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BASIC RESEARCH ON DESIGN ANALYSIS METHODS

FOR

ROTORCRAFT VIBRATIONS

NAG 1-1007

submitted by

S. Hanagud

Professor, School of Aerospace Engine

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332-0150
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INTRODUCTION

Background

The vast bulk of the work reported to date on identification of structural dynamic systems has focused on identifying mathematical models that reproduce test results, but little consideration has been given to the physical basis for the identified system equations. Typically, the identification procedures make systematic adjustments to the system equation, commonly to the stiffness and/or mass matrices but also to the damping matrix, so that the identified eigenvalues and eigenvectors reproduce as closely as possible the results measured in tests. The result of this process is almost inevitably identified mass, stiffness and damping matrices that are fully populated, that is, which have nonzero values for almost all elements. Such matrices, while capable of producing plausible eigenvalues and eigenvectors, can nonetheless be physically implausible in the sense that the large numbers of nonzero elements throughout the system matrices implies direct connectivity among the degrees of freedom that does not exist physically.

Identified mathematical models that are based on physically implausible system matrices may be quite acceptable if the objective of the study is to develop a *simulation* model. However, such results for analysis purposes are generally unsatisfactory because it is difficult or impossible to relate specific features of the physical system to the analysis results. This problem is particularly troublesome when the objective of the identification of a system model from experimental measurements is an accurate system model that, in turn, will be used to make modifications to or improvements in the original physical system. Such an example might be the modification of an existing aircraft structure to accommodate a new mission. In this case it would be desirable to formulate a structural model for the present structure, verify its accuracy against experimental measurements, and then use it as the basis for the modifications. When the verification process yields identified system matrices that are mathematically acceptable but physically implausible, the resulting model may be useless as the basis for future structural modifications.

The objective of the present work was to develop a method for identifying physically plausible finite element system models of airframe structures from test data. The assumed models were based on linear elastic behavior with general (nonproportional) damping. Physical plausibility of the identified system matrices was insured by restricting the identification process to designated physical parameters only and not simply to the elements of the system matrices themselves. For example, in a large finite element model the identified parameters might be restricted to the moduli for each of the different materials used in the structure. In the case of damping, a restricted set of damping values might be assigned to finite elements based on the material type and on the fabrication processes used. In this case, different damping values might be associated with riveted, bolted and bonded elements.

The method itself is developed first, and several approaches are outlined for computing the identified parameter values. The method is applied first to a simple structure for which the "measured" response is actually synthesized from an assumed model. Both stiffness and damping parameter values are accurately identified. The true test, however, is the application to a full-scale airframe structure. In this case, a NASTRAN model and actual measured modal parameters formed the basis for the identification of a restricted set of physically plausible stiffness and damping parameters.

Review of Previous Pertinent Work

Airframes are generally modelled using powerful finite element analysis packages such as *NASTRAN* that are capable of representing quite detailed aspects of the structural system. The accuracy of such models is determined by comparing the analytical results with flight or ground vibration test results. In the case of helicopter airframes, several recent efforts have focused on the correlation of *NASTRAN* model data with ground vibration test data[1-3]. The conclusions reached in these studies suggest that in cases where there is some degree of correlation, the model frequencies compare favorably with test frequencies, but generally only in the low frequency range

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below about 15 Hz [1-2]. The frequency response functions at selected locations also compare reasonably well in this range. Outside this range the comparisons are generally unsatisfactory, and the eigenvectors do not usually compare favorably in either range.

Although there have been numerous contributions to the literature in the area of the identification of structural dynamic systems[4-25], the majority of reported methods are based on simply adjusting the elements of one or more of the K, M, and C matrices. While this approach is capable of yielding a system matrix whose eigenvalues and eigenvectors suitably match measured results, the methods generally lose all physical interpretability inherent in the original K, M and C matrices by not maintaining relationships among elements dictated by the model topology. These difficulties are compounded for large-scale models with thousands of degrees of freedom.

Kuo and Wada[25] used nonlinear sensitivity coefficients (NSC) in the identification procedure. Their sensitivity coefficients are between the system parameters and eigenvalues. In the present work the interest is in the change of system matrices as a function of physical variables of the structure. A different type of sensitivity coefficient between system matrices and physical variables has therefore been developed.

