NASA-CR-192195

IN-63-CR
145803
P.25

SEMI - ANNUAL REPORT (1990-91 YEAR)
NAG - 1 - 863
ROBUST CONTROL WITH STRUCTURED PERTURBATIONS

by

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Submited to
NASA Langley Research Center
Hampton, Virginia 23665 - 5225

NASA Technical Monitor: Dr. Jer-Nan Juang, Spacecraft Dynamics Branch

(NASA-CR-192195) ROBUST CONTROL WITH STRUCTURED PERTURBATIONS Semiannual Report, 1990 - 1991 (Tennessee State Univ.) 25 p

N93-18579

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G3/63 0145803

March 1991

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I PROGRESS OF RESEARCH

A. Introduction

This semi-annual report describes continued progress on the research. Among several approaches in this area of research, our approach to the parametric uncertainties are being matured everyday. This approach deals with real parameter uncertainties which other techniques such as H^∞ optimal control, μ analysis and synthesis, and ℓ^1 optimal control cannot deal. The primary assumption of this approach is that the mathematical models are well obtained so that the most of system uncertainties can be translated into parameter uncertainties of their linear system representations. These uncertainties may be due to modeling, nonlinearity of the physical system, some time-varying parameters, etc.

In this report period of research, we are concentrating on implementing a computer aided analysis and design tool based on new results on parametric robust stability. This implementation will help us to reveal further details in this approach.

B. Computer Aided Analysis and Design: Parametric Robust Stability

There are two basic available frameworks: coefficients of the transfer function and parameters of the transfer function. If coefficients of the characteristic polynomial contain parameters of interest and these coefficients are subject to perturbations independently, we use the coefficient perturbation framework. However, this situation is very rare and not very realistic. More general setup is the case of coefficients being linear combinations of parameters of interests. This formulation fits the case of SISO, SIMO and MISO. Of course, in general a MIMO system provides the characteristic polynomial whose coefficients are nonlinear functions of parameters. So far there is no available result to directly handle this case. However, by accepting a reasonable amount of conservatism, one can easily reduce this situation to the second case we described above.

In this section some examples are given which demonstrate how this computer aided tool is working. The package is incorporated with the well known software MATLAB in order to improve portability. The package has not yet been completed, and is still under development.

1. Interlacing Property of Single Polynomial

Consider the following polynomial,

$$\begin{aligned}
\delta(s) : &= \delta_0 + \delta_1 s + \delta_2 s^2 + \cdots + \delta_{n-1} s^{n-1} + \delta_n s^n \\
&= \underbrace{\delta_0 + \delta_2 s^2 + \delta_4 s^4 + \cdots}_{\delta_{\text{even}}(s)} + \underbrace{\delta_1 s + \delta_3 s^3 + \delta_5 s^5 + \cdots}_{\delta_{\text{odd}}(s)} \\
\delta(j\omega) : &= \delta(s)|_{s=j\omega} \\
&= \underbrace{\delta_0 - \delta_2 \omega^2 + \delta_4 \omega^4 - \cdots}_{\delta_{\text{even}}(\omega)} + j\omega \underbrace{(\delta_1 - \delta_3 \omega^2 + \delta_5 \omega^4 - \cdots)}_{\frac{\delta_{\text{odd}}(j\omega)}{j\omega} := \delta_{\text{odd}}(\omega)}.
\end{aligned}$$

Theorem 1 *The polynomial $\delta(s)$ is Hurwitz stable if and only if*

a) $\delta_{\text{even}}(\omega)$ and $\delta_{\text{odd}}(\omega)$ have only simple roots and these roots interlace.

b) For all $\omega \in \mathcal{R}$, $\delta'_{\text{odd}}(\omega)\delta_{\text{even}}(\omega) - \delta_{\text{odd}}\delta'_{\text{even}}(\omega) > 0$.

The figure 1 shows the interlacing property of Hurwitz polynomial

$$\delta(s) = 6 + 49s + 155s^2 + 280s^3 + 331s^4 + 266s^5 + 145s^6 + 52s^7 + 11s^8 + s^9$$

Next some of the values of the coefficients are increased to observe how the graph changes. We selected δ_0,δ_2,δ_4,δ_6 and δ_8. The figure 2 shows the changes in the graph and the given polynomial becomes unstable when it violates the interlacing property.

2. Interlacing Property of Interval Polynomial

The well known Kharitonov's theorem[1] can be easily verified by the interlacing property. Consider the following

$$\delta(s) := \delta_0 + \delta_1 s + \delta_2 s^2 + \cdots + \delta_{n-1} s^{n-1} + \delta_n s^n$$

where

$$\underline{\delta}_i \leq \delta_i \leq \bar{\delta}_i \qquad \forall i$$

and its four Kharitonov's polynomials

$$\begin{aligned}
K_1(s) &:= \underline{\delta}_0 + \underline{\delta}_1 s + \bar{\delta}_2 s^2 + \bar{\delta}_3 s^3 + \underline{\delta}_4 s^4 + \underline{\delta}_5 s^5 + \bar{\delta}_6 s^6 + \cdots \\
&= \underbrace{\underline{\delta}_0 + \bar{\delta}_2 s^2 + \underline{\delta}_4 s^4 + \bar{\delta}_6 s^6 + \cdots}_{K_{\text{even}}^{\text{min}}(s)} + \underbrace{\underline{\delta}_1 s + \bar{\delta}_3 s^3 + \underline{\delta}_5 s^5 + \bar{\delta}_7 s^7 + \cdots}_{K_{\text{odd}}^{\text{min}}(s)} \\
K_2(s) &:= \underline{\delta}_0 + \bar{\delta}_1 s + \bar{\delta}_2 s^2 + \underline{\delta}_3 s^3 + \underline{\delta}_4 s^4 + \bar{\delta}_5 s^5 + \bar{\delta}_6 s^6 + \cdots \\
&= \underbrace{\underline{\delta}_0 + \bar{\delta}_2 s^2 + \underline{\delta}_4 + \bar{\delta}_6 s^6 + \cdots}_{K_{\text{even}}^{\text{min}}(s)} + \underbrace{\bar{\delta}_1 s + \underline{\delta}_3 s^3 + \bar{\delta}_5 s^5 + \underline{\delta}_7 s^7 + \cdots}_{K_{\text{odd}}^{\text{max}}(s)} \\
K_3(s) &:= \bar{\delta}_0 + \underline{\delta}_1 s + \underline{\delta}_2 s^2 + \bar{\delta}_3 s^3 + \bar{\delta}_4 s^4 + \underline{\delta}_5 s^5 + \underline{\delta}_6 s^6 + \cdots \\
&= \underbrace{\bar{\delta}_0 + \underline{\delta}_2 s^2 + \bar{\delta}_4 s^4 + \underline{\delta}_6 s^6 + \cdots}_{K_{\text{even}}^{\text{max}}(s)} + \underbrace{\underline{\delta}_1 s + \bar{\delta}_3 s^3 + \underline{\delta}_5 s^5 + \bar{\delta}_7 s^7 + \cdots}_{K_{\text{odd}}^{\text{min}}(s)} \\
K_4(s) &:= \bar{\delta}_0 + \bar{\delta}_1 s + \underline{\delta}_2 s^2 + \underline{\delta}_3 s^3 + \bar{\delta}_4 s^4 + \bar{\delta}_5 s^5 + \underline{\delta}_6 s^6 + \cdots \\
&= \underbrace{\bar{\delta}_0 + \underline{\delta}_2 s^2 + \bar{\delta}_4 s^4 + \underline{\delta}_6 s^6 + \cdots}_{K_{\text{even}}^{\text{max}}(s)} + \underbrace{\bar{\delta}_1 s + \underline{\delta}_3 s^3 + \bar{\delta}_5 s^5 + \underline{\delta}_7 s^7 + \cdots}_{K_{\text{odd}}^{\text{max}}(s)}
\end{aligned}$$

The figure 3 shows each tube constructed by $K_{\text{even}}^{\text{max}}(\omega)$ and $K_{\text{even}}^{\text{min}}(\omega)$, $K_{\text{odd}}^{\text{max}}(\omega)$ and $K_{\text{odd}}^{\text{min}}(\omega)$, respectively. According to the interlacing property, in order to ensure the stability of the family of the given polynomials, two tubes must interlace.

