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On Bilinearity of Manson-Coffin Low-Cycle-Fatigue Relationship

V.M. Radhakrishnan
Indian Institute of Technology
Madras–60 036, India

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ON BILINEARITY OF MANSON-COFFIN LOW-CYCLE-FATIGUE RELATIONSHIP

V.M. Radhakrishnan*
Indian Institute of Technology
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SUMMARY

Some alloy systems, such as aluminum-lithium alloys and dual-phase steels, have been found to show a bilinear Manson-Coffin low-cycle-fatigue relationship. This paper shows that such bilinear behavior is related to the cyclic stress-strain curve. A bilinear cyclic stress-strain curve is a likely indication of a bilinear Manson-Coffin relationship. It is shown that materials other than aluminum-lithium alloys and dual-phase steels also may exhibit bilinear Manson-Coffin behavior. Implications for design are discussed.

SYMBOLS

B, C	coefficients of elastic and plastic life relationships
b, c	exponents of elastic and plastic life relationships
E	modulus of elasticity
N_f	number of cycles to failure
N_T	transition life at which elastic strain range is equal to plastic strain range
n'	cyclic strain-hardening exponent
$\Delta\epsilon$	strain range
$\Delta\epsilon_{el}$	elastic strain range
$\Delta\epsilon_p$	plastic strain range
$\Delta\epsilon_t$	total strain range
$\Delta\epsilon_0$	constant
$\Delta\sigma$	stress range

*Work performed when National Research Council–NASA Research Associate.

$\Delta\sigma_{@10^6}$ fatigue strength corresponding to 10^6 cycles

$\Delta\sigma_{ys}$ cyclic yield strength corresponding to 0.2 percent $\Delta\epsilon_p$

INTRODUCTION

The Manson-Coffin (M–C) low-cycle-fatigue relationship between the plastic strain range $\Delta\epsilon_p$ and the number of fatigue cycles N_f is given in the form

$$\Delta\epsilon_p = C(N_f)^c \tag{1}$$

It has been found to be valid for many ductile metallic materials. The relationship is linear on the log-log plot of plastic strain range versus cycles to failure with a slope equal to c. In some materials, such as aluminum-lithium alloys and dual-phase steels, a transition occurs (refs. 1 to 3) in equation (1) and a bilinear relationship is obtained. A typical bilinear relationship is shown in figure 1 for an aluminum-lithium alloy (ref. 1). Various mechanisms have been proposed to explain the bilinear relationship. A change in the deformation behavior through the work-hardening characteristics has been proposed as a possible explanation (ref. 2). Development of different fracture morphology at different strain amplitudes (refs. 1 and 4) and environmentally assisted fatigue degradation (refs. 5 and 6) are suggested as alternative explanations for bilinearity. Similar bilinearity is not observed in the relationship between the elastic strain range $\Delta\epsilon_{el}$ and the life N_f, which is normally given in the form

$$\Delta\epsilon_{el} = B(N_f)^b \tag{2}$$

where B and b are constants.

It will be shown herein that the bilinear relationship between $\log \Delta\epsilon_p$ and $\log N_f$ is a more general case than for a particular class of materials, such as aluminum-lithium alloys or dual-phase steels. The bilinearity can be encountered in all metallic materials for which the transition life N_T is comparatively small. The transition in linearity occurs in the region beyond the transition life wherein the elastic strain range dominates the plastic strain range.

ANALYSIS

It can be noted in figure 1 that the deviation from linearity in the $\Delta\epsilon_p$-N_f relationship occurs at values of fatigue life greater than the transition life of 340 cycles. The transition is complete at a plastic strain range of approximately 0.002, which corresponds to the cyclic offset yielding condition. The relationships of elastic strain range $\Delta\epsilon_{el}$ and total strain range $\Delta\epsilon_t$ to fatigue life are also shown in figure 1. Here $b = -0.07$ and $c = -0.5$. For lives above 340 cycles the plastic strain range $\Delta\epsilon_p$ decreases rapidly, approaching very small values (10^{-6}) as the fatigue life reaches 1 million cycles.