The most significant achievement in the present work[30] is to preserve the physical interpretability of the M, C, K matrices so that the identification can provide evidence of possible sources of erroneous modeling and point to specific regions of the model that are unduly sensitive and need additional consideration in modeling. The identification procedure developed in this paper is capable of adjusting physical quantities such as boundary conditions, moments of inertia, stiffnesses, damping or other selected physical parameters.

PROGRESS DURING THIS REPORTING PERIOD

Our previous work was tested on simple analytical models, simple structural systems like beams, and test results from AH1G Helicopter. The method yielded reasonably accurate identification of models and preserved physical interpretability of the system matrices. However, the use of proportional or nonproportional damping and nonlinear sensitivity coefficients did not adjust the model in certain regions. A careful examination suggested that we will need a more general representation of the damping behavior. In order to accommodate damping parameters, other than the linear viscous damping, we have studied the possible use of Hammerstein integral equation based models. These models can accommodate both linear and nonlinear systems. They have been used to consider nonlinearities in forcing terms. We have modified this approach to include linear and nonlinear damping. We have studied linear (nonproportional damping) and Coulomb damping. The appendix to this progress report contains some the details of our approach.

WORK PLANNED FOR THE NEXT REPORTING PERIOD

As a next step, we will study methods of including structural damping. Following this work, we would like to include the modified Hammerstein approach in our identification procedure that can preserve the physical interpretability of system matrices.

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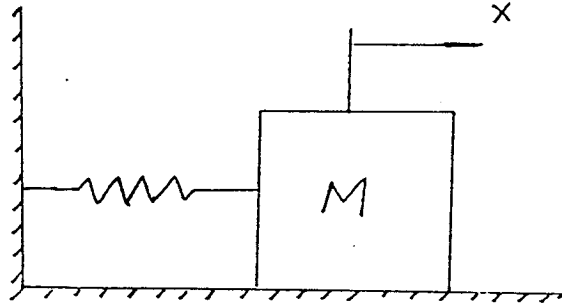
THE APPENDIX

Figure 1.1: A mass-spring system

The identifying the constant c of a Single Degree of Freedom (SDOF) nonlinear dynamic system and the estimating the parameters of Multiple Degree of Freedom (MDOF) nonlinear dynamic system have been illustrated in this report.

These identification procedures are based on various models of nonlinear dynamical systems. Usually, a nonlinear system is represented by a set of nonlinear differential or integral equations. In many practical applications, an input-output approach of a nonlinear dynamical system is a means of describing a relationship between the input and the output of the system in some straightforward way and is considered to be more useful.

An approach for modeling a nonlinear dynamical system is by the use of Volterra Series

[1],[2].

$$
\begin{aligned}
x(t) \;=\; & \int_0^t h_1 u(t - \tau)d\tau \\
+ \; & \int_0^t \int_0^t h_2(\tau_1, \tau_2) u(t - \tau_1) u(t - \tau_2) d\tau_1 d\tau_2 \\
+ \; & \int_0^t \int_0^t \int_0^t h_3(\tau_1, \tau_2, \tau_3) u(t - \tau_1) u(t - \tau_2) \\
& u(t - \tau_3) d\tau_1 d\tau_2 d\tau_3 + \cdots
\end{aligned}
\tag{1.0.3}
$$

The Volterra Series, Eq (1.0.3), is a functional series, It maps past inputs into the present output. This means that many kernel values are required to estimate. Several techniques have been presented [3],[4], [5]. Because we have to decide which terms of Volterra Series are necessary for a given practical problem and to estimate many kernel values, the procedure of identification is usually a difficult procedure.

Several other simple block-oriented input-output models for representing nonlinear dynamical systems are as folloows. [7].

- Simple Hammerstein Model.

- Generalized Hammerstein Model.

- Simple Wiener Approach.

- Generalized Wiener Approach.

- Extended Wiener Approach.

- Simple Wiener-Hammerstein Approach.

- Generalized Wiener-Hammerstein Model.

- Extended Wiener-Hammerstein Model.

