows a clear relationship between the back scatter coefficients and the RMS short wave height. The physical optics integral accurately represents the relative relationship between the Ku band back scatter coefficient and the short wave RMS height. The measured C band back scatter coefficient versus RMS short wave height has a slightly smaller slope, especially for smaller RMS short wave heights, than the physical optics integral estimate.

Now that a relationship between scatter coefficient and RMS short wave hight has been established, the effect of short wave modulation will be investigated. We begin by examining in detail one ten minute data record. A thirty second time series of measured wave displacement is shown in figure 9, along with the envelope of the short waves (frequency > 0.88 Hz). A visual inspection shows the short wave amplitude being modulated by the long wave displacement. The short waves are clearly larger at the crests of the long waves than in the troughs. The modulation can also be seen by looking at the RMS short wave height versus the long wave displacement as in figure 10. It is seen that within two standard deviations of the mean sea level, the modulation appears to be linear with wave displacement.

The relative back scatter coefficient profile can be estimated from the short wave modulation profile by using the physical optics integral of equation (9) and the short wave model of section 1.1. The estimated and measured relative back scatter coefficient profiles for C and Ku bands are shown in figure 11. The estimated profiles show that the short wave modulation correctly predicts more scatter from the troughs of the long waves than the crests, which is in good agreement with the measured profiles.

As noted above, the short wave modulation can be represented by a linear profile as given in equation (18). This allowed the modulation strength, defined as $M = m/4$ where m is defined in equation (18), to be computed for each remaining record for the 7 days of the experiment. The modulation strength is measured using the wire wave gauge and is measured independently from the scatterometer measurements. Hourly averages of the modulation strength and normalized bias are shown in figure 12. It is easily observed that the normalized bias increases as the modulation strength increases. The bias is given by equation (22) in terms of the short wave modulation, which can be rewritten in terms of the significant wave height as

$$\epsilon - \alpha M H_{\frac{1}{3}} \qquad (24)$$

This shows the normalized bias is proportional to the modulation strength as observed in figure 12.

As seen in figure 12, the normalized bias and modulation strength are correlated with wind speed. However, at low wind speeds near days 3 and 7, the modulation strength has sudden drops, and the corresponding short wave modulation profiles become more random. This probably indicates that, at low wind speeds, the dominant scatterers are waves with lengths less than one meter. The cause of the differences between the bias and modulation strength at the beginning of day one is not known.

The C band bias versus modulation strength is shown in figure 13. The normalized bias appears to be a linear function of the modulation strength as predicted by equations (22) and (29). The constant α_C for 1.7 cm global RMS short wave height (the average, short wave RMS height for the 7 days of data) is 1.47 and 1.23 for $p = 2.5$ and 3.0 respectively. The lines in figure 13 show the ideal dependence of the bias on modulation strength according to equation (29) and the stated values of α_C. Figure 14 shows the Ku band bias versus the modulation strength. The lines in figure 14 correspond to $\alpha_{\mathrm{Ku}} =$ 1.39 and 1.15 for $p = 2.5$ and 3.0 respectively. Some of the scatter in figures 13 and 14 is due to the sudden drops in the modulation strength at low wind speeds noted earlier.

Hourly averages of significant wave height, wind speed and measured and estimated bias are shown in figure 15. The estimated biases were computed from equation (29), equation (23), and the measured modulation strength. The estimated bias is in good agreement with the measured bias except near the beginning of days 1, 3, and 7, which is due to the difference in modulation strength as noted earlier.

A more direct comparison of the estimated and measured bias is shown in figure 16. The Ku bias is underestimated for low values of bias. This is at least partly due to the low modulation strength at low wind speeds as noted earlier.

