

1992

NASA/ASEE Summer Faculty Fellowship Program

Marshall Space Flight Center  
The University of Alabama

PROBLEMS INVOLVING COMBINED LOADING

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The first problem was to determine the capability of a ground support equipment (GSE) rack knee bracket for handling a spacelab rack. The geometric center of gravity was calculated for the upper and lower part of the rack and found to be in the center of gravity's allowable envelope (Spacelab Payload Accommodation Handbook, SPAH, Appendix B page B3-154). Using the corners of the center of gravity envelopes, free body diagrams (FBD) were set up to represent each of the four cases. Moments about point P are caused by the force at G and the weight of the upper and lower part located at the respective centers of gravity (figures 1a-b). The greatest moment would occur when the centers of gravity are located at point 1 for the upper part and point 2 or 3 for the lower part (figure 1a). These locations are at the greatest distance from point P thus causing the greatest moment (figure 1b). Using basic static procedures and a safety factor of 5, this case will give  $R_{sy} = R_y = 2944$  lbs and  $R_z = 7500$  lbs for a maximum upper rack weight of 375 lbs and a maximum lower rack weight 1125 lbs. In the X-Z plane, the center of gravity may be 2 inches off center (figure 1c). This will give  $R_y \text{ max} = 1614$  lbs and  $R_z \text{ max} = 4112$  lbs.

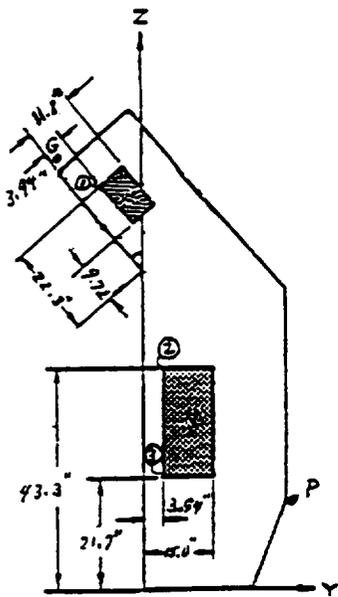


Figure 1a  
Side View

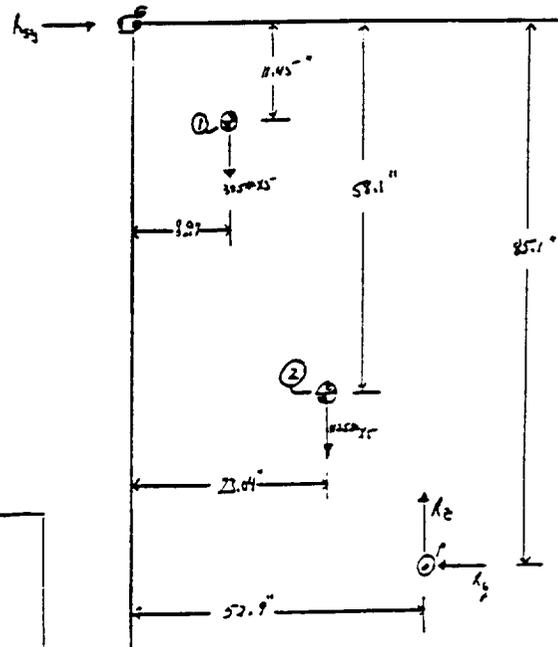


Figure 1b  
FBD Side View

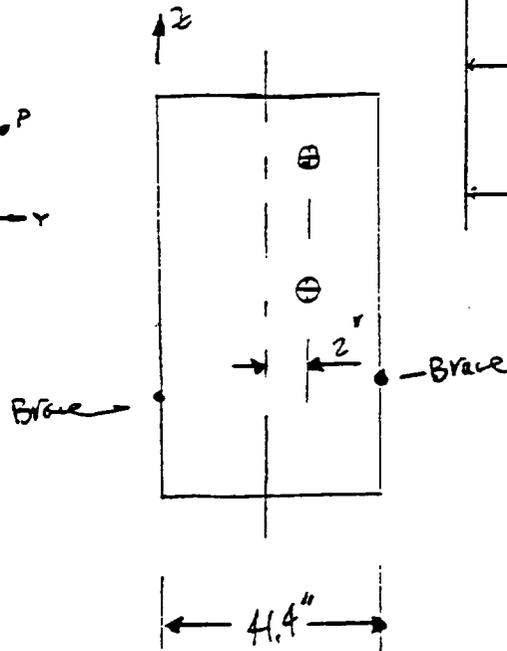


Figure 1c  
Front View

The second problem was to determine the exact margin of safety for an axial load and a shear load on a bolt. The equation for failure is axial load squared plus shear load cubed equal one. This gives a curve shaped like the following:

$$a^2 + v^3 = 1$$

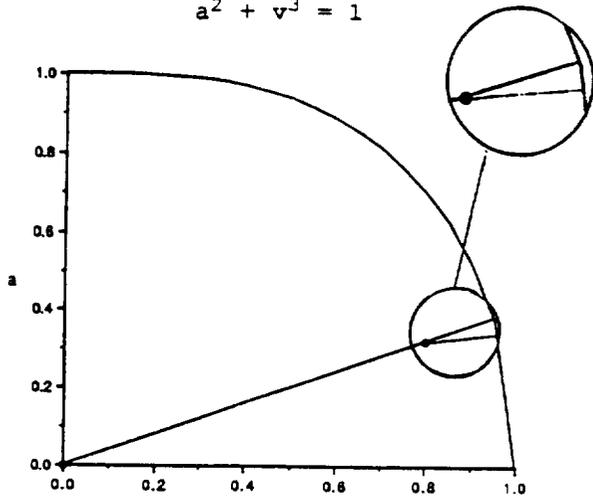


Figure 2

With the performance level located under the curve, the margin of safety is the shortest distance to the curve. The traditional method for selecting a margin of safety is the distance from the performance level to the curve located on a straight line through the origin. This is a good enough approximation of the margin of safety except when the bolt is close to failure. Example; close to the curve (figure 2). The shortest distance would always be the minimum distance from the point of performance level to the curve. Using calculus to minimize distance, a seventh degree equation emerges. If specific points of performance are known, it would be

possible to solve. Clearly, additional study is needed.

The third problem was to simplify an expression for stress on a generic non-symmetrical bolt configuration to a form familiar to "bolt people." This form is

$$P_b = PLD + \eta \phi P_{ext} \quad [1]$$

Where  $P_b$  is the total load,  $PLD$  is the initial bolt preload,  $\eta$  is the loading plane factor,  $\phi$  is the stiffness factor, and  $P_{ext}$  is the external load.

$$P_b = PLD + \{P_{ext}\} \frac{\frac{A D k_a k_b}{A D k_a + (A + D) k_b}}{\frac{B C k_a}{B + C} + \frac{A D k_a k_b}{A D k_a + (A + D) k_b}} \quad [2]$$

Using conditions given by the problem, this equation was algebraically reduced to

$$P_b = PLD + \{P_{ext}\} \left( \frac{B + C}{B C} \right) \frac{k_b}{k_a + k_b} \quad [3]$$

which is in the same form as equation [1]. This will be presented in a paper by Henry Lee, MSFC.

The final problem was the structural analysis of the spacelab rack corner posts. The minimum crippling strength of front and rear corner posts (FCP and RCP) was calculated by two methods. The Gerard Method which gives a single value and the Alcoa method which should give values at specific points. The Gerard method equation is

$$\frac{F_{cs}}{F_{cy}} = 0.56 \left[ \frac{gt^2}{A} \left( \frac{E}{F_{cy}} \right)^{0.5} \right]^{0.85} \quad [4]$$

(TM-SEAD-85039A page A.3-21) where  $F_{cs}$  is minimum crippling strength (ksi),  $F_{cy}$  is the minimum allowable compressive yield stress (55 lbs for T73), and  $E$  is the modulus of elasticity in compression ( $10.5 \times 10^3$  ksi). The allowable design stress is  $0.9F_{cs}$ . The Alcoa equations are

$$B_p = F_{cy} \left[ 1 + \frac{(F_{cy})^{\frac{1}{3}}}{11.4} \right] \quad [5]$$

$$D_p = \frac{B_o}{10} \left( \frac{B_o}{E} \right)^{\frac{1}{2}} \quad [6]$$

$$C_p = 0.409 \frac{B_o}{D_p} \quad [7]$$

$$\lambda_p \frac{t}{b} = 2.89 \text{ no free edge or } 1.24 \text{ one free edge} \quad [8]$$

$$\lambda_p \leq C_p; F_{cs} = B_p - \lambda_p D_p \quad [9]$$

$$\lambda_p > C_p; F_{cs} = \frac{\Pi^2 E}{(\lambda_p)^2} \quad [10]$$

(TM-SEAD-850039A page A.3-23 - a.3-24). These values have been calculated for all rack posts and are ready for the test data correlation.