The block-oriented models have been widely used because of their simplicity.

In 1985, a nonlinear difference equation model NARMAX (Nonliear Autoregressive Moving Average Models with inputs) was presented by Leontaritis and Billings [9],[10]

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. The NARMAX model is considered as an unified representation of a finitely realizable nonlinear system. The finitely realizable nonlinear system in essence means that the state space of the system can not be infinite dimensional. This model maps the past inputs and outputs to current output. For the SISO (single input and single output) nonlinear dynamical system with white noise, it can be denoted by [11]

$$x(k) = F[x(k-1), ..., x(k-n_x), u(k-1), ..., u(k-n_u)] \qquad (1.0.4)$$

Where $F(*)$ is an unknown nonlinear function. In general, it will be determined for a given real sampled nonlinear system. Leontaritis and Billings proved that a nonlinear discrete time invariant system can always be denoted by Eg.(1.0.4) in a region around an equilibrium point, if the response function of system is finitely realizable and a linearized model exists at the chosen equilibrium.

The NARMAX model is derived assuming zero initial state response, but it can be carried over to the non-zero-initial-state cases. The response functions of a system are different for different initial condition, but the input-output NARMAX model for the system will always be the same within a region around an equilibrium point. Several simple forms of the NARMAX model have been proposed for nonlinear dynamic system identification, such as the Bilinear Model.[11],[12]

$$\begin{aligned} x(k) &= a_0 + \sum_{i=1}^{n} a_i x(k-i) + \sum_{i=1}^{n} b_i u(k-i) \\ &+ \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x(k-i) u(k-j) \end{aligned} \qquad (1.0.5)$$

the frational model.[11], [13],[14]

$$x(k) = \cfrac{b[x(k-1), \cdots, \\ x(k-r), u(k-1), \cdots, u(k-r)]}{a[x(k-1), \cdots, \\ x(k-r), u(k-1), \cdots, u(k-r)]} \qquad (1.0.6)$$

Haber and Unbehauen [7] prefer the NARMAX model, because the NARMAX model is parametric and has fewer parameters than the Volterra series.

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In aerospace engineering applications, a nonlinear structural dynamical system is usually described by a system of nonlinear differential equations. In SISO case , the nonlinear differential equation of a system is of the form

$$\ddot{x} + b\dot{x} + cx + f(\dot{x}, x) = u(t) \tag{1.0.7}$$

where $f(*)$ is a nonlinear fuction of $\dot{x}$,x. If $f(*)$ is represented by a polynomial extension for simplicity, Eq.(1.0.7) becomes

$$\ddot{x} + b\dot{x} + cx + \alpha_2 x^2 + \alpha_3 x^3 + ... +$$
$$\beta_2 \dot{x}^2 + \beta_3 \dot{x}^3 + ... = u(t) \tag{1.0.8}$$

Every term in Eq.(1.0.8) has a distinct physical meaning. Identifying the parameters of Eq.(1.0.8) are useful for dynamic analysis, structural dynamic design, control and design modification. If the nonlinear structural dynamic system is modeled by using Eq.(1.0.8), the problem of the identification of a system is to estimate the parameters : $b, c, \alpha_2, \cdots, \beta_2, \cdots$.

Many techniques for estimating these parameters have been proposed. Hanagud, Meyyappa and Craig (1985) [15] used the method of multiple scales to formulate a procedure for identification of parameters of Eq.(1.0.8). Mook(1988) [16] used a model error method to find the model error $d(t)$ which represents the nonlinear terms of the nonlinear dynamic system and then estimated the nonlinear parameters from $d(t)$ by using a least square method. Yun and Shinozuka [17] proposed an approach that is based on two versions of Kalman filter for identifying the parameters. Ibanez [18] used an approach for estimating parameters in which it is assumed that the system response is dominated by a periodic response at the forcing frequency and an approximate transfer function is constructed. Broersen [19] replaced nonlinear terms in the equation by a series expansion for a system subjected to random excitation. Distefano and Rath, Yun and Shinozuka [20] [21] described several methods of of identification and applied nonlinear Kalman filtering techniques for estimation.

