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# A GENERAL INTRODUCTION TO AEROACOUSTICS AND ATMOSPHERIC SOUND

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# A GENERAL INTRODUCTION TO AEROACOUSTICS AND ATMOSPHERIC SOUND

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## ABSTRACT

This paper uses a single unifying principle (based upon the nonlinear "momentum-flux" effects produced when different components of a motion transport different components of its momentum) to give a broad scientific background to several aspects of the interaction between airflows and atmospheric sound. First, it treats the generation of sound by airflows of many different types. These include, for example, jet-like flows involving convected turbulent motions - with the resulting aeroacoustic radiation sensitively dependent on the Mach number of convection -- and they include, as an extreme case, the supersonic "boom" (shock waves generated by a supersonically convected flow pattern). Next, the paper analyses sound propagation through nonuniformly moving airflows, and quantifies the exchange of energy between flow and sound; while, finally, it turns to problems of how sound waves "on their own" may generate the airflows known as acoustic streaming.

## 1. Broad Overview

This general introductory paper is devoted to Interactions of Sound with Air, including transmission through the atmosphere and

$$\left.\begin{array}{l}\text{both generation of sound by}\\\text{and propagation of sound in}\end{array}\right\}\text{airflows}$$

(e.g., manmade flows – around aircraft or air machinery – or natural winds) as affected by the air's boundaries and atmospheric composition; with (conversely) generation of airflows by sound (acoustic streaming).

From linear acoustics I utilize the properties of the wave equation, including

(i) the short-wavelength ray-acoustics approximation (Lighthill, 1978b, hereinafter denoted WF, pp. 67-) and

(ii) the theory of multipole sources (WF, pp. 31-) – with the long-wavelength "compact-source" approximation (source region of size $\ell$ with $\omega\ell/c$ small, where $\omega$ = radian frequency, radiates like a concentrated source);

while from nonlinear acoustics I use (WF, pp. 150-) the physics of waveform shearing and shock formation.

Techniques special to aeroacoustics and atmospheric sound are centered on the momentum equation for air. Its difference from a wave-equation approximation include

A. Linear effects, of gravity acting on air stratified as meteorologists observe; effects which allow independent propagation (WF, pp. 292-) of "internal" gravity waves and of sound, except at wavelengths of many kilometers when the atmosphere becomes a waveguide (WF, pp. 425-) for global propagation of interactive acoustic-gravity waves;

and (still more importantly) include

B. Nonlinear effects, of the momentum flux $\rho u_i u_j$; i.e. the flux – rate of transport across unit area – of any $\rho u_i$ momentum component by any $u_j$ velocity component. This term, neglected in linear acoustics, acts like a stress (i.e. force per unit area – since rate of change of momentum is force). In particular,

(i) an airflow's momentum flux $\rho u_i u_j$ generates sound like a distribution of (time-varying) imposed stresses; thus not only do forces between the airflow and its boundary radiate sound as distributed dipoles, ($\rightarrow \mathbf{F} \simeq - + \mathbf{F}$) but also such stresses (acting on fluid elements with equal and opposite dipole-like forces) radiate (WF, pp. 63-) as distributed quadrupoles; ($\rightarrow\leftarrow \simeq - + + -$) (Lighthill, 1952 and Lighthill, 1962)

(ii) the mean momentum flux $< \rho u_i u_j >$ in any sound waves propagating through a sheared flow (with shear $\partial V_i/\partial x_j$) is a stress on that flow (Lighthill, 1972 and WF, pp. 329-), and the consequent energy exchange (from sound to flow when positive, vice versa when negative) is

$$< \rho u_i u_j > \partial V_i/\partial x_j; \tag{1}$$

(iii) even without any pre-existing flow, energy-flux attenuation in a sound wave allows streaming to be generated (WF, pp. 337-) by unbalanced stresses due to a corresponding attenuation in acoustic momentum flux – essentially, then, as acoustic energy flux is dissipated into heat, any associated acoustic momentum flux is transformed into a mean motion (Lighthill, 1978a and WF).

And another (less crucial) momentum-equation/wave-equation difference is

C. Nonlinear deviation of pressure excess $p - p_0$ from a constant multiple, $c_0^2(\rho - \rho_0)$, of density excess.

(i) For sound generation by airflows, this adds an isentropic term to the quadrupole strength per unit volume (WF, pp. 60-)

$$T_{ij} = \rho u_i u_j + \left[(p - p_0) - c_0^2(\rho - \rho_0)\right]\delta_{ij}, \tag{2}$$

the last term being considered important mainly for flows at above-ambient temperatures (Lighthill, 1952 and Lighthill, 1963);

(ii) for propagation of sound with energy density $E$ through flows of air with adiabatic index $\gamma$, the mean deviation is about $\left(\frac{1}{2}\right)(\gamma - 1)E$, and the total radiation stress (Bretherton and Garrett, 1968)

$$< \rho u_i u_j > +\frac{1}{2}(\gamma - 1)E\delta_{ij} \tag{3}$$

adds an isotropic pressure excess to the mean momentum flux (although the energy exchange (1) is unchanged in typical cases with $\partial V_i/\partial x_i$ essentially zero).

(And we note that the very special case of sound waves interacting on themselves [in other words, nonlinear acoustics] may be interpreted (WF, p. 148) as a combined operation of the "self-convection effect" B and the (smaller) "sound-speed deviation" C.)

## 2. Compact Source Regions

### 2.1. <u>Sound generation by low-Mach-number airflows</u>

The main nondimensional parameters governing airflows of characteristic speed $U$ and length-scale $L$ are Mach number $M = U/c$ (where, in aeroacoustics, $c$ is taken as the sound speed in the atmosphere into which sound radiates) and Reynolds number $R = UL/\nu$ where kinematic viscosity. Low-Mach-number airflows are compact sources of sound, with frequencies

$$\left.\begin{array}{c} \text{narrow-banded} \\ \text{at moderate } R \\ \text{broad-banded} \\ \text{at high } R \end{array}\right\} \begin{array}{c} \text{when flow} \\ \text{instabilities} \\ \text{lead to} \end{array} \left\{\begin{array}{l} \text{regular flow} \\ \quad \text{oscillations;} \\ \text{extremely irregular} \\ \quad \text{turbulence.} \end{array}\right.$$

Since, in either case, a typical frequency $\omega$ scales as $U/L$ (Strouhal scaling), the compactness condition $\omega L/c$ small is satisfied if $M = U/c$ is small (Lighthill, 1962).

A solid body which, because of flow instability, is subjected to a fluctuating aerodynamic force $F$ scaling as $\rho U^2 L^2$ (at frequencies scaling as $U/L$), radiates as an acoustic dipole of strength $F$, with mean radiated power $< \dot{F}^2 > /12\pi\rho c^3$.

This acoustic power scales as $\rho U^6 L^2 / c^3$ (a sixth-power dependence on flow speed). Therefore acoustic efficiency, defined as the ratio of acoustic power to a rate of delivery (scaling as $\rho U^3 L^2$) of energy to the flow, scales as $(U/c)^3 = M^3$.

(Exceptions to compactness include bodies of high aspect-ratio; thus, a long wire in a wind [where the scale $L$ determining frequency is its diameter] radiates as a lengthwise distribution of dipoles.)

Away from any solid body a compact flow (oscillating or turbulent, with frequencies scaling as $U/L$) leads to quadrupole radiation (see B(i) above) with total quadrupole strength scaling as $\rho U^2 L^3$. Acoustic power then scales as $\rho U^8 L^2 / c^5$: an eighth-power dependence (Lighthill, 1952 and Lighthill, 1962) on flow speed. In this case acoustic efficiency (see above) scales as $(U/c)^5 = M^5$.

Such quadrupole radiation, though often important, may become negligible near a solid body when dipole radiation due to fluctuating body force (with its sixth-power dependence) is also present (Curle, 1955 and Ffowcs Williams and Hawkings, 1969).

Near not necessarily compact bodies a more refined calculation – using Green's functions not for free space but for internally bounded space – leads in general to the same conclusion: that quadrupole radiation with its eighth-power dependence is negligible alongside the sixth-power dependence of dipole radiation due to fluctuating body forces; but important exceptions to this rule include sharp-edged bodies, where features of the relevant Green's function imply a fifth-power dependence on flow speed of acoustic radiation from turbulence

(Ffowcs Williams and Hall, 1970; Crighton and Leppington, 1970; Crighton and Leppington, 1971; and Crighton, 1981).

## 2.2. <u>Sound generation by turbulence at not so low Mach number</u>

The chaotic character of turbulent flow fields implies that velocity fluctuations at points $P$ and $Q$, although they are well correlated when $P$ and $Q$ are very close, become almost uncorrelated when $P$ and $Q$ are not close to one another.

