NASA
Technical
Memorandum

NASA TM - 108377

THE MECHANISM OF BOLT LOADING

By H.M. Lee

Structures and Dynamics Laboratory
Science and Engineering Directorate

October 1992

NASA
National Aeronautics and
Space Administration

George C. Marshall Space Flight Center

REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 1992	3. REPORT TYPE AND DATES COVERED Technical Memorandum

4. TITLE AND SUBTITLE

The Mechanism of Bolt Loading

5. FUNDING NUMBERS

6. AUTHOR(S)

H.M. Lee

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM-108377

11. SUPPLEMENTARY NOTES

Prepared by Structures and Dynamics Laboratory, Science and Engineering Directorate.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified — Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This report shows that the mechanism of bolt loading for preloaded fasteners can be effectively portrayed through simple spring models and some algebraic manipulations. Understanding schematically what is involved in such joints provides insight into the distribution of loads. The equations developed confirm that for both symmetric and nonsymmetric joints the loading plane factor (η) and the stiffness factor (ϕ) directly affect the load seen in preloaded fasteners. The manner in which an external loading is transferred through the joint can be explained as energy dissipated in the various springs of both the abutment and the bolt itself.

14. SUBJECT TERMS

Preloaded Bolt, Loading Plane Factor, Stiffness Factor, Symmetric Joint, Nonsymmetric Joint

15. NUMBER OF PAGES
14

16. PRICE CODE
NTIS

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

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INTRODUCTION

For years engineers have discussed the effects that a preloaded bolt has on structural joints. Great debates would arise over the mechanisms and the mathematics involved in relating the subsequent loading of the fastener to the actual applied external load. Some have believed that through simple statics a bolt will "feel" all external tensile load coming into the joint, while others have maintained that the bolt "sees" no load increase until the external tensile load exceeds the magnitude of the initially applied preload. The actual fact is that either of these scenarios is possible, but probably not likely. In the real world of preloaded bolts, the bolt will usually carry only a portion of the total external tensile load. The actual percentage of load transferred to the bolt is dependent on the relative stiffness of the bolt and its associated joint, and the way in which the external load is distributed to the joint.

The intent of this report is to attempt to relate what is happening in such a preloaded joint from a schematic and mathematical perspective. It will be shown that if the proper schematic representation of the joint is developed then, with relatively simple mathematics, some light will be shed on the mechanism of bolt loading.

THE SYMMETRIC JOINT

Figure 1 depicts a sectional view of a typical symmetric bolted joint (sometimes called a concentric joint). It is termed symmetric because the abutment is structurally symmetric about the interface plane. In other words, both abutment components are of the same material and have the same dimensional thicknesses. This fact assures us that the same mechanical characteristics will be displayed on both sides of the interface. Notice, too, that the hardware in the figure is represented as sturdy and would allow very little prying of the bolt head or any other less predictable nonlinear behavior.

Figure 2 schematically reflects the hardware for such a joint, first with no preload (fig. 2a) and then the same joint with an initial preload (fig. 2b). Looking at the joint without preload and applying equal but opposite external forces (P_{ext}) at location 2, it has been assumed that the sturdy hardware has uniformly transmitted the load throughout the abutment. This particular case means that the loading plane factor (η) is equal to $1/2$. With the loading plane factor at this value, the effective abutment loading is occurring halfway through the component thicknesses.

The external load will essentially meet only the resistance of the abutment stiffness from locations 2 to 3 (k_{2-3}) plus the bolt stiffness (k_b). These springs are in series and will act as an effective stiffness (k_{eff}):