The C band versus Ku band bias is shown in figure 17. As found earlier in chapter 2, the Ku band bias is larger than the C band bias for small values of bias and smaller for large values of bias. The biases, as predicted from the modulation strength, show the C band bias to be slightly larger than the Ku bias. The differences between the measured and predicated biases are partly explained by figure 8. The parameter α is the local slope of σ^0 versus $\sigma_m k$. The Ku band data is accurately represented by the physical optics integral of equation (9) indicated by the solid line, but the C band data has a smaller slope. This causes the physical optics scattering theory and the short wave modulation model to overestimate the C band bias at small short wave heights which correspond to small biases.

4. Discussion

The validity of using physical optics scattering for the employed short wave spectrum was established by comparison with method of moment calculations as shown in figures 3 and 4. The relationship between the high frequency wave energy and the back scatter coefficient as described by equation (9) was established by the results shown in figures 7 and 8. This led to the most fundamental result of this research, namely, the prediction of the electromagnetic bias based on the modulation of the short waves.

The effect of the short wave modulation on the bias is shown in figures 9 thru 12. These results show the short wave modulation to be the dominant cause of the electromagnetic bias at C and Ku bands for moderate wind and wave conditions. The results of figures 13 and 14 show a linear dependence of the normalized bias on short wave modulation strength. The observed linear dependence is described by the theoretical bias of equation (29).

The EM bias at C and Ku bands was found to depend on wave height and wind speed by *Melville et al.* [1991] and *Walsh et al.* [1991], where the dependence was found empirically to be of the form

$$\frac{\epsilon}{H_{\frac{1}{3}}} = a_0 + a_1 U + \cdots \tag{25}$$

The results of this paper show the cause of these observed dependencies. From equation (29) the normalized bias is given by

$$\frac{\epsilon}{H_{\frac{1}{3}}} = -\alpha M \tag{26}$$

As indicated in figure 12, the short wave modulation strength has the same dependence on wind speed as the observed normalized bias. Thus, the dependence of the bias on wave height and wind speed can be attributed to the short wave modulation.

The frequency dependence of the EM bias has only been partially addressed. A better explanation lies in a better short wave modulation model. The employed short wave modulation model contains no dependence on wave number. This is clearly an oversimplification. For example, consider the measurement on EM bias at ultraviolet by *Walsh et al.* [1989]. They found the UV EM bias to be biased above mean sea level rather than below, as in the case of microwave frequencies. This implies that the modulation of short capillary waves is opposite in sign to the modulation of short gravity waves. The much smaller Ka band bias as measured by *Walsh et al.* [1989, 1991] might also imply a decrease in modulation with increasing wave number, causing a larger decrease in bias with increasing electromagnetic frequency.

Other limitations of this work include the spectral model employed, the neglect of long wave tilt and curvature and the unidirectional wave assumption. Figures 13 and 14 show the sensitivity of the bias to the choice of the power spectrum exponent in k^{-p}. The unidirectional wave assumption could cause an underestimation of the bias. In the high frequency limit σ° is inversely proportional to the standard deviation and variance of the surface height respectively for a unidirectional and isotropic surface. For small modulation strengths this will cause the bias for an isotropic surface to have twice the bias as a unidirectional surface.

In light of the results presented in this paper two concerns are raised in the theoretical EM bias papers of *Jackson* [1979], *Huang* [1984], *Barrick and Lipa* [1985] and *Srokosz* [1986]. First, the spectral filter function used by those studies is of questionable validity. The common practice of equally weighting waves having wave number less than the electromagnetic wave number, may lead to overemphasizing the high frequency waves as indicated in figure 4. Second, since these theories depend on the joint height slope probability density function, the development of this function should include the effect of the short wave modulation.

The are several differences between this work and the theoretical bias paper of *Rodriguez et al.* [1992]. First and most important is the difference between the measured dependence of the bias on short wave modulation and that predicted by *Rodriguez et al.* Figure 12, 13, and 14 of this work show that the observed normalized bias increases with short wave modulation strength. However, figures 4 and 9d of *Rodriguez et al.,* show the normalized bias remaining the same or decreasing with increasing short wave modulation strength.