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If a structural control is considered, an input- output approach of nonlinear structural dynamic system in time domain and its parameter identification is useful. For this purpose, the Hammerstein Feedback Model (HFM) has been considered here.

Example

In practical egineering, the real damping usually is different from design damping. Identification of the difference is useful for analysis, design, and control. If the mass matrix $[K]$, stiffness matrix $[K]$, and damping matrix $[C]$ are known, the defference of damping can be estimated by using Hammerstein Feedback Model. The defference of damping is assumed to be $[dC]$. We assume a linear dynamic system as following deferential equation.

$$[M][\ddot{x}] + ([C] + [dC])[\dot{x}] + [K][x] = [F]$$

where diference of damping is assumed as

$$[dC] = \begin{bmatrix} 0.05 & -0.005 \\ -0.005 & 0.05 \end{bmatrix}$$

The resposes of displacement $x_1(t)$, $x_2(t)$ and velocity $\dot{x}_1(t)$, $\dot{x}_2(t)$ are obtained by using Runge-Kutta method and shown in Fig. 1
The HFM of the system is assumed as

$$x_1(k) + a_1 x_1(k-1) + a_2 x_1(k-2) + a_3 x_2(k-2) + \cos 0.5(k-2)$$
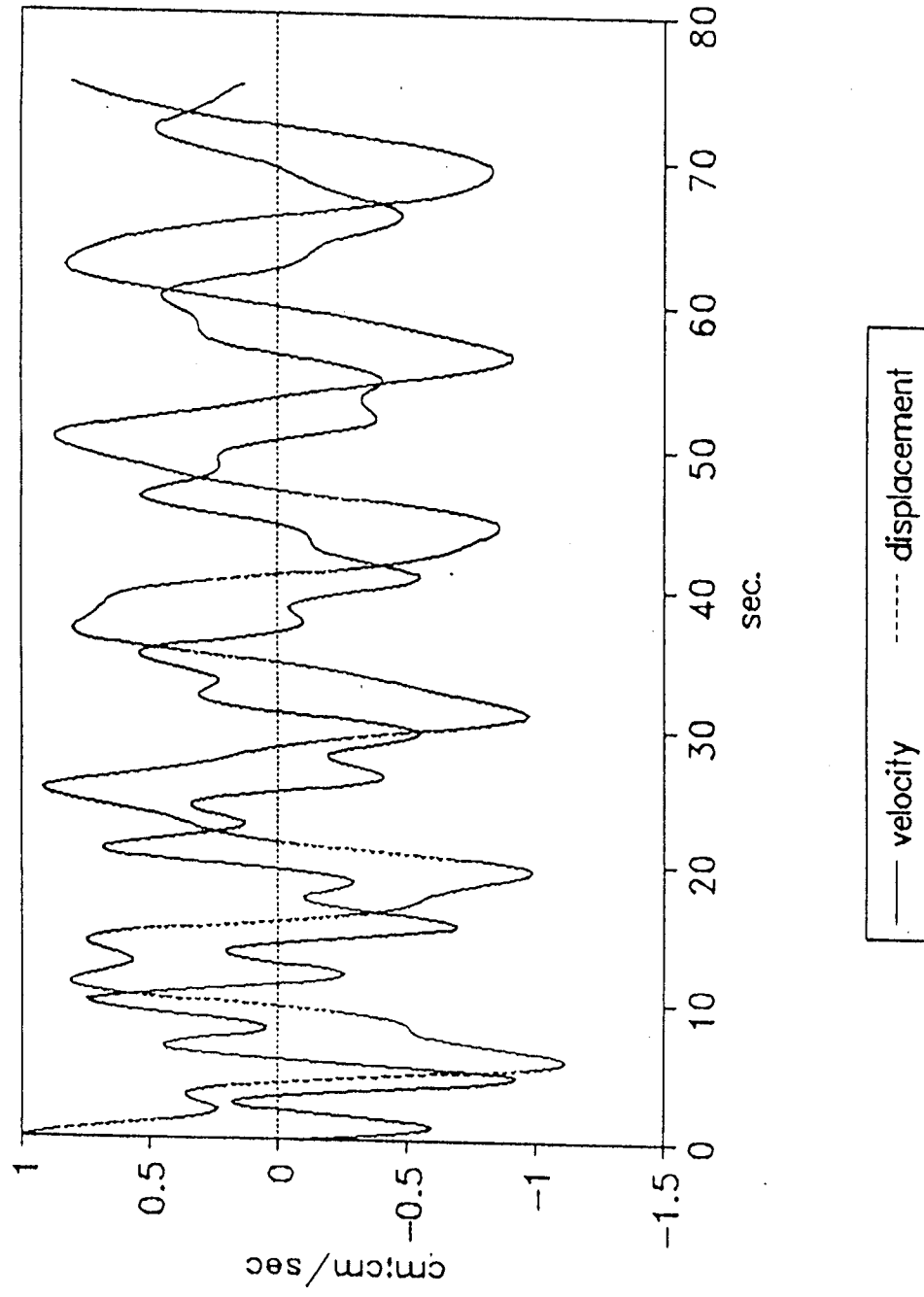$$= a_4 \dot{x}_1(k-2) + a_5 \dot{x}_2(k-2)$$