The fatigue life as a function of strain range has been analyzed for the following materials to verify the bilinearity or otherwise of the M–C relationship. The base data points have been taken from references 7 and 8. The materials analyzed were

(1) 52100 Steel
(2) 4340 Steel

(3) 4130 Steel
(4) Inconel X
(5) Ti-6Al-4V
(6) 2014 T6 Aluminum alloy
(7) 4340 Steel (annealed)
(8) 1100 Aluminum

The following assumptions have been made in analyzing the data for these materials:

(1) The exponent b is constant over the life range investigated.
(2) A plastic strain of 10^{-6} is representative of negligible plastic strain at a life of 10^6 cycles to failure.

Figure 2 shows the relationship between the elastic strain range $\Delta\epsilon_{el}$ and the plastic strain range $\Delta\epsilon_p$, on a log-log plot for the materials indicated. For materials (1) to (6) the relationship shows two regions. The first region is below the plastic strain range of 0.003 to 0.005, in which the slope of the line is very small and the plastic strain tends to 10^{-6} when the elastic strain range reaches its value at 10^6 cycles to failure. In the second region the plastic strain is greater than the elastic strain and the slope of the line is larger. The relationship between the elastic and plastic strain ranges in a region is given in the form

$$\Delta\epsilon_{el} = \epsilon_0(\Delta\epsilon_p)^{n'} \tag{3}$$

The exponent n' has a comparatively low value in the plastic strain range from 10^{-6} to approximately 0.003 and a high value above 0.005. The value of n' in the higher strain ranges is approximately 0.15 to 0.2. In lower strain ranges the value of n' (denoted here as n'_1) depends on the high-cycle fatigue strength (defined as the elastic strain range at a cyclic life of 10^6) and lies in the range 0.035 to 0.06 for materials (1) to (6).

Materials (7) and (8) form a second set. Their relationship between $\Delta\epsilon_{el}$ and $\Delta\epsilon_p$ could be obtained only down to a plastic strain range of 0.001. For plastic strain ranges just below this the fatigue life has been observed to reach 1 million cycles. This indicates that the stress range at 1 million cycles is very nearly equal to the cyclic yield strength range $\Delta\sigma_{ys}$ of the material. The solid symbols in the figure are interpolated values that are used in the subsequent analysis.

From equations (1) and (2) it can be shown that the cyclic strain-hardening exponent n' can be related to the exponents b and c in the form

$$n' = b/c \tag{4}$$

The data showed that when n' decreased, the value of the exponent c increased correspondingly, with b remaining constant.

Figures 3(a) and (b) schematically show the relationship between the plastic strain range and the elastic strain range for the two sets of materials just described. For the first set the relationship is divided into two regions as discussed previously — one with a low plastic strain range where the elastic strain range is higher, and the other with a plastic strain range that is comparable to or higher than the elastic strain range, with the transition occurring around a plastic strain range of 0.002. In such materials the 10^6 cycle fatigue strength, $\Delta\sigma_{@10^6}$ will be below the cyclic yield strength range $\Delta\sigma_{ys}$. The plastic strain range will tend to 10^{-6}, or less, near $\Delta\sigma_{@10^6}/E$. Hence the slope of the $\Delta\epsilon_{el}$-$\Delta\epsilon_p$ line will be smaller in the lower region than in the higher strain range region. The hysteresis loops that could be obtained in the two regions are indicated schematically in the figure. For the second set there will be only one region and here the plastic strain range will be comparable to or higher than the elastic strain range. In these materials the stress range at 10^6 cycles will be very nearly equal to the cyclic yield strength range.