$$\frac{1}{k_{eff}} = \frac{1}{4k_a} + \frac{1}{4k_a} + \frac{1}{k_b} \, ,$$
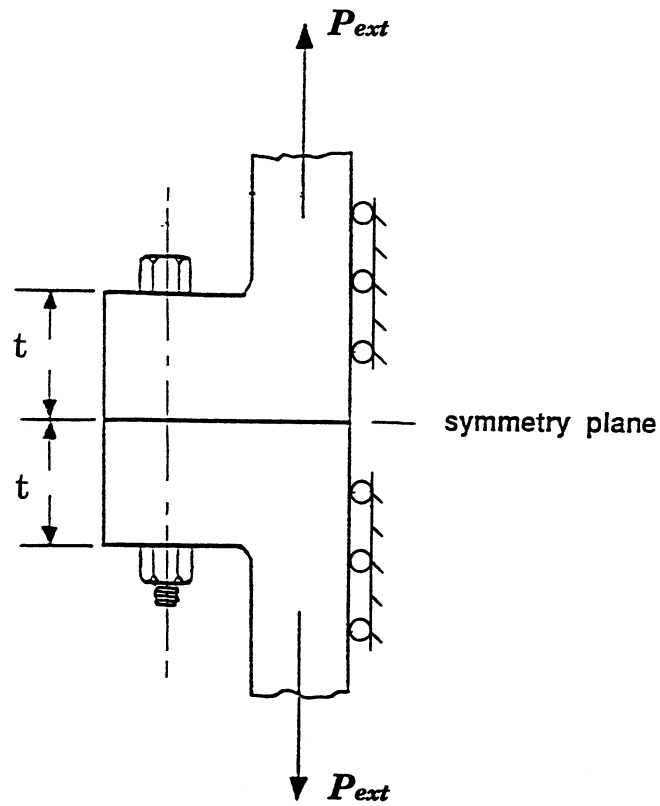
$$k_{eff} = \frac{2k_a k_b}{(2k_a + k_b)} \, .$$

Figure 1. Bolted joint section.

(a)
no preload
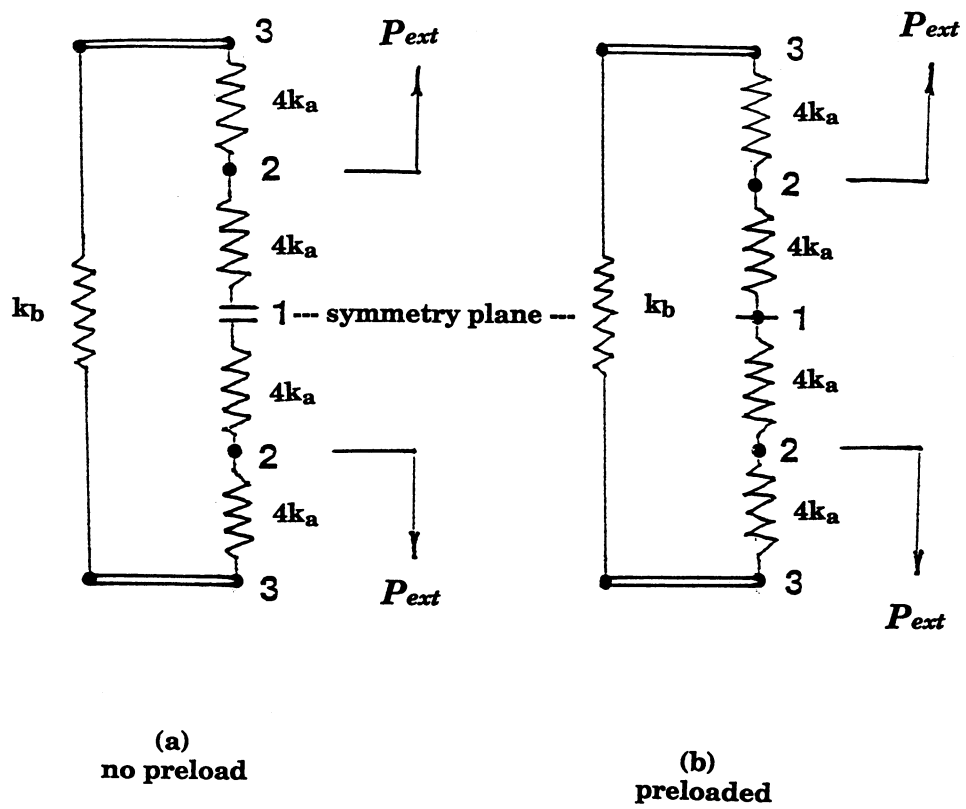
(b)
preloaded

Figure 2. Symmetric stiffness models.

Since the effective stiffness is only on one side of location 2 and is a series-derived one, both the abutment (P_a) and the bolt (P_b) will carry the entire external loading:

$$P_a = P_b = P_{ext} \ .$$

This case, quite simply, obeys what one would expect from a basic static viewpoint.