The conclusion of *Rodriguez et al.,* concerning the cause of the wind speed and frequency dependence of the bias, is also different from this work. They found the wind speed dependence to be due to the increased modulation of large surface tilt as a function of wind speed, and the frequency dependence of the EM bias was explained in terms of the sensitivity of radar cross section to surface tilt and the modulation of tilt variance. This work has explained both the wind speed and frequency dependence of the bias in terms of the short wave modulation. The experimental evidence shown indicates that the model presented in this work gives better agreement with measured data at C and Ku bands than does the model of *Rodriguez et al.* However, a combination of the two theories would likely lead to better result.

Appendix A: Correlation Coefficient

For a surface spectrum given by

$$S_p(k,\eta) \;=\; \begin{cases} (p-1)\sigma_s^2(\eta)k_s^{p-1}k^{-p} & k \geq k_s = \frac{2\pi}{L} \\ 0 & k > k_s \end{cases} \tag{A.1}$$

for p greater than one, the correlation coefficient is given by

$$C_p(x) = \int_{k_s}^{\infty} dk(p-1)k_s^{p-1}k^{-p}\cos kx \tag{A.2}$$

Making a change of variable gives the correlation coefficient as

$$C_p(z) = (p-1)z^{p-1}\int_{z}^{\infty} du\; u^{-p}\cos u \tag{A.3}$$

where $z = k_s x$. The last equation can be written in terms of the incomplete gamma function

$$\gamma(a,x) = \int_0^x dt\; e^{-t} t^{a-1} \tag{A.4}$$

as

$$C_p(z) = (p-1)\cos[\frac{\pi}{2}(p-1)]\Gamma(1-p) - \frac{1}{2}(p-1)z^{p-1}[i^{1-p}\gamma(1-p,-iz)+(-i)^{1-p}\gamma(1-p,iz)] \tag{A.5}$$

Expanding the incomplete gamma function about $x = 0$ gives

$$\gamma(a,x) = x^a \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!(a+n)} \tag{A.6}$$

Substituting the last equation into equation (A.5) gives an expression for the correlation function for small argument as

$$C_p(z) = 1 + (p-1)\cos[\frac{\pi}{2}(p-1)\Gamma(1-p)|z|^{p-1} + (1-p)\sum_{m=1}^{\infty} \frac{(-1)^m z^{2m}}{(2m-p+1)(2m)!} \tag{A.7}$$

The last expression is valid for fractional p. For p equal to an even integer, the appropriate limit of the cosine and gamma term be taken, yielding

$$C_{2n}(z) = 1 + \frac{(-1)^n \pi |z|^{2n-1}}{2(2n-2)!} + (1-2n)\sum_{m=1}^{\infty} \frac{(-1)^m z^{2m}}{(2m-2n+1)(2m)!} \tag{A.8}$$

For p equal to an odd integer, equation (A.3) can be expressed in terms of an exponential integral [*Abramowitz and Steagen*, 1975] and represented in terms of its series expansion about zero as

$$C_{2n+1}(z) = 1 + \frac{(-1)^n z^{2n}}{(2n-1)!}[-\gamma + \sum_{k=1}^{2n} \frac{1}{k} - 1n|z|] - \sum_{m=1, m\neq n}^{\infty} \frac{n(-1)^m z^{2m}}{(m-n)(2m)!} \tag{A.9}$$

For large argument, equation (A.3) can be integrated using integration by parts, giving

$$C_p(z) = -(p-1)\frac{\sin z}{z}\left[1 - \frac{p(p+1)}{z^2} + \cdots\right] + (p-1)\frac{\cos z}{z}\left[\frac{p}{z} - \frac{p(p+1)(p+2)}{z^3} + \cdots\right] \tag{A.10}$$