Figures 4(a) and (b) show the strain-range-versus-fatigue-life relationships for these two sets of materials. For the first set, where the 10^6 cycle fatigue strength $\Delta\sigma_{@10^6}$ is less than the cyclic yield strength $\Delta\sigma_{ys}$, it can be seen that, in the lower strain regions where the fatigue life is around 10^4 to 10^6 cycles, the plastic strain range will be quite small, and that the plastic strain range data fall well below the extended M–C line. At higher strain ranges where $\Delta\epsilon_p$ is greater than or equal to $\Delta\epsilon_{el}$, the points fall on the M–C line. Thus there will be a bilinear M–C relationship if the strain range covers the lower strain regions sufficiently. The transition life N_T will be small for these materials. For the second set of materials, where $\Delta\sigma_{@10^6}$ is very nearly equal to $\Delta\sigma_{ys}$, the plastic strain range dominates over the elastic strain range over a long lifespan. Hence the transition life N_T will be large. In such cases a linear relationship between the plastic strain range and the fatigue life can be expected over a long lifespan without deviation from linearity.

CASE STUDY

Figures 5(a) to (f) show the strain-range-versus-fatigue-life relationship for the first set of materials. Both the elastic strain range and the plastic strain range life relationships are shown. The plastic strain range values in the lower regions (solid symbols) have been obtained from figure 2. For these materials the transition life N_T is comparatively small. Just at or slightly beyond the transition life, a break in the $\Delta\epsilon_p$-N_f relationship can be noted (i.e., bilinearity of the M–C relationship occurs for these materials).

Figures 5(g) and (h) show the strain-range-versus-fatigue-life relationship for the second set of materials. Here the N_T value is comparatively large and the plastic strain range in the entire lifespan is either higher than or comparable to the elastic strain range. No break is observed in the M–C relationship in these materials.

From equations (1) and (2) the transition life N_T can be shown to be

$$N_T = (C/B)^{1/(c-b)} \tag{5}$$

Table I gives the values of the constants C, B, b, and c and the calculated values of the transition life N_T. For the first six materials investigated, excepting 4130 steel, the transition life is comparatively small. For the 4340 steel (annealed) and the 1100 aluminum alloy it is large. The 4130 steel, with a comparatively large value of the transition life N_T, may be a borderline case.

High values of C and low values of B give large N_T values and in such cases the bilinearity in the M–C relationship cannot be experienced in the possible lifespan. The M–C relationship as given by equation (1) with a single value for both C and c, will be valid over the lifespan of practical importance.

Table II gives the values of the exponents n' and c in both the higher strain range and the lower strain range regions (denoted here as n'_l and c_l). Both calculated and experimental values are shown.

The agreement appears to be good. Thus a change in the value of n' reflects itself in a change in the slope c of the M–C relationship.

For materials that exhibit bilinearity in the M–C low-cycle-fatigue relationship, extrapolation of the experimental data obtained in the high plastic strain ranges will lead to overestimation of fatigue life in the lower ranges because the slope c_l is steeper in the lower ranges. A typical overestimation in the case of the aluminum-lithium alloy that is discussed in conjunction with figure 1 is shown in figure 6. Life is overestimated by nearly an order of magnitude at a plastic strain range of 5×10^{-4}. Thus caution should be exercised in using the M–C relationship, especially for material whose transition life is small and where the cyclic stress range at 1 million cycles is less than the cyclic yield stress of the material. Furthermore the plastic strain range values beyond the transition life will rapidly decrease (below that corresponding to yielding), making it difficult for practical measurement or for use as design input. In such cases it is better to take the total strain range $\Delta\epsilon_t$ for design consideration of the fatigue life because this relationship is mainly governed by the dominant elastic strain component beyond the transition life and is not much influenced by the nonlinearity of the M–C low-cycle-fatigue relationship. Figure 6 also shows the relationship between the total strain range and the fatigue life with and without bilinearity in the M–C relationship. It can be observed that the $\Delta\epsilon_t$-N_f relationship is influenced only slightly by the bilinearity in the M–C relationship and hence is a better criterion for design purposes.