A review of the preloaded model (fig. 2b) shows a slightly different, but crucial, arrangement. Because the abutment is under an initial compressive preload, the two abutment components cannot be easily separated. There exists a formidable amount of compressive strain energy between them. The two components will, in effect, act as though there is only one component in the abutment. Location 1 in the figure is merely the location of the plane of symmetry and is not truly an interface until separation occurs. The law of superposition can now be utilized since location 1 is so loaded in compression. When an external tensile load is applied at location 2, energy must be expended in three areas: (1) reduction of the compressive forces in the abutment from location 1 to 2 ($2k_a$), (2) an increase in the compressive forces in the abutment from location 2 to 3 ($2k_a$), and (3) an increase in the magnitude of the tensile force in the bolt (k_b).

Mathematically this appears as:

$$\frac{1}{k_{1-2}} = \frac{1}{4k_a} + \frac{1}{4k_a} \ , \qquad k_{1-2} = 2k_a \tag{1}$$

$$\frac{1}{k_{2-3}} = \frac{1}{4k_a} + \frac{1}{4k_{aa}} + \frac{1}{k_b} \ , \qquad k_{2-3} = \frac{2k_a k_b}{2k_a + k_b} \tag{2}$$

so that

$$k_{eff} = k_{1-2} + k_{2-3} = \frac{2k_a + 2k_a k_b}{2k_a + k_b} \ . \tag{3}$$

Figure 3 shows a model of what resistance this externally applied force (P_{ext}) will see when it is introduced at location 2. Since the system is a parallel stiffness one, part of the external load will go into the abutment and the remaining portion into the fastener. These magnitudes can be calculated as:

$$P_b = P_{ext} \frac{k_{2-3}}{k_{eff}} = P_{ext} \frac{2k_a k_b/(2k_a + k_b)}{2k_a + 2k_a k_b/(2k_a + k_b)} \ .$$

For the bolt, this simply reduces to

$$P_b = \{1/2\} \left[\frac{k_b}{k_a + k_b} \right] P_{ext} \qquad \text{symmetric joint}$$

and for the abutment

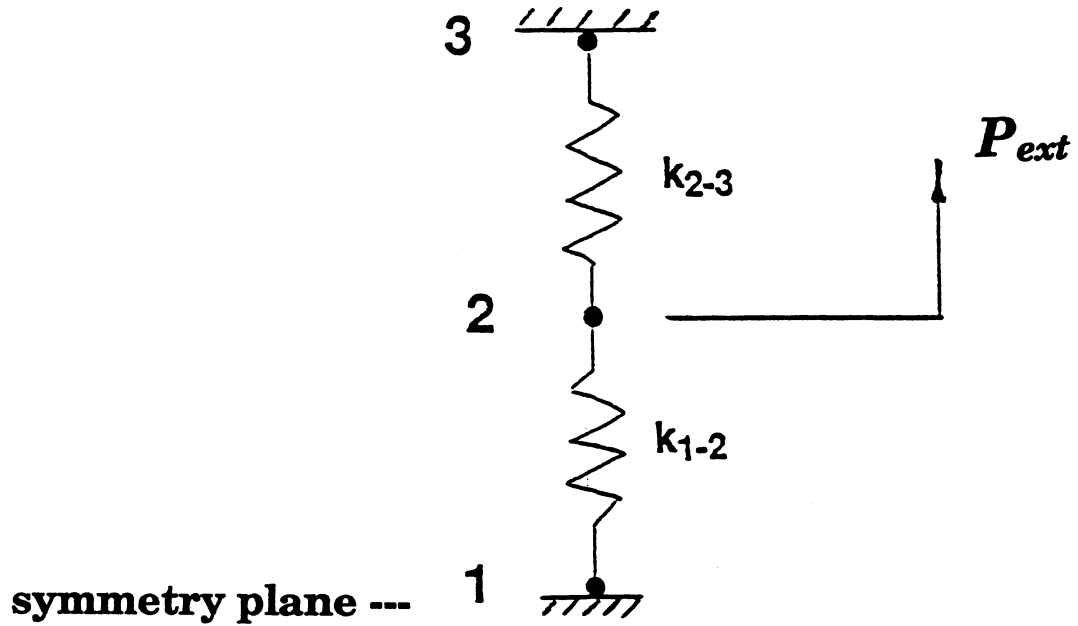$$P_a = P_{ext} \frac{k_{1-2}}{k_{eff}} = P_{ext} \frac{2k_a}{2k_a + 2k_a k_b/(2k_a + k_b)} \ .$$