Appendix B: Physical Optics Formulation

Physical optics is described by *Beckman and Spizzichino* [1963], *Tsang et al.* [1985], *Kong* [1986] and *Holliday et al.* [1986], but since the derivation is short it is given here for application to a unidirectional surface and to illustrate the approximations that are made. For a perfectly conducting surface, the back scattered electric field at normal incidence from and L by L patch on a unidirectional surface ($\delta\eta/\delta y = 0$) is found from the electric surface current K in terms of a Green's function as

$$\overline{E}(r) = ik\eta_0 L \int_{-L/2}^{L/2} dx' \Big(1 + \Big[\frac{\partial\eta(x')}{\partial x}\Big]^2\Big)^{\frac{1}{2}} g(r,x')\overline{K}(x') \qquad (B.1)$$

The far filed ($r \gg x'$) Green's function is giving by

$$g(r,x') = e^{-ikz'} \frac{e^{ikr}}{4\pi r}\Big|_{z'=\eta(x')} \qquad (B.2)$$

The electromagnetic wave number is given by k, the surface displacement by η and the impedence of free space by η_0. The physical optics or tangent plane approximation is used to estimate the surface current as

$$\overline{K}(x') = 2\hat{n} \times \overline{H}_i = \frac{2E_0}{\eta_0}\Big(1 + \Big[\frac{\partial\eta(x')}{\partial x}\Big]^2\Big)^{-\frac{1}{2}} e^{-ik\eta(x')} \qquad (B.3)$$

The unit vector normal to the surface is given by n, and the incident electric and magnetic fields are given by E_0 and H_i. For this approximation to provide an accurate estimate of the surface current, the surface must be sufficiently smooth. A criteria for the surface smoothness is given by equation 10.

The back scatter coefficient is given by

$$\sigma^2 = \frac{1}{L^2}\langle\frac{4\pi r^2 \overline{E}\cdot\overline{E}^*}{E_0^2}\rangle = \frac{k^2}{\pi}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} dx dx' \langle e^{i2k[\eta(x)-\eta(x')]}\rangle \qquad (B.4)$$

Assuming the surface displacement has a Gaussian probability density function, the average term can be expressed in terms of the surface correlation function and variance σ^2 as [*Tsang et al.,* 1985, p. 79 or *Kong,* 1986, p. 535]

$$\langle e^{i2k[\eta(x)-\eta(x')]}\rangle = e^{-4\sigma^2k^2[(1-C(x-x')]} \qquad (B.5)$$

The double integral can be reduced to a single integral as

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} dx dx' f(x-x') \;=\; L\int_{-L}^{L} dx\,(1-|\alpha|/L)f(\alpha) \qquad (B.6)$$

The back scatter coefficient is given by

$$\sigma^0 = \frac{k^2 L}{\pi}\int_{-L}^{L} dx(1-|x|/L)\, e^{-4\sigma^2 k^2[1-C(x)]} \qquad (B.7)$$

By making the integration variable nondimensional and rearranging, the back scatter coefficient is given by

$$\sigma^0 = (\frac{k^2 L^2}{\pi})\int_{-1}^{1} du(1-|u|)\, e^{-4\sigma^2 k^2[1-C(Lu)]} \qquad (B.8)$$

REFERENCES

Arnold, D.V., J.A. Kong, W.K. Melville, and R.W. Stewart, "Theoretical prediction of EM bias," Proc. Progress in Electromagnetics Research Symposium, 354-355, Boston, Mass., 1989.

Arnold, D.V., W.K. Melville, and J.A. Kong, "Theoretical prediction of EM bias," Conference Proc. Oceans 90, 253-256, Washington, D.C., 1990.

Arnold, D.V., J.A. Kong, and W.K. Melville, "Physical optics prediction of EM bias," Proc. Progress in Electromagnetics Research Symposium, 169, Boston, Mass., 1991.

Axline, R.M. and A.K. Fung, "Numerical computation of scattering from a perfectly conducting rough surface," IEEE Trans. Antennas and Propag., 26, 482-488, 1978.

Banner, M.L., I.S.F. Jones, and J.C. Trinder, "Wavenumber spectra of short gravity waves," J. Fluid Mech., 198, 321-344,1989.