**Reminder:** statisticians define correlation coefficient $C$ for the velocities $\mathbf{u}_P$ and $\mathbf{u}_Q$ as $C = <\mathbf{v}_P \cdot \mathbf{v}_Q> / <\mathbf{v}_P^2>^{1/2} <\mathbf{v}_Q>^{1/2}$ in terms of the deviations, $\mathbf{v}_P = \mathbf{u}_P - <\mathbf{u}_P>$ and $\mathbf{v}_Q = \mathbf{u}_Q - <\mathbf{u}_Q>$, from their means. When two uncorrelated quantities are combined, their mean square deviations are added up:

$$< \mathbf{v}_P + \mathbf{v}_Q >^2 = <\mathbf{v}_P^2> + 2 <\mathbf{v}_P \cdot \mathbf{v}_Q> + <\mathbf{v}_Q^2>$$

$$= <\mathbf{v}_P^2> + <\mathbf{v}_Q^2> \text{ if } C = 0.$$

Theories of turbulence define a correlation length $\ell$, with

$$\mathbf{u}_P \text{ and } \mathbf{u}_Q \left\{ \begin{array}{c} \text{well correlated} \\ \text{uncorrelated} \end{array} \right\} - -C \text{ close to } \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\} - -$$

$$\text{when } PQ \text{ is substantially} \left\{ \begin{array}{c} < \ell \\ > \ell \end{array} \right\}.$$

Roughly speaking, different regions of size $\ell$ ("eddies") generate sound independently, and the mean square radiated noise is the sum of the mean square outputs from all the regions (Lighthill, 1954). Typical frequencies in the turbulence are of order $\omega = v/\ell$, where $v$ is a typical root mean square velocity deviation $<\mathbf{v}^2>^{1/2}$, so that for each region the compactness condition $\omega\ell/c$ small is satisfied if $v/c$ is small. Compactness, then, requires only that a r.m.s. velocity deviation $v$ (rather than a characteristic mean velocity $U$) be small compared with $c$ – which is less of a restriction on $M = U/c$ and can be satisfied at "not so low" Mach number (Lighthill, 1963).

## 3. Doppler Effect

How is the radiation from such "eddies" modified by the fact that they are being convected at "not so low" Mach number? The expression Doppler effect, covering all aspects of how the movement of sources of sound alters their radiation patterns, comprises (i) frequency changes (WF), (ii) volume changes (Lighthill, 1952 and Lighthill, 1962), (iii) compactness changes (Lighthill, 1963 and Ffowcs Williams, 1963).

## 3.1. Frequency changes

When a source of sound at frequency $\omega$ approaches an observer at velocity $w$, then in a single period $T = 2\pi/\omega$ sound emitted at the beginning travels a distance $cT$ while at the end of the period sound is being emitted from a source that is closer by a distance $wT$. The wavelength $\lambda$ (distance between crests) is reduced to

$$\lambda = cT - wT = 2\pi(c - w)/\omega \tag{4}$$

(Figure 1) and the frequency heard by the observer ($2\pi$ divided by the time $\lambda/c$ between arrival of crests) is increased to the Doppler-shifted value (WF)

$$\omega_r = \frac{\omega}{1 - (w/c)} : \text{the relative frequency} \tag{5}$$

that results from relative motion between source and observer. For an observer located on a line making an angle $\theta$ with a source's direction of motion at speed $V$, the source's velocity of approach towards the observer is $w = V\cos\theta$ (Figure 1) and the relative frequency becomes

$$\omega_r = \frac{\omega}{1 - (V/c)\cos\theta} : \left\{ \begin{array}{c} \text{augmented} \\ \text{diminished} \end{array} \right\} \text{ when } \theta \text{ is an } \left\{ \begin{array}{c} \text{acute} \\ \text{obtuse} \end{array} \right\} \text{angle.} \tag{6}$$

Such Doppler shifts in frequency are familiar everyday experiences.

## 3.2. Volume changes

When an observer is approached at velocity $w$ by a source whose dimension (in the direction of the observer) is $\ell$, sounds arriving simultaneously (Figure 2) from the source's $\left\{ \begin{array}{c} \text{far} \\ \text{near} \end{array} \right\}$ sides have been emitted $\left\{ \begin{array}{c} \text{earlier} \\ \text{later} \end{array} \right\}$ by a time $\tau$ (say).

In the time $t$ for sound from the far side to reach the observer, after travelling a distance $ct$, the relative distance of the near side in the direction of the observer was increased from $\ell$ to $\ell + w\tau$ before it emitted sound which then travelled a distance $c(t - \tau)$. Both sounds arrive simultaneously if

$$ct = \ell + w\tau + c(t - \tau), \text{ giving } \tau = \frac{\ell}{c - w}$$

$$\text{and } \ell + w\tau = \frac{\ell}{1 - (w/c)} = \ell\omega_r/\omega. \tag{7}$$

The source's effective volume during emission is increased, then, by the Doppler factor $\omega_r/\omega$ (since dimension in the direction of the observer is so increased whilst other dimensions are unaltered: Lighthill, 1952 and Lighthill, 1962).

If turbulent "eddies" are effectively being convected, relative to the air into which they are radiating, at velocity $V$, then Equation (6) gives, for radiation at angle $\theta$, the Doppler factor $\omega_r/\omega$ which modifies both the frequencies at which they radiate and the effective volume occupied by a radiating eddy.

But Equation (2) specifies the quadrupole strength $T_{ij}$ per unit volume for such an eddy. Without convection the pattern of acoustic intensity around a compact eddy of volume $\ell^3$ and quadrupole strength $\ell^3 T_{ij}$ would be

$$< (\ell^3 \ddot{T}_{ij} x_i x_j r^{-2})^2 > /16\pi^2 r^2 \rho_0 c^5; \tag{8}$$

and, since different eddies of volume $\ell^3$ radiate independently, we can simply add up mean squares in the corresponding expressions for their far-field intensities. This gives

$$\ell^3 < (\ddot{T}_{ij} x_i x_j r^{-2})^2 > /16\pi^2 r^2 \rho_0 c^5 \tag{9}$$

as the intensity pattern radiated by unit volume of turbulence. The Doppler effect modifies this, when the compactness condition is satisfied, by five factors $\omega_r/\omega$ (one for the change in source volume $\ell^3$ and four for the frequency change as it affects the mean square of a multiple of the second time-derivative of $T_{ij}$) and this intensity modification by a factor

$$[1 - (V/c)\cos\theta]^{-5} \tag{10}$$

brings about an important preference for forward emission (Ffowcs Williams, 1963).

### 3.3. Compactness changes

As $(V/c)$ increases, however, the Doppler effect tends to degrade the compactness of aeroacoustic sources in relation to forward emission. Not only does $\omega\ell/c$ increase in proportion to Mach number, but an even greater value is taken by $\omega_r \ell/c$, the ratio which must be small if convected sources are to be compact. A restriction on the extent (10) of intensity enhancement for forward emission as $V/c$ increases is placed by these tendencies (Lighthill, 1963; Ffowcs Williams, 1963; and Dowling et al., 1978).

They can develop, indeed, to a point where the compact-source approximation may appropriately be replaced by its opposite extreme: the ray-acoustics approximation (WF). Thus, for supersonic source convection ($V/c > 1$), the relative frequency (6) becomes infinite in

$$\text{the Mach direction } \theta = \cos^{-1}(c/V), \tag{11}$$

and radiation from the source proceeds (Figure 3) along rays emitted at this angle (Ffowcs Williams and Maidanik, 1965).

**Explanatory note**: the source's velocity of approach $w$ towards an observer positioned at an angle (11) to its direction of motion is the sound speed $c$; thus, not only is the generated wavelength (4) reduced indefinitely (the ray-acoustics limit) but, essentially, different parts of a signal are observed simultaneously: the condition of stationary phase satisfied on rays (WF). Sounds emitted (WF, p. 196) by a source approaching an observer at a speed $w$ exceeding $c$ are heard by him in reverse order ("pap pep pip pop pup" becomes "pup pop pip pep pap"!) but when $w = c$ all the sound (vowels and consonants!) are heard together as one single "boom."

**Further note**: the influences placing a limit on the signal propagated along rays may include the duration $\delta$ of well-correlated emission from turbulent "eddies"; and, also, may include nonlinear effects (see §4.2. Supersonic booms).

### 3.4. Uniformly valid Doppler-effect approximations

Just as a correlation length $\ell$ for turbulence was specified in §2.2, so a correlation duration $\delta$ can be characterized by the requirement that moving eddies have $\left\{ \begin{array}{c} \text{well correlated} \\ \text{uncorrelated} \end{array} \right\}$ velocities at times differing by substantially $\left\{ \begin{array}{c} < \delta \\ > \delta \end{array} \right\}$. Combined use of correlation length $\ell$ and duration $\delta$ affords an approximation to the radiation pattern from convected "eddies" that has some value at all Mach numbers, spanning the areas of applicability of the compact-source and ray-acoustics approximations.

Figure 4 uses space-time diagrams where the space-coordinate (abscissa) is distance in the direction of the observer. Diagram (a) for unconvected "eddies" approximates the region of good correlation as an ellipse with axes $\ell$ (in the space direction) and $\delta$ (in the time direction). Diagram (b) shows such a region for convected "eddies" whose velocity of approach towards the observer is $w$; thus, it is Diagram (a) sheared by distance $w$ per unit time.

Signals from far points $F$ and near points $N$, in either case, reach the observer simultaneously – as do signals from other points on the line $FN$ – if this line slopes by distance $c$ (the sound speed) per unit time.

Compact-source case (i) with $w/c$ small: the space component of FN in Diagram (b) is $\ell[1 - (w/c)]^{-1}$, just as in Equation (7) for normal Doppler effect (neglecting finite $\delta$).

Ray-acoustics case (ii) with $w/c = 1$: the space component of $FN$ is $c\delta$.

Intermediate case (iii) with $w/c$ "moderately" $< 1$: the space component of $FN$ is $\ell$ multiplied by an enhancement factor

$$[(1 - w/c)^2 + (\ell/c\delta)^2]^{-1/2} \tag{12}$$

which represents the effective augmentation of source volume due to convection (Ffowcs

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Williams, 1963).