CONCLUSIONS

From this study on the bilinearity of the Manson-Coffin (M–C) low-cycle-fatigue relationship in metallic materials, the following conclusions were drawn:

1. Metallic materials whose cyclic stress range corresponding to 1 million cycles is below the cyclic yield strength range of the material will tend to exhibit a bilinear M–C relationship. The transition in the relationship between the plastic strain range and the number of cycles to failure $\Delta\epsilon_p$-N_f occurs at $\Delta\epsilon_p$ of approximately 0.003 to 0.005. In such materials the relationship between the elastic and the plastic strain ranges, given in the form

$$\Delta\epsilon_{el} = \epsilon_0(\Delta\epsilon_p)^{n'}$$

will exhibit two values for the exponent n': In the lower ranges of the strain n' will be smaller; in the higher ranges n' will be larger. The change in the value of n' will be reflected in the slope c of the $\Delta\epsilon_p$-N_f relationship because n' is related to the slope b in the form $n' = b/c$ with the slope b remaining constant.

2. In some materials, such as 4340 steel (annealed) and 1100 aluminum, the elastic stress range at 1 million cycles will be very nearly equal to the cyclic yield strength range. Such materials may not exhibit a bilinearity in the M–C relationship.

3. The smaller the transition life N_T, the higher are the chances of bilinearity in the M–C relationship.

4. Total strain range is best to use in longer life regimes because the plastic strain range can become too small to be determined accurately, especially if a bilinear M–C relationship exists.

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TABLE I.—CALCULATION OF TRANSITION LIFE N_T

[$N_T = (C/B)^{1/(c-b)}$.]

Material	C	B	c	b	N_T
(1) 52100 Steel	0.13	0.02	−0.45	−0.06	120
(2) 4340 Steel	.60	.015	−.55	−.08	2 560
(3) 4130 Steel	1.00	.013	−.55	−.08	10 300
(4) Inconel X	.60	.02	−.60	−.12	1 200
(5) Ti-6Al-4V	1.00	.033	−.66	−.10	445
(6) 2014 T6 Aluminum	.60	.02	−.63	−.08	485
(7) 4340 Steel (annealed)	.65	.013	−.55	−.11	11 000
(8) 1100 Aluminum	2.0	.0045	−.66	−.066	30 000

TABLE II.—CALCULATION OF c_1 IN LOWER STRAIN RANGE REGION

[c is exponent in higher strain range region; c_1 is exponent in lower strain range region; n' is exponent in higher strain range region; n'_1 is exponent in lower strain range region; $n' = b/c$; and $c_1 = b/n'_1$.]

Material	n'		n'_1		c_1	
	Experiment	Calculated	Experiment	Calculated	Experiment	Calculated
(1) 52100 Steel	0.15	0.14	0.06	0.054	−1.12	−1.0
(2) 4340 Steel	.15	.15	.06	.057	−1.4	−1.33
(3) 4130 Steel	.15	.15	.035	.035	−2.3	−2.3
(4) Inconel X	.2	.2	.06	.067	−1.8	−2.0
(5) Ti-6A1-4V	.15	.15	.04	.050	−2.0	−2.5
(6) 2014 T6 Aluminum	.14	.13	.045	.053	−1.5	−1.77
(7) 4340 Steel (annealed)	.2	.18	------	------	------	-------
(8) 1100 Aluminum	.15	.1	------	------	------	-------

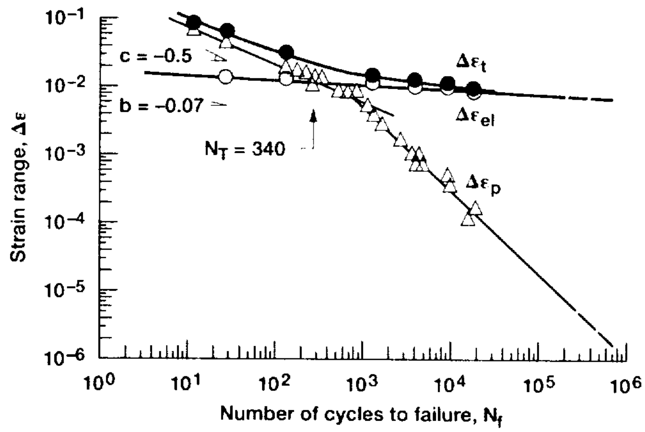


Figure 1.—Relationship between strain range and fatigue life of an aluminum-lithium alloy. Data from reference 1. (The transition life N_T shown is calculated based on equation (5).)