Barrick, D.E., "Rough surface scattering based on the specular point theory," IEEE Trans. Ant. Propag., 16, 449-454, 1968.

Barrick, D.E., "Unacceptable height correlation coefficients and the quasi-specular component in rough surface scattering," Radio Science, 5, 647-654, 1970.

Barrick, D.E. and E. Bahar, "Rough surface scattering using specular point theory," IEEE Trans. Ant. Propag., 29, 798-800, 1981.

Barrick, D.E. and B.J. Lipa, "Analysis and interpretation of altimeter sea echo," Advances in Geophysics, 27, 61-100, 1985.

Beckmann, P. and A. Spizzichino, "The scattering of electromagnetic waves from rough surfaces," Macmillan Co., New York, N.Y., 503 pp., 1963.

Born, G.H., M.A. Richards and G.W. Rosborough, "An empirical determination of the effects of sea state bias on SEASAT altimetry," J. Geophys. Res., 87, 3221-3226, 1982.

Chen, M.F. and A.K. Fung, "A numerical study of the regions of validity of the Kirchhoff and the small-perturbation rough surface scattering models," Radio Science, 23, 163-170, 1988.

Chelton, D.B. and P.J. McCabe, "A review of satellite altimeter measurement of sea surface wind speed: With a proposed new algorithm," J. Geophys. Res., 90, 4707-4720, 1985.

Choy, L.W., D.L. Hammond and E.A. Uliana, "Electromagnetic bias of 10 GHz radar altimeter measurements of MSL," Mar. Geod., 8(1-4), 297-312, 1984.

Desanto, J.A. and G.S. Brown, "Analytical techniques for multiple scattering from rough surfaces," Progress in Optics, 23, 1-62, 1986.

Donelan, M.A., J. Hamilton and W.H. Hui, "Directional spectra of wind-generated waves," Phil. Trans. R. Soc. Lond. A, 315, 509-562, 1985.

Donelan, M.A. and W.J. Pierson, Jr., "Radar scattering and equilibrium ranges in wind-generated waves with application to scatterometry," J. Geophys. Res., 92, 4971-5029, 1987.

Douglas, B.C. and R.W. Agreen, "The sea state correction for GEOS 3 and SEASAT satellite altimeter data," J. Geophys. Res., 88, 1655-1661, 1983.

Doviak, R.J. and D.S. Zrnic, "Meteorological radar signal processing," in Doppler Radar and Weather Observations, 91-120, Academic, San Diego, Calif., 1984.

Evans, D.D. and O.H. Shemdin, "An investigation of the modulation of capillary and short gravity waves in the open ocean," J. Geophys. Res., 85, 5019-5024, 1980.

Fu, L. and R. Glazman, "The effect of the degree of wave development on the sea state bias in radar altimetry measurement," J. Geophys. Res., 96, 829-834, 1991.

Fung, A.K. and R.K. Moore, "The correlation function in Kirchhoff's method of solution of scattering of waves from statistically rough surfaces," J. Geophys. Res., 71, 2939-2943, 1966.

Fung, A.K. and H.L. Chan, "On the integral for backscattering from a randomly rough surface," Proc. IEEE, 59, 1280-1281, 1971.

Fung, A.K. and H.J. Eom, "Note on the Kirchhoff rough surface solution in backscattering,' Radio Science, 16, 299-302, 1981.

Hagfors, T., "Relationship of geometric optics and autocorrelation approaches to the analysis of lunar and planetary radar," J. Geophys. Res., 71, 379-382, 1966.

Hayne, G.S. and D.W. Hancock, III, "Sea-state-related altitude errors in the SEASAT radar altimeter," J. Geophys. Res., 87, 3227-3231, 1982.

Hoge, F.E., W.B. Krabill and R.N. Swift, "The reflection of airborne UV laser pulses from the ocean," Mar. Geod., 8(1-4), 313-344, 1984.