The figure 4 shows the development of instability in some members in the family when selected coefficients are increased.

These can also be shown by plotting perturbation boxes in the complex plane. The figure 5 show the traces of boxes while ω moves 0 to ∞. Each box has its vertices at $K_{\text{even}}^{\max}(\omega)$, $K_{\text{even}}^{\min}(\omega)$, $K_{\text{odd}}^{\max}(\omega)$ and $K_{\text{odd}}^{\min}(\omega)$. If any box contains the origin, it means that there is at least one unstable polynomial in the family. The figure 5 shows the Hurwitz stable polynomial and the figure 6 shows that the family becomes unstable when certain coefficients are increased.

3. ℓ_2 Stability Margin in Coefficient Space

For the given Hurwitz polynomial, one often wants to know how much coefficient perturbation can be allowed while the family of polynomials maintains Hurwitz stability[2]. This can conveniently be measured in terms of ℓ_2 norm such as:

$$\rho := \|\Delta\delta\|_2$$

where

$$\Delta\delta := \begin{bmatrix} \Delta\delta_1 & \Delta\delta_2 & \cdots & \cdots & \Delta\delta_n \end{bmatrix}^T$$

and

$$\delta(s, \Delta\delta_i) := (\delta_0 \pm \Delta\delta_0) + (\delta_1 \pm \Delta\delta_1)s + \cdots + (\delta_n \pm \Delta\delta_n)s^n$$

For the case of nonmonic polynomials (i.e., $\Delta\delta_n \neq 0$), the ℓ_2 stability margin is given

$$\rho = \min\{\delta_0, \delta_n, \delta_\omega\}.$$

The expression of δ_ω is found in [2]. The figure 7 shows the graph of δ_ω and $*$ indicates the minimum value for our example

$$\delta(s) = 6 + 49s + 155s^2 + 280s^3 + 331s^4 + 266s^5 + 145s^6 + 52s^7 + 11s^8 + s^9$$

If we consider the monic polynomial (i.e., $\delta_n = 1$, $\Delta\delta_n = 0$), we have

$$\rho = \min\{\delta_0, \delta_w\}$$

Again, the expression of δ_ω for the monic case is found in [2].

4. Generalized Kharitonov's Theorem

Kharitonov's theorem is powerful and elegant, but it is not very useful for studying control systems because it assumes that all coefficients of the characteristic polynomial perturb independently. An improved version of this theorem was introduced in [3]. For the illustrative purpose, we give the simplest version.

For the given interval polynomials

$$\begin{aligned} P_1(s) &= \alpha_0^1 + \alpha_1^1 s + \alpha_2^1 s^2 + \alpha_3^1 s^3 + \cdots + \alpha_{p_1}^1 s^{p_1} \\ P_2(s) &= \alpha_0^2 + \alpha_1^2 s + \alpha_2^2 s^2 + \alpha_3^2 s^3 + \cdots + \alpha_{p_2}^2 s^{p_2} \end{aligned}$$

where

$$\underline{\alpha}_i^j \leq \alpha_i^j \leq \bar{\alpha}_i^j, \quad i = 0, 1, \cdots, p_1, j = 1, 2$$

and the give fixed polynomials

$$\begin{aligned} Q_1(s) &= \beta_0^1 + \beta_1^1 s + \beta_2^1 s^2 + \beta_3^1 s^3 + \cdots + \beta_{q_1}^1 s^{q_1} \\ Q_2(s) &= \beta_0^2 + \beta_1^2 s + \beta_2^2 s^2 + \beta_3^2 s^3 + \cdots + \beta_{q_2}^2 s^{q_2} \end{aligned}$$

The problem is to check the stability of the family of polynomials

$$Q_1(s)P_1(s) + Q_2(s)P_2(s).$$

Let us first define

$$\begin{aligned} K_1^j(s) &= K_{\text{even,min}}^j(s) + K_{\text{odd,min}}^j(s) \\ K_2^j(s) &= K_{\text{even,min}}^j(s) + K_{\text{odd,max}}^j(s) \\ K_3^j(s) &= K_{\text{even,max}}^j(s) + K_{\text{odd,min}}^j(s) \\ K_4^j(s) &= K_{\text{even,max}}^j(s) + K_{\text{odd,max}}^j(s) \end{aligned}$$

Step 1: Set $P_1(s) = K_1^1(s)$.

Step 2: check the stability of each following segment.

$$\begin{aligned}
S_{11}^1 &= Q_1(s)P_1(s) + Q_2(s)[(1-\lambda)K_1^2(s) + \lambda K_2^2(s)] \\
S_{12}^1 &= Q_1(s)P_1(s) + Q_2(s)[(1-\lambda)K_1^2(s) + \lambda K_3^2(s)] \\
S_{13}^1 &= Q_1(s)P_1(s) + Q_2(s)[(1-\lambda)K_2^2(s) + \lambda K_4^2(s)] \\
S_{14}^1 &= Q_1(s)P_1(s) + Q_2(s)[(1-\lambda)K_3^2(s) + \lambda K_4^2(s)]
\end{aligned}$$

$$\begin{aligned}
S_{11}^1 &= Q_1(s)P_1(s) + Q_2(s)[(1-\lambda)K_1^2(s) + \lambda K_2^2(s)] \\
&= Q_1(s)P_1(s) + Q_2(s)K_1^2(s) - \lambda Q_2(s)K_1^2(s) + \lambda Q_2(s)K_2^2(s) \\
&= (1-\lambda)\underbrace{[Q_1(s)P_1(s) + Q_2(s)K_1^2(s)]}_{\delta_1(s)} + \lambda\underbrace{[Q_1(s)P_1(s) + Q_2(s)K_2^2(s)]}_{\delta_2(s)}
\end{aligned}$$

then call the "segment lemma" with $\delta_1(s)$ and $\delta_2(s)$.

Step 3: Set $P_1(s) = K_2^1(s)$, repeat Step 2 for S_{2k}^1 with $k = 1, 2, 3, 4$.

Step 4: Set $P_1(s) = K_3^1(s)$, repeat Step 2 for S_{3k}^1 with $k = 1, 2, 3, 4$.