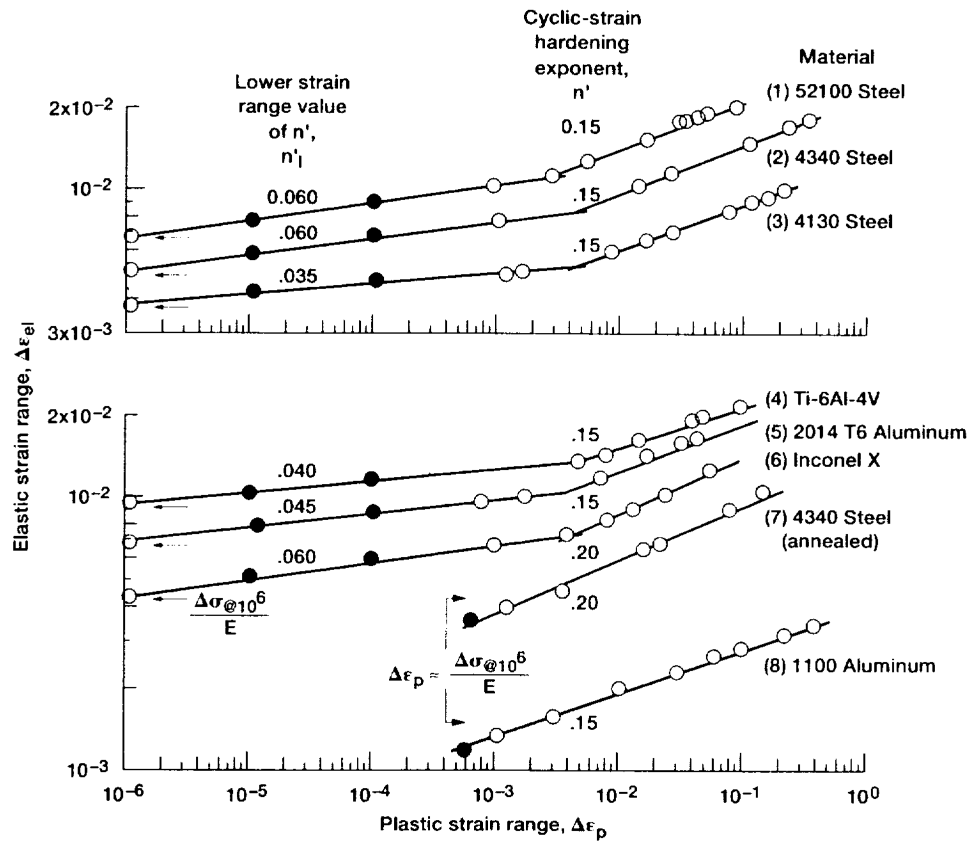
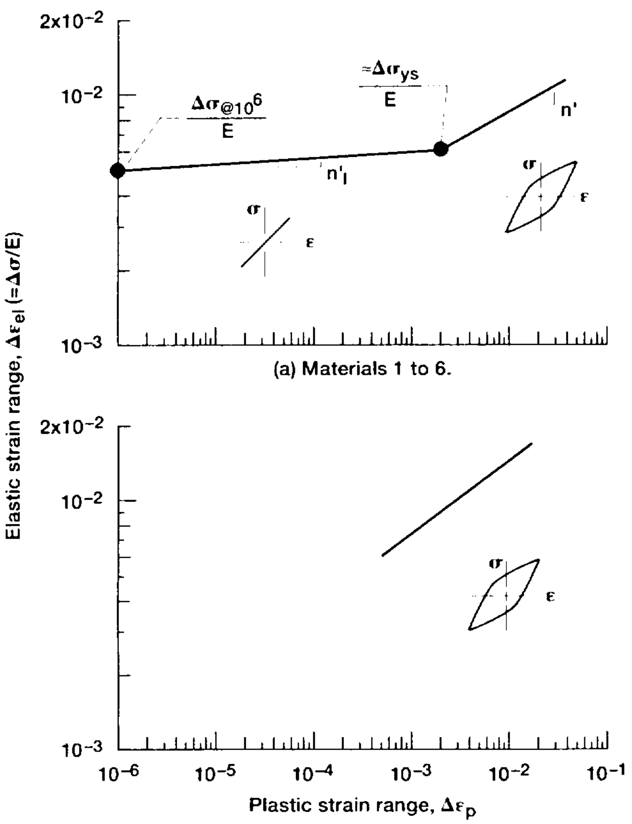
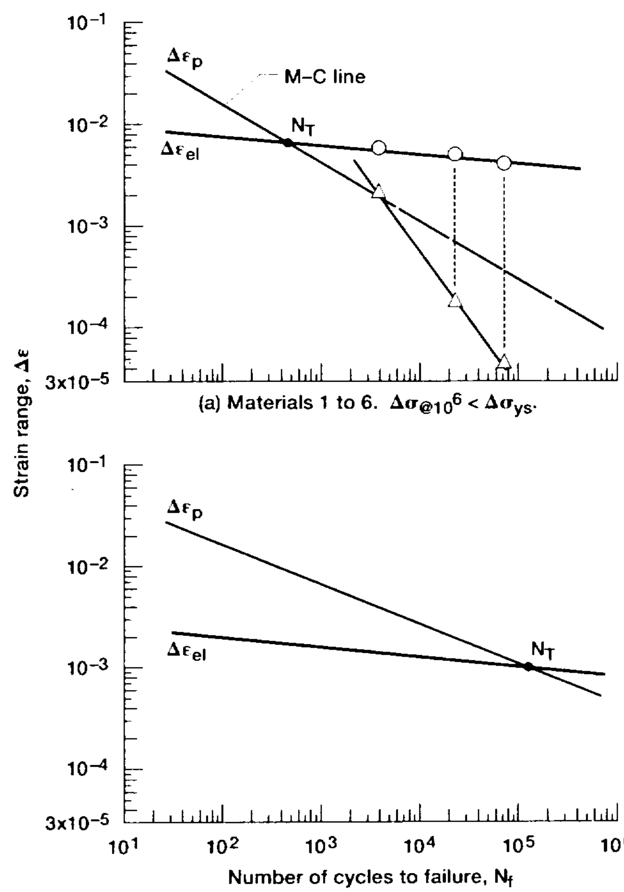