This enhancement factor (12) is applied not only to the volume term $\ell^3$ in the quadrupole field (9) but also twice to each of the pair of twice-differentiated terms inside the mean square; essentially, because time-differentiations in quadrupole fields arise (WF) from differences in the time of emission by different parts of the quadrupole source region (and the time component of FN in Diagram (b) is simply the space component divided by $c$). As before, then, five separate factors (12) enhance the intensity field; and, with $w$ replaced by $V\cos\theta$, expression (10) for the overall intensity modification factor is replaced by

$$\{[1 - (V/c)\cos\theta]^2 + (\ell/c\delta)^2\}^{-5/2}. \tag{13}$$

This modification factor (13) affords us an improved description of the influence of Doppler effect not only on the preference for forward emission but also on the overall acoustic power output from convected turbulence (Lighthill, 1963 and Ffowcs Williams, 1963). For example, Diagram (c) gives (plain line) a log-log plot of the average (spherical mean) of (13) as a function of $V/c$ on the reasonable assumption that $\ell = 0.6V\delta$. As $V/c$ increases this average modification factor rises a little at first, but falls drastically like $5(V/c)^{-5}$ for $V/c$ significantly $> 1$.

Now low-Mach-number turbulence away from solid boundaries (§2.1) should radiate sound with an acoustic efficiency scaling as $(U/c)^5$ where $U$ is a characteristic velocity in the flow. With $V$ taken as that characteristic velocity (although in a jet a typical velocity $V$ of eddy convection would be between 0.5 and 0.6 times the jet exit speed), the modification of (say) an acoustic efficiency of $10^{-3}(V/c)^5$ for low Mach number by the average modification factor would cause acoustic efficiency to follow the broken-line curve in Diagram (c), tending asymptotically to a constant value, 0.005, (aeroacoustic saturation) at high Mach number. Such a tendency is often observed for sound radiation from "properly expanded" supersonic jets (see below).

## 4. Introduction to Aircraft Noise

### 4.1. Aero-engine and airframe noise

How are aeroacoustic principles applied to practical problems – such as those of studying aircraft noise with a view to its reduction (Crighton, 1975; Goldstein, 1976; and Goldstein, 1984)?

In any analysis of the generation of sound by airflows, we may need first of all to ask whether the geometry of the problem has features that tend to promote resonance. For example, a long wire in a wind (§2.1) generates most sound when vortex-shedding frequencies

$\left\{ \begin{array}{c} \text{are fairly close to} \\ \text{and so can "lock on" to} \end{array} \right\}$ the wire's lowest natural frequency of vibration; giving good correlation of sideforces, and so also of dipole strengths, all along the wire.

Again, a jet emerging from a thin slit may interact with a downstream edge (parallel to the slit) in a resonant way (Curle, 1953 and Powell, 1953b); with very small directional disturbances at the jet orifice being amplified by flow instability as they move downstream to the edge, where they produce angle-of-attack variations. Dipole fields associated with the resulting sideforces can at particular frequencies renew the directional disturbances at the orifice with the right phase to produce a resonant oscillation. Some musical wind instruments utilize such jet-edge resonances, reinforced by coincidence with standing-wave resonances in an adjacent pipe.

But in the absence of such resonances (leading to enhanced acoustic generation at fairly well defined frequencies) airflows tend to generate acoustic "noise" whose reaction on the flow instability phenomena themselves is negligible.

Resonances analogous to the above which need to be avoided in aircraft design include, for example,

(a) panel flutter, generated at a characteristic frequency as an unstable vibration of a structural panel in the presence of an adjacent airflow (Dowell, 1975);

(b) screeching of supersonic jets from nozzles which, instead of being "properly expanded" so that an essentially parallel jet emerges, produce a jet in an initially non-parallel form followed by shock waves in the well known recurrent "diamond" shock-cell pattern; the first of these, replacing the edge in the above description, can through a similar feedback of disturbances to the jet orifice generate a powerful resonant oscillation (Powell, 1953a; Powell, 1953b; and Howe and Ffowcs Williams, 1978).

And undesirable resonances may also be associated with aeroengine combustion processes (Candel and Poinsot, 1988). But we turn now to the aircraft noise of a broad-banded character that remains even when resonances have been avoided.

Then aero-engine jet noise proper (Lighthill, 1963) (that is, the part unrelated to any interaction of jet turbulence with solid boundaries) tends to follow a broad trend similar to that in Figure 4; where, however, because the eddy convection velocity $V$ is between 0.5 and 0.6 times the jet exit speed $U$, the acoustic efficiency makes a transition between a value of around $10^{-4}M^5$ in order-of-magnitude terms for subsonic values of $M = U/c$ and an asymptotically constant value of $10^{-2}$ or a little less for $M$ exceeding about 2.

The above tendency for $M < 1$ implies that noise emission from jet engines may be greatly diminished if a given engine power can be achieved with a substantially lower jet

exit speed, requiring of course a correspondingly larger jet diameter, $L$. Furthermore, with acoustic power output scaling as $\rho U^8 L^2 / c^5$ (§2.1) and jet thrust as $\rho U^2 L^2$, noise emission for given thrust can be greatly reduced if $U$ can be decreased and $L$ increased by comparable factors.

Trends (along these lines) in aero-engine design towards large turbofan engines with higher and higher bypass ratios, generating very wide jets at relatively modest mean Mach numbers, have massively contributed to jet noise suppression (whilst also winning advantages of reduced fuel consumption). On the other hand, such successes in suppressing jet noise proper (originally, the main component of noise from jet aircraft) led to needs for a dedicated focusing of attention upon parallel reductions of other aircraft-noise sources (Crighton, 1972):

(a) those associated with the interaction of jet turbulence with solid boundaries — where sharp-edged boundaries (§2.1) pose a particular threat;

(b) fan noise emerging from the front of the engine and turbine noise from the rear;

(c) airframe noise including acoustic radiation from boundary-layer turbulence and from interaction of that turbulence with aerodynamic surfaces for control purposes or lift enhancement.

Some key areas of modern research on aero-engine and airframe noise are:

for jet noise, techniques for relating acoustic output to vorticity distributions (Powell, 1964 and Mohring, 1978), and to any coherent structures (Ribner, 1964 and Ffowcs Williams and Kempton, 1978), in jet turbulence; and for taking into account (cf §5.4) propagation through the sheared flow in a wide jet (Phillips, 1960 and Mani, 1976);

for noise from fans and propellers, mathematically sophisticated ways of reliably estimating the extent of cancellation of dipole radiation from different parts of a rotating-blade system (alongside a good independent estimate of quadrupole radiation: Parry and Crighton, 1991);

for airframe noise, a recognition (Powell, 1960 and Crighton, 1984) that massive cancellations act to minimize noise radiation from boundary-layer turbulence on a flat surface of uniform compliance – and, therefore, that avoidance of sharp nonuniformities in airframe skin compliance may promote noise reduction.

## 4.2. Supersonic booms

In addition to aero-engine and airframe noise, any aircraft flying at a supersonic speed $V$ emits a concentrated "boom"-like noise along rays (Figure 3) in the Mach direction (11). I sketch the theory of supersonic booms with the atmosphere approximated as isothermal

(so that the undisturbed sound speed takes a constant value $c$ even though the undisturbed density $\rho$ varies with altitude): a case permitting quite a simple extension of the nonlinear analysis of waveform shearing and shock formation (WF and Whitham, 1956). Then the rays continue as straight lines at the Mach angle for reasons summarized in the explanatory note below expression (11). (Actually, the slight refraction of rays by temperature stratification in the atmosphere, when taken into account in a generalized version of the theory, produces only somewhat minor modifications of the results.)

As such straight rays stretch out from a straight flight path along cones with semi-angle (11), any narrow tube of rays has its cross-sectional area $A$ increasing in proportion to distance $r$ along the tube (WF and Lighthill, 1956). On linear theory (WF), acoustic energy flux $u^2 \rho c A$ is propagated unchanged along such a ray tube (so that $u(\rho r)^{1/2}$ is unchanged) where $u$ is air velocity along it. On nonlinear theory, $u(\rho r)^{1/2}$ is propagated unchanged but at a signal speed altered to

$$c + \frac{\gamma + 1}{2} u \tag{14}$$

by self-convection and excess-wavespeed effects.