Step 5: Set $P_1(s) = K_4^1(s)$, repeat Step 2 for S_{4k}^1 with $k = 1, 2, 3, 4$.

Step 6: Set $P_2(s) = K_1^2(s)$.

Step 7: check the stability of each following segment.

$$\begin{aligned}
S_{11}^2 &= Q_1(s)[(1-\lambda)K_1^1(s) + \lambda K_2^1(s)] + Q_2(s)P_2(s) \\
S_{11}^2 &= Q_1(s)[(1-\lambda)K_1^1(s) + \lambda K_3^1(s)] + Q_2(s)P_2(s) \\
S_{11}^2 &= Q_1(s)[(1-\lambda)K_2^1(s) + \lambda K_4^1(s)] + Q_2(s)P_2(s) \\
S_{11}^2 &= Q_1(s)[(1-\lambda)K_3^1(s) + \lambda K_4^1(s)] + Q_2(s)P_2(s)
\end{aligned}$$

Step 8: Set $P_2(s) = K_2^2(s)$, repeat Step 7 for S_{2k}^2 with $k = 1, 2, 3, 4$.

Step 9: Set $P_2(s) = K_3^2(s)$, repeat Step 7 for S_{3k}^2 with $k = 1, 2, 3, 4$.

Step 10: Set $P_2(s) = K_4^2(s)$, repeat Step 7 for S_{4k}^2 with $k = 1, 2, 3, 4$.

This algorithm can be easily extended to the case of $\sum_i Q_i(s)P_i(s)$. The following figures show the family of polynomials as functions of ω. The entire family is Hurwitz stable if and only if the family does not contain the origin for all ω. Clearly Figure 9 shows the family is stable and Figure 10 shows some members in the family are unstable.

5. ℓ_2 Stability Margin in Parameter Space

Consider the following polynomial with parameters $p_1, p_2, \cdots, p_l$

$$\begin{aligned}
\delta(s, \mathbf{p}) : &= [a_{01}(p_1 + \Delta p_1) + a_{02}(p_2 + \Delta p_2) + \cdots + a_{0l}(p_l + \Delta p_l)] + \\
&\quad [a_{11}(p_1 + \Delta p_1) + a_{12}(p_2 + \Delta p_2) + \cdots + a_{1l}(p_l + \Delta p_l)]s + \\
&\quad \cdots + [a_{n1}(p_1 + \Delta p_1) + a_{n2}(p_2 + \Delta p_2) + \cdots + a_{nl}(p_l + \Delta p_l)]s^n \\
&= \underbrace{(a_{01}p_1 + a_{02}p_2 + \cdots + a_{0l}p_l) + \cdots (a_{n1}p_1 + a_{n2}p_2 + \cdots + a_{nl}p_l)s^n}_{\delta^o(s)} \\
&\quad \underbrace{+ \sum_{k=1}^{l} \underbrace{(a_{0k} + a_{1k}s + \cdots + a_{nk}s^n)}_{a_k(s)} \Delta P_k}_{\Delta\delta(s, \Delta \mathbf{p})}
\end{aligned}$$

Then the following algorithm provides the ℓ_2 stability margin in the parameter space.

$$\begin{aligned}
a_k(j\omega) &= a_{0k} + ja_{1k}\omega - a_{2k}\omega^2 - ja_{3k}\omega^3 + a_{4k}\omega^4 + ja_{5k}\omega^5 - a_{6k}\omega^6 - \cdots \\
&= \underbrace{(a_{0k} - a_{2k}\omega^2 + a_{4k}\omega^4 - a_{6k}\omega^6 + \cdots)}_{a_{kr}(j\omega)} + j\underbrace{(a_{1k}\omega - a_{3k}\omega^3 + a_{5k}\omega^5 - a_{7k}\omega^7 + \cdots)}_{a_{ki}(j\omega)}
\end{aligned}$$

Let

$$A(j\omega) := \begin{bmatrix} a_{1r}(j\omega) & a_{2r}(j\omega) & a_{3r}(j\omega) & \cdots & a_{lr}(j\omega) \\ a_{1i}(j\omega) & a_{2i}(j\omega) & a_{3i}(j\omega) & \cdots & a_{li}(j\omega) \end{bmatrix} \qquad b(j\omega) := \begin{bmatrix} -\delta_r^o(j\omega) \\ -\delta_i^o(j\omega) \end{bmatrix}$$

where

$$\delta^o(j\omega) := \delta^o(s)|_{s=j\omega} = \delta^o_r(j\omega) + j\delta^o_i(j\omega)$$

If $A(j\omega)A(j\omega)^T$ is invertible,

$$t_\lambda := A(j\omega)^T[A(j\omega)A(j\omega)^T]^{-1}b(j\omega).$$

If $A(j\omega)A(j\omega)^T$ is not invertible,

$$t_\lambda := A(j\omega)^T[A(j\omega)A(j\omega)^T]^{-1}b(j\omega).$$

where

$$A(j\omega) := \begin{bmatrix} a_{1r}(j\omega) & a_{2r}(j\omega) & a_{3r}(j\omega) & \cdots & a_{lr}(j\omega) \end{bmatrix}$$

Finally, the l^2 stability margin in parameter space is computed by

$$\rho := \min_{\omega \in (0,\infty)} |t_\lambda|_2^2$$

where

$$|t_\lambda|_2^2 = t_{\lambda 1}^2 + t_{\lambda 2}^2 + \cdots + t_{\lambda l}^2$$

<u>Design Example</u>

$$\begin{aligned} J_1\ddot{\theta}_1 + p_1(\dot{\theta}_1 - \dot{\theta}_2) + p_2(\theta_1 - \theta_2) &= T_c \\ J_2\ddot{\theta}_2 + p_1(\dot{\theta}_2 - \dot{\theta}_1) + p_2(\theta_2 - \theta_1) &= T_c \end{aligned}$$

$$G(s) = \frac{s^2 + p_1 s + p_2}{s^2(s^2 + 2p_1 s + 2p_2)}$$

$$0.18 \leq p_2 \leq 0.3$$

$$0.04\sqrt{\frac{p_2}{10}} \leq p_1 \leq 0.2\sqrt{\frac{p_2}{10}}$$

In the following figure, the dotted box indicates the range of parameter perturbations to be tolerated. The circles indicate the ℓ_2 stability margin of the characteristic polynomial. If a circle completely covers the box, the stability of the closed loop system is guaranteed under the given parameter perturbations. Figures show that several circles that correspond to different controllers.