Figure 3. Preloaded symmetric joint model.

$$P_a = \{1/2\} \, [(2k_a+k_b)/(k_a+k_b)] \, P_{ext} \qquad \text{symmetric joint}$$

For those familiar with bolted joint hand analysis, the equation for the bolt load (P_b) is a familiar one. The factor $^1/_2$ represents what was referred to as earlier as the loading plane factor (η), and the term $k_b/(k_a+k_b)$ is known as the stiffness factor (ϕ). Hence, the total load in the preloaded bolt becomes:

$$P_b = PLD+\eta\phi P_{ext} \, ,$$

where PLD = initial bolt preload.

A more general solution involving symmetry can also be obtained for preloaded configurations where the loading plane factor may not be equal to $^1/_2$. Shown in figure 4 is a symmetric situation in which schematically the external load is enforced at some arbitrary point (location 2). Using the same analysis technique as before, it can be shown that:

$$k_{1-2} = \frac{Bk_a}{2} \, ,$$

$$k_{2-3} = \frac{Ak_a k_b}{Ak_a+2k_b} \, ,$$

and
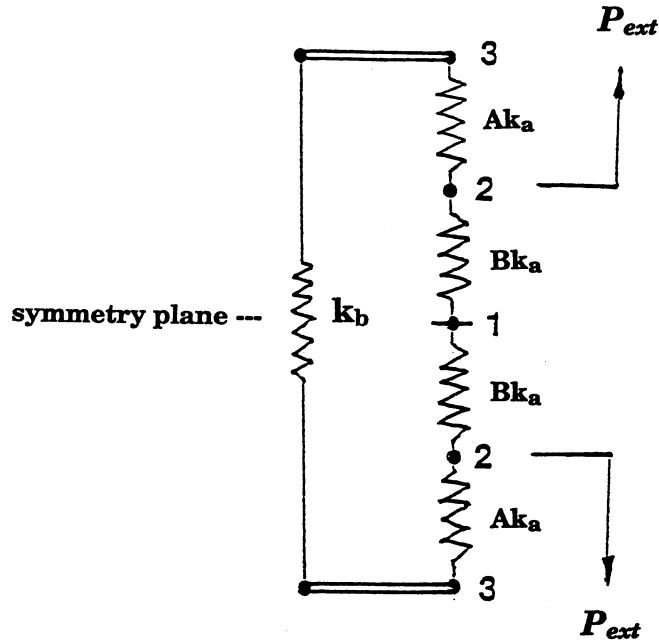
$$k_{eff} = \frac{Ak_a k_b}{Ak_a+2k_b} + \frac{Bk_a}{2} \, ,$$

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Figure 4. General symmetric stiffness joint.

knowing that

$$\frac{1}{Ak_a}+\frac{1}{Ak_a}+\frac{1}{Bk_a}+\frac{1}{Bk_a}=\frac{1}{k_a} \ ,$$

leads to $B = 2A/A–2$ (where: $A\rightarrow\infty$). Substituting this into the equation

$$P_b = P_{ext}[k_{2-3}/k_{eff}] \ ,$$

$$P_b = \left\{\frac{A–2}{A}\right\}\left[\frac{k_b}{k_a+k_b}\right]P_{ext} \qquad \text{general symmetric joint}$$

Once again the familiar bolt load equation is evident. The quantity $\{(A–2)/A\}$ now defines a more general loading plane factor (η).