This property can be described (WF) by an equation

$$\left[ \left\{ \frac{1}{c} - \frac{\gamma + 1}{2} \frac{u}{c^2} \right\} \frac{\partial}{\partial t} + \frac{\partial}{\partial r} \right] u(\rho r)^{1/2} = 0 \tag{15}$$

(WF, pp. 187-) where the quantity in braces is the altered value of the reciprocal of the signal speed (14). Now a simple transformation of variables

$$x_1 = r - ct, \quad t_1 = \int_0^r (\rho r)^{-1/2} dr, \quad u_1 = \frac{\gamma + 1}{2} \frac{u}{c} (\rho r)^{1/2} \tag{16}$$

is found to convert Equation (15) into the familiar form

$$\frac{\partial u_1}{\partial t_1} + u_1 \frac{\partial u_1}{\partial x_1} = 0 \tag{17}$$

which describes the waveform shearing at a uniform rate that is associated with shock formation and propagation in nonlinear plane-wave acoustics (Figure 5)

From amongst this equation's physically relevant solutions – namely, those with area-conserving discontinuities (representing shocks) – the famous $N$-wave solution is the one produced by an initial signal (such as an aircraft's passage through the air) that is first compressive and then expansive. The rules (WF) governing $N$-wave solutions of Equation (17) are that the discontinuity $\Delta u_1$ at each shock falls off like $t_1^{-1/2}$ while the space (change $\Delta x_1$ in $x_1$) between shocks increases like $t_1^{1/2}$. These rules for the transformed variables (16) have the following consequences for the true physical variables: at a large distance $r$ from

the flight path the velocity change $\Delta u$ at each shock and the time interval $\Delta t$ between the two shocks vary as

$$\Delta u \approx \left[(\rho r) \int_0^r (\rho r)^{-1/2} dr\right]^{-1/2} \quad \text{and} \quad \Delta t \approx \left[\int_0^r (\rho r)^{-1/2} dr\right]^{1/2}. \tag{18}$$

On horizontal rays (at the level where the aircraft is flying), $\rho$ is independent of $r$ and the Equations (18) take the greatly simplified form

$$\Delta u \approx r^{-3/4} \quad \text{and} \quad \Delta t \approx r^{1/4} \tag{19}$$

appropriate to conical $N$-waves in a homogeneous atmosphere (which supersonically convected eddies may also generate). Actually, the rules (19) apply also to the propagation of cylindrical blast waves generated by an exploding wire; since, here also, ray tube areas increase in proportion to $r$.

On downward pointing rays in an isothermal atmosphere $\rho$ increases exponentially in such a way that the time interval $\Delta t$ between shocks approaches the constant value obtained in (18) by making the integral's upper limit infinite (WF and Lighthill, 1956). On the other hand the shock strength (proportional to the velocity change $\Delta u$) includes the factor $(\rho r)^{-1/2}$ where the large increase in $\rho$ from the flight path to the ground (as well as in $r$) enormously attenuates the supersonic boom. Below Concorde cruising at Mach 2, for example, an observer on the ground hears two clear shocks with an interval of around 0.5s between them, and yet with strengths $\Delta p/p$ only about 0.001.

## 5. Propagation of Sound through Steady Mean Flows

### 5.1. Adaptations of ray acoustics

Useful information on sound propagation (including sound of aeroacoustic origin) through steady mean flows (Blokhintsev, 1956 and Lighthill, 1972) can be obtained by adaptations of the ray-acoustics approximation. I sketch these here before, first, applying them (in §5.3 below) to propagation through sheared stratified winds and, secondly, giving indications of how effects of such parallel mean flows are modified at wavelengths too large for the applicability of ray acoustics.

Sound propagation through a steady airflow represents an autonomous mechanical system: one governed by laws that do not change with time. Then small disturbances can be Fourier-analyzed in the knowledge that propagation of signals with different frequencies $\omega$ must proceed without exchange of energy between them.

# General Theory of Such Systems (WF, pp. 317-)

Such disturbances of frequency $\omega$ involve pressure changes in the form $P\cos\alpha$ where $P$ varies with position and the phase $\alpha$ is a function of position and time satisfying

$$\frac{\partial\alpha}{\partial t} = \omega : \text{ the frequency; and } -\frac{\partial\alpha}{\partial x_i} = k_i : \text{ the wavenumber,} \tag{20}$$

a vector with its direction normal to crests and its magnitude $2\pi$ divided by a local wavelength.

In ray theory for any wave system (WF and Lighthill, 1972), we assume that the wavelength is small enough (compared with distances over which the medium – and its motion, if any – change significantly) for a well-defined relationship

$$\omega = \Omega(k_i, x_i) \tag{21}$$

to link frequency with wavenumber at each position. Equations (20) and (21) require that

$$-\frac{\partial k_j}{\partial t} = \frac{\partial^2\alpha}{\partial x_j \partial t} = \frac{\partial\omega}{\partial x_j} = \frac{\partial\Omega}{\partial k_i}\left(-\frac{\partial^2\alpha}{\partial x_i \partial x_j}\right) + \frac{\partial\Omega}{\partial x_j} = \frac{\partial\Omega}{\partial k_i}\frac{\partial k_j}{\partial x_i} + \frac{\partial\Omega}{\partial x_j}, \tag{22}$$

yielding the basic law (in Hamiltonian form) for any wave system:

$$\text{on rays satisfying } \frac{dx_i}{dt} = \frac{\partial\Omega}{\partial k_i} \text{ wavenumbers vary as } \frac{dk_j}{dt} = -\frac{\partial\Omega}{\partial x_j}; \tag{23}$$

equations easy to solve numerically for given initial position and wavenumber. However, the variations (23) of wavenumber ("refraction") produce no change of frequency along rays:

$$\frac{d\omega}{dt} = \frac{\partial\Omega}{\partial k_i}\frac{dk_i}{dt} + \frac{\partial\Omega}{\partial x_i}\frac{dx_i}{dt} = \frac{\partial\Omega}{\partial k_i}\left(-\frac{\partial\Omega}{\partial x_i}\right) + \frac{\partial\Omega}{\partial x_i}\frac{\partial\Omega}{\partial k_i} = 0, \tag{24}$$

so that rays are paths of propagation of the excess energy, at each frequency, associated with the waves' presence.

For sound waves we write $k$ as the magnitude of the wavenumber vector, expecting that at any point the value of the relative frequency in a frame of reference moving at the local steady flow velocity $u_{fi}$ will be $c_f k$ (the local sound speed times $k$); this implies (WF and Lighthill, 1972) that

$$\omega_r = \frac{\partial\alpha}{\partial t} + u_{fi}\frac{\partial\alpha}{\partial x_i} = \omega - u_{fi}k_i, \text{ giving } \omega = \omega_r + u_{fi}k_i = c_f k + u_f k_i \tag{25}$$

as the acoustic form of the relationship (21).

[Note: this rule (25) for relative frequency agrees with the Doppler rule (6), since the velocity of a source of frequency $\omega$ relative to stationary fluid into which it radiates is minus the velocity of the fluid relative to a frame in which the acoustic frequency is $\omega$.]

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Use of this form (25) of the relationship (21) in the basic law (23) tells us that

$$\frac{dk_j}{dt} = -k\frac{\partial c_f}{\partial x_j} - k_i\frac{\partial u_{fi}}{\partial x_j} \text{ on rays with } \frac{dx_i}{dt} = c_f\frac{k_i}{k} + u_{fi}; \tag{26}$$

where the last terms in these equations represent adaptations of ray acoustics associated with the mean flow. For example, the velocity of propagation along rays is the vector sum of the mean flow velocity $u_{fi}$ with a wave velocity of magnitude $c_f$ and direction normal to crests.

## 5.2. Energy exchange between sound waves and mean flow

The excess energy (say, $E$ per unit volume) associated with the presence of sound waves is propagated along such rays; in particular, if attenuation of sound energy is negligible, then

$$\text{flux of excess energy along a ray tube} = \text{constant.} \tag{27}$$

Note: this excess energy density $E$ is by no means identical with the sound waves' energy density

$$E_r = <\frac{1}{2}\rho_f u_{si}u_{si}> + <\frac{1}{2}c_f^2\rho_f^{-1}\rho_s^2> = c_f^2\rho_f^{-1} <\rho_s^2> \tag{28}$$

(where the subscript $s$ identifies changes due to the sound waves and the equality of the kinetic and potential energies makes $E_r$ simply twice the latter) in a frame of reference moving at the local flow velocity (compare the definition (25) of $\omega_r$). The kinetic-energy part of the excess energy density $E$ is

$$<\frac{1}{2}(\rho_f + \rho_s)(u_{fi} + u_{si})^2> - \frac{1}{2}\rho_f u_{fi}u_{fi}, \tag{29}$$

which includes an extra term

$$<\rho_s u_{fi}u_{si}> = <\rho_s u_{fi}\frac{c_f}{\rho_f}\rho_s\frac{k_i}{k}> = E_r\frac{u_{fi}k_i}{c_f k} = E_r\left(\frac{\omega}{\omega_r} - 1\right); \tag{30}$$

and $E$ is the sum of expressions (28) and (30), giving

$$E = E_r\frac{\omega}{\omega_r}; \text{ or, equivalently, either } E_r = E\frac{\omega_r}{\omega} \text{ or } \frac{E_r}{\omega_r} = \frac{E}{\omega}. \tag{31}$$

The quantity $E/\omega$, called action density in Hamiltonian mechanics, is identical in both frames of reference, and Equations (24) and (27) tell us that its flux along a ray is constant (WF and Lighthill, 1972).

But Equation (31) shows too that energy is exchanged between (i) the acoustic motions relative to the mean flow and (ii) the mean flow itself. For example, where sound waves of frequency $\omega$ enter a region of opposing flow (or leave a region where the mean flow is along

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their direction of propagation) the ratio $\omega_r/\omega$ increases and so therefore does $E_r/E$: the sound waves gain energy at the expense of the mean flow.

The rate of exchange of energy takes the value (1) written down in §1. This is readily seen from the laws governing motion in an accelerating frame of reference, which feels

$$\text{an inertial force} = -(\text{mass}) \times (\text{acceleration of frame}). \tag{32}$$

If we use at each point of space a local frame of reference moving with velocity $u_{fi}$ then fluid in that frame has velocity $u_{si}$ but is subject to an additional force (32); where, per unit volume, mass is $\rho_f$ and the frame's acceleration is

$$u_{sj}\frac{\partial u_{fi}}{\partial x_j} \text{ giving force } -\rho_f u_{sj}\frac{\partial u_{fi}}{\partial x_j} \text{ doing work } -\rho_f < u_{si}u_{sj} > \frac{\partial u_{fi}}{\partial x_j} \tag{33}$$

per unit time on the local relative motions. This rate of energy exchange (33) proves to be consistent with the fact that it is the flux, not of $E_r$ but of action $E_r/\omega_r$, that is conserved along ray tubes.