C. Appendix

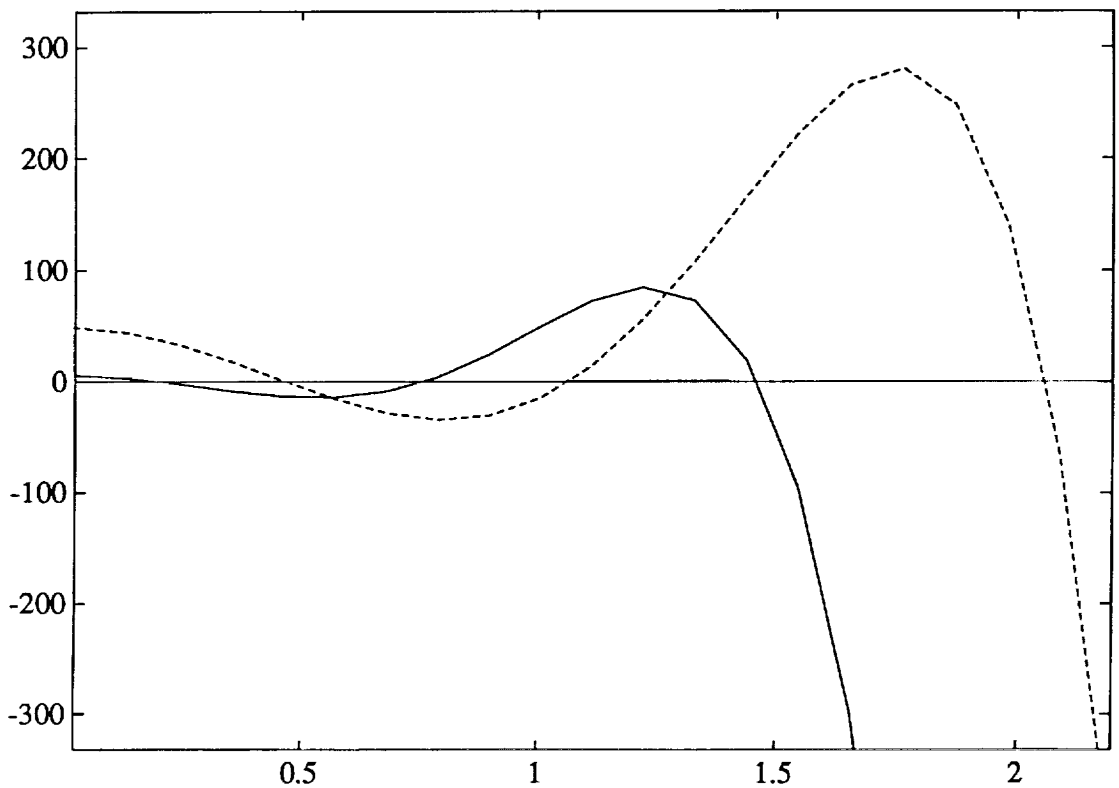


Figure 1

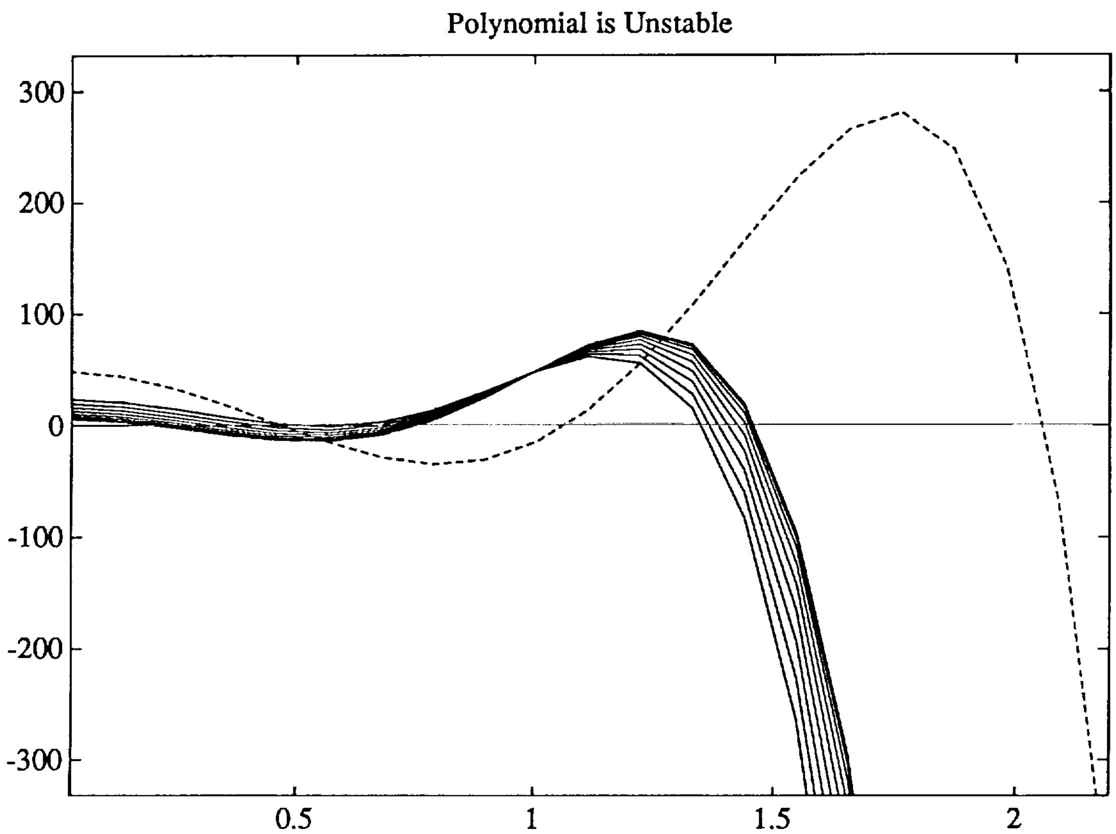


Figure 2

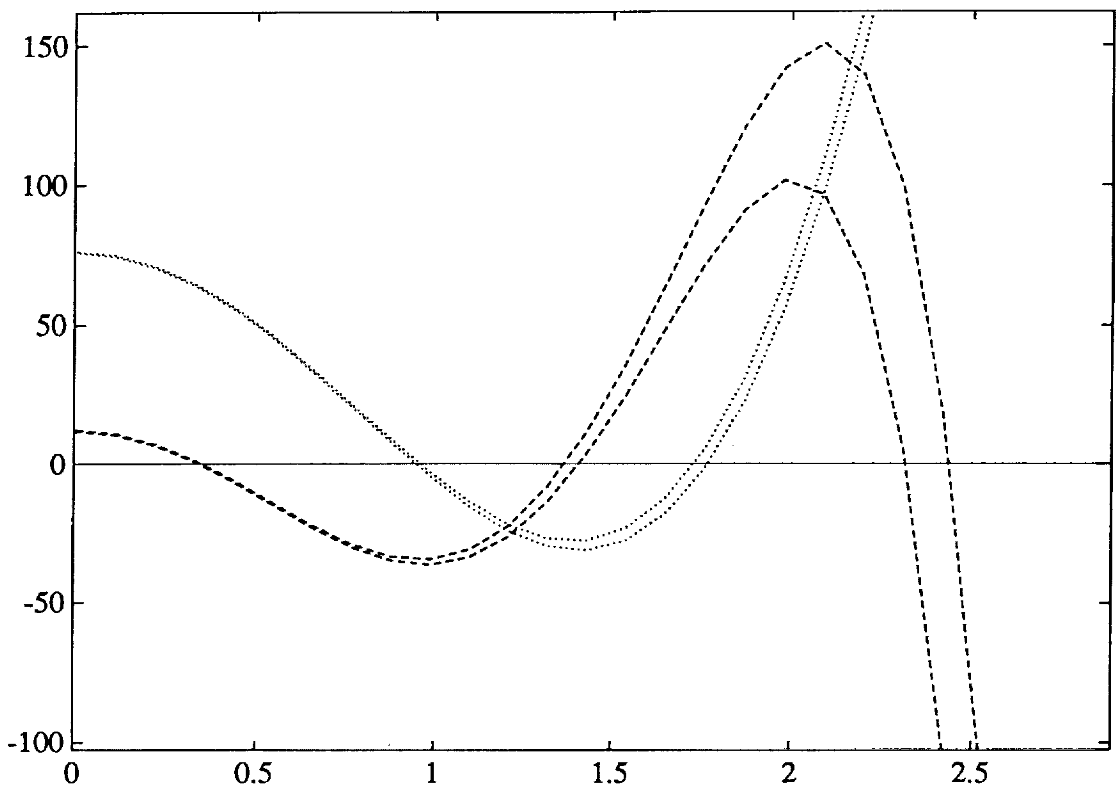