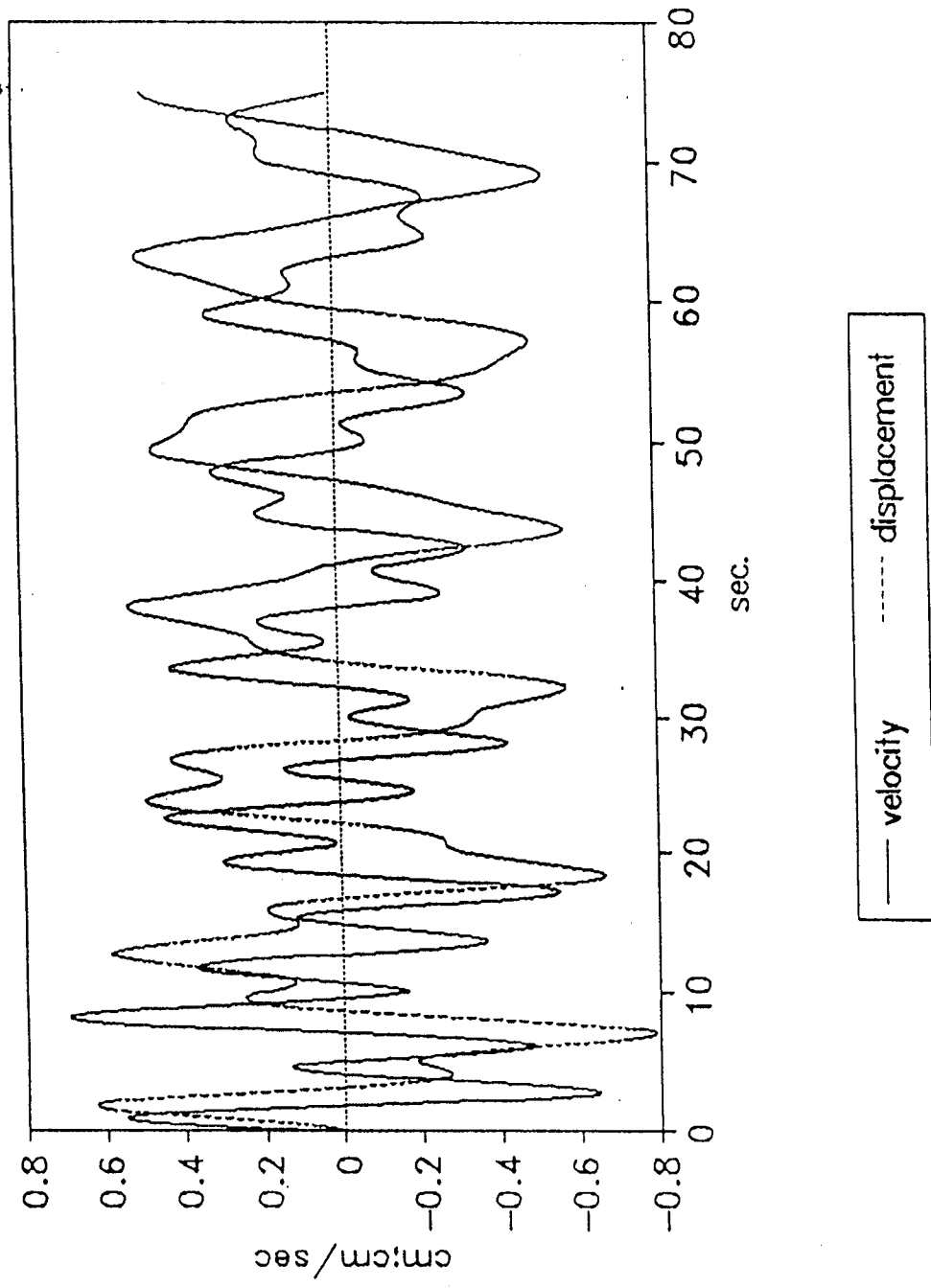Figure   The responses x_1 and $\dot{x}_1$ of two degree of freedom nonlinear dynamic system with Coulomb damping