THE NONSYMMETRIC JOINT

Figure 5 details the view of a nonsymmetric bolted joint. It is said to be nonsymmetric because the abutment components have different thicknesses and/or use different materials. In this case, there is no longer a symmetry plane, but an interface plane. Figure 6 shows the spring schematic for such a joint under an initial preload.
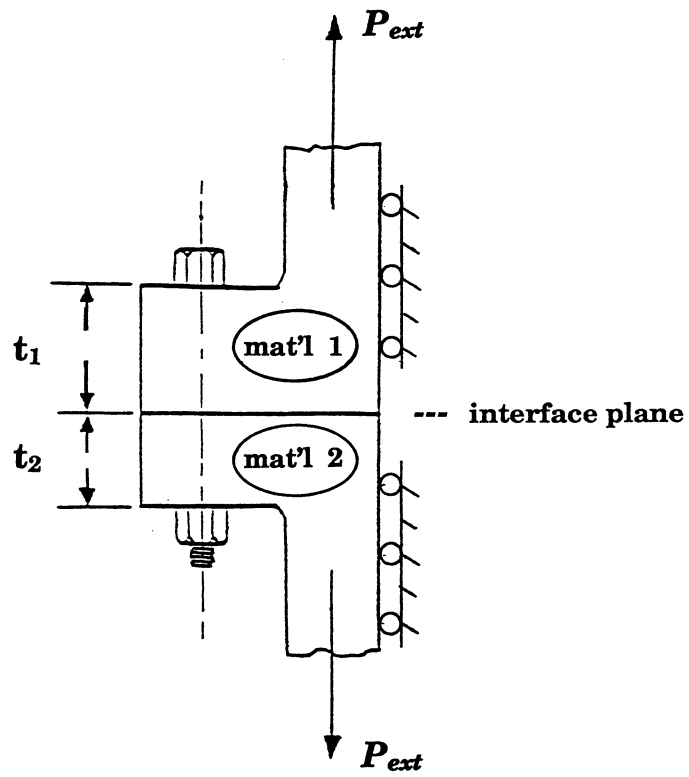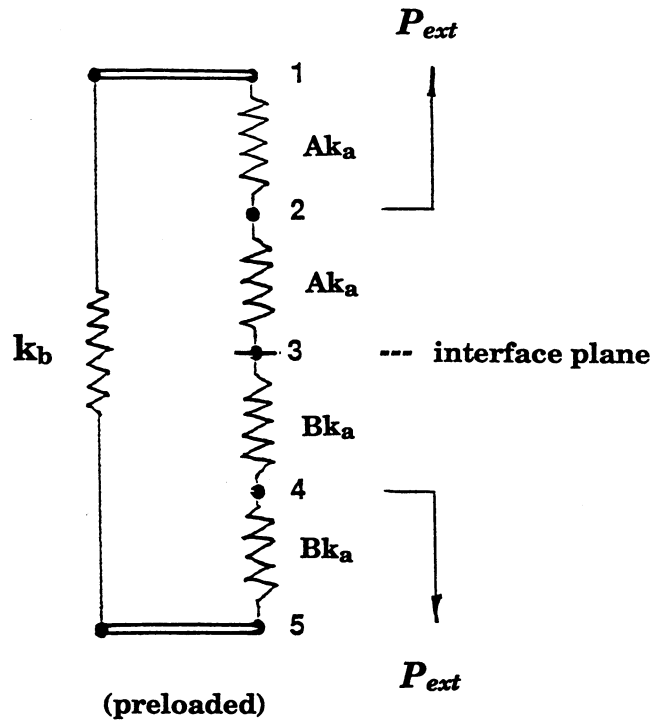
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Figure 5. Nonsymmetric bolted joint section.

Figure 6. Nonsymmetric stiffness model.

This particular configuration again assumes that the hardware transmit all loadings uniformly throughout the abutment (i.e., $\eta = 1/2$). Using the same logic as before, this mathematically becomes:

$$\frac{1}{k_{2-3,3-4}} = \frac{1}{Ak_a} + \frac{1}{Bk_a} \ , \qquad k_{2-3,3-4} = \frac{ABk_a}{A+B} \qquad (4)$$

$$\frac{1}{k_{1-2,4-5}} = \frac{1}{Ak_a} + \frac{1}{Bk_a} + \frac{1}{k_b} \ , \qquad k_{1-2,4-5} = \frac{ABk_ak_b}{ABk_a+Ak_b+Bk_b} \qquad (5)$$

so that

$$k_{eff} = k_{2-3,3-4} + k_{1-2,4-5} \qquad (6)$$

$$k_{eff} = \frac{ABk_a}{A+B} + \frac{ABk_ak_b}{ABk_a+Ak_b+Bk_b} \ .$$

The magnitude of the bolt loading for this case is easily calculated as before:

$$P_b = P_{ext}\frac{k_{1-2,4-5}}{k_{eff}} \ ,$$

$$P_b = \frac{ABk_ak_b/(ABk_a+Ak_b+Bk_b)}{ABk_a/(A+B)+ABk_ak_b/(ABk_a+Ak_b+Bk_b)} \ .$$