Holliday, D., G. St-eyr and N.E. Woods, "A radar ocean imaging model for small to moderate incidence angles," Int. J. Remote Sensing, 7, 1809-1834, 1986.

Huang, N.E., S.R. Long, L.F. Bliven and C. Tung, "The non-gaussian joint probability density function of slope and elevation for a nonlinear gravity wave field," J. Geophys. Res., 89, 1961-1972, 1984.

Jackson, F.C., "The reflection of impulses from a nonlinear random sea," J. Geophys. Res., 84, 4939-4943, 1979.

Jahne, B. and K.S. Riemer, "Two-dimensional wave number spectra of small-scale water surface waves," J. Geophys. Res., 95, 11531-11546, 1990.

Jessup, A.T., "Detection and characterization of deep water wave breaking using moderate incidence angle microwave backscatter from the sea surface," Ph.D. thesis, 344pp, Joint Program in Oceanogr. and Oceanogr. Eng., Mass. Inst. of Technol./Woods Hole Oceanogr. Inst., Woods, Hole, Mass., 1990.

Jessup, A.T., W.K. Melville and W.C. Keller, "Breaking waves affecting microwave backscatter," J. Geophys. Res., 96, 20547-20569, 1991.

Kodis, R.D., "A note on the theory of scattering from an irregular surface," IEEE Trans. Ant. Propag., 14, 77-82, 1966.

Lipa, B.J. and D.E. Barrick, "Ocean surface height-slope probability density function from SEASAT altimeter echo," J. Geophys. Res., 86, 10921-10930, 1981.

Longuet-Higgins, M.S., "The effect of non-linearities on statistical distributions in the theory of sea waves," J. Fluid Mech., 17, 459-480, 1963.

Melville, W.K., D.V. Arnold, R.H. Stewart, W.C. Keller, J.A. Kong, A.T. Jessup and E. Lamarre, "Measurements of EM bias at Ku and C bands," Conf. Proc. Oceans 90, 181-186, Washington, D.C., 1990.

Melville, W.K., R.H. Stewart, W.C. Keller, J.A. Kong, D.V. Arnold, A.T. Jessup, M.R. Loewen and A.M. Slinn, "Measurements of electromagnetic bias in radar altimetry," J. Geophys. Res., 96, 4915-4924, 1991.

Nerem, R.S., B.D. Tapley and C.K. Shum, "Determination of the ocean circulation using GEOSAT altimetry," J. Geophys. Res., 95, 3163-3179, 1990.

Reece, A.M., Jr., "Modulation of short waves by long waves," Boundary Layer Meteorol., 13, 203-214, 1978.

Rodriguez, E., Y. Kim and J.M. Martin, "The effect of small-wave modulation on the electromagnetic bias," J. Geophys. Res., 97, 2379-2389, 1992.

Shemdin, O.H., H.M. Tran and S.C. Wu, "Directional measurement of short ocean waves with stereophotography," J. Geophys. Res., 93, 13891-13901, 1988.

Slinn, A., M.S. Thesis, 1990.

Srokosz, M.A., "On the joint distribution of surface elevation and slopes for a nonlinear random sea with application to radar altimetry," J. Geophys. Res., 91, 995-1006, 1986.

Tyler, G.L., "Wavelength dependence in radio-wave scattering and specular-point theory," Radio Science, 11, 83-91, 1976.

Valenzuela, G.R., "Theories for the interaction of electromagnetic and ocean waves - a review," Boundary Layer Meteorol., 13, 61-85, 1978.

Walsh, E.J., D.W. Hancock, D.E. Hines and J.E. Kenney, "Electromagnetic bias of 36 GHz radar altimeter measurements of MSL," Mar. Geod., 8, 265-296, 1984.

Walsh, E.J., F.C. Jackson, E.A. Uliana and R.N. Swift, "Observations on electromagnetic bias in radar altimeter sea surface measurements," J. Geophys. Res., 94, 14575-14584, 1989.