Figure .: The responses x_2 and $\dot{x}_2$ of two degree of freedom nonlinear dynamic system with Coulomb damping.

and

$$x_2(k) - b_1 x_2(k-1) - b_2 x_2(k-2) + b_3 x_1(k-1)$$

$$= b_4 \dot{x}_2(k-2) + b_5 \dot{x}_1(k-2)$$

where a_1, a_2, a_3, b_1, b_2, b_3 are calculated from $[M]$, $[K]$. The $[dC]$ has elements:

$$dc_{11} = \frac{a_4}{(\Delta t)^2}$$

$$dc_{12} = \frac{a_5}{(\Delta t)^2}$$

$$dc_{21} = \frac{b_5}{(\Delta t)^2}$$

$$dc_{22} = \frac{b_4}{(\Delta t)^2}$$

$\Delta t = 0.05$ and 500 samples of the input and output are considered, then the estimated parameters are shown in table 9.

Table 1

Exact	Estimated by Mook	Estimated by HFM
0	0.00001	0.00002
0.05	0.0492 error:1.6%	0.0499547 error:0.09%

Table 2

Exact	Identified	Error
0	0.000012	
0.0025	0.0025030	0.12%

Table 3

Exact	Estimated	Error
0	0.0000064	
0.0025	0.00256	2.4 %

Table 4

P	True P	Est. P	Error
a1	-1.995	-1.995	0
a2	1	1	0
a3	1	0.99864	0.136 %
a4	-0.2	-0.1994	0.06 %
a5	0.3	0.298934	0.35 %
a6	-0.3	-0.298984	0.35 %
a7	0.1	0.099702	0.29 %

Table 5

P	True P	Est. P	Error
b1	-1.995	-1.995	0
b2	1	1	0
b3	1	0.99952	0.048%
b4	-0.2	-0.19919	0.07%
b5	0.3	0.298169	0.6 %
b6	-0.3	-0.298756	0.4 %
b7	0.1	0.099566	0.4 %

Table 6

P	True P	Est. P	Error
a1	-1.99	-1.99002	0.005 %
a2	0.995	0.995012	0.0012%
a3	2	1.98684	0.66 %
a4	-1	-0.994756	0.5 %
a5	-0.2	-0.198628	0.69 %
a6	0.3	0.302082	0.69%
a7	-0.3	-0.3061	2 %
a8	0.1	-0.946464	5 %

Table 7

P	True P	Est. P	Error
b1	-1.99	-1.99004	0.002%
b2	0.995	0.995016	0.0016%
b3	2	1.99138	0.43 %
b4	- 1	-0.99566	0.43 %
b5	-0.2	-0.20172	0.8 %
b6	0.3	0.301755	0.6 %
b7	-0.3	-0.300028	0.01 %
b8	0.1	0.099048	0.95 %

Table 8

P	Real Parameters	Est. Parameters
C_{11}	0.2	0.19888
C_{22}	0.2	0.2000 4

Table 9

	Real Change	Est. Change.
dc_{11}	0.05	0.0500384
dc_{12}	-0.005	-0.00505286
dc_{21}	-0.005	-0.00495963
dc_{22}	0.05	0.0499845

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