This can be reduced to

$$P_b = P_{ext}\frac{(A+B)k_b}{ABk_a+2(A+B)k_b} \ .$$

Again, knowing that

$$\frac{1}{Ak_a} + \frac{1}{Ak_a} + \frac{1}{Bk_a} + \frac{1}{Bk_a} = \frac{1}{k_a} \ ,$$

which leads to

$$B = 2A/(A-2) \ ,$$

from these equations we get:

$$P_b = P_{ext}\frac{[1+2/(A-2)]k_a}{[A/(A-2)]k_a+[1+2/(A-2)]k_b} \ ,$$

$$P_b = P_{ext}\{1/2\}\frac{[1+2/(A-2)]k_a}{[A/(A-2)]k_a+[1+2/(A-2)]k_b} \ ,$$

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which reduces to:

$$P_b = \{1/2\}\left[\frac{k_b}{k_a+k_b}\right]P_{ext} \qquad \text{nonsymmetric joint}.$$

The general nonsymmetric joint equation has four variable spring stiffness modifiers (A,B,C,D) and is shown below in its unreduced form:

$$P_b = P_{ext}\frac{(ADk_ak_b)/[ADk_a+(A+D)k_b]}{BCk_d/(B+C)+ADk_ak_b/[ADk_a+(A+D)k_b]}.$$

This expression, too, can be reduced to:

$$P_b = \left\{\frac{B+C}{BC}\right\}\left[\frac{k_b}{k_a+k_b}\right]P_{ext} \qquad \text{general nonsymmetric joint},$$

where

$$\frac{B+C}{BC} = 1 - \frac{A+D}{AD}.$$

Utilizing a classical bolt diagram, figure 7 depicts the bolt loading (P_b) versus the externally applied loading (P_{ext}) for all four cases described in this report; namely (1) symmetric joints, (2) general symmetric joints, (3) nonsymmetric joints, and (4) general nonsymmetric joints. As can be seen by the plot, any increase in the bolt load above the initial preload (PLD) is dependent on the ratio of the bolt and abutment stiffnesses, and on the location within the abutment at which the external load effectively transfers through the abutment (η). In addition, the graph reveals that the bolt load for each of the four cases can be described by the same basic equation:

$$P_b = PLD + \{\eta\}[\phi]P_{ext}.$$

CONCLUSIONS

This report has shown that the equation for relating the load in a bolt to the initial preload and the externally applied load can be derived through some relatively simple joint schematics and straightforward stiffness manipulations. The usefulness of the schematics is to show conceptually how and why the energy from the external loading may transfer into a bolt at a much reduced level. The technique is validated by proving that the loading plane factor is equal to $1/2$ when assuming that the external loads propagate through the joint in a totally uniform manner both for the symmetric and nonsymmetric cases. A final chart relates the classical bolt load diagram for each of the four joint cases considered: symmetric, general symmetric, nonsymmetric, and general nonsymmetric. This chart also shows the relative