Energy can be extracted from a mean flow, then, not only by turbulence but also by sound waves; and, in both cases, the rate of extraction takes the same form (33) in terms of perturbation velocities $u_{si}$. It represents the effect (§1) of the

$$\text{mean momentum flux } \rho_f < u_{si}u_{sj} > \tag{34}$$

or Reynolds stress (Reynolds, 1895) with which either the sound waves or the turbulent motions act upon the mean flow. For sound waves, by Equation (28) for $E_r$ and by the substitution

$$u_{si} = \frac{c_f}{\rho_f}\rho_s\frac{k_i}{k}, \text{ mean momentum flux } = E_r\frac{k_i}{k_j}k^2; \tag{35}$$

so that the Reynolds stress is a uniaxial stress in the direction of the wavenumber vector having magnitude $E_r$.

[Note: strictly speaking, the complete

$$\text{radiation stress } E_r\left(\frac{k_ik_j}{k^2} + \frac{\gamma - 1}{2}\delta_{ij}\right) \text{ for sound waves} \tag{36}$$

includes not only the momentum flux (35) but also the waves' mean pressure excess

$$\frac{1}{2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{\rho=\rho_f} < \rho_s^2 > = \frac{\gamma - 1}{2}\frac{c_f^2}{\rho_f} < \rho_s^2 > = \frac{\gamma - 1}{2}E_r \tag{37}$$

acting equally in all directions (Bretherton and Garrett, 1968); however (§1) this isotropic component produces no energy exchange with solenoidal mean flows.]

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## 5.3. Propagation through sheared stratified winds

The extremely general ray-acoustics treatment outlined above for sound propagation through fluids in motion has far-reaching applications (in environmental and, also, in engineering acoustics) which, however, are illustrated below only by cases of propagation through parallel flows, with stratification of velocity as well as of temperature (WF and Lighthill, 1972). The $x_1$-direction is taken as that of the mean flow velocity $V(x_3)$ which, together with the sound speed $c(x_3)$, depends only on the coordinate $x_3$. Thus, $V$ replaces $u_{f1}$ in the general theory while $c$ replaces $c_f$ (and, for atmospheric propagation, $x_3$ is altitude). [Note: the analysis sketched here is readily extended to cases of winds veering with altitude, where $u_{f2}$ as well as $u_{f1}$ is nonzero.]

Either the basic law (23) or its ray-acoustics form (26) provides, in general, "refraction" information in the form of three equations for change of wavenumber; while the single, far simpler, Equation (24) is a consequence of, but is by no means equivalent to, those three. By contrast, in the particular case when $u_{fi}$ and $c_f$ are independent of $x_1$ and $x_2$, Equations (24) and, additionally, (26) in the cases $j = 1$ and 2 give three simple results,

$$\omega = \text{ constant}, \quad k_1 = \text{ constant and } k_2 = \text{ constant along rays}, \tag{38}$$

that may be shown fully equivalent to the basic law.

If now we write the wavenumber (a vector normal to crests) as

$$(k_1, k_2, k_3) = (\kappa \cos \psi, \kappa \sin \psi, \kappa \cot \theta), \tag{39}$$

so that $\kappa$ is its constant horizontal resultant, $\psi$ its constant azimuthal angle to the wind direction, and $\theta$ its variable angle to the vertical (Figure 6), and use Equation (25) in the form

$$\omega = c(x_3)k + V(x_3)k_1 = c(x_3)\kappa \text{cosec}\theta + V(x_3)\kappa \cos \psi, \tag{40}$$

we obtain the important

$$\text{extension } \sin \theta = \frac{c(x_3)}{\omega \kappa^{-1} - V(x_3) \cos \psi} \text{ to Snell's Law} \tag{41}$$

from the classical case ($V = 0$) when the denominator is a constant. This extended law (41) tells us how $\theta$ varies with $x_3$ along any ray – whose path we can then trace, using Equations (26) in the form

$$\frac{dx_1}{dt} = c(x_3) \cos \psi \sin \theta + V(x_3), \quad \frac{dx_2}{dt} = c(x_3) \sin \psi \sin \theta,$$

$$\frac{dx_3}{dt} = c(x_3) \cos \theta, \tag{42}$$

by simply integrating $dx_1/dx_3$ and $dx_2/dx_3$ with respect to $x_3$.

It follows that a ray tube covers the same horizontal area at each altitude, so that conservation of the flux of wave action $E_r/\omega_r$ along it implies that the vertical component

$$(E_r/\omega_r)(dx_3/dt) = E_r \kappa^{-1} \sin\theta \cos\theta \tag{43}$$

of wave action flux is constant along rays; from which, with Equation (28), sound amplitudes are readily derived.

Wind shear is able to reproduce all the main types of ray bending (WF) associated with temperature stratification, and often to an enhanced extent. Roughly, the downward curvature of near-horizontal rays in $(km)^{-1}$ comes to

$$3 \{V'(x_3)\cos\psi + c'(x_3)\} \tag{44}$$

where the velocity gradients are in $s^{-1}$ and the factor $3(km)^{-1}s$ outside the braces is an approximate reciprocal of the sound speed (WF and Lighthill, 1972).

Cases when (44) is negative: curvature is upward; its magnitude with zero wind is at most $0.018 \ (km)^{-1}$ (because temperature lapse rate in stable atmospheres cannot exceed $10°C$ per km, giving $c' = -0.006 s^{-1}$) but with strong wind shear can take much bigger values for upwind propagation ($\psi = \pi$). In either case Figure 7(a) shows how the lowest ray emitted by a source "lifts off" from the ground, leaving below it a zone of silence (on ray theory – actually, a zone where amplitudes decrease exponentially with distance below that ray).

Cases when (44) is positive: curvature is downward, as found with zero wind in temperature-inversion conditions (e.g., over a calm cold lake) and even more with strong wind shear for downwind propagation ($\psi = 0$). Figure 7(b) shows how this leads to signal enhancement through multiple-path communication.

In summary, then, the very familiar augmentation of sound levels downwind, and diminution upwind, of a source are effects of the wind's shear (increase with altitude).

## 5.4. Wider aspects of parallel-flow acoustics

The propagation of sound through parallel flows at wavelengths too great for the applicability of ray acoustics can be analyzed by a second-order ordinary differential equation. Thus, a typical Fourier component of the sound pressure field takes the form

$$p_s(x_3)e^{i(\omega t - k_1 x_1 - k_2 x_2)} \text{ with } \rho \frac{d}{dx_3}\left[\frac{1}{\rho(\omega - Vk_1)^2}\frac{dp_s}{dx_3}\right]$$
$$+ \left[\frac{1}{c^2} - \frac{k_1^2 + k_2^2}{(\omega - Vk_1)^2}\right] p_s = 0. \tag{45}$$

Equation (45) can be used to improve on ray acoustics

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(a) near caustics (envelopes of rays) where it allows a uniformly valid representation of amplitude in terms of the famous Airy function, giving "beats" between superimposed waves on one side of the caustic, and exponential decay on the other (WF);

(b) at larger wavelengths by abandoning ray theory altogether in favor of extensive numerical solutions of Equation (45); and

(c) to obtain waveguide modes for sound propagation in a two-dimensional duct (between parallel planes) (Pridmore-Brown, 1958; Mungur and Gladwell, 1969; and Shankar, 1971).

On the other hand, in the case of a three-dimensional duct carrying parallel flow $V(x_2, x_3)$ in the $x_1$-direction, Equation (45) is converted into a partial differential equation (the first term being supplemented by another with $d/dx_2$ replacing $d/dx_3$, while $k_2$ is deleted) which is used

(d) to obtain waveguide modes in such ducts;

(e) in calculations of propagation of sound through the wide jets – modelled as parallel flows – typical (§4.1) of modern aero-engines; and,

(f) with aeroacoustic source terms included, in certain enterprising attempts at modelling jet noise generation and emission (Phillips, 1960 and Mani, 1976).


## 6. Acoustic Streaming

### 6.1. Streaming as a result of acoustic attenuation

Sound waves act on the air with a Reynolds stress (34) even when mean flow is absent (so that subscript $f$ becomes subscript zero). The $j$-component of force acting on unit volume of air (WF, pp. 337-) is then

$$F_j = -\frac{\partial}{\partial x_i} < \rho_0 u_{si} u_{sj} >:$$  (46)

the force generating acoustic streaming (Lighthill, 1978a).

However, the force (46) could not produce streaming for unattenuated sound waves; indeed, their linearized equations can be used to show that

$$\text{if } p^M = < \frac{1}{2} c_0^2 \rho_0^{-1} \rho_s^2 - \frac{1}{2} \rho_0 u_{si} u_{si} > \text{ then } F_j - \frac{\partial p^M}{\partial x_j} = < \frac{\partial}{\partial t}(\rho_s u_{sj}) >$$  (47)

which is necessarily zero (as the mean value of the rate change of a bounded quantity). Accordingly, the fluid must remain at rest, responding merely by setting up the distribution $p^M$ of mean pressure whose gradient can balance the force. [Note: actually, on the ray-acoustic approximation (28), $p^M$ is itself zero, but the above argument does not need to use this approximation.]