Figure 2.—Relationship between elastic strain range and plastic strain range for materials investigated. (Points corresponding to 10^{-6} plastic strain are obtained from the stress range at 1 million cycles to failure. The solid points are interpolated values that are used in the extrapolation of the Manson-Coffin relationship in figure 4.)



(b) Materials 7 and 8.

Figure 3.—Schematic presentation of relationship between elastic and plastic strain.



(b) Materials 7 and 8. $\Delta\sigma_{@10^6} \approx \Delta\sigma_{ys}$.

Figure 4.—Schematic presentation of relationship between strain range and fatigue life.

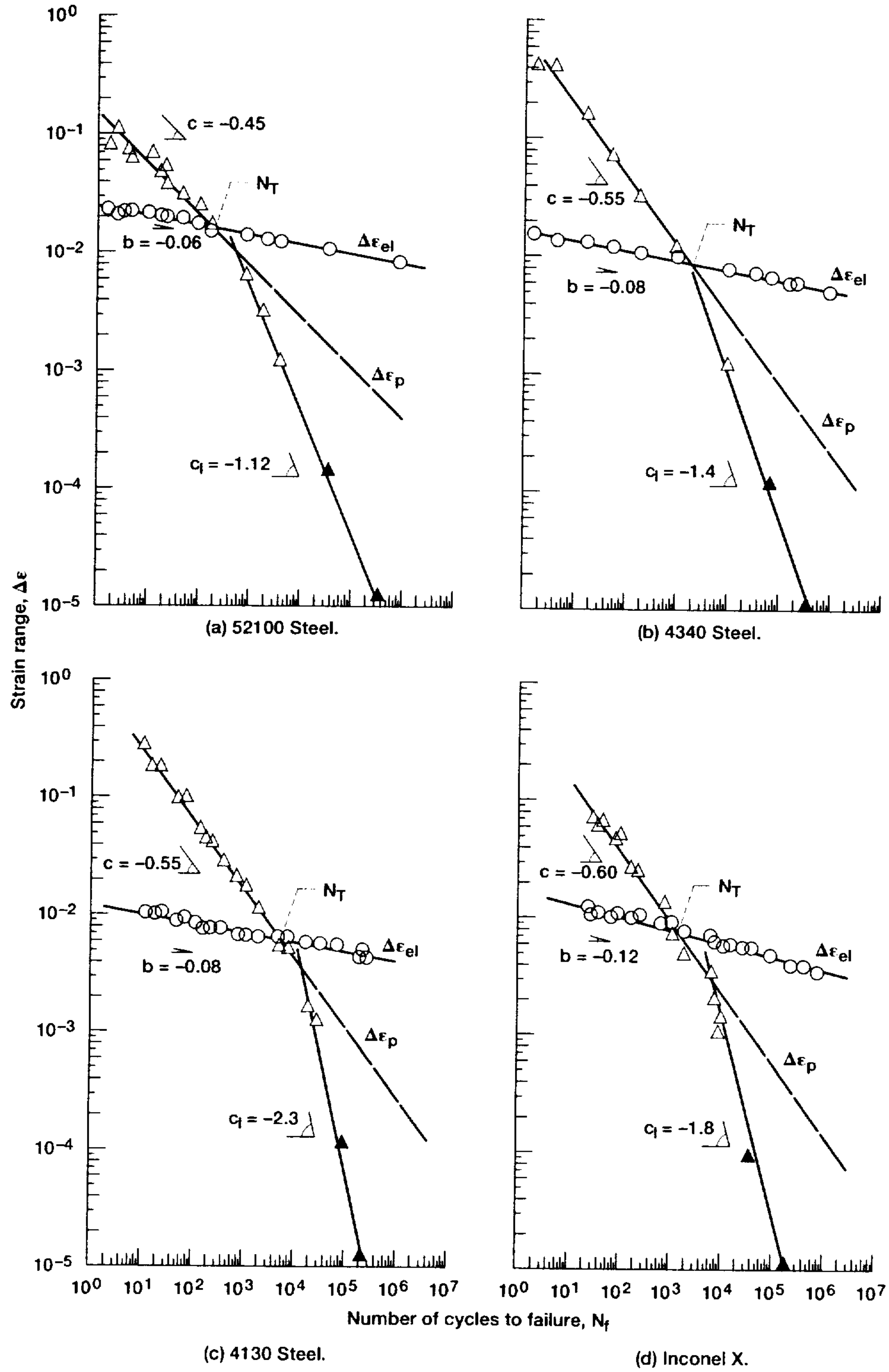
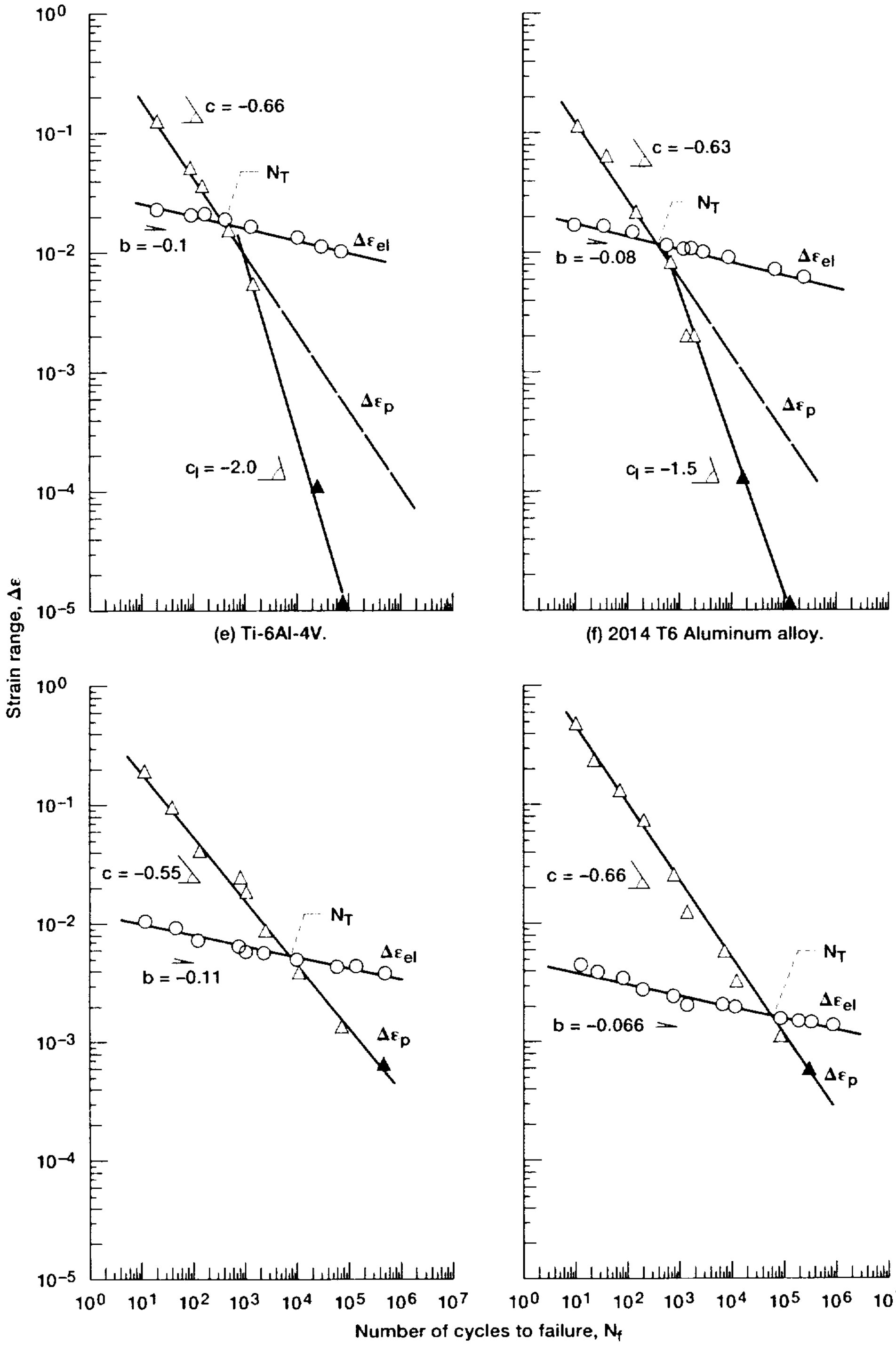