In order to get crippling values from test data, both axial loads and moment values must be applied. The loads were applied as FPA at the center of gravity,  $FM_x$  on the y axis, and  $FM_y$  on the x axis.  $F$  is the axial load,  $M_x$  is the moment in the x direction,  $M_y$  is the moment in the y direction, and  $L$  is the distance from the center of gravity where  $FM_x$  and  $FM_y$  are applied. Therefore,

$$M_x = FM_x(L) \quad [11]$$

$$M_y = FM_y(L) \quad [12]$$

$$F = FPA(L) - M_y + M_x \quad [13]$$

(SL-DEV-ED92-012 figure 3 page 10).

A sample test specimen of a FCP of actual flight hardware was used to correlate measured strain gage data with calculated strain values. No measurements were taken of the sample specimen. The strain values were calculated using the following formulas and SPAH B3-195.18 for minimum FCP gross cross section area,  $I_{x_o}$ ,  $I_{y_o}$ , and  $I_{x_o y_o}$  values. Axial stress is

$$\sigma_a = \frac{F}{A} \quad [14]$$

where F is axial load and A is cross section area. Bending stress is

$$\sigma_b = \frac{M_x}{I_x I_y - I_{xy}^2} (I_{xy} X - I_y Y) + \frac{M_y}{I_x I_y - I_{xy}^2} (I_x X - I_{xy} Y) \quad [15]$$

(SPAH B3-195.12)

$$\sigma_{TOT} = \sigma_a + \sigma_b \quad [16]$$

To convert stress to strain:  $\epsilon = \frac{\sigma_{TOT}(1-\nu^2)}{E}$  Adjustments of 0.03 inch had to be made in both the  $X_0$  and  $Y_0$  directions to get a good curve shape between the test data and the theory. When the measurements from SPAH B3-195.18 were used, the correlation on the 10,000 lb axial load was very good, but the moment values were off. After a study of the FBD, it was determined the moment was not equally applied over the length of the specimen as originally thought, but decayed in a triangular loading such that the moment at the strain gage locations was approximately 57% of the moment at the top. When this adjustment was made, the correlation between experiment data and theory was excellent (figure 3). Another problem was

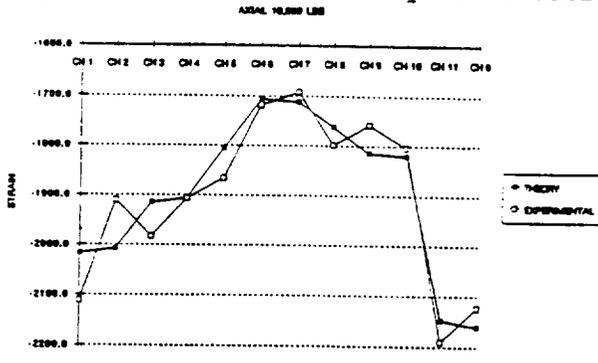


figure 3

the correlation of data at two of the channels. After examining the test specimen, it was discovered that these two channels were next to a bolt hole. This would cause the stress to be concentrated at a higher value (up to 35% more) at that location (Timoshenko, Str. of Mat. Part II page 304). A 10% adjustment in the theory values at these two channels brought them in line with the experimental data.

The testing will continue for some months more. The plans are to correlate the experimental data for the other test specimens using strain gages with the theoretical calculations. The experimental data for crippling values will then be compared with the theoretical crippling data. The goal of the tests is to be able to predict with some accuracy the crippling values of the rack posts.

#### REFERENCES

1. Spacelab Payload Accommodation Handbook, SLP/2104-2 Appendix B Structure Interface Definition NASA MSFC (1985)154-195.18
2. Spacelab Rack Corner Post Load Characterization Test and Checkout Procedure SL-DEV-ED92-012 (1992) Figure 3, page 10
3. Timoshenko, S., Strength of Materials Part II, third edition D. Van Nostrand Co. Inc., New York (1956) 304
4. TM-SEAD-85039A NASA Contract NAS8-32350, DR SE-12,WBS 4.2.21 (1986) A.3-21 - A.3.24