Attenuation of sound waves takes place

(a) in the bulk of the fluid through the action of viscosity, thermal conductivity and lags in attaining thermodynamic equilibrium (Ch. 6); and

(b) near solid walls by viscous attenuation in Stokes boundary layers.

All these effects produce forces (46) which act to generate acoustic streaming. It is important to note, furthermore, that even the forces due solely to viscous attenuation – being opposed just by the fluid's own viscous resistance – generate mean motions which do not disappear as the viscosity $\mu$ tends to zero (Rayleigh, 1896; Nyborg, 1953; Westervelt, 1953; and Nyborg, 1965).

## 6.2. Jets generated by attenuated acoustic beams

Attenuation of type (a) produces a streaming motion $u_{fj}$ satisfying

$$\rho u_{fi}\partial u_{fj}/\partial x_i = F_j - \partial p/\partial x_j + \mu\nabla^2 u_{fj}. \tag{48}$$

Substantial streaming motions can be calculated from this equation only with the left-hand side included (Stuart, 1966); although in pre-1966 literature it was misleadingly regarded as "a fourth-order term" and so ignored – thus limiting all the theories to uninteresting cases when the streaming Reynolds number would be of order 1 or less.

We can use streaming generated by acoustic beams to illustrate the above principles. If acoustic energy is attenuated at a rate $\beta$ per unit length, then a source at the origin which beams power $P$ along the $x_1$-axis transmits a distribution of

$$\text{power } Pe^{-\beta x_1}, \text{ and therefore energy per unit length } c^{-1}Pe^{-\beta x_1}; \tag{49}$$

which is necessarily the integral of energy density, and so also of the uniaxial Reynold stress (35), over the beam's cross-section. It follows by differentiation that the force per unit volume (46), integrated over a cross-section, produces (Lighthill, 1978a)

$$\text{a force } c^{-1}P\beta e^{-\beta x_1} \text{ per unit length in the } x_1\text{-direction.} \tag{50}$$

At high ultrasonic frequencies the force distribution (50) is rather concentrated, the distance of its center of application from the origin being just $\beta^{-1}$ (which at 1 MHz, for example, is

24mm in air). Effectively (WF, p. 345), the beam applies at this center a total force $c^{-1}P$ (integral of the distribution (50)).

The type (WF and Lighthill, 1978a) of streaming motion generated by this concentrated force $c^{-1}P$ depends critically on the value of $\rho c^{-1}P\mu^{-2}$: a sort of Reynolds number squared, which is about $10^7 P$ in atmospheric air (with $P$ in watts). Streaming of the low-Reynolds-number "stokeslet" type predicted (for a concentrated force) by Equation (48) with the left-hand side suppressed is a good approximation only for $P < 10^{-6}W$. For a source of power $10^{-4}W$, by contrast, the force $c^{-1}P$ generates quite a narrow laminar jet with momentum transport $c^{-1}P$, and at powers exceeding $3 \times 10^{-4}W$ this jet has become turbulent, spreading conically with semi-angle about $15°$ and continuing to transport momentum at the rate $c^{-1}P$. Such turbulent jets generated by sound are strikingly reciprocal to a classical aeroacoustic theme!

At lower frequencies an acoustic beam of substantial power delivers a turbulent jet with a somewhat more variable angle of spread — but one which

$$\text{at each point } x_1 \text{ carries momentum transport } c^{-1}P(1 - e^{-\beta x_1}), \tag{51}$$

generated by the total force (50) acting up to that point. This momentum transport in the jet represents the source's original rate of momentum delivery minus the acoustic beam's own remaining momentum transport (49). In summary, as acoustic power is dissipated into heat, the associated acoustic momentum transport is converted into a mean motion (which, at higher Reynolds numbers, is turbulent: Lighthill, 1978a).

## 6.3. Streaming around bodies generated by boundary-layer attenuation

Sound waves of frequency $\omega$ well below high ultrasonic frequencies have their attenuation concentrated, if solid bodies are present, in thin Stokes boundary layers attached to each body (WF). Then the streaming generated near a particular point on a body surface is rather simply expressed by using local coordinates with that point as origin, with the $z$-axis normal to the body and the $x$-axis in the direction of the inviscid flow just outside the boundary layer: the exterior flow. The Stokes boundary layer for an exterior flow

$$(U(x,y), V(x,y))e^{i\omega t} \text{ has interior flow}$$

$$(U(x,y), V(x,y))e^{i\omega t}\left[1 - e^{-z\sqrt{(i\omega\rho/\mu)}}\right]. \tag{52}$$

[Note that my choice of coordinates makes $V(0,0) = 0$, and that the expressions (52) become identical outside the layer.] The streaming (WF and Lighthill, 1978a) is calculated from the equation

$$F_j^{INT} - F_j^{EXT} + \mu\partial^2 u_{jj}/\partial z^2 = 0, \tag{53}$$

with certain differences from Equation (48) explained as follows:

(a) the first term is the force (46) generating streaming within the boundary layer;

(b) we are free, however, to subtract the second, since (see §6.1) it can produce no streaming, and conveniently, the difference is zero outside the layer;

(c) gradients in the $z$-direction are so steep that the third term dominates the viscous force – and, indeed, in such a boundary layer, dominates also the left-hand side of Equation (48).

The solution of Equation (53) which vanishes at $z = 0$ and has zero gradient at the edge of the layer is obtained by two integrations, and its exterior value is

$$u_{fj}^{\text{EXT}} = \mu^{-1} \int_0^\infty \left( F_j^{\text{INT}} - F_j^{\text{EXT}} \right) z\, dz; \tag{54}$$

where integration extends in practice, not to "infinity," but to the edge of the layer within which the integrand is nonzero. Expression (54) for the exterior streaming is yet again (see §6.1) independent of the viscosity $\mu$ since Equation (52) makes $z\, dz$ of order $\mu/\rho\omega$; and it is easily evaluated.

At $x = y = 0$ (in the coordinates specified earlier) the exterior streaming (54) has

$$x-\text{component} \ -U\frac{3\partial U/\partial x + 2\partial V/\partial y}{4\omega} \quad \text{and} \quad y-\text{component} \ -U\frac{\partial V/\partial x}{4\omega}, \tag{55}$$

with zero $z$-component. This is a generalized form of the century-old Rayleigh law of streaming (which covers cases when $V$ is identically zero).

For the complete streaming pattern, expressions (55) are, effectively, boundary values for its tangential component at the body surface (because the Stokes boundary layer is so thin). Therefore, any simple solver for the steady-flow Navier-Stokes equations with specified tangential velocities on the boundary allows the pattern to be determined. Important note: here, the inertial terms in the Navier-Stokes equations must not be neglected, unless the Reynolds number $R_s$ based on the streaming velocity (55) be of order 1 or less; when, however, the corresponding streaming motions would (as in §6.2) be uninterestingly small.

In the other extreme case when $R_s$ is rather large (at least $10^3$) the streaming motion remains quite close to the body (Stuart, 1966) within a steady boundary layer whose dimension (relative to that of the body) is of order $R_s^{-1/2}$. This layer is by no means as thin as the Stokes boundary layer, but it does confine very considerably the acoustic streaming motion. Equations (55) direct this motion towards one of the exterior flow's stagnation points, whence the steady-boundary-layer flow emerges as a jet (Figure 8) – yet another jet generated by sound (Riley, 1987 and Amin and Riley, 1990).

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## Acknowledgements

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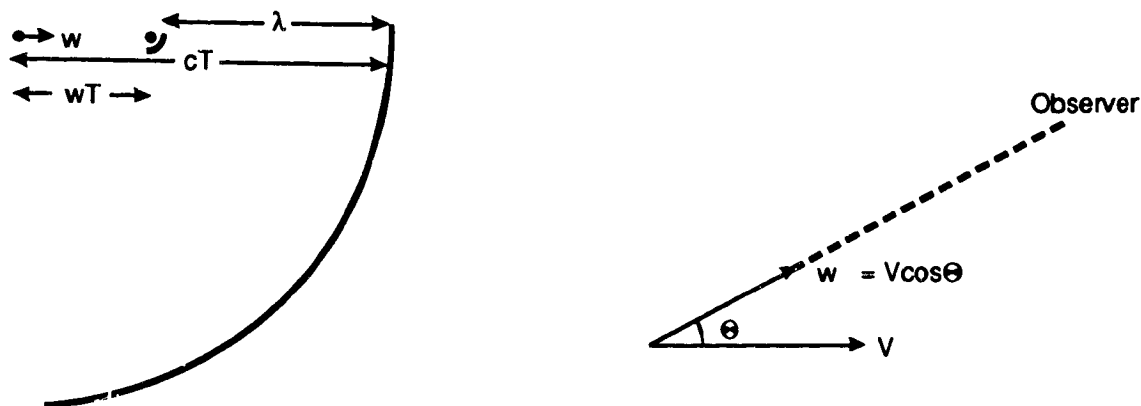
Figure 1

(a) When a source of sound of period $T$ approaches an observer at velocity $w$, successive wave crests are emitted towards him with separation $\lambda = cT - wT$.