Figure 3

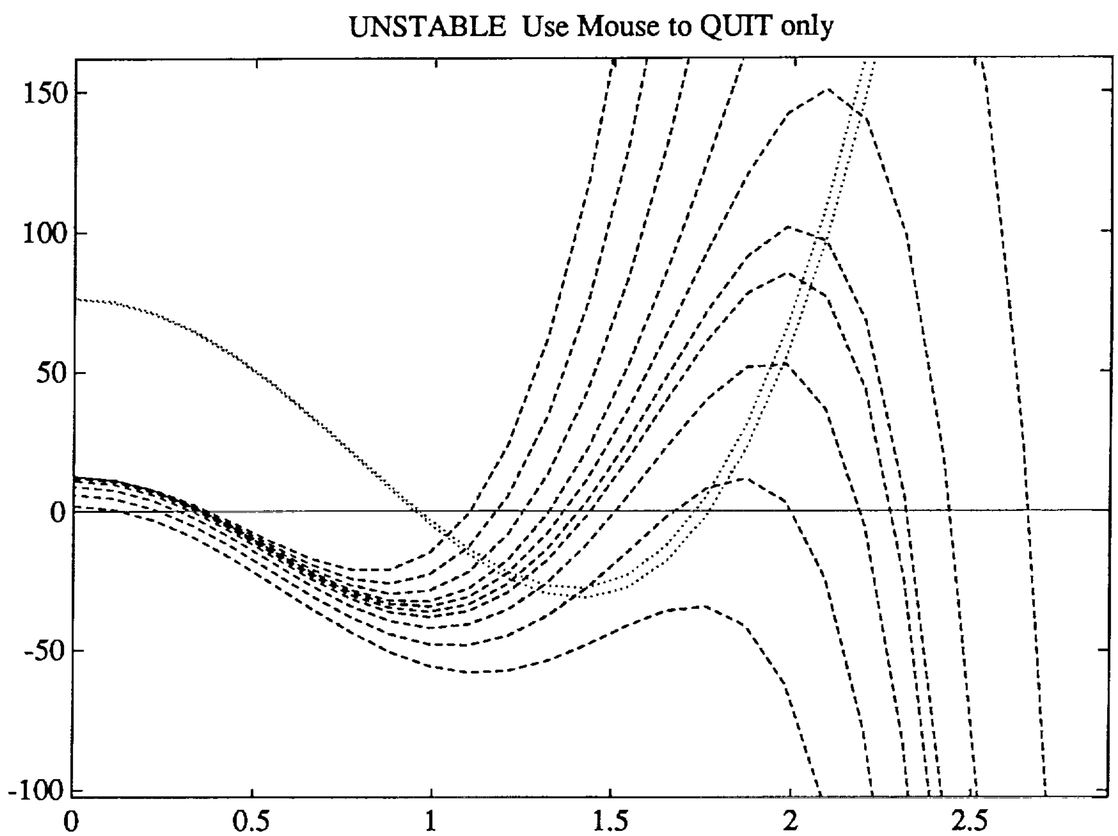


Figure 4

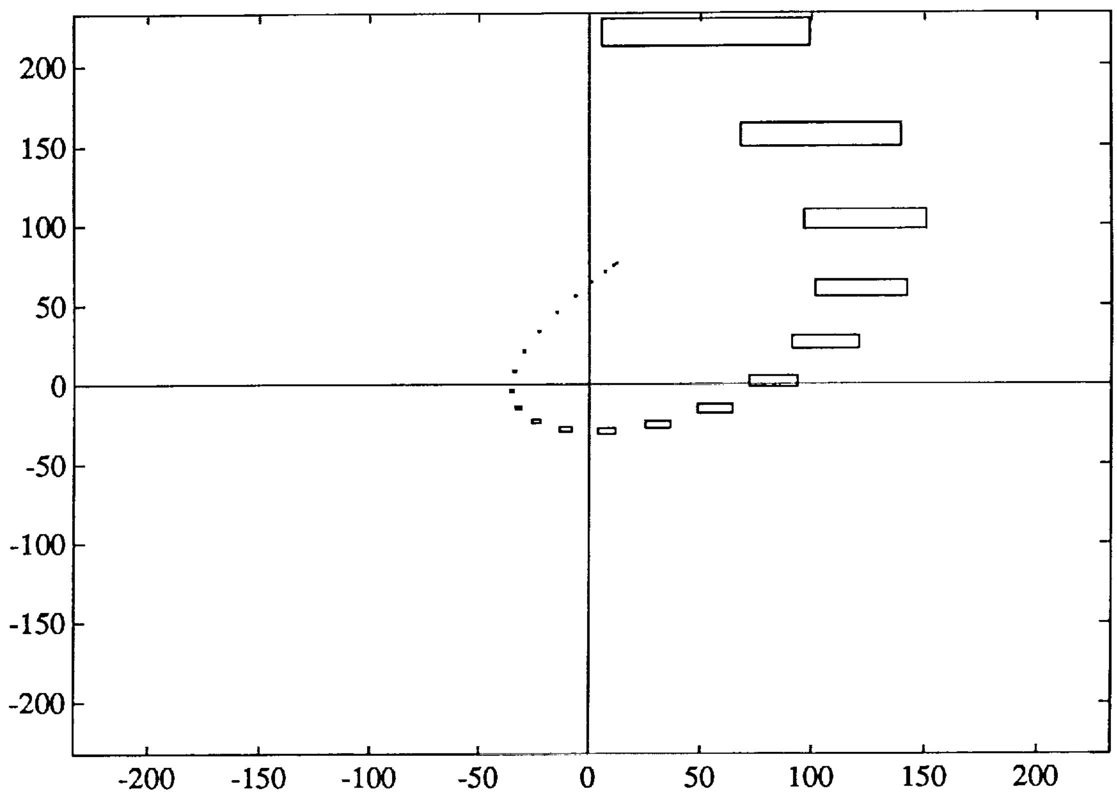
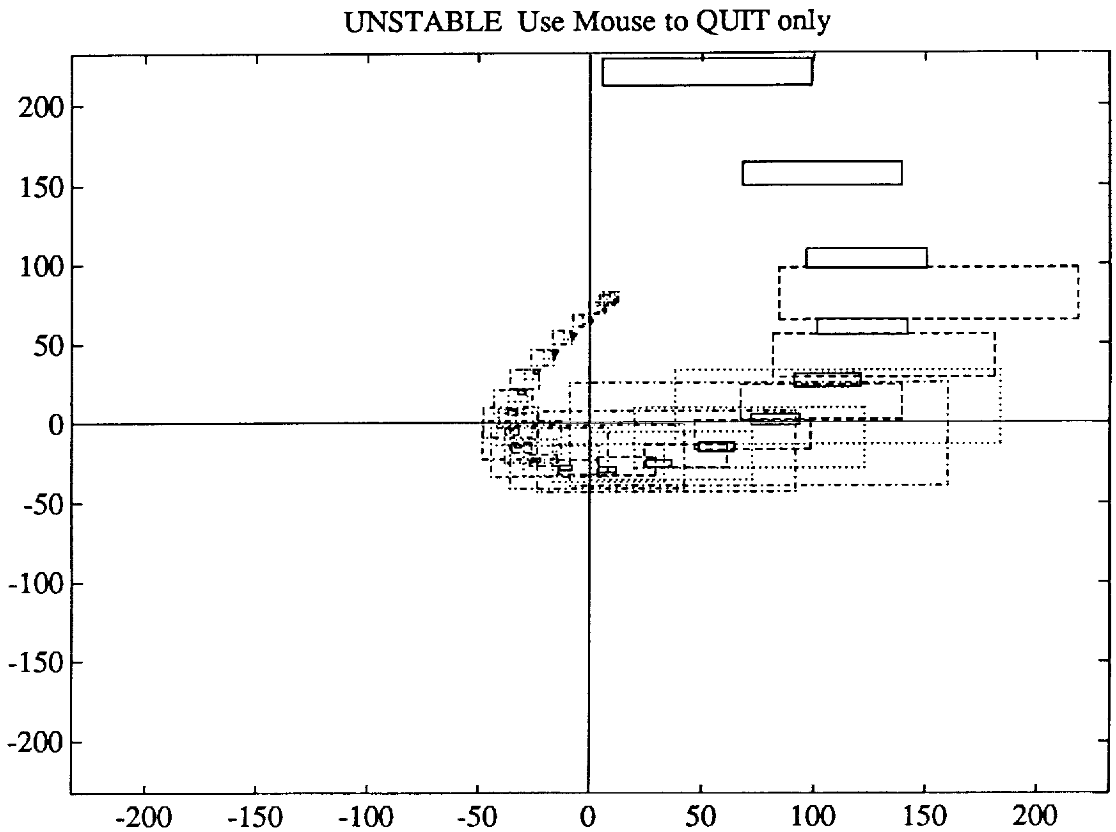