Walsh, E.J., F.C. Jackson, D.E. Hines, C. Piazza, L.G. Hevizi, D.J. McLaughlin, R.E. McIntosh, R.N. Swift, J.F. Scott, J.K. Yungel and E.B. Frederick, "Frequency dependence of electromagnetic bias in radar altimeter sea surface range measurements," J. Geophys. Res., 96, 20571-20583, 1991.

Yaplee, B.S., A. Shapiro, D.L. Hammond, B.D. Au and E.A. Uliana, "Nanosecond radar observations of the ocean surface from a stable platform," IEEE Trans. Geosci. Electron., 9, 170-174, 1971.

Figure Captions

Figure 1: Ku band relative back scatter coefficient as a function of displacement from mean sea level in standard deviations.

Figure 2: Short wave modulation model parameters. L is the separation wavelength corresponding to the illuminated spot size. λ_{EM} is the electromagnetic wavelength. λ_0 is the dominant ocean wave length. σ_1 is the long wave RMS height. σ_s is the local short wave RMS height. $\sigma_{3\mathrm{dB}}$ is the beamwidth of the scatterometers.

Figure 3: Back scatter coefficient as a function of $\sigma_s k$. The solid line is the numerical evaluation of the physical optics integral. The circles are physical optics scattering coefficients computed using a Monte Carlo simulation. The triangles are the exact scattering coefficients, computed using a method of moments technique.

Figure 4: The left axis and the decreasing curves show the backscatter coefficient normalized by the back scatter coefficient for an effective infinite k_{h}, as a function of k_{h}, where k_{h} is the high wave number cutoff. The right axis and the increasing curves are the ratio of the electromagnetic wavelength to the average radius of curvature.

Figure 5: Back scatter coefficient as a function of $\sigma_{\mathrm{s}} k$. The solid lines were computed numerically using the physical optics integral of equation (9). The dashed curves were computed using the asymptotic solution of equations (15)-(17).

Figure 6: α of equation (23) as a function of $\sigma_{\mathrm{m}} k$. The dashed curves are given by the asymptotic solution of equation (17).

Figure 7: Time series of relative back scatter coefficient, short wave RMS height (m) and wind speed (m/s) recorded during the 7 days of the experiment.

Figure 8: Back scatter coefficient as a function of $\sigma_{\mathrm{m}} k$. The solid line was computed numerically using the physical optics integral of equation (9). The circles and triangles are measurements made using the scatterometers for σ^0 and the wire wave gauge for σ_{m}.

Figure 9: Time series of long wave displacement and short wave envelope for a large EM bias case.

Figure 10: Short wave RMS height as a function of surface displacement from mean sea level in standard deviations for a large EM bias case.

Figure 11: Ku band (top) and C band (bottom) relative back scatter coefficient as a function of surface displacement from mean sea level in standard deviations for a large EM bias case. The solid curves were measured. The dashed curves were estimated using physical optics scattering and the measured short wave RMS height profile.

Figure 12: Time series of normalized electromagnetic bias, short wave modulation strength, and wind speed (m/s) recorded during the 7 days of the experiment.

Figure 13: C band normalized electromagnetic bias as a function of short wave modulation strength. The lines show the ideal dependence of the bias on the modulation strength (see the text for details).

Figure 14: Ku band normalized electromagnetic bias as a function of short wave modulation strength. The lines show the ideal dependence of the bias on the modulation strength (see the text for details).

Figure 15: Time series of significant wave height, wind speed, and C and Ku band biases for the 7 days of the experiment. The predicted electromagnetic bias is found using the short wave modulation strength and equations (23) and (29).

Figure 16: A comparison of the measured and predicted electromagnetic bias. The predicted bias is found using the short wave modulation strength and equations (23) and (29).

Figure 17: A comparison of the Ku and C band electromagnetic biases. The predicted bias is found using the short wave modulation strength and equation (23) and (29).