(b) Case of observer at angle $\theta$ to source's direction of motion at speed $V$; then velocity of approach is $w = V \cos \theta$.
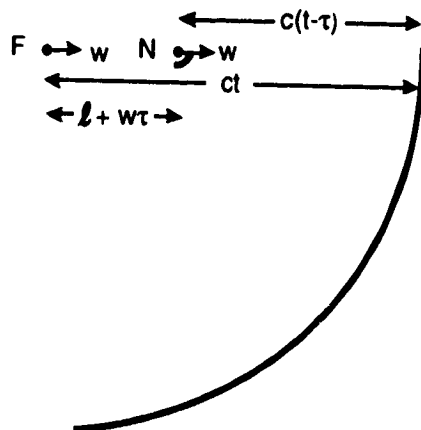


Figure 2. Sounds arriving simultaneously from a source's far side $F$ and near side $N$ took times $t$ and $t - \tau$ (say) to reach the observer, while the distance $FN$ increased from $\ell$ to $\ell + w\tau$ during the time $\tau$ between emissions from $F$ and $N$.
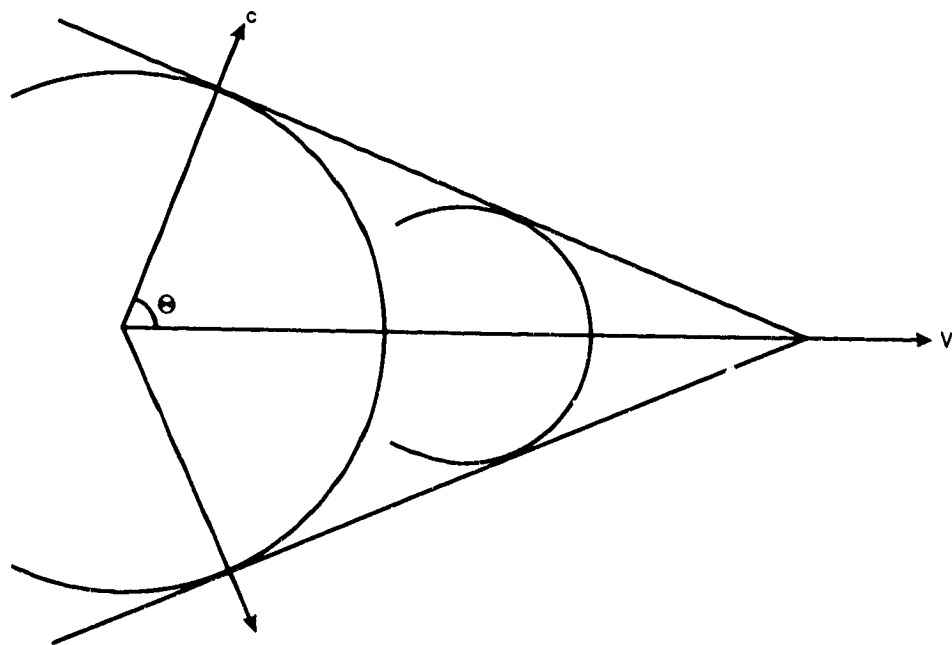
Figure 3. Supersonic source convection produces radiation along rays in the Mach direction (11).

(a)  (b)  (c)
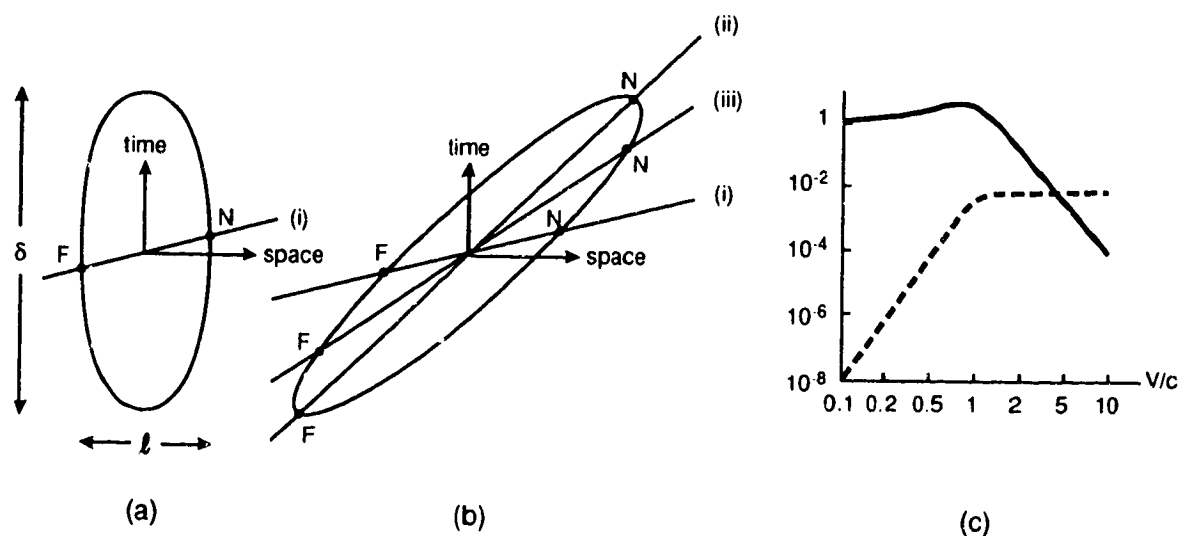
Figure 4. A uniformly valid Doppler-effect approximation.

Diagram (a) Space-time diagram for unconvected "eddies" of correlation length $\ell$ and duration $\delta$.

Diagram (b) Case of "eddies" convected towards observer at velocity $w$; being Diagram (a) sheared by a distance $w$ per unit time. Here, lines sloping by a distance $c$ per unit time represent emissions received simultaneously by observer.

Case (i): $w/c$ small. Case (ii): $w/c = 1$. Case (iii): intermediate value of $w/c$.

Diagram (c) ――――― Average modification factor (13). - - - - - - - - - Acoustic efficiency, obtained by applying this factor to a low-Mach-number "quadrupole" efficiency of (say) $10^{-3}(V/c)^5$

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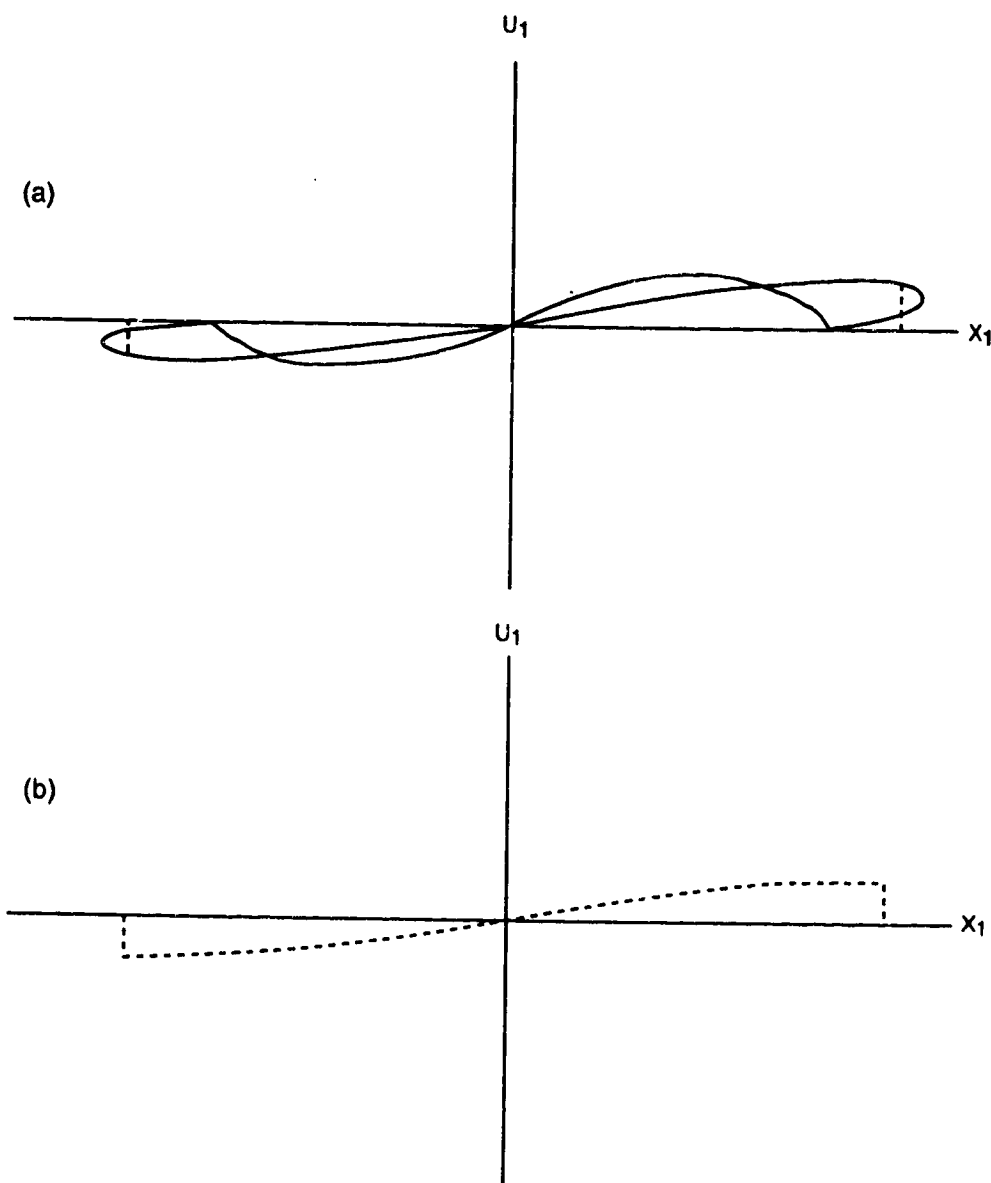
Figure 5. Equation (17) tells us that any value of $u_1$ propagates unchanged along a characteristic $dx_1/dt_1 = u_1$. This implies (WF, p. 151) waveform shearing at a uniform rate (Diagram (a)). According to nonlinear acoustics (WF, pp. 170-) area-conserving discontinuities, shown as broken lines in Diagram (a), have to be incorporated wherever necessary to keep the solution one-valued; leading, in the case illustrated, to the $N$-wave form (Diagram (b)).
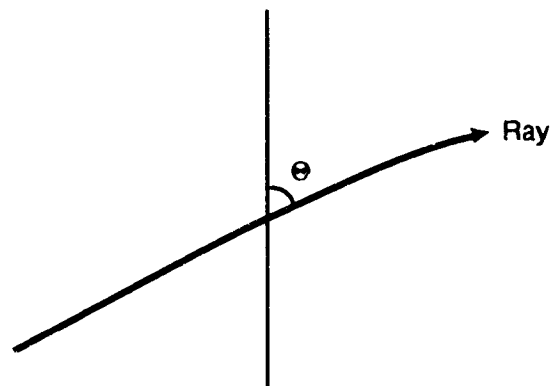
Figure 6. The angle $\theta$ between a ray and the vertical varies in accordance with the extension (41) to Snell's law.