Figure 5


UNSTABLE Use Mouse to QUIT only
200
150
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-100
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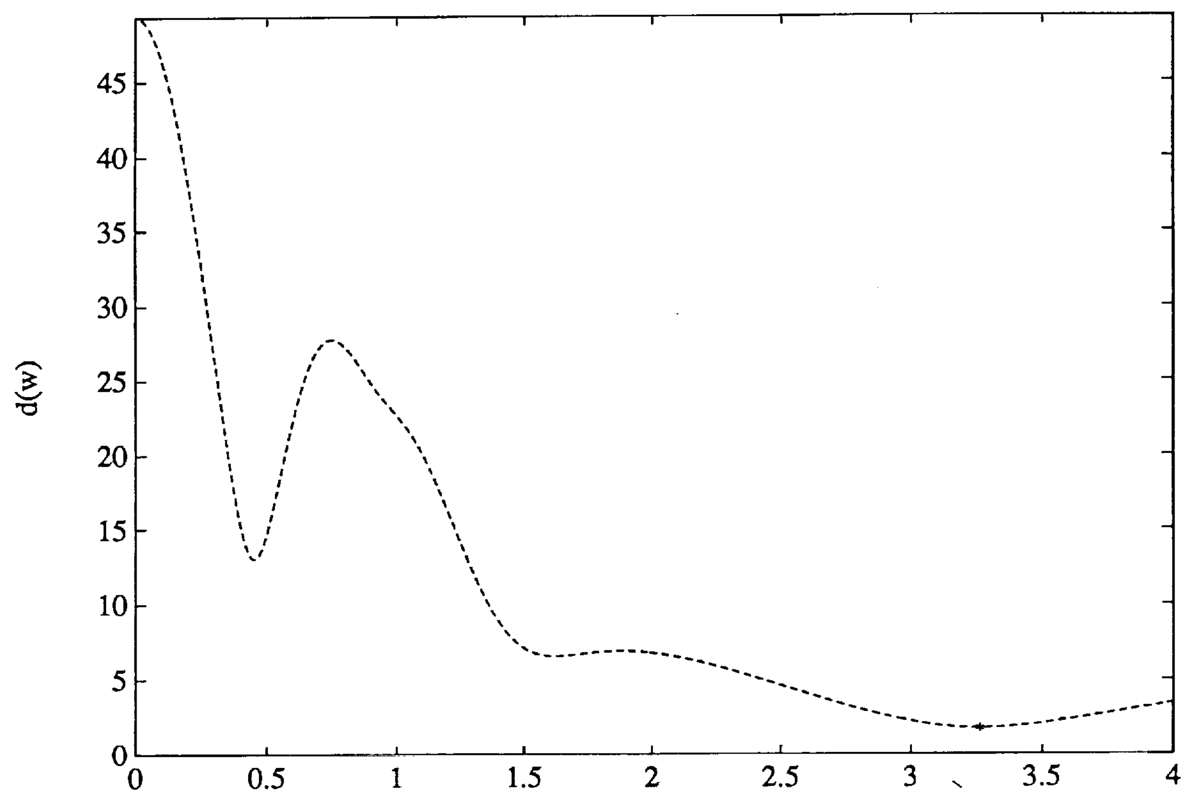
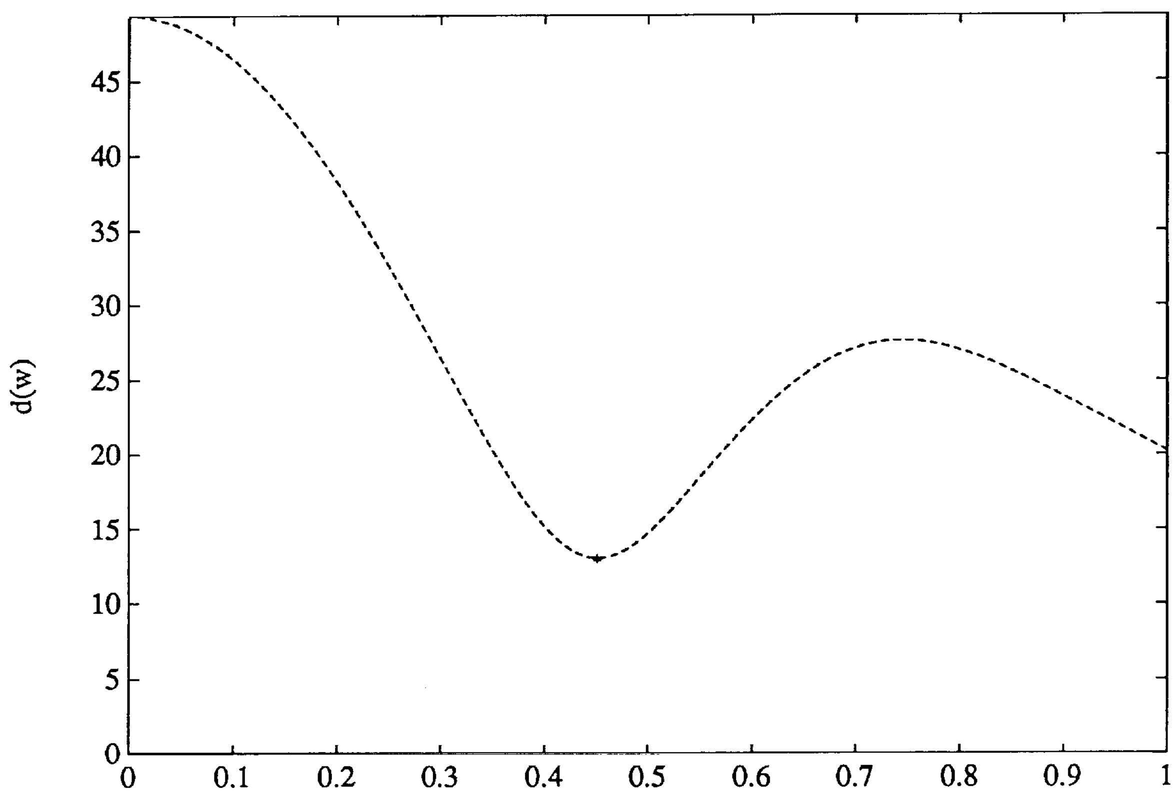
50
100
150
200