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importance of both the loading plane factor $\{\eta\}$ and the stiffness factor $[\phi]$ for each case. The technique used in this report clearly shows that for each of the cases analyzed the classical bolt equation relating bolt load to preload and external load applies.
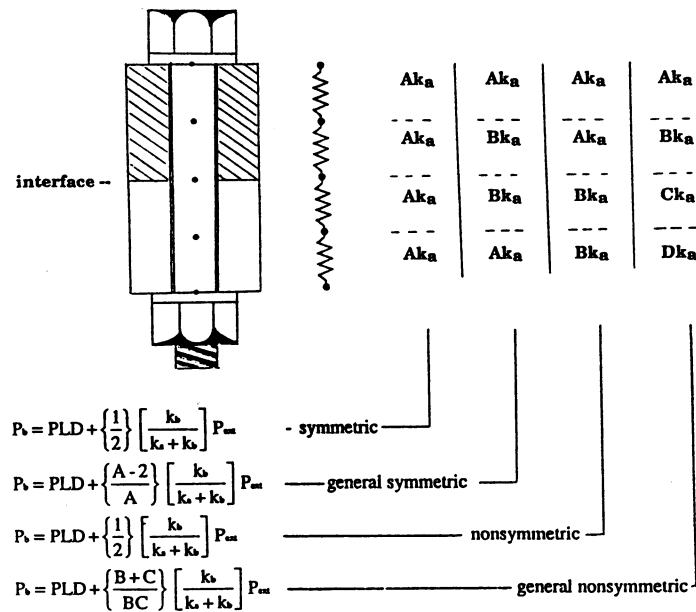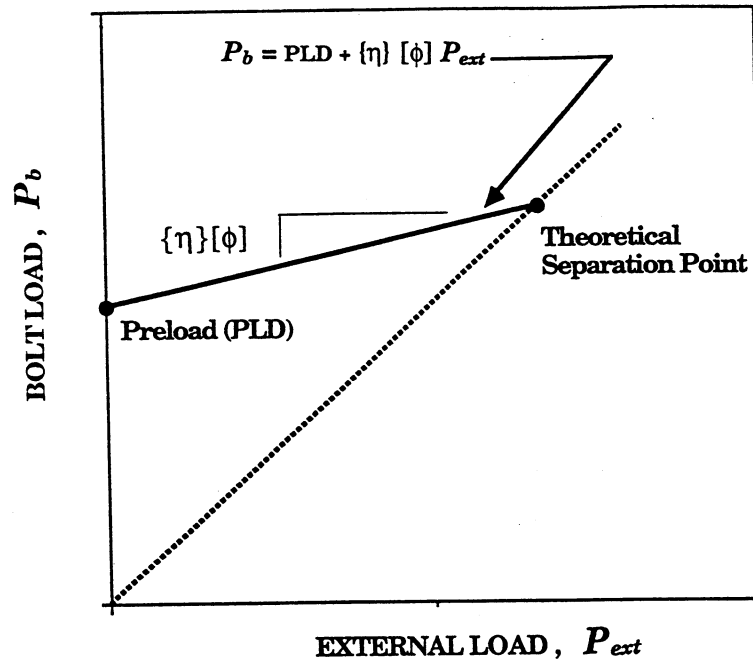

$$P_b = \text{PLD} + \{\eta\}\ [\phi]\ P_{ext}$$

BOLT LOAD, P_b

$\{\eta\}[\phi]$

Preload (PLD)

Theoretical Separation Point

EXTERNAL LOAD, P_{ext}

interface –

Ak_a	Ak_a	Ak_a	Ak_a
- - -	- - -	- - -	- - -
Ak_a	Bk_a	Ak_a	Bk_a
- - -	- - -	- - -	- - -
Ak_a	Bk_a	Bk_a	Ck_a
- - -	- - -	- - -	- - -
Ak_a	Ak_a	Bk_a	Dk_a

$P_b = \text{PLD} + \left\{\dfrac{1}{2}\right\}\left[\dfrac{k_b}{k_a + k_b}\right]P_{ext}$ — symmetric

$P_b = \text{PLD} + \left\{\dfrac{A-2}{A}\right\}\left[\dfrac{k_b}{k_a + k_b}\right]P_{ext}$ — general symmetric

$P_b = \text{PLD} + \left\{\dfrac{1}{2}\right\}\left[\dfrac{k_b}{k_a + k_b}\right]P_{ext}$ — nonsymmetric

$P_b = \text{PLD} + \left\{\dfrac{B+C}{BC}\right\}\left[\dfrac{k_b}{k_a + k_b}\right]P_{ext}$ — general nonsymmetric

Figure 7. Classical bolt load diagram.

APPROVAL

THE MECHANISM OF BOLT LOADING

By H.M. Lee

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

J.C. BLAIR
Director, Structures and Dynamics Laboratory