Figure 5.—Relationship between plastic strain range and fatigue life and between elastic strain range and fatigue life for different materials investigated. (The filled points were obtained from interpolation in figure 2.)



(g) 4340 Steel (annealed).

(h) 1100 Aluminum.

Figure 5.—Concluded.

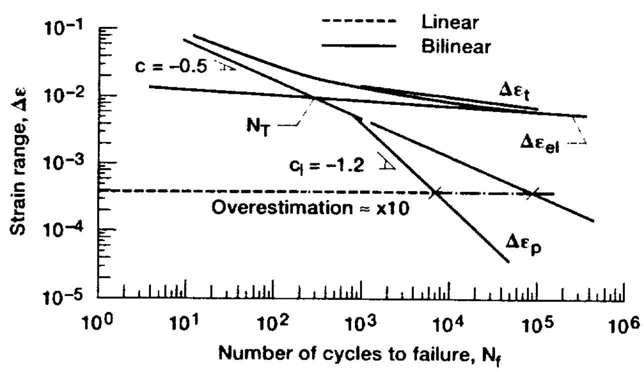


Figure 6.—Relationship between strain range and fatigue life for an aluminum-lithium alloy.

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V.M. Radhakrishnan, National Research Council–NASA Research Associate at Lewis Research Center; present address: Indian Institute of Technology, Madras–60 036, India. Work performed when National Research Council–NASA Research Associate. Responsible person, Gary Halford, (216) 433-3265.

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