Figure 6



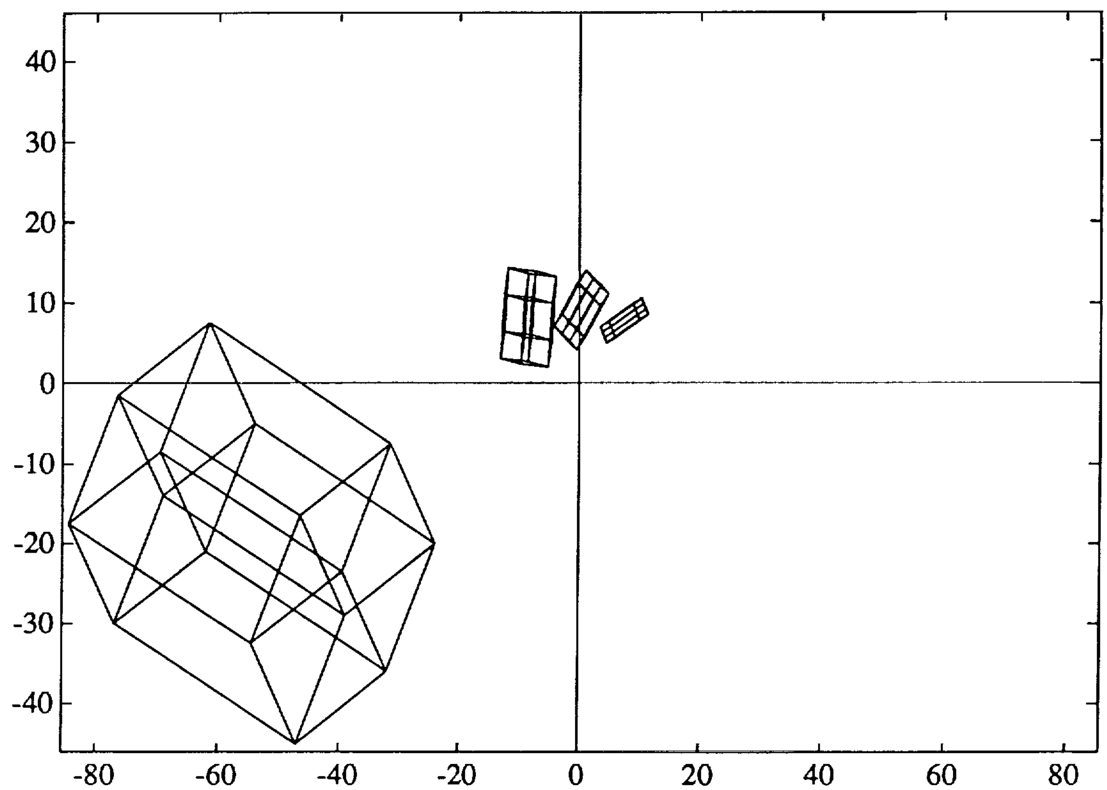
inf d(w)= 1.766 at Frequency = 3.266
L-2 Stability = min(d0,dn,dw) = 1