(a)  (b)
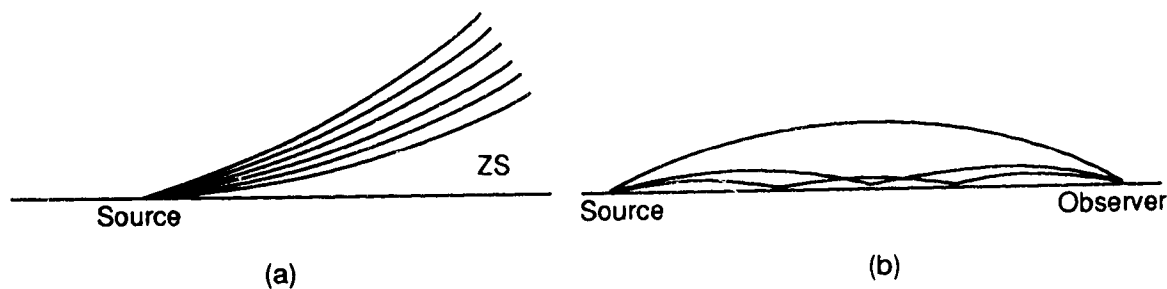
Figure 7. Effects of ray curvature (44) on propagation from a source on horizontal ground.

Diagram (a) Rays of given upward curvature (due to temperature lapse or upwind propagation) can leave a Zone of Silence (ZS) below the ray emitted horizontally.

Diagram (b) Rays of given downward curvature (due to temperature inversion or downwind propagation) can enhance received signals through multiple-path communication.
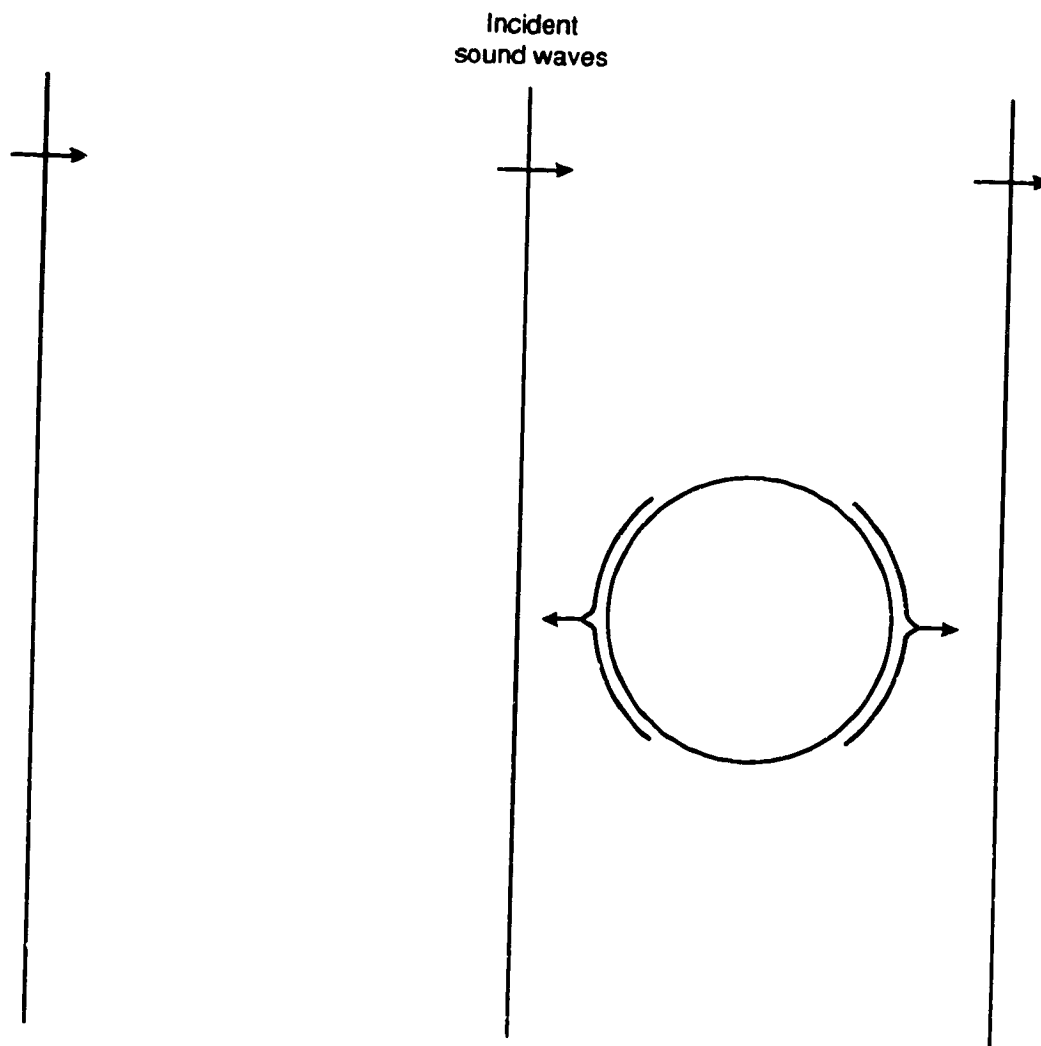
Figure 8. Plane sound waves incident on a sphere may generate a steady streaming motion (Amin and Riley, 1990), concentrated in a relatively thick boundary layer, and directed towards one of the exterior flow's stagnation points – whence it emerges as a jet.

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13. ABSTRACT (Maximum 200 words)

This paper uses a single unifying principle (based upon the nonlinear "momentum-flux" effects produced when different components of a motion transport different components of its momentum) to give a broad scientific background to several aspects of the interaction between airflows and atmospheric sound. First, it treats the generation of sound by airflows of many different types. These include, for example, jet-like flows involving convected turbulent motions -- with the resulting aeroacoustic radiation sensitively dependent on the Mach number of convection -- and they include, as an extreme case, the supersonic "boom" (shock waves generated by a supersonically convected flow pattern). Next, the paper analyses sound propagation through nonuniformly moving airflows, and quantifies the exchange of energy between flow and sound; while, finally, it turns to problems of how sound waves "on their own" may generate the airflows known as acoustic streaming.

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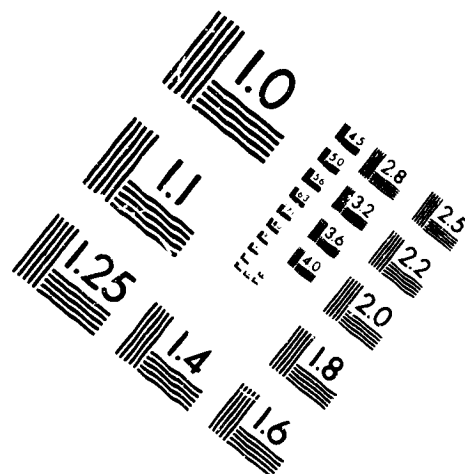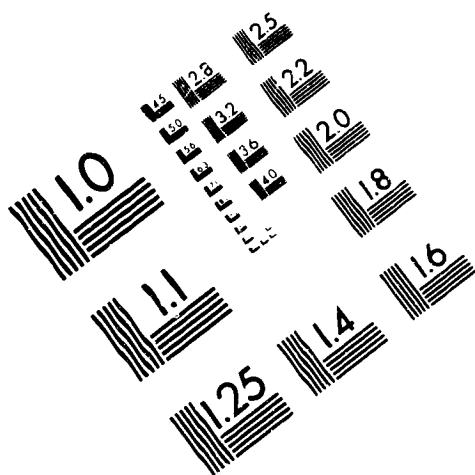
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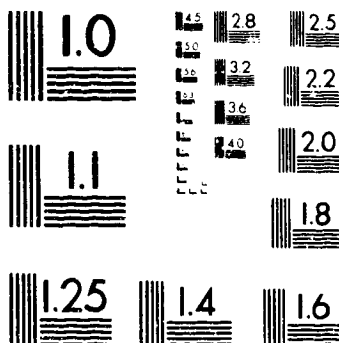
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