Figure 7



inf d(w)= 13.02 at Frequency = 0.451
L-2 Stability = min(d0,dw) = 6

Figure 8



Press Enter to Continue

Figure 9

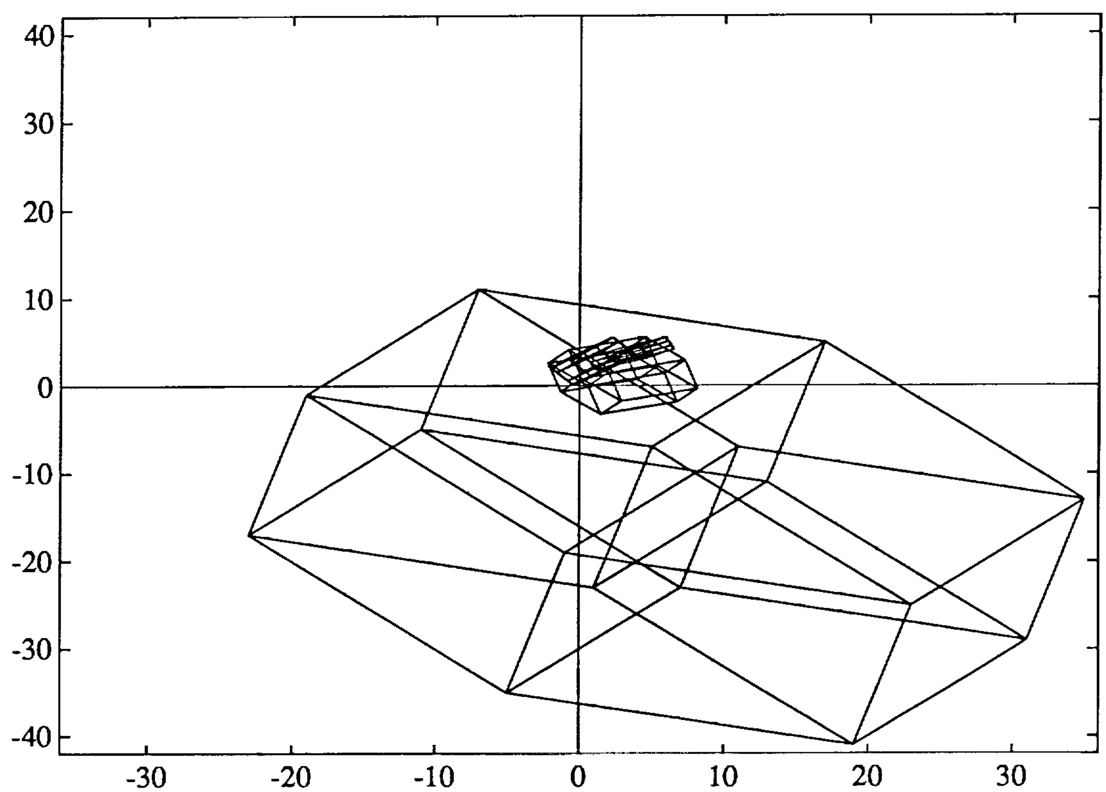


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Figure 10

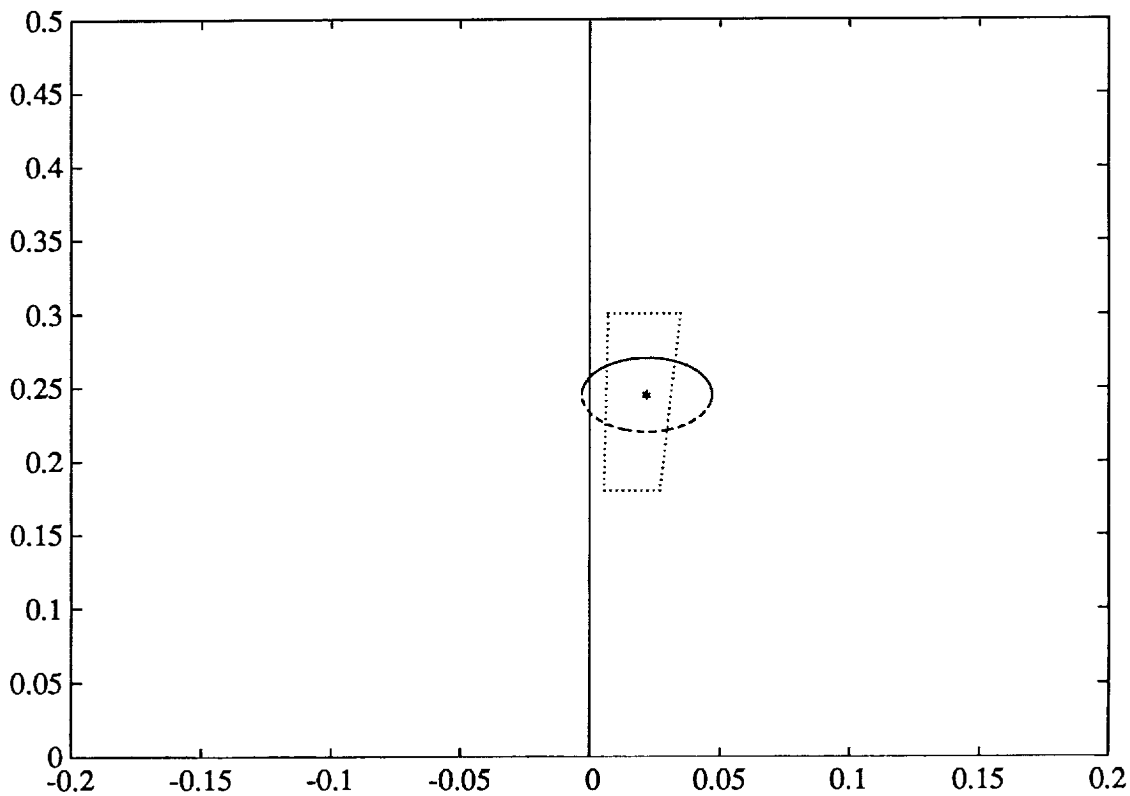

Figure 11

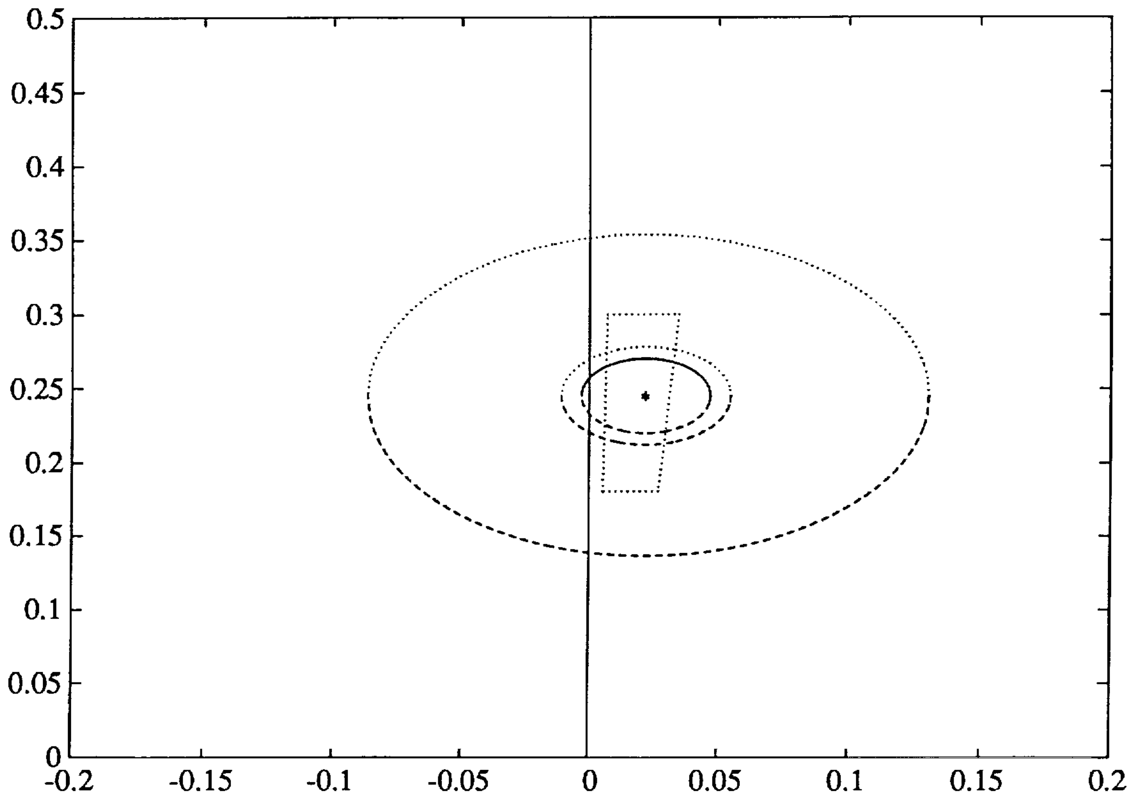

Figure 12

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[2] H. Chapellat, S. Bhattacharyya, and L. Keel, "Stability margin for hurwitz polynomials," in *Proceedings of the 27th IEEE Conference on Decision and Control*, pp. 1392 – 1398, December 1988. Austin, TX.

[3] H. Chapellat and S. Bhattacharyya, "A generalization of kharitonov's theorem: robust stability of interval plants," *IEEE Transactions on Automatic Control*, vol. AC - 34, pp. 306 – 311, March 1989.

II DISCUSSION AND DIRECTION OF RESEARCH

The computer aided tool currently under implementation is very useful for future research on this topic. In addition, it is expected that the tool will be valuable for engineers who actually perform design and analysis of systems.

Different aspects of robust control research are also under study. While dealing with various forms of system uncertainties is important, the problems currently under study are also important and meaningful in practical control systems. Two problems currently under investigation are, *zero assignment and LQG/LTR* and *the sensor failure problem.* Details of these problems will be discussed later.